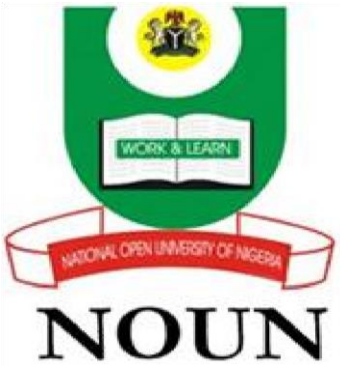

NATIONAL OPEN UNIVERSITY OF NIGERIA

INTRODUCTION TO ECONOMETRICS I

ECO 355

DEPARTMENT OF ECONOMICS
FACULTY OF SOCIAL SCIENCES

COURSE GUIDE

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Introduction

Welcome to ECO: 355 INTRODUCTION TO ECONOMETRICS I.

ECO 355: Introduction to Econometrics I is a three-credit and one-semester undergraduate course for Economics student. The course is made up of nineteen units spread across fifteen lectures weeks. This course guide gives you an insight to introduction to econometrics and how it is applied in economics. It tells you about the course materials and how you can work your way through these materials. It suggests some general guidelines for the amount of time required of you on each unit in order to achieve the course aims and objectives successfully. Answers to your tutor marked assignments (TMAs) are therein already.

Course Content

This course is basically an introductory course on Econometrics. The topics covered include the econometrics analysis, single-equation (regression models), Normal linear regression model and practical aspects of statistics testing.

Course Aims

The aims of this course is to give you in-depth understanding of the macroeconomics as regards

- Fundamental concept of econometrics
- To familiarize students with single-equation of regression model
- To stimulate student‘s knowledge on normal linear regression model
- To make the students to understand some of the practical aspects of econometrics test.
- To expose the students to rudimentary analysis of simple and multiple regression analysis.

Course Objectives

To achieve the aims of this course, there are overall objectives which the course is out to achieve though, there are set out objectives for each unit. The unit objectives are included at the beginning of a unit; you should read them before you start working through the unit. You may want to refer to them during your study of the unit to check on your progress. You should always look at the unit objectives after completing a unit. This is to assist the students in accomplishing the tasks entailed in this course. In this way, you can be sure you have done what was required of you by the unit. The objectives serves as study guides, such that student could know if he is able to grab the knowledge of each unit through the sets of objectives in each one. At the end of the course period, the students are expected to be able to:

- to understand the basic fundamentals of Econometrics distinguish between Econometrics and Statistics.
- know how the econometrician proceed in the analysis of an economic problem.
- know how the econometrician make use of both mathematical and statistical analysis in solving economic problems.
- understand the role of computer in econometrics analysis identify/explain the types of econometrics analysis.
- understand the basic Econometrics models
- differentiate between Econometrics theory and methods
- know the meaning of Econometrics and why Econometrics is important within Economics.
- know how to use Econometrics for Assessing Economic Model understand what is Financial Econometrics.
- examine the linear regression model

- understand the classical linear regression model
- be able to differentiate the dependant and independent variables.
- prove some of the parameters of ordinary least estimate.
- know the alternative expression for ˆ
- understand the assumptions of classical linear regression model.
- know the properties that our estimators should have
- know the proofing of the OLS estimators as the best linear unbiased estimators (BLUE).
- examine the Goodness fit
- understand and work through the calculation of coefficient of multiple determination
- identify and know how to calculate the probability normality assumption for U_i
- understand the normality assumption for U_i
- understand why we have to conduct the normality assumption.
- identify the properties of OLS estimators under the normality assumption
- understand what is probability distribution
- understand the meaning of Maximum Likelihood Estimation of two variable regression Model.
- understand the meaning of Hypothesis
- know how to calculate hypothesis using confidence interval analyse and interpret hypothesis result.
- understand the meaning of accepting and rejecting an hypothesis identify a null and alternative hypothesis.
- understand the meaning of Level of significance
- understand the Choice between confidence-interval and test-of-significance Approaches to hypothesis testing
- understand the meaning of regression analysis and variance
- know how to calculate the regression analysis and analysis of variance

Working Through The Course

To successfully complete this course, you are required to read the study units, referenced books and other materials on the course.
Each unit contains self-assessment exercises called Student Assessment Exercises (SAE). At some points in the course, you will be required to submit assignments for assessment purposes. At the end of the course there is a final examination. This course should take about 15weeks to complete and some components of the course are outlined under the course material subsection.

Course Material

The major component of the course, What you have to do and how you should allocate your time to each unit in order to complete the course successfully on time are listed follows:

1. Course guide
2. Study unit
3. Textbook
4. Assignment file
5. Presentation schedule

Study Unit

There are 19 units in this course which should be studied carefully and diligently.

MODULE ONE ECONOMETRICS ANALYSIS

Unit 1: Meaning Of Econometrics
Unit 2: Methodology of Econometrics
Unit 3: Computer and Econometrics
Unit 4: Basic Econometrics Models: Linear Regression
Unit 5: Importance of Econometrics

MODULE TWO SINGLE- EQUATION (REGRESSION MODELS)

Unit One: Regression Analysis
Unit Two: The Ordinary Least Square (OLS) Method Estimation
Unit Three: Calculation of Parameter β and the Assumption of Classical Least Regression Method (CLRM)
Unit Four: Properties of the Ordinary Least Square Estimators
Unit Five: The Coefficient of Determination (R^2): A measure of ―Goodness of fit‖

MODULE THREE NORMAL LINEAR REGRESSION MODEL (CNLRM)

Unit One: Classical Normal Linear Regression Model
Unit Two: OLS Estimators Under The Normality Assumption
Unit Three: The Method Of Maximum Likelihood (ML)
Unit Four: Confidence intervals for Regression Coefficients β_1 and β_2 Unit Five: Hypothesis Testing

MODULE FOUR PRACTICAL ASPECTS OF ECONOMETRICS TEST

Unit One Accepting & Rejecting an Hypothesis

Unit Two The Level of Significance
Unit Three Regression Analysis and Analysis of Variance
Unit Four Normality tests

Each study unit will take at least two hours, and it include the introduction, objective, main content, self-assessment exercise, conclusion, summary and reference. Other areas border on the Tutor-Marked Assessment (TMA) questions. Some of the self-assessment exercise will necessitate discussion, brainstorming and argument with some of your colleges. You are advised to do so in order to understand and get acquainted with historical economic event as well as notable periods.

There are also textbooks under the reference and other (on-line and off-line) resources for further reading. They are meant to give you additional information if only you can lay your hands on any of them. You are required to study the materials; practice the selfassessment exercise and tutor-marked assignment (TMA) questions for greater and indepth understanding of the course. By doing so, the stated learning objectives of the course would have been achieved.

Textbook and References

For further reading and more detailed information about the course, the following materials are recommended:

Adesanya, A.A., (2013). Introduction to Econometric, 2nd edition, Classic Publication limited, Lagos Nigeria.
Adekanye, D. F., (2008). Introduction to Econometrics, 1st edition, Addart Publication limited, Lagos Nigeria.
Begg, Iain and Henry, S. G. B. Applied Economics and Public Policy. Cambridge University Press, United Kingdom: 1998.
Bello, W.L., (2015). Applied Econometrics in a large Dimension, Fall Publication, Benini, Nigeria.
Cassidy, John. *The Decline of Economics.*
Dimitrios, A & Stephen, G., (2011). Applied Econometrics, second edition 2011, first edition 2006 and revised edition 2007.
Emmanuel, E.A., (2014). Introduction to Econometrics, 2nd edition, World gold Publication limited.
Faraday, M.N., (2014) Applied Econometrics, 1st Edition, Pentagon Publication limited.

Friedland, Roger and Robertson, A. F., eds. Beyond the Marketplace: Rethinking

Economy and Society. Walter de Gruyter, Inc. New York: 1990

Gordon, Robert Aaron. *Rigor and Relevance in a Changing Institutional Setting.*

Kuhn, Thomas. *The Structure of Scientific Revolutions*.

Gujarat, D. N. (2007) Basic Econometrics, 4th Edition, tata Mcgraw – Hill publishing company limited, New Delhi.

Hall, S. G., & Asterion, D. (2011) Applied Econometrics, 2nd Edition, Palgrave Macmillian, New York city, USA

Medunoye, G.K., (2013). Introduction to Econometrics, 1st edition, Mill Publication limited.

Olusanjo, A.A. (2014). Introduction to Econometrics, a broader perspective, 1st edition, world press publication limited, Nigeria.

Parker, J.J., (2016). Econometrics and Economic Policy, Journal vol 4, pg 43-73, Parking & Parking Publication limited.

Warlking, F.G., (2014). Econometrics and Economic theory, 2nd edition, Dale Press limited.

Assignment File

Assignment files and marking scheme will be made available to you. This file presents you with details of the work you must submit to your tutor for marking. The marks you obtain from these assignments shall form part of your final mark for this course. Additional information on assignments will be found in the assignment file and later in this Course Guide in the section on assessment.

There are four assignments in this course. The four course assignments will cover:

Assignment 1 - All TMAs‘ question in Units 1 – 5 (Module 1)

Assignment 2 - All TMAs' question in Units 6 – 10 (Module 2)

Assignment 3 - All TMAs' question in Units 11 – 15 (Module 3)

Assignment 4 - All TMAs' question in Unit 16 – 19 (Module 4).

Presentation Schedule

The presentation schedule included in your course materials gives you the important dates for this year for the completion of tutor-marking assignments and attending

tutorials. Remember, you are required to submit all your assignments by due date. You should guide against falling behind in your work.

Assessment

There are two types of the assessment of the course. First are the tutor-marked assignments; second, there is a written examination.

In attempting the assignments, you are expected to apply information, knowledge and techniques gathered during the course. The assignments must be submitted to your tutor for formal Assessment in accordance with the deadlines stated in the Presentation Schedule and the Assignments File. The work you submit to your tutor for assessment will count for 30 % of your total course mark.

At the end of the course, you will need to sit for a final written examination of three hours' duration. This examination will also count for 70% of your total course mark.

Tutor-Marked Assignments (TMAs)

There are four tutor-marked assignments in this course. You will submit all the assignments. You are encouraged to work all the questions thoroughly. The TMAs constitute 30% of the total score.

Assignment questions for the units in this course are contained in the Assignment File. You will be able to complete your assignments from the information and materials contained in your set books, reading and study units. However, it is desirable that you demonstrate that you have read and researched more widely than the required minimum. You should use other references to have a broad viewpoint of the subject and also to give you a deeper understanding of the subject.

When you have completed each assignment, send it, together with a TMA form, to your tutor. Make sure that each assignment reaches your tutor on or before the deadline given in the Presentation File. If for any reason, you cannot complete your work on time, contact your tutor before the assignment is due to discuss the possibility of an extension. Extensions will not be granted after the due date unless there are exceptional circumstances.

Final Examination and Grading

The final examination will be of three hours' duration and have a value of 70% of the total course grade. The examination will consist of questions which reflect the types of self-assessment practice exercises and tutor-marked problems you have previously encountered. All areas of the course will be assessed

Revise the entire course material using the time between finishing the last unit in the module and that of sitting for the final examination to. You might find it useful to review your self-assessment exercises, tutor-marked assignments and comments on them before the examination. The final examination covers information from all parts of the course.

Course Marking Scheme

The Table presented below indicates the total marks (100%) allocation.

Assignment	**Marks**
Assignments (Best three assignments out of four that is marked)	30%
Final Examination	70%
Total	**100%**

Course Overview

The Table presented below indicates the units, number of weeks and assignments to be taken by you to successfully complete the course, Introduction to Econometrics (ECO 355).

Units	**Title of Work**	**Week's Activities**	**Assessment (end of unit)**
	Course Guide		
	Module 1 ECONOMETRICS ANALYSIS		
1	Meaning Of Econometrics	Week 1	Assignment 1
2	Methodology of Econometrics	Week 1	Assignment 1
3	Computer and Econometrics	Week 2	Assignment 1
4.	Basic Econometrics Models: Linear Regression.	Week 2	Assignment 1
5.	Importance Of Econometrics		
	Module 2 SINGLE- EQUATION (REGRESSION MODELS)		
1.	Regression Analysis	Week 3	Assignment 2
2.	The Ordinary Least Square (OLS) Method Estimation	Week 3	Assignment 2
3.	Calculation of Parameter β and the Assumption of Classical Least Regression Method (CLRM)	Week 4	Assignment 2

4.	Properties of the Ordinary Least Square Estimators	Week 5	Assignment 2
5.	The Coefficient of Determination (R^2): A measure of ―Goodness of fit‖	Week 6	Assignment 3
Modu le 3 NORMAL LINEAR REGRES SION MODEL (CNLRM)			
1.	Classical Normal Linear Regression Model	Week 7	Assignment 3
2.	OLS Estimators Under The Normality Assumption	Week 8	Assignment 3
3.	The Method Of Maximum Likelihood (ML)	Week 9	Assignment 3
4.	Confidence intervals for Regression Coefficients β_1 and β_2	Week 10	Assignment 4
5.	Hypothesis Testing	Week 11	
Module 4 PRACTICAL ASPECTS OF ECONOM ETRICS TEST			
1.	Accepting & Rejecting an Hypothesis	Week 12	Assignment 4
2.	The Level of Significance	Week 13	Assignment 4
3.	Regression Analysis and Analysis of Variance	Week 114	Assignment 4
4.	Normality tests	Week 15	Assignment 4
	Total	**15 Weeks**	

How To Get The Most From This Course

In distance learning the study units replace the university lecturer. This is one of the great advantages of distance learning; you can read and work through specially designed study materials at your own pace and at a time and place that suit you best. Think of it as reading the lecture instead of listening to a lecturer. In the same way that a lecturer might set you some reading to do, the study units tell you when to read your books or other material, and when to embark on discussion with your colleagues. Just as a lecturer might give you an in-class exercise, your study units provides exercises for you to do at appropriate points.

Each of the study units follows a common format. The first item is an introduction to the subject matter of the unit and how a particular unit is integrated with the other units and the course as a whole. Next is a set of learning objectives. These objectives let you know what you should be able to do by the time you have completed the unit.

You should use these objectives to guide your study. When you have finished the unit you must go back and check whether you have achieved the objectives. If you make a habit of doing this you will significantly improve your chances of passing the course and getting the best grade.

The main body of the unit guides you through the required reading from other sources. This will usually be either from your set books or from a readings section. Some units require you to undertake practical overview of historical events. You will be directed when you need to embark on discussion and guided through the tasks you must do.
The purpose of the practical overview of some certain historical economic issues are in twofold. First, it will enhance your understanding of the material in the unit. Second, it will give you practical experience and skills to evaluate economic arguments, and understand the roles of history in guiding current economic policies and debates outside your studies. In any event, most of the critical thinking skills you will develop during studying are applicable in normal working practice, so it is important that you encounter them during your studies.

Self-assessments are interspersed throughout the units, and answers are given at the ends of the units. Working through these tests will help you to achieve the objectives of the unit and prepare you for the assignments and the examination. You should do each selfassessment exercises as you come to it in the study unit. Also, ensure to master some major historical dates and events during the course of studying the material.

The following is a practical strategy for working through the course. If you run into any trouble, consult your tutor. Remember that your tutor's job is to help you. When you need help, don't hesitate to call and ask your tutor to provide it.

1. Read this Course Guide thoroughly.
2. Organize a study schedule. Refer to the `Course overview' for more details. Note the time you are expected to spend on each unit and how the assignments relate to the units. Important information, e.g. details of your tutorials, and the date of the first day of the semester is available from study centre. You need to gather together all this information in one place, such as your dairy or a wall calendar. Whatever method you choose to use, you should decide on and write in your own dates for working breach unit.
3. Once you have created your own study schedule, do everything you can to stick to it. The major reason that students fail is that they get behind with their course work. If you get into difficulties with your schedule, please let your tutor know before it is too late for help.
4. Turn to Unit 1 and read the introduction and the objectives for the unit.

5. Assemble the study materials. Information about what you need for a unit is given in the `Overview' at the beginning of each unit. You will also need both the study unit you are working on and one of your set books on your desk at the same time.
6. Work through the unit. The content of the unit itself has been arranged to provide a sequence for you to follow. As you work through the unit you will be instructed to read sections from your set books or other articles. Use the unit to guide your reading.
7. Up-to-date course information will be continuously delivered to you at the study centre.
8. Work before the relevant due date (about 4 weeks before due dates), get the Assignment File for the next required assignment. Keep in mind that you will learn a lot by doing the assignments carefully. They have been designed to help you meet the objectives of the course and, therefore, will help you pass the exam. Submit all assignments no later than the due date.
9. Review the objectives for each study unit to confirm that you have achieved them. If you feel unsure about any of the objectives, review the study material or consult your tutor.
10. When you are confident that you have achieved a unit's objectives, you can then start on the next unit. Proceed unit by unit through the course and try to pace your study so that you keep yourself on schedule.
11. When you have submitted an assignment to your tutor for marking do not wait for it return `before starting on the next units. Keep to your schedule. When the assignment is returned, pay particular attention to your tutor's comments, both on the tutor-marked assignment form and also written on the assignment. Consult your tutor as soon as possible if you have any questions or problems.
12. After completing the last unit, review the course and prepare yourself for the final examination. Check that you have achieved the unit objectives (listed at the beginning of each unit) and the course objectives (listed in this Course Guide).

Tutors and Tutorials

There are some hours of tutorials (2-hours sessions) provided in support of this course. You will be notified of the dates, times and location of these tutorials. Together with the name and phone number of your tutor, as soon as you are allocated a tutorial group.

Your tutor will mark and comment on your assignments, keep a close watch on your progress and on any difficulties you might encounter, and provide assistance to you during the course. You must mail your tutor-marked assignments to your tutor well

before the due date (at least two working days are required). They will be marked by your tutor and returned to you as soon as possible.

Do not hesitate to contact your tutor by telephone, e-mail, or discussion board if you need help. The following might be circumstances in which you would find help necessary. Contact your tutor if.

- You do not understand any part of the study units or the assigned readings
- You have difficulty with the self-assessment exercises
- You have a question or problem with an assignment, with your tutor's comments on an assignment or with the grading of an assignment.

You should try your best to attend the tutorials. This is the only chance to have face to face contact with your tutor and to ask questions which are answered instantly. You can raise any problem encountered in the course of your study. To gain the maximum benefit from course tutorials, prepare a question list before attending them. You will learn a lot from participating in discussions actively.

Summary

The course, Introduction to Econometrics II (ECO 355), expose you to the field of Econometrics analysis such as Meaning of Econometrics, Methodology of Econometrics, Computer and Econometrics, and Basic Econometrics Models: Linear Regression, Importance of Econometrics etc. This course also gives you insight into Single- Equation (Regression Models) such as; Regression Analysis, the Ordinary Least Square (OLS) Method Estimation, Calculation of Parameter β and the Assumption of Classical Least Regression Method (CLRM), Properties of the Ordinary Least Square Estimators and the Coefficient of Determination (R^2): A measure of ―Goodness of fit‖. The course shield more light on the Normal Linear Regression Model (CNLRM) such as Classical Normal Linear Regression Model, OLS Estimators Under The Normality Assumption, the Method Of Maximum Likelihood (ML). However, Confidence intervals for Regression Coefficients β_1 and $\beta_2$and Hypothesis Testing were also examined. Furthermore the course shall enlighten you about the Practical Aspects of Econometrics Test such accepting & Rejecting an Hypotheses, the Level of Significance, regression Analysis and Analysis of Variance and Normality tests.

On successful completion of the course, you would have developed critical thinking skills with the material necessary for efficient and effective discussion on Econometrics Analysis, Single- Equation (Regression Models), Normal Linear Regression Model (CNLRM) and Practical Aspects of Econometrics. However, to gain a lot from the course please try to apply anything you learn in the course to term papers writing in other economic development courses. We wish you success with the course and hope that you will find it fascinating and handy.

Module 1: Econometrics Analysis

This module introduces you to Econometrics analysis. The module consists of 5 units which include: meaning of econometrics, methodology, computer and econometrics, basic econometrics models: linear regression and importance

Unit One: Meaning of Econometrics
Unit Two: Methodology of Econometrics
Unit Three: Computer and Econometrics
Unit Four: Basic Econometrics Models: Linear Regression
Unit Five: Importance of Econometrics

Unit One: Meaning of Econometrics

Unit Structure

1.1. Introduction
1.2. Learning Outcome
1.3. Definition/Meaning of Econometrics
1.4. Why is Econometrics a Separate Discipline?
1.5. Summary
1.6. References/Further Readings/Web Resources
1.7. Possible Answers to Self-Assessment Exercises (SAEs)

1.1 INTRODUCION

The study of econometrics has become an essential part of every undergraduate course in economics, and it is not an exaggeration to say that it is also an essential part of every economist‘s training. This is because the importance of applied economics is constantly increasing and the ability to quantity and evaluates economic theories and hypotheses constitutes now, more than ever, a bare necessity. Theoretical economies may suggest that there is a relationship between two or more variables, but applied economics demands both evidence that this relationship is a

real one, observed in everyday life and quantification of the relationship, between the variable relationship using actual data is known as econometrics.

1.2. Learning Content

At the end of this unit, you should be able to:

i. understand the basic fundamentals of Econometrics

ii. Distinguish between Econometrics and Statistics.

1.3. Definition/Meaning of Econometrics

Literally econometrics means measurement (the meaning of the Greek word metrics) in economic. However econometrics includes all those statistical and mathematical techniques that are utilized in the analysis of economic data. The main aim of using those tools is to prove or disprove particular economic propositions and models.
Econometrics, the result of a certain outlook on the role of economics consists of the application of mathematical statistics to economic data to tend empirical support to the models constructed by mathematical economics and to obtain numerical results.
Econometrics may be defined as the quantitative analysis of actual economic phenomena based on the concurrent development of theory and observation, related by appropriate methods of inferences.
Econometrics may also be defined as the social sciences in which the tools of economic theory, mathematics and statistical inference are applied to the analysis of economic phenomena.
Econometrics is concerned with the empirical determination of economic laws.

Self-Assessment Exercises 1

Define Econometrics

1.4 Why Is Econometrics A Separate Discipline?

Based on the definition above, econometrics is an amalgam of economic theory, mathematical economics, economic statistics and mathematical statistics. However, the course (Econometrics) deserves to be studied in its own right for the following reasons:

1. Economic theory makes statements or hypotheses that are mostly qualitative in nature. For example, microeconomics they states that, other thing remaining the same, a reduction in the price of a commodity is expected to increase the quantity demanded of that commodity. Thus, economic theory postulates a negative or inverse relationship between the price and quantity demanded of a commodity. But the theory itself does not provide any numerical measure of the relationship between the two\; that is it does not tell by how much the quantity will go up or down as a result of a certain change in the price of the commodity. It is the job of econometrician to provide such numerical estimates. Stated differently, econometrics gives empirical content to most economic theory.
2. The main concern of mathematical economics is to express economic theory in mathematical form (equation) without regard to measurability or mainly interested in the empirical verification of the theory. Econometrics, as noted in our discussion above, is mainly interested in the empirical verification of economic theory. As we shall see in this course later on, the econometrician often uses the mathematical equations proposed by the mathematical economist but puts these equations in such a form that they lend themselves to empirical testing and this conversion of mathematical and practical skill.
3. Economic statistics is mainly concerned with collecting, processing and presenting economic data in the form of charts and tables. These are the jobs of the economic statistician. It is he or she who is primarily responsible for collecting data on gross national product (GNP) employment, unemployment, price etc. the data on thus collected constitute the raw data for econometric work, but the economic statistician does not go any further, not being concerned with using the collected data to test economic theories and one who does that becomes an econometrician.
4. Although mathematical statistics provides many tools used in the trade, the econometrician often needs special methods in view of the unique nature of the most economic data, namely, that the data are not generated as the result of a controlled experiment. The econometrician, like the meteorologist, generally depends on data that cannot be controlled directly.

Self-Assessment Exercises 2

Why Is Econometrics A Separate Discipline?

1.5 Summary

In econometrics, the modeler is often faced with observational as opposed to experimental data. This has two important implications for empirical modeling in econometrics. The modeler is required to master very different skills than those needed for analyzing experimental data and the separation of the data collector and the data analyst requires the modeler familiarize himself/herself thoroughly with the nature and structure of data in question. The units vividly look at the meaning of econometrics which is different from the modern day to day calculation or statistical analysis we are all familiar with. However, the units also discuss the reasons why econometrics is studied differently from other disciplines in economics and how it is so important in formulating and forecasting the present to the future.

1.6. References/Further Readings/Web Resources

Gujarat, D. N. (2007). *Basic Econometrics*, 4th Edition, tata Mcgraw – Hill publishing company limited, New Delhi.

Hall, S. G., & Asterion, D. (2011). *Applied Econometrics*, 2nd Edition, Palgrave Macmillian, New York city, USA.

2.7 Possible Answers to SAEs

These are the possible answers to the SAEs within the content.

Answers to SAEs 1

Econometrics uses economic theory, mathematics, and statistical inference to quantify economic phenomena. In other words, it turns theoretical economic models into useful tools for economic policymaking.

Answers to SAEs 2

The course deserves to be studied in its own right for the following reasons: • Economic theory makes statements or hypotheses that are mostly qualitative in nature (the law of demand), the law does not provide any numerical measure of the relationship.

UNIT 2 METHODOLOGY OF ECONOMETRICS

Unit Structure

2.1. Introduction
2.2. Learning Outcomes
2.3. Traditional Econometrics Methodology
2.4. Summary

2.1. INTRODUCTION

One may ask question that how economists justifies their argument with the use of statistical, mathematic and economic models to achieve prediction and policy recommendation to economic problems. However, econometrics may also come inform of applied situation, which is called applied econometrics. Applied econometrics works always takes (or, at least, should take) as its starting point a model or an economic theory. From this theory, the first task of the applied econometrician is to formulate an econometric model that can be tested empirically and the next task is to collect data that can be used to perform the test and after that, to proceed with the estimation of the model. After this estimation, an econometrician performs specification tests to ensure that the model used was appropriate and to check the performance and accuracy of the estimation procedure. So these process keep on going until you are satisfied that you have a good result that can be used for policy recommendation.

2.2. Learning Outcomes

At the end of this unit, you should be able to:

i. Understand how the econometrician proceed in the analysis of an economic problem.
ii. Understand how the econometrician make use of both mathematical and statistical analysis in solving economic problems.

2.3. Traditional Econometrics Methodology

The traditional Econometrics methodology proceeds along the following lines:

1. Statement of theory or hypothesis.
2. Specification of the mathematical model of the theory.
3. Specification of statistical, or econometric, model.
4. Obtaining the data.
5. Estimation of parameters of the econometric model.
6. Hypothesis testing.
7. Forecasting or prediction.
8. Using the model for control or policy purposes.

However, to illustrate the proceeding steps, let us consider the well-known Keynesian theory of consumption.

1. Statement of the theory Hypothesis

Keynes stated:

The fundamental psychological law is that Men (Women) are disposed as a rule and on average, to increase their consumption as their income increases, but not as much as the increase in their income.

In short, Keynes postulated that the marginal propensity to consume (MPC), the rate of change of consumption for a unit change income is greater than zero but less than 1.

2. Specification of the mathematical model of consumption

Although Keynes postulated a positive relationship between consumption and income, he did not specify the precise form of the functional relationship between the two. However, a mathematical economist might suggest the following form of the Keynesian consumption function: $Y = \beta_1 + \beta_2 X$ where $0 < \beta_2 < 1$ _____(1).

Where Y = consumption expenditure and X = income and where β_1 and β_2, known as the parameters of the model, are respectively, the intercept and slope coefficients. The slope coefficient β_2 measures the MPC. In equation (1) above, which states that consumption is linearly related to income, is an example of a mathematical model of the relationship between consumption and income that is called consumption function in economics. A model is simply a set of mathematical equations, if the model had only one equation, as in the proceeding example, it is called a single equation model, whereas if it has more than one equation, it is known as a multiple-

equation model. In equation (1), the variable appearing on the left side of the equality sign is called the _dependent variable‘ and the variable(s) on the right side are called the independent or explanatory variables. Moreover, in the Keynesian consumption function in equation (1), consumption (expenditure) is the dependent variable and income is the explanatory variable.

3. Specification of the Econometric Model of Consumption

The purely mathematical model of the consumption function given in equation (1) is of limited interest to econometrician, for it assures that there is an exact or deterministic relationship between consumption and income. But relationships between economic variables are generally inexact. Thus, if we were to obtain data on consumption expenditure and disposable (that is after tax) income of a sample of, say, 500 Nigerians families and plot these data on a graph paper with consumption expenditure on the vertical axis And disposable income on the horizontal axis we would not expect all 500 observations to lie exactly on the straight line of equation (1) above, because, in addition to income other variables affect consumption expenditure. For example size of family, ages of the members in the family, family religion etc are likely to exert some influence on consumption.

To allow for the inexact relationships between economic variables, the econometrician would modify the deterministic consumption function in equation (1) as follows:

$$Y = \beta_1 + \beta_2 X + u _____ (2).$$

Where u, known as the disturbance, or error term, is a random (Stochastic) variable that has well-defined probabilistic properties. The disturbance term _u‘ may well represent all those factors that affect consumption but are not taken into account explicitly.

Equation (2) is an example of an econometric model. More technically, it is an example of a linear regression model, which is the major concern in this course.

4. Obtaining Data

To estimate the econometric model in equation (2), that is to obtain the numerical values of $\beta_1 and\ \beta_2$, we need data. Although will have more to say about the crucial importance of data for economic analysis. The data collection is used to analysis the equation (2) and give policy recommendation.

5. Estimation of the Econometric Model

Since from _obtaining the data‘ we have the data we needed, our next point of action is to estimate the parameters of the say consumption function. The numerical estimates of the parameters give empirical content to the consumption function. The actual mechanics of estimating the parameters will be discussed later in this course.

However, note that the statistical technique of regression analysis is the main tool used to obtain the estimates. For example assuming the data collected was subjected to calculation and we obtain the following estimates of $\beta_1 and \beta_2$, namely $-144.06\ and\ 0.8262$. Thus, the estimated consumption function is:

$\hat{y} = -144.06\ and\ 0.8262$_____(3)

The hat on the y indicated that it is an estimate. The estimated consumption function (that is regression line) is shown below.

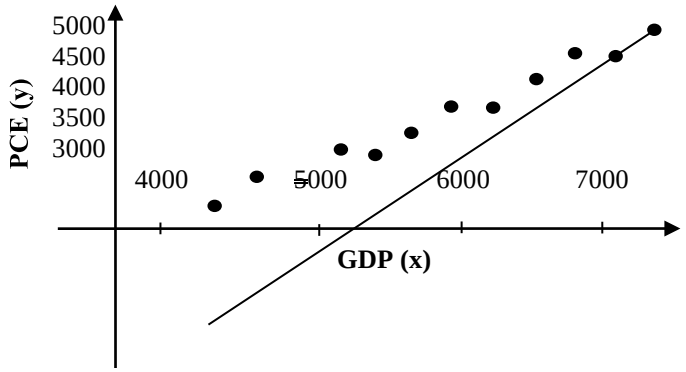


Figure 1: Showing personal consumption expenditure (y) in relation to GDP (x) from 1682 – 1996.

Moreover, the regression line fits the data quite well in that the data points are very close to the regression line.

6. Hypothesis Testing

Assuming that the fitted model is a reasonably good approximation of reality, we have to develop suitable criteria to find out whether the estimates obtained in equation (3) are in accord with the expectations of the theory that is being tested. According to ―positive‖ economists like Milton Freedman, a theory or hypothesis that is not verifiable by appeal to empirical evidence may not be admissible as a part of scientific enquiry.

As noted by Keynes that marginal propensity to consume (MPC) to be positive but less than 1. In equation (3) the MPC is 083. But before we accept this finding as confirmation of Keynesian consumption theory, we must enquire whether this estimate is sufficiently below unity to convince us that this is not a chance occurrence or peculiarity of the particular data we have used. In conclusion, 0.83 is statistically less than 1. If it is, it may support Keynes theory. This type of confirmation or refutation of the economic theories on the basis of sample evidence is based on a branch of statistical theory known as statistical inference (hypothesis testing).

7. Forecasting or Prediction

If the model we choose does not refute the hypothesis or theory under consideration, we may use it to predict the future value(s) of the dependent, or forecast variable y on the basis of known or expected future value(s) of the explanatory or predictor variable x.

Let us make use of equation (3) as an example. Suppose we want to predict the main consumption expenditure for 1997. The GDP value for 1997 (for example say is) 6158.7billion dollars. Putting this GDP figure on the right-hand side of equation (3), we obtain

$$\hat{}_{1997} = -144.06 + 0.8262\ X \quad \text{_____(3)}$$
$$_{1997} = -144.06 + 0.8262\ (6158.7) \quad \text{_____}$$
$$= -144.06 + 5088\ .31794$$
$$= -4944.2579$$

or about 4944 billion naira. Thus given the value of the GDP, the mean or average, forecast consumption expenditure is about 4944 billion naira. The actual value of the consumption expenditure reported in 1997 was 4913.5 billion naira. The estimated model (in equation 3) thus over predicted the actual consumption expenditure by about 30.76 billion naira. We could say that forecast error is about 30.76 billion naira, which is about 0.74 percent of the actual GDP value for 1997.

8. Use of the Model for Control or Policy Purpose

Let us assume that we have already estimated a consumption function given in equation (3). Suppose further the government believes that consumer expenditure of about say 4900 (billion of 1992 naira) will keep the unemployment rate at its current level of about 4.2 percent (early 2000). What level of income will guarantee the target amount of consumption expenditure? If the regression result given in equation (3) seem reasonable, sample arithmetic will show that 4900 = – 144.06 + 0.8262x _____ (5). Which gives x = 6105, approximately. That is, an income levels of about 6105 (billion) naira, given an MPC of about 0.83, will produce (10) an expenditure of about 4900 billion naira.

From the analysis above, an estimated model may be used for control or policy purposes. By appropriate fiscal and monetary policy mix, the government can manipulate the control variable x to produce the desired level of the target variable y.

Self-Assessment Exercises 1

Discuss four stages of econometrics?

2.4. Summary

Stages of econometrics analysis are the process of getting on economic theory, subject it to empirical model, and then make use of data, estimation, and hypothesis and policy recommendation. The unit has discussed attentively the stages econometrics analysis from the economic theory, mathematical model of theory, and econometric model of theory, collecting the data, estimation of econometric model, hypothesis testing, forecasting or prediction and using the model for control or policy purposes. Therefore at this end I belief you must have understand the stages of econometrics analysis.

2.5. References/Further Readings/Web Resources

Adekanye, D. F. (2008). *Introduction to Econometrics*, 1st edition, Addart Publisher

Dimitrios, A & Stephen, G. (2011). Applied Econometrics. 2nd Edition, Palgrave Macmillan

2.6. Possible Answers to SAEs

These are the possible answers to the SAEs within the content.

Answers to SAEs 1

4 stages of econometrics

i. Develop a theory or hypothesis. Econometricians first establish a hypothesis or theory to guide

data analysis.

ii. Specify a statistical model. In this step, econometricians identify a statistical model to examine

the relationship between variables. ...

iii. Estimate the model's variables. ...

iv. Perform a test.

UNIT 3 COMPUTER AND ECONOMETRICS

Unit Structure

3.1. Introduction
3.2. Learning Outcomes
3.3. Use of Computers in Economic Analysis and Forecasting
3.3.1. Forecasting
3.3.2. Approaches to Forecasting
3.3.3. Matching the Situations with Forecasting Methods
3.3.4. Six important characteristics or dimensions of planning and decision-making which Determine the choice of forecasting
3.3.5. Six major factors which are considered important in Forecasting
3.3.6. Forecasting Tools
3.3.7. Computers and Forecasting
3.5. Role of Computer in Econometrics analysis
3.6. Types of Econometrics Basic
3.7. Theoretical versus Applied Economics
3.8. The Differences between Econometrics Modeling and Machine Learning
3.9. Summary
3.10. References/Further Readings/Web Resources
3.11. Possible Answers to Self-Assessment Exercises (SAEs)

Computers and Forecasting

3.1. INTRODUCTION

In this unit, we are going to know briefly the role computer application in econometrics analysis and to be able to convinced people that are not economists that computer help in bringing the beauty of economic model to reality and prediction. The computer application are peculiar to social sciences techniques/analysis and economics in particular.

3.2. Learning Outcomes

At the end of this unit, you should be able to:

i. understand the role of computer in econometrics analysis
ii. identify/explain the types of econometrics analysis.

3.3. Use of Computers in Economic Analysis and Forecasting

Computers are now-a-days often used in making complicated investment decisions. As we add more branches to the decision tree, we reduce our ability to analyse problems quickly. However, the rapid development of sophisticated computer equipment has increased the usefulness of computer-based analysis of complex investment decisions.

For example, the decision to build a nuclear power plant is a difficult one. It can take up to ten years to complete such a project. Along the way there may be threats of strikes, unanticipated cost increases, technical problems, and resistance from antinuclear groups. Revenues depend on future demand. Demand depends on production trends, income levels, energy use, and alternative sources of energy.

If we can determine probability distributions for each of these factors, we can programme a computer to simulate the future. The computer randomly selects a value from each of these distributions and simulates its effect on the firm's demand and cost functions. After hundreds of these simulations, the computer generates a distribution on the expected rates of return from this decision.

If there is a choice of projects, the firm can use the simulated rates of return to calculate both the expected return and the degree of risk involved. Using the certainty equivalent method or the risk-adjusted discount rate, the firm can compare these investments.

However, the success of this approach depends on the quality of the probability distributions of a large number of variables.

By using a similar but less-involved method, the firm can arbitrarily choose a best-guess value for each of the variables of importance.

For example, by recalculating the expected rate of return by varying each of these variables within a reasonable range, the firm can determine the sensitivity of the expected rates of return to changes in these variables. The firm can then concentrate on finding more precise estimates of those variables that have the greatest influence on the expected rate of return of the project.\

Using sophisticated in-house technology a major computer manufacturer developed a United States-based rein voicing center that controls literally all of the company's cross-border transactions.

The computer system provides daily worldwide exposure reports, facilitating centralized exposure management, aggressive leading and lagging strategies and substantial savings on holding costs each year which is an impossible chore without a computer.

1. Computers Streamline Operations:

In today's highly competitive business world, firms strive to increase productivity and slash costs. In fact, a growing number of companies are instituting austerity programmes to cut layers of corporate management, especially on the international side.

Computers play a critical role in this effort. By automating finance, companies can reduce labour costs and dramatically improve the speed and accuracy of many routine tasks.

For example, the controller of a leading American automobile manufacturer believes that computers are essential for producing a cost-competitive car. By using computers it is possible to reduce labour costs considerably and produce less expensive cars.

2. Computers help Companies Manage Globalized Businesses:

As part of their drive to be competitive many companies now turn each of their component businesses as world-wide organisations, and plan their manufacturing and sourcing strategies on a global basis. To manage their far-flung operations effectively, firms increasingly turn to computers.

As one financial executive of a large multinational noted, **"We receive data from over 50 markets. Without computers we couldn't possibly coordinate that volume of data quickly and efficiently."**

One main reason for the use of computers in economic analysis and forecasting is the widespread availability of in expense, convenient microcomputers. The personal computer (PC) has already become a fixture in financial departments the world over.

People are drawn by what PCs have to offer. For a small investment of time and effort one can now, perform various financial analyses more easily and quickly. The end result is increased productivity.

The capital budgeting process encompasses a variety of planning activities with a time horizon of more than one year, which is an increasingly difficult and critical exercise in today's environment. Extremely volatile currency and interest rates, political upheavals, and the sudden imposition of exchange controls all pose threats to what once were secure overseas investments. Now numerous fast- growing companies are turning to automation to cope with these uncertainties. As one financial planning manager explained, **"The biggest risks about projects now-a-days are the assumptions. By using computers, you can determine which of the assumptions are the most sensitive. This produces more and better data to use and rely on."**

It appears that over the next few years, global firms will more than double their use of computers for such key capital budgeting functions as project investment analysis and long-term portfolio planning, and will increasingly automate the forecasting of financial trends and political risk analysis to buttress their decisions.

Project investment analysis encompasses all DCF forecasting methods such as NPV, IRR, and payback period. Once a project has been proposed, finance staff must conduct sensitivity analysis. For example, what happens to the IRR on a project should prices degrade rather than hold constant? Other variables include price, market share, market volume, general economic conditions, and political risk.

According to a recent survey, the use of computers for analysing capital project proposals will rise dramatically over the next few years. This tremendous surge can be attributed in part to the spread of the PCs to all aspects of financial planning.

The use of PCs has enabled senior management to standardize new project analysis corporate-wide. The analysis process works as follows: Twice a year, the corporate planning department evaluates current costs of capital and, based on those figures, determines an appropriate hurdle rate for new projects.

In the past corporate economics department was seen as an aloof and separate entity. Now, through better integration of the economics department with the finance function, the corporate economic staff has become a more relevant factor in capital budgeting analysis.

They are involved in capital budgeting in two primary ways:

1. They prepare a summarized list of standardized economic assumptions, which are distributed corporate-wide. The economics department now concentrates on those parts of the economy perceived as most critical in the long run, such as real growth of the economy, interest rates and inflation. To distribute the forecast data, staff relies both on in-house publication and computer networks.

2. They respond to ad hoc queries from local project analysts. The use of computers has made it easier for corporate economists to get involved in the analysis for new project proposals. For example, if a review is under way to evaluate committing funds for a major plant expansion in a certain country or region, an economist may be required to estimate long-term project demand, inflation rates, or currency fluctuations.

3.3.1. Forecasting

In the present age of uncertainty and information revolution managerial focus has shifted to improving the decision-making process in business and government. The key point in decision-making is accurate forecasts. In the area of marketing, for instance, forecasts of market size and market characteristics must be reliable.

A company producing and selling refrigerators, T.Vs., etc., must make accurate forecasts of both regional market demand and types of customers. Based on this forecast, decisions regarding advertising and other sales promotion efforts are taken.

In the area of production management also there is need for forecasting. Product demand and product mix, production scheduling, inventory holding, labour scheduling, equipment purchase, plant capacity planning, maintenance, etc., are all based on such forecasts.

In finance and accounting, forecasting is of strategic importance in the area of cash flows, debt collection, capital expenditure rates, working capital management etc. Even the personnel department is required to make manpower planning which is nothing other than forecast for different types of human resources required in business now and in the future.

3.3.2. Approaches to Forecasting

Prior to 1950s there existed hardly any method for business forecasting. In the mid-1950s exponential smoothing technique was first used by the defence personnel for forecasting purposes. Subsequently, this technique was applied to business organisations.

In the 1960s the computer power became cheaper and techniques like multiple regression and econometric models were widely used to quantify and test economic theory with statistical data. As economics entered the age of computers in the 1970's the process was hastened by the availability of cheap computers.

In 1976 the Box-Jenkins method was developed. It is a systematic procedure for analysing time series data. In truth, the Box-Jenkins approach to time-series forecasting was as accurate as the econometric models and methods.

In the 1960s and 1970s technological forecasting methods were developed of which the Delphi method and cross-impact matrices were very popular. However, in 1970s it was first realised that forecasts were useless unless they were applied for planning and decision-making purposes.

3.3.3. Matching the Situations with Forecasting Methods

There are various methods of forecasting. Different methods are suitable for different situations and different purposes. A manager must select the appropriate forecasting technique, i.e., the one which answers his needs (or serves a particular purpose).

3.3.4 Six important characteristics or dimensions of planning and decision-making which determine the choice of forecasting methods are the following:

1. Time Horizon:

The period of time for which the decision is made will have an impact. It may be the immediate term (i.e., less than one month), short-term (up to 3 months), medium-term (up to-2 years) long-term (more than 2 years).

2. Level of Details:

While selecting a forecasting method for a particular situation, one must know the level of details which will be needed for the forecast to be useful for decision-making purposes. The need for detailed information varies from situation to situation and time to time.

3. The Number of Variables:

The number of variables to be forecast affects the need for detail which, its turn, determines the choice of appropriate methods even in the same situation. When forecast is to be made for a single variable, the procedures used can be more detailed and complex than when forecasts are made for a number of variables.

4. Constancy:

Forecasting a situation which does not change is different from forecasting a situation which is fairly unstable (i.e., a situation which often keeps on changing).

5. Control Vs. Planning:

The controlling function is performed by using a new technique called management by exception. Any forecasting method must be sufficiently flexible so that the changes in the basic patterns of behaviour of variables or relationships among them can be detected at an early stage.

6. Existing Planning Procedures:

For introducing new forecasting methods, often the existing planning and decision-making procedures have to be changed. Moreover, in case of any deviation from a set path it gives early warning and the managers face human resistance to such changes.

So the usual practice is to select those forecasting methods which are most closely related to the existing plans and procedures. In case of necessity, these methods can be improved later on.

3.3.5. Six major factors which are considered important in forecasting are given below:

1. Time Horizon:

Two aspects of the time horizon are related to most forecasting methods, viz., the span of time in future for which different methods are appropriate and that the number periods for which a forecast is required.

2. Data Pattern:

For matching forecasting methods with the existing pattern of data (i.e., seasonal/cyclical, time-series/cross section etc.) an appropriate method is to be selected.

3. Accuracy:

Forecasts must be as accurate as possible.

4. Cost:

In any forecasting procedure the following costs are generally involved:

(a) Development;

(b) Data preparation;

(c) Actual operation; and

(d) Cost of foregone opportunity.

5. Reliability:

Managers should not forecast anything based on data which is not reliable for the purpose of managerial decision making.

5. Availability of computer software:

It is not possible to apply any given quantitative forecasting method without an appropriate computer programme. Programmes must be "free" from major "bugs", well documented and easy to use, for getting satisfactory results.

3.3.6. Forecasting Tools

Economists have developed various forecasting tools to be able to foresee changes in the economy. In earlier times, economists used to look into the future by using easily available data on things like money supply, house construction, and steel production. For example, a sudden fall in steel production was a sign that businesses had reduced purchases and that the economy would soon slow down.

At a later stage this process was formalized by combining several different statistics into an 'index of leading indicators', which is now published every month by the US Department of Commerce. Although not very accurate, the index gives an early and mechanical warning on whether the economy is heading up or sliding down.

For a more accurate prediction of some key variables and for a more detailed look into the future, economists turn to computerised econometric forecasting models. Due to the pioneering works of Jan Tinbergen and L. R. Klein macroeconomic forecasting has gained much popularity and considerable reliability over the last 25 years.

The Wharton Forecasting Model developed in the Wharton Business School, Pennsylvania, is perhaps the most elaborated of all. Private consulting firms, such as Data Resources Inc., have developed models that are widely used by businesses and policy makers.

3.3.7. Computers and Forecasting

The commercial computers in the 1950s were very large, complicated, slow and expensive. Moreover, they had minimum storage capacity. In the 1960s substantial improvement on it was made.

The powerful microcomputers of today run faster, are comparatively cheap and contain more RAM memory. It is likely that there will be further improvement in speed memory and capacity of computers. It also seems that cost and size of the computers will also be reduced in the future.

Two major advantages of modern computers are the incredibly high speed and great accuracy with which they can do calculations. Hence any forecasting method can be programmed to run on a computer. Even the most calculation-intensive methods can be run on a micro-computer within a few minutes.

How are computer models of the economy constructed and used for forecasting purposes? As a general rule forecasters start with an analytical framework containing equations representing both aggregate demand and aggregate supply.

Using modern econometric techniques, each equation is 'fitted' to the historical data to obtain parameter estimates (such as the M PC, the shapes of the money- demand equations, the growth of potential GNP, etc.). Additionally, at each stage of the forecasting exercise modellers use their own judgement and experience to assess whether the results are reasonable.

Self-Assessment Exercises 1

1. What is the role of computer in forecasting?
2. What is the role of computer in econometrics?

3.5. Role of Computer in Econometrics Analysis

Regression analysis, the bread-and-better tool of econometrics, these days is unthinkable without the computer and some access to statistical software. However, several excellent regression packages are commercially available, both for the mainframe and the microcomputer and the lot is growing by the day.

Regression software packages such as SPSS, EVIENS, SAS, STATA etc. are few of the economic software packages use in conducting estimation analysis on economic equations and models.

Computers are used in the creation of complex forecasting models. As in computational finance, computer simulations and models can be used to predict how

markets will change. It offers good facilities for the analysis of cross-section and panel data. In economics a cross-section data set contains data on collection of economic agents at a given point in time. It also use for a more accurate prediction of some key variables and for a more detailed look into the future, economists turn to computerized econometric forecasting models.

Self-Assessment Exercises 2

What is the use of computer in forecasting?

3.6. TYPES OF ECONOMETRICS

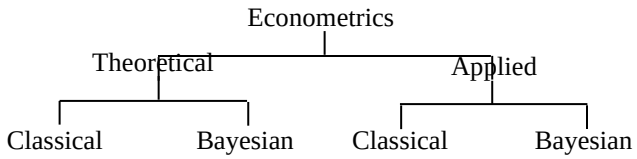


Figure 2: Showing categories of Econometrics.
As the classificatory scheme in figure 2 suggests, econometrics may be divided into two broad categories: **THEORETICAL ECONOMETRICS and APPLIED ECONOMETRICS.** In each category, one can approach the subject in the classical or Bayesian tradition.

Furthermore, theoretical econometrics is concerned with the development of appropriate methods for measuring economic relationships specified by econometrics models. In this aspect, econometrics leans heavily on mathematical statistics. Theoretical econometrics must spell out the assumptions of this method, its properties and what happens to these properties when one or more of the assumptions of the method are not fulfilled.
In applied econometrics we use the tools of theoretical econometrics to study some special field (s) of economics and business, such as the production function, investment function, demand and supply functions, portfolio theory etc.

3.7. Theoretical versus Applied Economics

The study of economics has taken place within a Kuhnian paradigm of perfect competition for years. Within this paradigm, the models of perfect competition,

rational expectations, supply and demand, and the other economic theories have been described. In recent years, there has been a strong movement towards mathematics and econometrics as a way to expound upon already established theories. This movement has come under some criticism, both from within the profession and without, as not being applicable to real world situations. There has been a push to move away from the econometric methods that lead to further theory explanation and to focus on applying economics to practical situations. While the theories are innately important to the study of any economic activity, the application of those theories in policy is also important.

There are many areas of applied economics, including environmental, agricultural, and transitional. However, the recent trends towards mathematical models has caused some to question whether or not expounding on the theories will help in the policy decisions of taxation, inflation, interest rates, etc. Solutions to these problems have been largely theoretical, as economics is a social science and laboratory experiments cannot be done.
However, there are some concerns with traditional theoretical economics that are worth mentioning. First, Ben Ward describes "stylized facts," or false assumptions, such as the econometric assumption that "strange observations do not count."

[1] While it is vital that anomalies are overlooked for the purpose of deriving and formulating a clear theory, when it comes to applying the theory, the anomalies could distort what *should* happen. These stylized facts are very important in theoretical economics, but can become very dangerous when dealing with applied economics. A good example is the failure of economic models to account for shifts due to deregulation or unexpected shocks.

[2] These can be viewed as anomalies that are unable to be accounted for in a model, yet is very real in the world today.

Another concern with traditional theory is that of market breakdowns. Economists assume things such as perfect competition and utility maximization. However, it is easily seen that these assumptions do not always hold. One example is the idea of stable preferences among consumers and that they act efficiently in their pursuit. However, people's preferences change over time and they do not always act rational nor efficient.

[3] Health care, for another example, chops down many of the assumptions that are crucial to theoretical economics. With the advent of insurance, perfect competition is no longer a valid assumption. Physicians and hospitals are paid by insurance companies, which assures them of high salaries, but which prevents them from being competitive in the free market. Perfect information is another market breakdown in health economics. The consumer (patient) cannot possibly know everything the

doctor knows about their condition, so the doctor is placed in an economically advantaged position. Since the traditional assumptions fail to hold here, a manipulated form of the traditional theory needs to be applied. The assumption that consumers and producers (physicians, hospitals) will simply come into equilibrium together will not become a reality because the market breakdowns lead to distortions. Traditional theorists would argue that the breakdown has to be fixed and then the theory can applied as it should be. They stick to their guns even when there is conflicting evidence otherwise, and they propose that the problem lies with the actors, not the theory.

[4] The third concern to be discussed here ties in with the Kuhnian idea of normal science. The idea that all research is done within a paradigm and that revolutions in science only occur during a time of crisis. However, this concerns a "hard" science, and economics is a social science. This implies that economics is going to have an effect on issues, therefore, economists are going to have an effect on issues. Value-neutrality is not likely to be present in economics, because economists not only explain what is happening, predict what will happen, but they prescribe the solutions to arrive at the desired solution. Economics is one of the main issues in every political campaign and there are both liberal and conservative economists. The inference is that economists use the same theories and apply them to the same situations and recommend completely different solutions. In this vein, politics and values drive what solutions economists recommend. Even though theories are strictly adhered to, can a reasonably economic solution be put forth that is not influenced by values? Unfortunately, the answer is no.

Theoretical economics cannot hold all the answers to every problem faced in the "real world" because false assumptions, market breakdowns, and the influence of values prevent the theories from being applied as they *should*. Yet, the Formalist Revolution or move towards mathematics and econometrics continues to focus their efforts on theories. Economists continue to adjust reality to theory, instead of theory to reality.

[5] This is Gordon's "Rigor over Relevance." The concept that mathematical models and the need to further explain a theory often overrides the sense of urgency that a problem creates. There is much literature about theories that have been developed using econometric models, but Gordon's concern is that relevance to what is happening in the world is being overshadowed.

[6] This is where the push for applied economics has come from over the past 20 years or so. Issues such as taxes, movement to a free market from a socialist system, inflation, and lowering health care costs are tangible problems to many people. The notion that theoretical economics is going to be able to develop solutions to these problems seems unrealistic, especially in the face of stylized facts and market breakdowns. Even if a practical theoretical solution to the problem of health care

costs could be derived, it would certainly get debated by economists from the left and the right who are sure that this solution will either be detrimental or a saving grace.

Does this mean that theoretical economics should be replaced by applied economics? Certainly not. Theoretical economics is the basis from which economics has grown and has landed us today. The problem is that we do not live in a perfect, ideal world in which economic theory is based. Theories do not allow for sudden shocks nor behavioral changes.

[7] This is important as it undercuts the stable preferences assumption, as mentioned before. When the basic assumptions of a theory are no longer valid, it makes very difficult to apply that theory to a complex situation. For instance, if utility maximization is designed as maximizing my income, then it should follow that income become the measuring stick for utility. However, if money is not an important issue to someone, then it may appear as if they are not maximizing their utility nor acting rationally. They may be perfectly happy giving up income to spend time with their family, but to an economist they are not maximizing their utility. This is a good example of how theory and reality come into conflict.
The focus in theoretical economics has been to make reality fit the theory and not viceversa. The concern here is that this version of problem-solving will not actually solve any problems. Rather, more problems may be created in the process. There has been some refocusing among theoreticians to make their theories more applicable, but the focus of graduate studies remains on econometrics and mathematical models. The business world is beginning to take notice of this and is often requiring years away from the academic community before they will hire someone. They are looking for economists who know how to apply their knowledge to solve real problems, not simply to expound upon an established theory. It is the application of the science that makes it important and useful, not just the theoretical knowledge.

This is not to say that theoretical economics is not important. It certainly is, just as research in chemistry and physics is important to further understand the world we live in. However, the difference is that economics is a social science with a public policy aspect. This means that millions of people are affected by the decisions of policy-makers, who get their input from economists, among others. Legislators cannot understand the technical mathematical models, nor would they most likely care to, but they are interested in policy prescriptions. Should health care be nationalized? Is this the best solution economically? These are the practical problems that face individuals and the nation every day. The theoreticians provide a sturdy basis to start from, but theory alone is not enough. The theory needs to be joined with practicality that will lead to reasonable practical solutions of difficult economic problems. Economics cannot thrive without theory, and thus stylized facts

and other assumptions. However, this theory has to explain the way the world actually is, not the way economists say it should be.

[8] Pure economic theory is a great way to understand the basics of how the market works and how the actors *should* act within the market. False assumptions and market breakdowns present conflict between theory and reality. From here, many economists simply assume is not the fault of the theory, but rather the economic agents in play. However, it is impossible to make reality fit within the strict guidelines of a theory; the theory needs to be altered to fit reality. This is where applied economics becomes important. Application of theories needs to be made practical to fit each situation. To rely simply on theory and models is not to take into account the dynamic nature of human beings. What is needed is a strong theoretical field of economics, as well as a strong applied field. This should lead to practical solutions with a strong theoretical basis.

3.8. THE DIFFERENCE BETWEEN ECONOMETRICS MODELING AND MACHINE LEARNING

Econometric models are statistical models used in econometrics. An econometric model specifies the statistical relationship that is believed to be held between the various economic quantities pertaining to a particular economic phenomenon under study.

On the other hand- Machine learning is a scientific discipline that explores the construction and study of algorithms that can learn from data. So that makes a clear distinction right? If it learns on its own from data it is machine learning. If it is used for economic phenomenon it is an econometric model. However the confusion arises in the way these two paradigms are championed. The computer science major will always say machine learning and the statistical major will always emphasize modeling. Since computer science majors now rule at face book, Google and almost every technology company, you would think that machine learning is dominating the field and beating poor old econometric modeling.

But what if you can make econometric models learn from data?

Lets dig more into these algorithms. The way machine learning works is to optimize some particular quantity, say cost. A loss function or cost function is a function that maps a value(s) of one or more variables intuitively representing some ―cost‖ associated with the event. An optimization problem seeks to minimize a loss function. Machine learning frequently seek optimization to get the best of many alternatives.

Now, cost or loss holds different meanings in econometric modeling. In econometric modeling we are trying to minimize the error (or root mean squared error). Root mean squared error means root of the sum of squares of errors. An error is defined as the difference between actual and predicted value by the model for previous data.

The difference in the jargon is solely in the way statisticians and computer scientists are trained. Computer scientists try to compensate for both actual error as well as computational cost – that is the time taken to run a particular algorithm. On the other hand statisticians are trained primarily to think in terms of confidence levels or error in terms or predicted and actual without caring for the time taken to run for the model.

That is why data science is defined often as an intersection between hacking skills (in computer science) and statistical knowledge (and math). Something like K Means clustering can be taught in two different ways just like regression can be based on these two approaches. I wrote back to my colleague in Marketing – we have data scientists. They are trained in both econometric modeling and machine learning. I looked back and had a beer. If university professors don‘t shed their departmental attitudes towards data science, we will have a very confused set of students very shortly arguing without knowing how close they actually are.

3.9. Summary

Computer and Econometrics have a long history in econometrics analysis. The use of software to calculate data in economics analysis is very important in econometrics analysis and it has shown and gives the way forward in forecasting and policy recommendations to the stakeholders, private companies and government. The unit discussed extensively on the role of computer in econometrics. When equations in economics are turn to mathematical equations and becomes a model in economics, the computer software or what are economists called econometrics packages to solve/run the analysis for forecast and policy recommendation.

3.10. References/Further Readings/Web Resources

Begg, I. & Henry, S. G. (1998). *Applied Economics and Public Policy*. Cambridge University Press, United Kingdom.

Dimitrios, A & Stephen, G., (2011). *Applied Econometrics*. 2nd Edition, revised edition 2007.

Friedland, Roger and Robertson, A. F. (1990). *Beyond the Marketplace: Rethinking Economy and Society*. Walter de Gruyter, Inc. New York: Gordon, Robert Aaron. *Rigor and Relevance in a Changing Institutional Setting.*

Kuhn, Thomas. *The Structure of Scientific Revolutions*.

3.11 Possible Answers to SAEs

These are the possible answers to the SAEs within the content.

Answers to SAEs 1

1. Computers hold numerical modeling data for weather forecasting models. These computers make use of virtually all observational data that the NWS collects. This data comes from satellites, weather balloons, buoys, radar, and more.
2. It offers good facilities for the analysis of cross-section and panel data. In economics a cross-section data set contains data on collection of economic agents at a given point in time

Answers to SAEs 2

Computer-Based forecasting system incorporates forecasting techniques such as regression analysis, curve fitting, evaluation of closeness to a fit, moving averages (simple, exponential and weighted) and seasonal adjustments.

UNIT 4: BASIC ECONOMETRICS MODELS: LINEAR REGRESSION

Unit Structure

4.1. Introduction
4.2. Learning Outcomes
4.3. Econometrics Theory
4.4. Econometrics Methods
4.5. Examples of a Relationship in Econometrics
4.6. Limitations and Criticism
4.7. Summary
4.8. References/Further Readings/Web Resources
4.9. Possible Answers to Self-Assessment Exercises (SAEs)

4.1 INTRODUCTION

The basic tool for econometrics is the linear regression model. In modern econometrics, other statistical tools are frequently used, but linear regression is still the most frequently used starting point for an analysis. Estimating a linear regression on two variables can be visualized as fitting a line through data points representing paired values of the independent and dependent variables.

Okun's law representing the relationship between GDP growth and the unemployment rate. The fitted line is found using regression analysis. For example, consider Okun's law, which relates GDP growth to the unemployment rate. This relationship is represented in a linear regression where the change in unemployment rate (Δ Unemployment) is a function of an intercept ( ), a given value of GDP growth multiplied by a slope coefficient and an error term, :

$$\Delta \text{ Unemployment} = \beta_0 + \beta_1 \text{Growth} + \varepsilon.$$

The unknown parameters and can be estimated. Here is estimated to be −1.77 and is estimated to be 0.83. This means that if GDP growth increased by one percentage point, the unemployment rate would be predicted to drop by 1.77 points. The model could then be tested for statistical significance as to whether an increase in growth is associated with a decrease in the unemployment, as hypothesized. If the estimate of were not significantly different from 0, the test would fail to find evidence that changes in the growth rate and unemployment rate were related. The variance in a prediction of the dependent variable (unemployment) as a function of the independent variable (GDP growth) is given in polynomial least squares.

4.2. Learning Outcomes

At the end of this unit, you should be able to:

i. To understand the basic Econometrics models

ii. To be able to differentiate between Econometrics theory and methods

4.3. Econometric Theory

Econometric theory uses statistical theory to evaluate and develop econometric methods. Econometricians try to find estimators that have desirable statistical properties including unbiasedness, efficiency, and consistency. An estimator is unbiased if its expected value is the true value of the parameter; it is consistent if it converges to the true value as sample size gets larger, and it is efficient if the estimator has lower standard error than other unbiased estimators for a given sample size. Ordinary least squares (OLS) is often used for estimation since it provides the BLUE or "best linear unbiased estimator" (where "best" means most efficient, unbiased estimator) given the Gauss-Markov assumptions. When these assumptions are violated or other statistical properties are desired, other estimation techniques such as maximum likelihood estimation, generalized method of moments, or generalized least squares are used. Estimators that incorporate prior beliefs are advocated by those who favor Bayesian statistics over traditional, classical or "frequents" approaches. However, Econometrics uses economic theory, mathematics, and statistical inference to quantify economic phenomena. In other words, it turns theoretical economic models into useful tools for economic policymaking.

Self-Assessment Exercises 1

What is econometric theory?

4.4. Econometrics Methods

Applied econometrics uses theoretical econometrics and real-world data for assessing economic theories, developing econometric models, analyzing economic history, and forecasting.

Econometrics may use standard statistical models to study economic questions, but most often they are with observational data, rather than in controlled experiments. In this, the design of observational studies in econometrics is similar to the design of studies in other observational disciplines, such as astronomy, epidemiology, sociology and political science. Analysis of data from an observational study is guided by the study protocol, although exploratory data analysis may be useful for generating new hypotheses. Economics often analyzes systems of equations and inequalities, such as supply and demand hypothesized to be in equilibrium. Consequently, the field of econometrics has developed methods for identification and estimation of simultaneous-equation models. These methods are analogous to methods used in other areas of science, such as the field of system identification in systems analysis and control theory. Such methods may allow researchers to estimate models and investigate their empirical consequences, without directly manipulating the system.

One of the fundamental statistical methods used by econometricians is regression analysis. Regression methods are important in econometrics because economists typically cannot use controlled experiments. Econometricians often seek illuminating natural experiments in the absence of evidence from controlled experiments. Observational data may be subject to omitted-variable bias and a list of other problems that must be addressed using causal analysis of simultaneous-equation models.

4.4.1. Examples of a Relationship in Econometrics

A simple example of a relationship in econometrics from the field of labor economics is:

$$\ln(\text{wage}) = \beta_0 + \beta_1(\text{years of education}) + \varepsilon.$$

This example assumes that the natural logarithm of a person's wage is a linear function of the number of years of education that person has acquired. The parameter β_1 measures the increase in the natural log of the wage attributable to one more year of education. The term ε is a random variable representing all other factors that may have direct influence on wage. The econometric goal is to estimate the parameters, β_0 and β_1, under specific assumptions about the random variable ε. For example, if is uncorrelated with years of education, then the equation can be estimated with ordinary least squares.

If the researcher could randomly assign people to different levels of education, the data set thus generated would allow estimation of the effect of changes in years of education on wages. In reality, those experiments cannot be conducted. Instead, the econometrician observes the years of education of and the wages paid to people who differ along many dimensions. Given this kind of data, the estimated coefficient on Years of Education in the equation above reflects both the effect of education on wages and the effect of other variables on wages, if those other variables were correlated with education. For example, people born in certain places may have higher wages and higher levels of education. Unless the econometrician controls for place of birth in the above equation, the effect of birthplace on wages may be falsely attributed to the effect of education on wages.

The most obvious way to control for birthplace is to include a measure of the effect of birthplace in the equation above. Exclusion of birthplace, together with the assumption that ϵ is uncorrelated with education produces a misspecified model. Another technique is to include in the equation additional set of measured covariates which are not instrumental variables, yet render β_1 identifiable. An overview of econometric methods used to study this problem was provided by Card (1999).

Self-Assessment Exercises 2

What is econometric methods example?

4.5. LIMITATIONS AND CRITICISMS OF ECONOMETRICS MODEL

Like other forms of statistical analysis, badly specified econometric models may show a spurious relationship where two variables are correlated but causally unrelated. In a study of the use of econometrics in major economics journals, McCloskey concluded that some economists report p values (following the Fisherian tradition of tests of significance of point null-hypotheses) and neglect concerns of type II errors; some economists fail to report estimates of the size of effects (apart from statistical significance) and to discuss their economic importance. Some economists also fail to use economic reasoning for model selection, especially for deciding which variables to include in a regression. It is important in many branches of statistical modeling that statistical associations make some sort of theoretical sense to filter out spurious associations (e.g., the collinearity of the number of Nicolas Cage movies made for a given year and the number of people who died falling into a pool for that year).

In some cases, economic variables cannot be experimentally manipulated as treatments randomly assigned to subjects. In such cases, economists rely on observational studies, often using data sets with many strongly associated covariates, resulting in enormous numbers of models with similar explanatory ability but different covariates and regression estimates. Regarding the plurality of models compatible with observational data-sets, Edward learner urged that "professionals ... properly withhold belief until an inference can be shown to be adequately insensitive to the choice of assumptions".

Like other forms of statistical analysis, badly specified econometric models may show a spurious correlation where two variables are correlated but causally unrelated. Economist Ronald Coase is widely reported to have said "if you torture the data long enough it will confess". McCloskey argues that in published econometric work, economists tend to rely excessively on statistical techniques and often fail to use economic reasoning for including or excluding variables.Economic variables are not readily isolated for experimental testing, but Edward Leamer argues that there is no essential difference between econometric analysis and randomized trials or controlled trials provided judicious use of statistical techniques eliminates the effects of collinearity between the variables.

Economists are often faced with a high number of often highly collinear potential explanatory variables, leaving researcher bias to play an important role in their selection. Leamer argues that economists can mitigate this by running statistical tests with different specified models and discarding any inferences which prove to be "fragile", concluding that "professionals properly withhold belief until an inference can be shown to be adequately insensitive to the choice of assumptions

Econometrics is sometimes criticized for relying too heavily on the interpretation of raw data without linking it to established economic theory or looking for causal mechanisms. Also, It is crucial that the findings revealed in the data are able to be adequately explained by a theory, even if that means developing your own theory of the underlying processes. Regression analysis also does not prove causation, and just because two data sets show an association, it may be spurious. For example, drowning deaths in swimming pools increase with GDP. Does a growing economy cause people to drown? This is unlikely, but perhaps more people buy pools when the economy is booming. Econometrics is largely concerned with correlation analysis, and it is important to remember that correlation does not equal causation.

Self-Assessment Exercises 3

1. What are the limitation of econometrics model?
2. Briefly discuss the criticism of econometrics model.
3. Discuss the argument by some economist why most econometric models are wrong.

4.6. Summary

The unit discussed extensively on basic Econometrics models of linear regression analysis such as Econometrics theory, Econometrics Methods, Examples of econometrics modeling and the limitations and criticism of the models. Therefore, the unit conclude that basic econometrics models is the basis of econometrics and from the simple straight line graph we can see that a simple/linear regression equation is derived from the graph and from there, the model of econometrics started to emanate to becomes higher level of econometrics model which is called multiple regression analysis.

4.7. References/Further Readings/Web Resources

Akin, A. A. (2020). *Introduction to Econometrics*, 1st Edition, Mill World Publication.

Olusanjo, A. A. (2014). *Introduction to Econometrics, a Broader Perspective*, 1st Edition, World Press Publication.

Warlking, F.G., (2014). *Econometrics and Economic Theory*, 2nd edition, Dale Press Limited.

4.8. Possible Answers to SAEs

These are the possible answers to the SAEs within the content.

Answers to SAEs 1

Econometrics uses economic theory, mathematics, and statistical inference to quantify economic phenomena. In other words, it turns theoretical economic models into useful tools for economic policymaking.

Answers to SAEs 2

An example of the application of econometrics is to study the income effect using observable data. An economist may hypothesize that as a person increases their income, their spending will also increase.

Answers to SAEs 3

1. Econometrics is sometimes criticized for relying too heavily on the interpretation of raw data without linking it to established economic theory or looking for causal mechanisms.

2. McCloskey argues that in published econometric work, economists tend to rely excessively on statistical techniques and often fail to use economic reasoning for including or excluding variables.

3. Since econometrics does not content itself with only making optimal predictions, but also aspires to explain things in terms of causes and effects, econometricians need loads of assumptions tthat is most important of these are additivity and linearity. Important, simply because if they are not true, your model is invalid and descriptively incorrect. And when the model is wrong, then it is wrong.

Limiting model assumptions in economic science always have to be closely examined since if we are going to be able to show that the mechanisms or causes that we isolate and handle in our models are stable in the sense that they do not change when we 'export' them to our 'target systems,' we have to be able to show that they do not only hold under *ceteris paribus* conditions and *a fortiori* only are of limited value to our understanding, explanations or predictions of real economic systems.

Our admiration for technical virtuosity should not blind us to the fact that we have to have a cautious attitude towards probabilistic inferences in economic contexts. We should look out for causal relations, but econometrics can never be more than a starting point in that endeavour since econometric (statistical) explanations are not explanations in terms of mechanisms, powers, capacities or causes. Firmly stuck in an empiricist tradition, econometrics is only concerned with the measurable aspects of reality. But there is always the possibility that there are other variables of vital importance and although perhaps unobservable and non-additive, not necessarily epistemologically inaccessible that were not considered for the model. Those that can never be guaranteed to be more than potential causes, and not real causes. A rigorous application of econometric methods in economics really presupposes that the phenomena of our real world economies are ruled by stable causal relations between variables. A perusal of the leading econometric journals shows

that most econometricians still concentrate on fixed parameter models and that parameter-values estimated in specific spatio-temporal contexts are presupposed to be exportable to totally different contexts. To warrant this assumption one, however, has to convincingly establish that the targeted acting causes are stable and invariant so that they maintain their parametric status after the bridging. The endemic lack of predictive success of the econometric project indicates that this hope of finding fixed parameters is a hope for which there really is no other ground than hope itself.

Real-world social systems are not governed by stable causal mechanisms or capacities. The kinds of 'laws' and relations that econometrics has established, are laws and relations about entities in models that presuppose causal mechanisms being atomistic and additive. When causal mechanisms operate in real-world systems they only do it in ever-changing and unstable combinations where the whole is more than a mechanical sum of parts. If economic regularities obtain they do it (as a rule) only because we engineered them for that purpose. Outside man-made 'nomological machines' they are rare, or even non-existent. Unfortunately, that also makes most of the achievements of econometrics as most of the contemporary endeavours of mainstream economic theoretical modelling rather useless.

Even in statistics, the researcher has many degrees of freedom. In statistics as in economics and econometrics the results we get depend on the assumptions we make in our models. Changing those assumptions that is playing a more important role than the data we feed into our models leads to far-reaching changes in our conclusions. Using statistics is no guarantee we get at any 'objective truth.'

UNIT FIVE: IMPORTANCE OF ECONOMETRICS

Unit Structure

5.1. Introduction
5.2. Learner Outcomes
5.3. Why is Econometrics important within economics?
5.4. Meaning of Modern Econometrics
5.5. Using Econometrics for Assessing Economic Model
5.6. Financial Economics
5.6.1. Relationship with the capital Asset pricing model
5.7. Summary
5.8. References/Further Readings/Web Resources
5.9. Possible Answers to Self-Assessment Exercises (SAEs)

5.1. INTRODUCTION

Econometrics contains statistical tools to help you defend or test assertions in economic theory. For example, you think that the production in an economy is in Cobb-Douglas form. But do data support your hypothesis? Econometrics can help you in this case.

To be able to learn econometrics by yourself, you need to have a good mathematics/statistics background. Otherwise it will be hard. Econometrics is the application of mathematics, statistical methods, and computer science to economic data and is described as the branch of economics that aims to give empirical content to economic relations.

5.2. Learning Outcomes

At the end of this unit, you should be able to:

i. Explain the meaning of Econometrics and why Econometrics is important within Economics.
ii. Explain how to use Econometrics for Assessing Economic Model
iii. Understand what Financial Econometrics is.

5.3. WHY IS ECONOMETRICS IMPORTANT WITHIN ECONOMICS?

So Econometrics is important for a couple of reasons though I would strongly urge you to be very wary of econometric conclusions and I will explain why in a minute.

1. It provides an easy way to test statistical significance so in theory, if we specify our econometric models properly and avoid common problems (i.e. heteroskedasticity or strongly correlated independent variables etc.), then it can let us know if we can say either, no there is no statistical significance or yes there is. That just means that for the data set we have at hand, we can or cannot rule out significance. Problems with this: Correlation does not prove causality. It is theory which we use to demonstrate causality but we most definitely cannot use it to "discover" new relationships (only theory can be used to tell us what causes what, for example, we may find a strong statistical significance between someone declaring red is their favorite color and income, but this
obviously not an important relationship just chance)

Another problem is that many times people run many regressions until they find one that "fits" their idea. So think about it this way, you use confidence intervals in economics, so if you are testing for a 95% confidence interval run 10 different regressions and you have a 40% chance of having a regression model tell you there is statistical significance when there isn't. Drop this number to 90% and you have a 65% chance. Alot of shady researchers do exactly this, play around with data series and specifications until they get something that says their theory is right then publish it. So remember, be wary of regression analysis and really only use it to refute your hypotheses and never to "prove" something.

Regression analysis is your friend and you will see how people love to use it. If you don't understand econometrics very well, particularly how to be able to sift through the different specifications so that you rule out any poorly specified models, and so that you understand what all these crazy numbers they are throwing at you mean. If you don't know econometrics yet try reading some papers using regression analysis and notice how
you don't know what any of the regression analysis means. This should give you an idea of why you need to learn it.

However, many people use it, and believe me many people get undergraduate degrees in economics without knowing econometrics and this makes you less capable then those peers of yours who did learn it.

Self-Assessment Exercises 1

What is the contribution of econometrics in solving economic issues?

5.4. MEANING OF MODERN ECONOMETRICS

Modern econometrics is the use of mathematics and statistics as a way to analyze economic phenomena and predict future outcomes. This is often done through the use of complex econometric models that portray the cause and effect of past or current economic stimuli. Econometric analysts can plug new data into these models as a means of predicting future results. One of the distinguishing features of modern econometrics is the use of complex computer algorithms that can crunch tremendous amounts of raw data and create a concise and coherent overview of some aspect of the economy.

For a long period of time in the past, economists could make hypotheses and guesses about the economy but couldn‘t prove their theories without some sort of obvious sea change in the economy as an indicator. As a result, many started to use mathematics and statistics to give proof about their different ideas. Some began to realize that these same tools could actually give accurate assessments about future economic events, which is how the field of modern econometrics first came into being.

Although it can be defined in many different ways, modern econometrics essentially boils down to plugging statistical information about an economy into mathematical formulas. When that happens, the results can show cause and effect about certain economic characteristics. For example, when interest rates rise, it might affect employment levels, inflation, economic growth, and so on. Using econometrics, an analyst might be able to pinpoint exactly how and to what extent this occurs.

Economic models are a huge part of the field of modern econometrics. This is where the leaps and bounds made by computer technology in the modern era come into play. Sophisticated programs devised by analysts can take all of the information that is entered, analyze the relationships between the numerical data, and come up with specific information about how certain economic stimuli affect the overall picture. It is an effective way for those practicing econometrics to use the past to predict the future.

Proponents of modern econometrics should factor in those unforeseen circumstances that can trigger huge negative changes in an economy. One way to do this is to simulate worst-case scenarios for an economy. By doing this, analysts can see what the potential damage done by hypothetical economic catastrophes might be. In addition, models can be used to show the ways out of such dire occurrences. The boundaries for econometrics are practically limitless, but using them can be fruitless without sound economic theories as their basis.

Self-Assessment Exercises 2

What is the contribution of econometrics in solving economic issues?

5.5. USING ECONOMETRICS FOR ASSESSING ECONOMIC MODELS

Econometrics is often used passively to provide the economist with some parameter estimates in a model which from the outset is assumed to be empirically relevant. In this sense, econometrics is used to illustrate what we believe is true rather than to find out whether our chosen model needs to be modified or changed altogether. The econometric analyses of this special issue should take its departure from the latter more critical approach. We would like to encourage submissions of papers addressing questions like whether a specific economic model is empirically relevant in general or, more specifically, in a more specific context, such as in open, closed, deregulated, underdeveloped, mature economies, etc. For example, are models which were useful in the seventies still relevant in the more globalized world of today? If not, can we use the econometric analysis to find out why this is the case

and to suggest modifications of the theory model? We encourage papers that make a significant contribution to the discussion of macroeconomics and reality, for example, by assessing the empirical relevance of influential papers, or the robustness of policy conclusions to econometric misspecification and the ceteris paribus clause, or by comparing different expectation‘s schemes, such as the relevance of forward versus backward expectations and of model consistent rational expectations versus imperfect/incomplete knowledge expectations, etc.

Self-Assessment Exercises 2

How does the econometric model relate to the economic model?

5.6. FINANCIAL ECONOMETRICS

Financial econometrics is the subject of research that has been defined as the application of statistical methods to financial market data. Financial econometrics is a branch of financial economics, in the field of economics. Areas of study include capital markets , financial institutions, corporate finance and corporate governance. Topics often revolve around asset valuation of individual stocks, bonds, derivatives, currencies and other financial instruments. Financial econometrics is different from other forms of econometrics because the emphasis is usually on analyzing the prices of financial assets traded at competitive, liquid markets. People working in the finance industry or researching the finance sector often use econometric techniques in a range of activities – for example, in support of portfolio management and in the valuation of securities. Financial econometrics is essential for risk management when it is important to know how often 'bad' investment outcomes are expected to occur over future days, weeks, months and years.

5.6.1. Relationship with the capital asset pricing model

The APT along with the capital asset pricing model (CAPM) is one of two influential theories on asset pricing. The APT differs from the CAPM in that it is less restrictive in its assumptions. It allows for an explanatory (as opposed to statistical) model of asset returns. It assumes that each investor will hold a unique portfolio with its own particular array of betas, as opposed to the identical "market portfolio". In some ways, the CAPM can be considered a "special case" of the APT in that the securities market line represents a single-factor model of the asset price, where beta is exposed to changes in value of the market.

Additionally, the APT can be seen as a "supply-side" model, since its beta coefficients reflect the sensitivity of the underlying asset to economic factors. Thus, factor shocks would cause structural changes in assets' expected returns, or in the case of stocks, in firms' profitabilities.

On the other side, the capital asset pricing model is considered a "demand side" model. Its results, although similar to those of the APT, arise from a maximization problem of each investor's utility function, and from the resulting market equilibrium (investors are considered to be the "consumers" of the assets).

Self-Assessment Exercises 3

1. What is the importance of financial econometrics?
2. What is the difference between econometrics and financial econometrics?

5.7. Summary

The unit discussed extensively on the importance of Econometrics and why econometrics is very useful in our day to day activities and how the financial analyst also makes use of it to financial forecast and analysis. Conclude that econometrics is very important in Economics and financial analysis. Econometrics is the basis of using economics theories to justify a real life situation in the micro and macro economy of any nation.

5.8. References/Further Readings/Web Resources

Gujarati, D. N. (2003). *Basic Econometrics*. 4th Edition, Macgraw-Hill Publisher

Molaolu, A. O. (2020). *Econometrics Analysis and Economic Theories*. 1st Edition, Indreg Publisher.

5.9. Possible Answers to SAEs

These are the possible answers to the SAEs within the content.

Answers to SAEs 1

Econometrics, the statistical and mathematical analysis of economic relationships, often serving as a basis for economic forecasting. Such information is sometimes used by governments to set economic policy and by private business to aid decisions on prices, inventory, and production.

Answers to SAEs 2

Econometrics uses economic theory, mathematics, and statistical inference to quantify economic phenomena. In other words, it turns theoretical economic models into useful tools for economic policymaking.

Answers to SAEs 3

Financial econometrics and statistics have become very important tools for empirical research in both finance and accounting. Econometric methods are important tools for asset-pricing, corporate finance, options, and futures, and conducting financial accounting research.

Module 2: Single-Equation (Regression Models)

This module introduces you to single-equation (regression models). The module consists of 5 units which include: regression analysis, the ordinary least square (OLS) method estimation, Calculation of Parameter $\boldsymbol{\beta}$ and the Assumption of Classical Least Regression Method (CLRM), Properties of the Ordinary Least Square Estimators and the Coefficient of Determination (R^2): A measure of "Goodness of fit"

Unit One: Regression Analysis
Unit Two: The Ordinary Least Square (OLS) Method Estimation

Unit Three: Calculation of Parameter $\boldsymbol{\beta}$ and the Assumption of Classical Least Regression Method (CLRM)
Unit Four: Properties of the Ordinary Least Square Estimators
Unit Five: The Coefficient of Determination (R^2): A measure of "Goodness of fit"

Unit One: Regression Analysis

Unit Structure

1.1. Introduction
1.2. Learning Outcomes
1.3. The Linear Regression Model
1.3.1. The Classical Linear Regression Model
1.4. Regression Vs Causation
1.4.1. Regression
1.4.2. Causation
1.5. Regression Vs Correlation
1.6. Summary
1.7. References/Further Readings/Web Resources
1.8. Possible Answers to Self-Assessment Exercises (SAEs)

1.1. INTRODUCTION

The term regression was introduced by Francis Galton. In a famous paper, Galton found that, although there was a tendency for tall parents to have tall children and for short parents to have short children, the average height of children born of parents of a given height tended to move to ―regress‖ toward the average height in the population as a whole. In other words, the height of the children of unusually tall or unusually short parents tends to move toward the average height of the population. Galton's law of universal regression was confirmed by his friend Karl Pearson who collected more than a thousand records of heights of members of family groups. He found that the average height of sons of a group of tall fathers was less than their father's height and the average height of sons of a group of short fathers was greater than their fathers' height, thus ―regressing‖ tall and short sons alike toward the

average height of all men. In the word of Galton, this was ―regression to mediocrity‖.

1.2. Learning Outcomes

At the end of this unit, you should be able to:

i. Understand the meaning linear regression model
ii. Explain the meaning of classical linear regression model

1.3. The Linear Regression Model

We can ask ourselves a question that why do we regress? Econometric methods such as regression can help to overcome the problem of complete uncertainty and guide planning and decision-making. Of course, building a model is not an easy task. Models should meet certain criteria (for example a model should not suffer from serial correlation) in order to be valid and a lot of work is usually needed before we achieve a good model. Furthermore, much decision making is required regarding which variables to include in the model. Too many may cause problems (unneeded variables misspecification), while too few may cause other problems (omitted variables misspecification or incorrect functional form).

When modeling the relationship between a scalar answer and one or more explanatory variables in statistics, linear regression is a linear method (also known as dependent and independent variables). Simple linear regression is used when there is only one explanatory variable, and multiple linear regression is used when there are numerous variables. As opposed to multivariate linear regression, which predicts numerous correlated dependent variables as opposed to a single scalar variable, this phrase is more specific.

In linear regression, linear predictor functions are used to model relationships, with the model's unknown parameters being estimated from the data. These models are referred to as linear models. The conditional mean of the response is typically considered to be an affine function of the values of the explanatory variables (or predictors); the conditional median or another quantile is occasionally employed. In common with all other types of regression analysis, linear regression concentrates on the conditional probability distribution of the response given the values of the predictors rather than the joint probability distribution of all these variables, which is the purview of multivariate analysis.

The first regression analysis method that received a great deal of attention from researchers and was widely applied in real-world scenarios was linear regression. This is due to the fact that models with linear rather than non-linear dependence on their unknown parameters are simpler to fit and that it is simpler to ascertain the statistical characteristics of the resulting estimators.

There are numerous applications for linear regression. Most applications fit into one of the two broad groups listed below:
1. Linear regression can be used to fit a predictive model to an observed data set of values for the response and explanatory variables if the goal is error reduction in forecasting or prediction. The fitted model can be used to forecast the response if further values of the explanatory variables are collected after constructing such a model but without an accompanying response value.
2. If the objective is to quantify the strength of the relationship between the response and the explanatory variables, in particular to ascertain whether some explanatory variables may have no linear relationship with the response at all, or to determine which subsets of explanatory variables may contain redundant information, linear regression analysis can be applied.
The least squares approach is frequently used to fit linear regression models, but there are other methods as well. For example, least absolute deviations regression minimizes the "lack of fit" in another norm, while ridge regression (L2-norm penalty) and lasso regression minimize a penalized version of the least squares cost function (L1-norm penalty). On the other hand, models that are not linear can be fitted using the least squares method. Consequently, despite their close relationship, the phrases "least squares" and "linear model" are not interchangeable.

1.3.1. The classical linear regression model

The classical linear regression is a way of examining the nature and form of the relationships among two or more variables. In this aspect we will consider the case of only two variables. One important issue in the regression analysis is the direction of causation between the two variables; in other words, we want to know which variable is affecting the other. Alternatively, this can be stated as which variable depends on the other. Therefore, we refer to the two variables as the dependent variable (usually denoted by Y) and the independent or explanatory variable (usually denoted by X). We want to explain /predict the value of Y for different values of the explanatory variable X. Let us assume that X and Y are linked by a simple linear relationship:

$$\in(Y_t) = a + \beta X_t \quad -_____(I)$$

Where $\in (Y_t)$ denotes that average value ofY_t for given X_t and unknown population parameters ‗a' and β (the subscript t indicates that we have time series data). Equation (1) is called the population regression equation. The actual value of (Y_t) will not always equal its expected value $\in (Y_t)$. There are various factors that can ‗disturb' its actual behaviour and therefore we can write actualY_t as:

$$Y_t = \in (Y_t) + U_t - - - - - - - (II)$$

or

$$Y_t = a + \beta X_t + U_t - - - - - - (III)$$

Where U_t is a disturbance. There are several reasons why a disturbance exiaists:

1. Omission of explanatory variables: There might be other factors (other than X_t) affecting $Y_t$that have been left out of equation (III). This may be because we do not know. These factors, or even if we know them we might be unable to meaure them in order to use them in a regression analysis.
2. Aggregation of variables: In some cases it is desirable to avoid having too many variables and therefore we attempt to summarize in aggregate a number of relationships in only one variable. Therefore, eventually we have only a good approximation $Y_t$of with discrepanceis that are captured by the disturbance term.
3. Model misspecification: We might have a smisspecified model in terms of its structure. For example, it might be that $Y_t$is not affected by X_t, but it is affected by the valaue of X in the previous period (that is X_{t-i}). In this case, if $X_t$and X_{t-i} are closely related, the estimation of equation (III) will lead to discrepancies that are again captured by the error term.
4. Functional Misspecification: The relationship between X and Y might be nonlinear.
5. Measurement Errors: If the measurement of one or more variables is not correct then errors appear in the relationship and these contribute to the disturbance term.

Self-Assessment Exercises 1

What is the difference between classical linear regression model and OLS?

1.4. Regression Vs Causation

1.4.1. Regression

Although regression analysis deals with the dependence of one variable on other variables, it does not necessarily imply causation. In other words of Kendall and Stuart, ―A statistical relationship, however strong and however suggestive, we can never establish cuasal connection: our ideas of causation must come from outside statistics, ultimately from some theory or other. In the crop-yield, there is not statistical reason to assume that rainfall does not depend on crop yeild. The fact that we treat crop yeeld as dependent on rainfall (among other things) is due to nonstatistical considerations: common sense suggests that the relationship cannot be reversed for we cannot control rainfall by barying crop yield. Regression analysis is a group of statistical procedures for estimating the relationships between a dependent variable (often referred to as the "outcome" or "response" variable, or a "label" in the language of machine learning), and one or more independent variables (often referred to as "predictors," "covariates," "explanatory variables," or "features"). In linear regression, the most typical type of regression analysis, the line (or a more complicated linear combination) that most closely matches the data in terms of a given mathematical criterion is found. By using the ordinary least squares method, for instance, the specific line (or hyperplane) that minimizes the sum of squared differences between the genuine data and that line is computed (or hyperplane).
This enables the researcher to estimate the conditional expectation (or population average value) of the dependent variable when the independent variables take on a specified set of values for precise mathematical reasons (see linear regression). Quantile regression, Required Condition Analysis, or estimating the conditional expectation across a larger collection of non-linear models are examples of less prevalent types of regression that employ somewhat different techniques to estimate alternative location parameters (e.g., nonparametric regression).
There are two main theoretically separate uses for regression analysis.

First, there is a significant overlap between the usage of regression analysis and machine learning in the areas of prediction and forecasting.

Second, regression analysis can be used to infer causal links between the independent and dependent variables in specific circumstances. Regressions by themselves, it should be noted, only illuminate connections between a dependent variable and a group of independent variables in a given dataset. Researchers must carefully explain why existing correlations have predictive value in a new context or why a link between two variables has a causal meaning before using regressions for prediction or to infer causal relationships, respectively. When attempting to estimate causal linkages using observational data, the latter is particularly crucial.

However, the history of method of least squares, first presented by Legendre in 1805 and Gauss in 1809, was the oldest type of regression. The approach was used by Legendre and Gauss to solve the issue of estimating the orbits of objects around the

Sun from astronomical measurements (mostly comets, but also later the then newly discovered minor planets). In 1821, Gauss presented a revised version of the theory of least squares that contained the Gauss-Markov theorem.

Francis Galton first used the word "regression" to describe a biological process in the 19th century. The issue was that descendants of tall ancestors typically have heights that are closer to the average (a phenomenon also known as regression toward the mean). Regression only had this biological significance for Galton, but Udny Yule and Karl Pearson later expanded on his work to include a broader statistical context. The joint distribution of the response and explanatory variables in Yule and Pearson's work is taken to be gaussian. In his works between 1922 and 1925, R.A. Fisher challenged this presumption.

Although the joint distribution need not be Gaussian, Fisher assumed that the conditional distribution of the response variable was. Fisher's premise is more similar to Gauss's statement from 1821 in this regard.

Economists computed regressions using electromechanical desk "calculators" in the 1950s and 1960s. Prior to 1970, it was not uncommon for one regression's results to take up to 24 hours to arrive.

A lot of research is still being done on regression algorithms. In the last few decades, new techniques have been developed for robust regression, regression involving correlated responses like time series and growth curves, regression where the predictor (independent variable) or response variables are curves, images, graphs, or other complex data objects, regression methods accommodating various types of missing data, nonparametric regression, Bayesian techniques for regression, regression where the predictor variables are measured with error.

1.4.2. Causation

The ability of one variable to impact another is known as causality, also known as causality. The first variable might create the second or might change the incidence of the second variable. Correlation, which shows how much two variables tend to rise or fall together, and causation are frequently misconstrued. Correlation does not, however, automatically imply causality. The variations in both variables could be caused by a third component, for instance. For instance, a statistically significant association has been shown between yellow cars and a reduced accident frequency. Just because there are fewer accidents involving yellow cars does not mean that they are any safer. It is more likely that a third component, such as the personality of the individual who buys a yellow car, is to blame than the actual paint color.

Self-Assessment Exercises 1

What is the difference between regression vs causation?

1.5. Regression Vs Correlation

Closely related to but conceptually very much different from regression analysis is correlation analysis, where the primary objective is to measure the strength or degree of linear association between two variables. The correlation coefficient measures the strength of (Linear) association. For example, we may be interested in finding the correlation (coefficient) between smoking and lung cancer, between scores on statistics and mathematics examinations, between high school grades and college grades and so on. In regression analysis, as already noted, we are not primarily interested in such a measure. Instead, we try to estimate or predict the average value of one variable on the basis of the fixed values of other variables. Thus, we may want to know whether we can predict the average score on a statistics examination by knowing a students'score on a mathematics examination. Correlation, as the name says, it determines the interconnection or a co-relationship between the variables. 'Regression' explains how an independent variable is numerically associated with the dependent variable. In Correlation, both the independent and dependent values have no difference.

Some of the differences are as follows:

1. Correlation, as the name says, it determines the interconnection or a co-relationship between the variables while regression' explains how an independent variable is numerically associated with the dependent variable.
2. In Correlation, both the independent and dependent values have no difference while in regression, both the dependent and independent variables are different.
3. The primary objective of Correlation is to find out a quantitative/numerical value expressing the association between the values while regression's main purpose is to calculate the values of a random variable based on the values of a fixed variable.
4. Correlation stipulates the degree to which both variables can move together while however, regression specifies the effect of the change in the unit in the known variable (p) on the evaluated variable (q).
5. Correlation helps to constitute the connection between the two variables while regression helps in estimating a variable's value based on another given value.

Self-Assessment Exercises 2

What is a difference between linear regression and correlation analysis?

1.6 SUMMARY

The key idea behind regression analysis is the statistical dependence of one variable, the dependent variable , on one or more of the variables, the explanatory variable. So, I believe in this unit you must have known the differences between regression and correlation and the classical linear regression analysis.

1.7. REFERENCES/Further Reading

Dimitrios, A & Stephen, G. (2011). Applied Econometrics, second edition 2011, first Edition 2006 and revised edition 2007.

Emmanuel, E. A. (2014). Introduction to Econometrics, 2nd Edition, World gold Publication limited.

1.8. Possible Answers to SAEs

These are the possible answers to the SAEs within the content.

Answers to SAEs 1

Yes, although 'linear regression' refers to any approach to model the relationship between one or more variables, OLS is the method used to find the simple linear regression of a set of data. Linear regression refers to any approach to model a LINEAR relationship between one or more variables

Answers to SAEs 2

Regression deals with dependence amongst variables within a model. But it cannot always imply causation. For example, we stated above that rainfall affects crop yield and there is data that support this. However, this is a one-way relationship: crop yield cannot affect rainfall.

Answers to SAEs 3

A correlation analysis provides information on the strength and direction of the linear relationship between two variables, while a simple linear regression analysis estimates parameters in a linear equation that can be used to predict values of one variable based on the other.

UNIT 2 THE ORDINARY LEAST SQUARE (OLS) METHOD OF ESTIMATION

Unit Structure

2.1. Introduction
2.2. Learning Outcome
2.3. The Method of Ordinary Least Square (OLS)
2.4. Properties of OLS
2.5. Summary
2.6. References/Further Readings/Web Resources
2.7. Possible Answers to Self-Assessment Exercises (SAEs)

2.1. INTRODUCTION

Ordinary Least Square (OLS) method is used extensively in regression analysis primarily because it is intuitively appealing and mathematically much simplier than the method of maximum likelihood.

2.2. Learning Outcome

At the end of this unit, you should be able to:

i. identify differentiate the dependant and independent variables.
ii. Explain some of the parameters of ordinary least estimate.

2.3. The method of Ordinary Least Squares (OLS)

The method of ordinary least squares is attributed to Carl Friedrich Gausss, a German mathematician. Under certain assumptions, the method of least square has some very attractive statistical properties that have made it one of the most powerful and popular methods of regression analysis.

In a linear regression model with fixed level-one effects and a set of explanatory variables, the ordinary least squares (OLS) method uses the principle of least squares to select the unknown parameters by minimizing the sum of the squares of the differences between the observed dependent variable's values in the input dataset and the result of the (linear) function of the independent variable.

The sum of the squared distances, measured perpendicular to the axis of the dependent variable, between each data point in the set and its corresponding point on the regression surface is how this is represented geometrically. The fewer the differences, the better the model matches the data. Particularly in the case of a basic linear regression, when there is only one regressor on the right side of the regression equation, the resultant estimator can be stated by a straightforward formula.

When the regressors are exogenous and form perfect colinearity (rank condition), the OLS estimator is consistent for the level-one fixed effects. It is also consistent for the variance estimate of the residuals when the regressors have finite fourth moments and by the Gauss-Markov theorem when the errors are homoscedastic and serially uncorrelated. When the errors have finite variances, the OLS method provides minimum-variance mean-unbiased estimate. OLS is the maximum likelihood estimator that outperforms any non-linear unbiased estimator with the additional assumption that the errors are normally distributed with zero mean.

To understand, we first explain the least square principle. Recall the two variable model.

$$Y_t = a + \beta X_t + U_t - - - - - - (I)$$

Where Y_t is called the dependent variable while $\beta_2 X_t + U_t$ are called independent or explanatory variables.

The equation is not directly observable. However, we can gather data and obtain estimates of a and β from a sample of the population. This gives us the following relationship, which is a fitted straight linw with intercept$\hat{}$ and $\hat{}$.

$$\hat{Y}_t = \hat{a} + \beta X_t \ - - - - - - - - (II)$$

Equation (II) can be referred to as the sample regression equation. Here $\hat{}$ and $\hat{\beta}$ are sample estimates of the population parameters a and β, and $\hat{Y}_t$ denotes the predicted value of Y. Once we have the estimated sample regression equation we can easily predict Y for various values of X.

When we fit a sample regression line to a scatter of points, it is obviously desirable to select the line in such a manner that it is as close as possible to the actual Y, or, in other words, that it provides the smallest possible number of residuals. To do this we adopt the following criterion: choose the sample regression function in such a way that the sum of the squared residuals is as small as possible (that is minimized).

Self-Assessment Exercises 1

What is ordinary least square method with example?

2.4. Properties of OLS

This method of estimation has some desirable properties that make it the most popular technique in uncomplicated applications of regression analysis, namely:

1. By using the squared residuals we eliminate the effect of the sign of the residuals, so it is not possible that a positive and negative residual will offset each other. For example, we could minimize the sum of the residuals by setting the forecast for Y ($\hat{}$). But this would not be a very well-fitting line at all. So clearly we want a transformation that gives all the residuals the same sign before making them as small as possible.
2. By squaring the residuals, we give more weight to the larger residuals and so, in effect, we work harder to reduce the very large errors.
3. The OLS method chooses $\hat{}$ and $\hat{}$ estimates that have certain numerical and statistical properties (such as unbiasedness and efficiency). Let us see how to derive the OLS estimators. Denoting by RSS the Residual Sum of square.

$$\text{Rss} = \widehat{U}_1^2 + U_2^2 + \ldots\ldots + \widehat{U}_n^2 = \sum_{t=i}^{n} \widehat{U}_t^2 \text{ ------ } (\mathbf{3})$$

However, we know that:

$$\widehat{U}_t^2 = (Y_t - Y_t) = \left(Y_t - \hat{a} - \hat{\beta}X_t\right) \text{ ------- } (4)$$

and therefore:

$$\text{Rss} = \sum_{t=1}^{n} \widehat{U}_t^2 = \sum_{t=1}^{n} (Y_t - \hat{Y}_t)^2 = \sum_{t=1}^{n} (Y_t - \hat{a} - \hat{\beta}X_t)^2 \text{ ------ } (5)$$

To minimum equation (V), the first order condition is to take the partial derivatives of Rss with respect to $\hat{}$ and $\hat{}$ and set them to zero.

Thus, we have:

$$\frac{\partial Rss}{\partial \hat{a}} = -2 \sum_{t=1}^{n} X_t(Y_t - \hat{a} - \hat{\beta}X_t) = 0 \text{ ------ } (6)$$

and

$$\frac{\partial Rss}{\partial \hat{\beta}} = -2 \sum_{t=1}^{n} X_t(Y_t - \hat{a} - \hat{\beta}X_t) = 0 \text{ ------ } (7)$$

The second – order partial derivatives are:

$$\frac{\partial^2 Rss}{\partial \hat{a}^2} = 2_n \text{ ------- } (8)$$

$$\frac{\partial^2 Rss}{\partial \hat{\beta}^2} = 2\sum_{t=1}^{n} X_t^2 - - - - - - - - (9)$$

$$\frac{\partial^2 Rss}{\partial \hat{\beta}^2} = 2\sum_{t=1}^{n} X_t - - - - - - - - (10)$$

Therefore it is easy to verify that the second-order conditions for a minimum are met. Since $\sum \hat{a} = n\hat{a}$ for simplicity on notation we omit the upper and lower limits of the summation

symbol), we can (by using that and rearranging) rewrite equation (6) and (7) as follows:

$$\sum Y_t = n\hat{a} - \hat{\beta} \sum X_t - - - - - - - - (11)$$

$$\sum X_t Y_t = \hat{a} \sum X_t - \hat{\beta} \sum X_t^2 - - - - - - - - (12)$$

The only unknowns in these two equations are ˆ and ˆ. Therefore, we can solve this system of two equations with two unknown to obtain ˆ and ˆ. First, we divide both sides of equation (11) by n to get;

$$\frac{\sum Y_t}{n} = \frac{n\hat{a}}{n} - \frac{\hat{\beta} \sum X_t}{n} - - - - - - - - (13)$$

Denoting $\frac{\sum Y_t}{n}$ by $\hat{Y}$ and $\frac{\sum Y_t}{n}$, by $\hat{X}$ and rearranging,

we obtain: $\hat{a} = \bar{Y} - \hat{\beta}\bar{X} - - - - - - - (14)$

Substituting equation (14) into equation (12), we get:

$$\sum X_t Y_t = \bar{Y} \sum X_t - \hat{\beta}\bar{X} \sum X_t + \hat{\beta} \sum X_t - - - - - - - (15)$$

or

$$\Sigma X_t Y_t = \frac{1}{n} \Sigma Y_t \Sigma X_t - \hat{\beta} \frac{1}{n} \Sigma X_t \Sigma X_t + \hat{\beta} \Sigma X_t^2 - - - - - - - (16)$$

And finally, factorizing the ˆ terms, we have:

$$\sum X_t Y_t = \frac{\Sigma y_t \Sigma x_t}{n} + \hat{\beta}\left[\Sigma X_t^2 - \frac{(\Sigma X_t)^2}{n}\right] - - - - - - - - - (17)$$

and finally, factorizing the ˆas:

$$\hat{\beta} = \frac{\Sigma X_t Y_t - \frac{1}{n} \Sigma Y_t \Sigma X_t}{\Sigma X_t^2 - \frac{1}{n} (\Sigma X_t)^2} - - - - - - - - - (18)$$

and given ˆ we can use equation (14) to obtain ˆ.

Properties of OLS Estimator are:

a. The regression model.
b. Matrix notation.

c. The estimator.
d. Writing the estimator in terms of sample means
e. Consistency of the OLS estimator
f. Asymptotic normality of the OLS estimator
g. Consistent estimation of the variance of the error terms
h. Consistent estimation of the asymptotic covariance matrix.

Self-Assessment Exercises 2

What are the properties of least square estimators?

2.5. Summary

In statistics, ordinary least squares (OLS) or linear least squares is a method for estimating the unknown parameters in a linear regression model, with the goal of minimizing the differences between the observed responses in some arbitrary dataset and the responses predicted by the linear approximation of the data (visually this is seen as the sum of the vertical distances between each data point in the set and the corresponding point on the regression line - the smaller the differences, the better the model fits the data). The resulting estimator can be expressed by a simple formula, especially in the case of a single regressor on the right-hand side.

2.6. REFERENCES/Further Reading

Dimitrios, A & Stephen, G., (2011). Applied Econometrics, 2nd, 1st Edition 2006 and revised

Edition 2007.

2.7. Possible Answers to SAEs

These are the possible answers to the SAEs within the content.

Answers to SAEs 1

Ordinary Least Squares regression (OLS) is a common technique for estimating coefficients of linear regression equations which describe the relationship between one or more independent quantitative variables and a dependent variable (simple or multiple linear regression).

Answers to SAEs 2

(a) The least squares estimate is unbiased: $E[\hat{\beta}] = \beta$. (b) The covariance matrix of the least squares estimate is $cov(\hat{\beta}) = \sigma 2(X X)-1$. 6.3 Theorem: Let $rank(X) = r<p$ and $P = X(X X)-X$, where $(X X)-$ is a generalized inverse of X X. (a) P and I − P are projection matrices.

UNIT 3: CALCULATION OF PARAMETER AND THE ASSUMPTION OF CLRM

Unit Structure

3.1. Introduction
3.2 Learning Outcome
3.3. Alternative Expression for β
3.4. The Assumptions of CLRM
3.4.1. The Assumptions
3.5. Summary
3.6. References/Further Readings/Web Resources
3.7. Possible Answers to Self-Assessment Exercises (SAEs)

3.1 INTRODUCTION

Based on Unit 2 we just discussed, we can also make an alternative expression for parameter$\hat{\beta}$ for residual sum of square and it can also be expressed further as co-variance analysis. In this unit, the assumptions of classical linear regression model will also be examined and you will be able to differentiate these assumptions from other economic analysis assumptions.

3.2. Learning Outcome

At the end of this unit, you should be able to:

i. Identify the alternative expression for β
ii. Explain the assumptions of classical linear regression model.

3.3. Alternative Expression for $\hat{\beta}$

We can express the numerator and denominator of equation (III) which is:

$$\hat{\beta} = \frac{\Sigma X_t Y_t - \frac{1}{n}\Sigma Y_t \Sigma X_t}{\Sigma {X_t}^2 - \frac{1}{n}(\Sigma X_t)^2} - - - - - - - - - (18)$$

as follows:

$$\Sigma(X_t - \bar{X})(Y_t - \bar{Y}) = \Sigma X_t Y_t - \frac{1}{n}\Sigma Y_t \Sigma X_t \text{ - - - - - (19)}$$

$$\Sigma\left(X_t - \hat{X}\right)^2 = \Sigma {X_t}^2 - \frac{1}{n}(\Sigma X_t)^2 \text{ - - - - - (20)}$$

So then we have:

$$\hat{\beta} = \Sigma(X_t - \bar{X})(Y_t - \bar{Y}) \text{ - - - - - -(21)}$$

or even

$$\hat{\beta} = \frac{\Sigma X_t Y_t}{\Sigma {X_t}^2} \text{ - - - - - - - - (22)}$$

Where obviously $X_t = (X_t - \bar{X})$ and $Y_t = (Y_t - \bar{Y})$, which are derivatives from their respective means.

We can use the definitions of Cov(X,Y) and Var(X) to obtain an alternative expression for $\hat{\beta}$ as:

1

$$\hat{\beta} = \frac{\Sigma X_t Y_t - \frac{}{n}\Sigma Y_t \Sigma X_t}{\Sigma {X_t}^2 - \frac{1}{n}(\Sigma X_t)^2} = \frac{\Sigma X_t Y_t - \bar{Y}\bar{X}}{\Sigma {X_t}^2 - (\bar{X})^2} \text{ - - - - - - -(23)}$$

or

$$\hat{\beta} = \frac{\Sigma(X_t - \bar{X})(Y_t - \bar{Y})}{\Sigma\left(X_t - \hat{X}\right)^2} \quad \text{ - - - - - - - - - - (24)}$$

If we further divide both nominator and denominator by $\frac{1}{n}$ we have:

$$\hat{\beta} = \frac{\frac{1}{n}\Sigma(X_t - \bar{X})(Y_t - \bar{Y})}{\frac{1}{n}\Sigma\left(X_t - \hat{X}\right)^2} \quad \text{ - - - - - - - - - - (25)}$$

and finally we can express $\hat{\beta}$ as

$$\hat{\beta} = \frac{\text{Cov}\left(X_t, Y_t\right)}{\text{Var}(X_t)} \text{ - - - - - - - - (26)}$$

Where $\text{COV}(X_t, Y_t)$ and $\text{Var}(X_t)$ are sample covariances and variances.

Self-Assessment Exercises 1

Discuss the alternative to Expression for $\hat{\beta}$

3.4. The Assumptions of CLRM

In a general term, when we calculate estimators of population parameters from sample data we are bound to make some initial assumptions about the population distribution. Usually, they amount to a set of statements about the distribution of the bariables we are investigating, without which our model and estimates cannot be justified. Therefore it is important not only to present the assumptions but also to move behond them, to the extent that we will at least study what happens when they go wrong and how we may test whether they have gone wrong.

3.4.1 The Assumptions

The CLRM consists of eight basic assumptions about the ways in which the observations are generated:

1. Linearity: The first assumption is that the dependent variable can be calculated as a linear function of a specific set of independent variables, plus a disturbance term. This can be expressed mathematically as follows: The regression model is linear
   in the unknown coefficients$\propto and\ \beta$ so that $Y_t = \propto + \beta X_t + U_t, for\ t =$ 1,2,3 ... . , n.
2. X_t has some variation: By this assumption we mean that not all observations of X_t are the same, at least one has to be different so that the sample $Var\ (X)$ is not 0. It is important to distinguish between the sample variance, which simply shows how much X varies over the particular sample, and the stochastic nature of X. in course, we shall make the assumption that X is non-stochastic. This means that the variance of X at any point in time is zero, so $Var(X_t)$ is not 0. It is important to distinguish between the sample variance, which simply shows how much X varies over the particular sampl, and the stochastice nature of X. In course, we shall make the assumption that X is non-stochastic. This means that the variance of X at any point in time is zero, so$Var(X_t) = 0$, and if we could somehow repeat the world over againX would always take exactly the same values. But of course, over any sample there will (indeed must) be some variations in X.
3. X_t is non-stochastic and fixed in repeated samples. By these assumptions we mean first that X_t is a variable whose value are not determined by some chance mechanism, that is they are determined by an experimenter or investigator, and seen that it is possible to repeat the sample with the same

independent variable values. This implies that $\text{Cov}(X_s U_t) = 0$for all s, and t =1, 2 ……., n: that is X_t and $U_t$are uncorrelated.

4. The expected value of the disturbance term is zero: - This means that the disturbance is a genuine disturbance, so that if we took a large number of samples the mean disturbance would be zero. This can be denoted as $\in (U_t) = 0$. We need this assumption in order to interpret the deterministic part of a regression model, $\propto and\ \beta X_t$ as a statistical average relation.
5. Homoskedasticity: This requires that all disturbance terms have the same variance, so that $Var(U_t) = \partial^2$= constant for all t.
6. Serial independence: This requires that all disturbance terms are independently distributed, or, more easily, are not correlated with one another, so that $\text{Cov}(U_t, U_s) = \in\ (U_t - \in U_t)(U_s - \in U_s) = \in (U_t U_s) = 0$ for all $t \neq$s. This assumption has a special significance in economics to grasp what it means in practice, recall that we nearly always obtain our data from time series in which each t is one year, or one quarter, or one week ahead of the last. The condition means, therefore, that the disturbance is one period should not be related to a disturbance in the next or previous periods. This condition is frequently violated since, if there is a disturbing effect at one time, it is likely to persist.
7. Normality of Residuals: The disturbance $U_1, U_2 \ldots \ldots, Un$ are assumed to be independently and identically normally distributed, with mean zero and common variance ∂^2.
8. n>2 and multicollinearity: This assumption says that the number of observations must be greater than two or in general than the relationships among the variables.

Self-Assessment Exercises 2

What are the assumptions of the CLRM?

3.5. SUMMARY

Classical linear regression model statistical-tool used in predicting future values of a target (dependent) variable on the basis of the behavior of a set of explanatory factors

(independent variables). A type of regression analysis model, it assumes the target variable is predictable, not chaotic or random.

3.6. REFERENCES/Further Reading

Dimitrios, A & Stephen, G., (2011). Applied Econometrics, 2nd Edition, Edition 2006 and

Revised Edition 2007.

3.7. Possible Answers to SAEs

These are the possible answers to the SAEs within the content.

Answers to SAEs 1

Ordinary Least Squares regression (OLS) is a common technique for estimating coefficients of linear regression equations which describe the relationship between one or more independent quantitative variables and a dependent variable (simple or multiple linear regression).

Answers to SAEs 2

(a) The least squares estimate is unbiased: E[ˆβ] = β. (b) The covariance matrix of the least squares estimate is cov(ˆβ) = σ2(X X)−1. 6.3 Theorem: Let rank(X) = r<p and P = X(X X)−X , where (X X)− is a generalized inverse of X X. (a) P and I − P are projection matrices.

Answers to SAEs 3

Assumption 1: The regression model is linear in the parameters as in Equation (1.1); it may or may not be linear in the variables, the Ys and Xs.
Assumption 2: The regressors are assumed fixed, or nonstochastic, in the sense that their values are fixed in repeated sampling.

UNIT 4 PROPERTIES OF THE ORDINARY LEAST SQUARE ESTIMATORS

Unit Structure

4.1. Introduction
4.2. Learning Outcome
4.3. Properties of the OLS Estimators
4.4. Advantage and Disadvantage Of OLS Estimator
4.5. Summary
4.6. References/Further Readings/Web Resources
4.7. Possible Answers to Self-Assessment Exercises (SAEs)

4.1. INTRODUCTION

The ordinary least square (OLS) estimator is consistent when the regressors are exogenous and there is no perfect multicollinearity, and optimal in the class of linear unbiased estimators when the errors are homoscedastic and serially uncorrelated. Under these conditions, the method of OLS provides minimum-variance mean-unbiased estimation when the errors have finite variances. Under the additional assumption that the errors be normally distributed, OLS is the maximum likelihood estimator. OLS is used in economics (econometrics), political science and electrical engineering (control theory and signal processing), among many areas of application. The Multi-fractional order estimator is an expanded version of OLS.

4.2. Learning Outcome

At the end of this unit, you should be able to:

i. know the properties that our estimators should have.

ii. Understand the proofing of the OLS estimators as the best linear unbiased estimators (BLUE).

iii. Understand the advantages and disadvantages of OLS Estimation.

4.3. Properties of the OLS Estimators

We now return to the properties that we would like our estimators to have. Based on the assumptions of the CLRM can prove that the OLS estimators are best linear unbiased estimators (BLUE). To do so, we first have to decompose the regression coefficients estimated under OLS into their random and non-random components. As a starting point, not that Y_t has a non-random component $(a + \beta X_t)$, as well as a random component, captured by the residual U_t. Therefore cov(X,Y) that is which depends on values of Y_t – will have a random and non-random component.

$$cov\ X,Y\ = cov\ X_t,\ a + \beta X_t + U\ (\quad)\qquad [(\qquad\qquad)]$$

$$cov(X,a) + cov(X,\beta X) + cov(X,u)$$

$$-------(27)$$

However, because $a\ and\ \beta$are constants we have that $cov(X,a) = 0$ and that $cov(X,\beta X) = \ cov\beta(X,X) = \ \beta Var(X)$. Thus

$$(\quad)\qquad(\ )$$

$$cov\ X,Y\ = \beta Var\ X\ + cov\ X,u\ \ -------(28)(\qquad)$$

and substituting that in equation (26) yields:

$$\beta = \frac{cov(X,Y)}{Var(X)} = \beta + \frac{cov(X,U)}{Var(X)} \ ---------- \ (29)$$

which says that the OLS coefficient $\widehat{\beta}$ estimated from any sample has a non-random component, $\beta\ and\ a$ random component which depends on $cov\ (X_t U_t)$.

(i). Linearity

Based on assumption 3, we have that X is non-stochastic and fixed in repeated samples. Therefore, the x values can be treated as cosntants and we need merely to concentrate on the Y values. If the OLS estimators are linear functions of the Y values then they are linear estimators.

From equation (22) we have that:

$$\hat{\beta} = \frac{\Sigma X_t Y_t}{\Sigma X_t{}^2} - - - - - - - - (30)$$

Since the X_t are regarded as constants, then the are regarded as constants as well, we have that:

$$\hat{\beta} = \frac{\Sigma X_t Y_t}{\Sigma X_t{}^2} = \frac{\Sigma X_t (Y_t - \bar{Y})}{\Sigma X_t{}^2} = \frac{\Sigma X_t - \bar{Y}\Sigma X_t}{\Sigma X_t{}^2} \quad - - - - - - - - (31)$$

but because $\bar{Y}\Sigma X_t = 0$, we have that:

$$\hat{\beta} = \frac{\Sigma X_t Y_t}{\Sigma X_t{}^2} = \Sigma Z_t Y_t \quad - - - - - - - - - - - (32)$$

Where $Z_t = X_t / \Sigma X_t{}^2$ can also be regarded as constant and therefore ˆ is indeed a linear estimator of the Y_t.

(ii) Unbiasedness.

(1) Unbiasedness of ˆ

To prove that ˆ is an unbiased estimator of ˆ we need to show that

$$\in (\hat{\beta}) = \in \left[\hat{\beta} + \frac{cov(X,U)}{Var(X)}\right] - - - - - - - \quad (33)$$

However, β is a constant, and using assumptions, that is X_t is non-random- we can take $Var(X)$ as a fixed constant to take them out of the expectation expression and have:

$$\in (\beta) = \in (\beta) + \frac{1}{Var(X)} \in \left[\left(cov(X,U)\right)\right] - - - - - - - - (34)$$

Therefore, it is enough to show that $\in \left[\left(cov(X,U)\right)\right] = \in$

$$\in \left[\left(cov(X,U)\right)\right] = \in \frac{1}{n}\sum_{t=1}^{n}(X_t - \bar{X})(U_t - \bar{U}) \; - - - - - - - - - - (35)$$

Where $\frac{1}{n}$ is constant, so we can take it out of the expectation expression and we can also break the sum down into the sum of its expectations to give:

$$\in \left[\left(cov(X_t, U_{\square})\right)\right] = \frac{1}{n}[\in (X_1 - \bar{X})(U_1 - \bar{U}) + \ldots\ldots + \in [X_n - \bar{X}](U_n - \bar{U})]$$

$$= \frac{1}{n}\sum_{t=1}^{n} \in [(X_t - \bar{X})(U_t - \bar{U})] - - - - - - - - - (36)$$

Furthermore, because X_t is non-random (again from assumption 3) we can take it out of the expression term to give:

$$\in \left[\left(cov(X,U)\right)\right] = \frac{1}{n}\sum_{t=1}^{n} \in [(X_t - \bar{X})(U_t - \bar{U})] - - - - - - - - (37)$$

Finally, using assumption 4, we have that $\in (U_t) = 0$ and therefore $\in (\bar{U}) = 0$.So $\in$

$\left[\left(cov(X,U)\right)\right] = 0$ and this proves that:

$$\in \left(\hat{\beta}\right) = \beta.$$

or, to put it in words, that ˆ is an unbiased estimator of the true population parameter β.

(b) Unbiasedness of ˆ. We know that

$\hat{a} = \bar{Y} - \hat{\beta}\bar{X}$, So;

$\in (\hat{a}) = \in (\bar{Y}) - \in$

$$\left(\hat{\beta}\right)\bar{X} \; - - - - - - - - \quad (38)$$

But we also have that:

$$\in (Y_t) = a + \beta X_t + \in (U_t) = a + \beta X_t$$

Where we eliminated the $\in (U_t) = 0, so; \in (\bar{Y}) = a + \beta\bar{X} - - - - - - (40)$

Substituting equation (40) into equation (38) gives:

$\in (\hat{a}) = a + \beta\bar{X} - \in \left(\hat{\beta}\right)\bar{X} - - - - - - (41)$

We have proved before that $E\left(\hat{\beta}\right) = \beta$; therefore:

$$E\left(\hat{\beta}\right) = a + \beta\bar{X} - \beta\bar{X} = a \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

Which proves that ˆ is an unbiased estimator of a

(iii) Efficiency and BLUEness

Under assumptions 5 and 6, we can then make a prove that the OLS estimators are the most efficient among all unbiased linear estimators. However, we can say that the OLS procedure yields BLU estimators.

The proof that the OLS estimators are BLU estimators is relatively complicated. It entails a procedure which goes the opposite way from that followed so far. We start the estimation from the beginning, trying to derive a BLU estimator of β based on the properties of linearity, unbiasedness and minimum variance one by one, and we will then check whether the BLU estimator derived by this procedure is the same as the OLS estimator. Thus, we want to derive the BLU estimator of β, say ˆ, concentrating first on the property of linearity. For ˆ to be linear we need to have:

$$\hat{\beta} = \delta_1 Y_1 + \delta_2 Y_2 \ldots\ldots + \delta_n Y_n = \sum \delta_t Y_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

Where the δ_t terms are constants, the values of which are to be determined proceeding with the property of unbiasedness, for $\hat{\beta}$to be unbiased, we must be able to have $\in \left(\hat{\beta}\right) = \beta$. However, we know that;

$$E\left(\hat{\beta}\right) = E\left(\sum \delta_t Y_t\right) = \sum \delta_t E\,(Y_t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

Therefore, let us substitute $\in (Y_t) = a + \beta X_t (because\ Y_t = a + \beta X_t + U_t$, and also because X_t is non-stochastic and $\in (U_t) = 0$, given by the basic assumptions of the model, we get;

$$E(\hat{\beta}) = \sum \delta_t(a + \beta X_t) = a\sum \delta_t + \beta \sum \delta_t Y_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

And therefore, in order to have unbiased $\hat{\beta}$, we need;

$$\sum \delta_t = 0 \text{ and } \sum \delta_t X_t = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (46)$$

I think you are learning through the process and you should know that econometric notation might show as if they are abstract but they have different meaning. Therefore, we can then proceed by deriving an expression for the variance (which we need to minimize) of β:

$$\begin{aligned} Var(\hat{\beta}) &= E[\hat{\beta} - \in(\hat{\beta})]^2 \\ &= E\left[\sum \delta_t Y_t - \in\left(\sum \delta_t Y_t\right)\right]^2 \\ &= E\left[\sum \delta_t Y_t - \sum \delta_t \in (Y_t)\right]^2 \\ &= E\left[\sum \delta_t Y_t - \in (Y_t)\right]^2 \ldots\ldots\ldots\ldots\ldots\ldots (47) \end{aligned}$$

From equation 47 above, we can use $Y_t = a + \beta X_t + U_t$ and $\in (Y_t) = a + \beta X_t$ respectively. Then:

$$Var(\hat{\beta}) = \in \left(\sum \delta_t U_t\right)^2$$

$$= E\delta_1{}^2 U_1{}^2 + \delta_2{}^2 U^2 + \delta_3{}^2 U_3{}^2 + \ldots. + \delta_n{}^2 U_n{}^2 + 2\,\delta_1\delta_2 U_1 U_2 + 2\delta_1\delta_3 U_1 U_3 + \cdots)$$

$$= \delta_1{}^2 E(U_1{}^2) + \delta_2{}^2 E(U_2{}^2) + \delta_3{}^2 \in$$

$$2\delta\ \delta\ E(U\ U\ ) + 2\ \delta\ \delta\ E(U\ U$$

$$(U_3{}^2) + \cdots. + \delta_n{}^2 (U_n{}^2) + \ldots..) \ldots\ldots\ldots\ldots\ldots\ldots\ldots.$$

$$1\ 2 \quad 1\ 2 \quad 1\ 3 \quad 1\ 3) \qquad .\ 48(\quad)$$

5 mov; (var (ut =

Let us use the assumptions ^{2})and 6; which is $(cov(U_t U_s) = E(u_t U_s) = 0)$ for all $t \neq s$ we obtain that:

$$Var(\hat{\beta}) = \sum \delta_t U_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (49)$$

We then need to choose δ_t in the linear estimator (equation 44 to be such as to minimize the variance (equation 49 subject to the constraints (equation 46) which ensure unbiasedness (with this then having a linear, unbiased minimum variance estimator). We formulate the Langrangian function:

$$L = {}^2\delta^2 - \lambda\ (_t \quad {}_1 \quad {}_t \quad {}_2 \quad {}_t\ {}_t \qquad (\quad)\delta\ ) - \lambda\ (\ \delta\ x\ - 1) \ldots\ldots\ldots\ldots\ldots\ldots.\ 50$$

Where λ_1 and $\lambda_2$are Langrangian multipliers.

However, following the regular procedure, which is to take the first-order conditions (that is the portal derivatives of Lwith respect to δ_t, λ_1 and λ_2) and set them equal to zero and after re-arrangement and mathematical manipulations (we omit the

mathematical details of the derivation because it is very lengthy and tedious and because it does not use any of the assumptions of the model in any case), we obtain the optimal δ_t as:

$$\delta_t = \frac{X_t}{\Sigma X_t^2} \dots\dots\dots\dots\dots\dots\dots\dots (51)$$

We can say that $\delta_t = z_t$ of the OLs expression given by Equation (32). so, substituting this into our linear estimators ˆ we have:

$$\hat{\beta} = \Sigma\delta_t Y_t = \Sigma z_t Y_t$$
$$= \Sigma z_t (Y_t - \bar{Y} + \bar{Y})^*$$
$$= \Sigma z_t (Y_t - \bar{Y}) + \bar{Y}\Sigma z_t$$
$$= \Sigma z_t Y_t = \frac{\Sigma x_t Y_t}{\Sigma {x_t}^2}$$
$$= \hat{\beta} \dots\dots\dots\dots\dots\dots\dots\dots\dots (52)$$

Therefore we can conclude that ˆ of the OLs is BLU. Let us then talk more about the advantage of the BLUEness: The advantages of the BLUEness condition is that it provides us with an expression for the variance by substituting the optional δ_t given in equation (51) into equation (49) and that will gives:

$$Var(\hat{\beta}) = Var\left(\hat{\beta}\right) = \sum \left(\frac{X_t}{\Sigma X_t^2}\right)^2 \sigma^2$$
$$= \frac{\Sigma X_t^2 \sigma^2}{(\Sigma X_t^2)^2} = \sigma^2 \frac{1}{\Sigma X_t^2} \dots\dots\dots\dots\dots\dots\dots (53)$$

(iv) CONSISTENCY

Consistency is the idea that, as the sample becomes infinitely large, the parameter estimate given by a procedure such as OLs converges on the true parameter value. This is obviously true when estimator is unbiased, as shown in our previous discussion above, as consistency is really just a weaker form of unbiasedness. However, the proof above rests on our assumption 3, that the X variables are fixed. If we relax this assumption it is no longer possible to prove the unbiasedness of OLs but we can still establish that it is a consistent estimator. That is, when we relax assumption 3, OLS is no longer a BLU estimator but it is still consistent. We showed in equation (29) in this module that ˆ $= \beta + cov\ (X, U)/var\ (X)$. Let us divide the top and the bottom of the last term by n, we have:

$$\hat{\beta} = \beta + \frac{Cor(X,U)/n}{Var\ X\ \ n} \dots\dots\dots\dots\dots\dots\dots\dots\dots (54)$$
$(\ )/$

Finally, using the law $Cor(X,U)/n$of large numbers, we know that coverages to its expectation, which is $Cor(X_t, U_t)$. Similarly $Var(X)/n$ converges to $Var(X_t)$. So as n tend to infinity (i.e. n →

$)\hat{\beta} \rightarrow \beta + Cor(X_t, U_t)/Var(X_t)$, which is equal to the true population parameter β if $Cor(X_t, U_t) = 0$ (that is if X_t and U_t are uncorrelated). Thus ˆ is a consistent estimator of the true population parameter β.

Self-Assessment Exercises 1

What is the blue property in econometrics?

4.4. ADVANTAGE AND DISADVANTAGE OF OLS ESTIMATOR

OLS estimation is a popular method for fitting linear models to data. It attempts to reduce the sum of squared errors between the observed and predicted values of the outcome variable. Is OLS, on the other hand, always the optimum option for statistical modeling? In this post, you'll learn about the benefits and drawbacks of OLS estimation, as well as when to use it and when to avoid it.

1. ADVANTAGES OF OLS

OLS estimation is a popular and widely used method for statistical modeling due to its simplicity, efficiency, and flexibility. It is easy to understand and implement, with a closed-form solution that can be computed analytically or numerically. OLS is the best linear unbiased estimator (BLUE) under the Gauss-Markov theorem, meaning that among all linear estimators that are unbiased, OLS has the smallest variance. It also has desirable properties such as consistency, asymptotic normality, and asymptotic efficiency. Moreover, OLS can be applied to various types of linear models and extended to nonlinear models by using a link function and a maximum likelihood approach.

2. DISADVANTAGES OF OLS

The usefulness and validity of OLS estimation may be constrained in some circumstances due to a number of flaws. It is sensitive to outliers, leverage points, and significant data that might skew the estimations and lessen their precision. Assumptions like linearity, independence, homoscedasticity, normalcy, and the absence of multicollinearity are also limitations on it. But actual data might not support these hypotheses. OLS estimation may result in inaccurate, inconsistent, or biased results in several circumstances. Alternative techniques and tests like transformation, regularization, generalized linear models, and hypothesis testing are available to overcome these problems.

Self-Assessment Exercises 1

What are the advantages and disadvantages of OLS?

4.5. SUMMARY

If an estimator's expected value corresponds to the population's parameter, it is considered impartial. It is clear that the econometric model's unknown regression coefficients and the mean values of the OLS estimators are in agreement.

4.6. REFERENCES/Further Reading

Aderemi, A. A. (2019). Introduction to Econometrics, 1st Edition, Pentagon Publisher

4.7. Possible Answers to SAEs

These are the possible answers to the SAEs within the content.

Answers to SAEs 1

OLS estimators are BLUE (i.e. they are linear, unbiased and have the least variance among the class of all linear and unbiased estimators). Amidst all this, one should not forget the Gauss-Markov Theorem (i.e. the estimators of OLS model are BLUE) holds only if the assumptions of OLS are satisfied

Answers to SAEs 2

Ordinary least squares (OLS) models

(i). Advantages: The statistical method reveals information about cost structures and distinguishes between different variables' roles in affecting output. ...

(ii). Disadvantages: Large data set is necessary in order to obtain reliable results.

UNIT 5: THE COEFFICIENT OF DETERMINATION(γ^2): A MEASURE OF "GOODNESS OF FIT".

Unit Structure

5.1. Introduction
5.2. Learning Outcome
5.3. Goodness of fit
5.4. Summary
5.5. References/Further Readings/Web Resources
5.6. Possible Answers to Self-Assessment Exercises (SAEs)

5.1. INTRODUCTION

In statistics, the coefficient of determination denoted R^2 or r^2 and pronounced R squared, is a number that indicates the proportion of the variance in the dependent variable that is predictable from the independent variable.

It is a statistic used in the context of statistical models whose main purpose is either the prediction of future outcomes or the testing of hypotheses, on the basis of other related information. It provides a measure of how well observed outcomes are replicated by the model, based on the proportion of total variation of outcomes explained by the model.

There are several definitions of R^2 that are only sometimes equivalent. One class of such cases includes that of simple linear regression where r^2 is used instead of R^2. In this case, if an intercept is included, then r^2 is simply the square of the sample correlation coefficient (i.e., r) between the outcomes and their predicted values. If additional explanator are included, R^2 is the square of the coefficient of multiple correlations. In both such cases, the coefficient of determination ranges from 0 to 1.

Important cases where the computational definition of R^2 can yield negative values, depending on the definition used, arise where the predictions that are being compared to the corresponding outcomes have not been derived from a model-fitting procedure using those data, and where linear regression is conducted without including an intercept.
Additionally, negative values of R^2 may occur when fitting non-linear functions to data. In cases where negative values arise, the mean of the data provides a better fit to the outcomes than do the fitted function values, according to this particular criterion.

5.2. Learning Outcome

At the end of this unit, you should be able to:

i. understand the Goodness fit
ii. Explain the assumptions of classical linear regression model.

5.3. GOODNESS OF FIT

So, far, we have been dealing with the problem of estimating regression coefficients, and some of their properties, we now consider the GOODNESS OF FIT of the fitted regression line to a set of data: that is we will find out how ―well‖ the sample regression

line fits the data. Let us consider a least square graph given below:

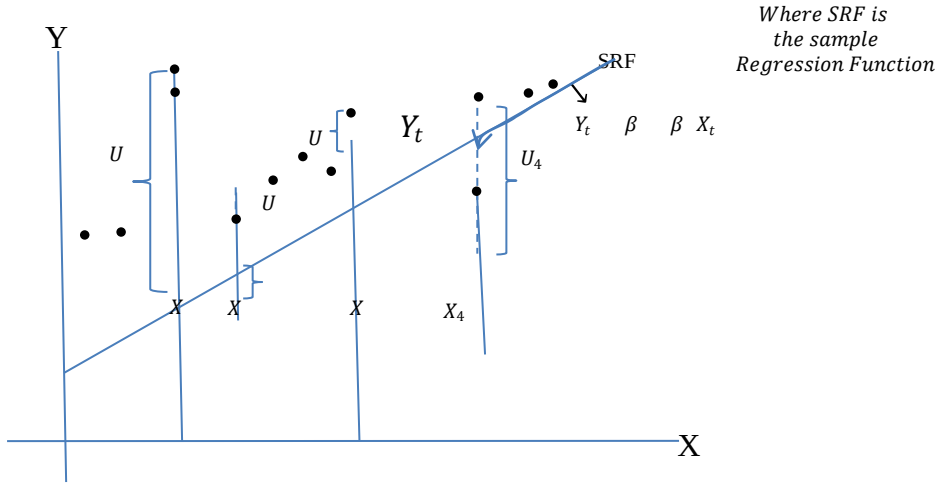


Figure 5.1 Showing least square Criterion.

From the graph it is clear that if all the observations were to lie on the regression line we would obtain a ―perfect‖ fit, but this is rarely the case. Generally, there will be some positive $\widehat{U}_t$ and some negative $\widehat{U}_t$. What we hope for is that these residuals around the regression line are as small as possible. The coefficient of determination r^2 (two-variable case) or R^2 (multiple regressions) is a summary measure that tells how well the sample regression line fits the data.

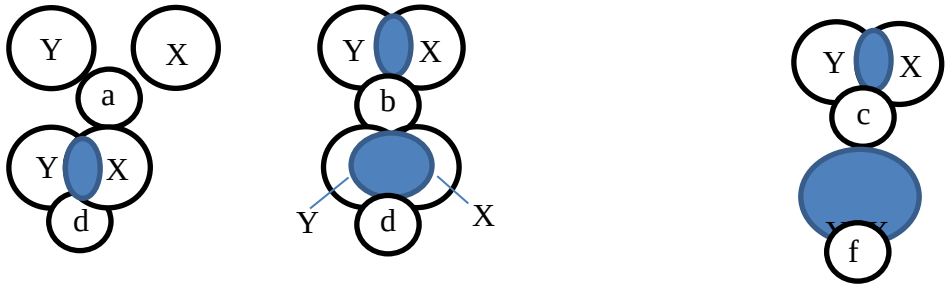


Figure 5.2 Showing the Ballentine view of r^2(a) $r^2 = 0$: if $r^2 = 1$·

Before we go on to show how r^2 is computed, let us consider a heuristic explanation of r^2 in terms of a graphical device, known as Venn diagram, or The Ballentine shown above.

However, in this figure the circle Y represents variation in the dependent variable Y and the circle X represent variation in X (say, via an OLS regression). The greater the extent of the overlap, the greater the variation in Y is explained by X. The r^2 is

simply a numerical measure of this overlap. In the figure as we move from left to right, the area of the overlap increases, that is, successively a greater proportion of the variation in Y is explained by X. In conclusion, r^2 increases. When there is no overlap, r^2 is obviously zero, but is explained by X. However, let us consider:

$$Y_t = \hat{Y}_t + \hat{U}_t \dots\dots\dots\dots\dots\dots\dots\dots (55)$$

Or in the deviation form

$$y_t = \hat{y}_t + \hat{u}_t \dots\dots\dots\dots\dots\dots\dots\dots (56)$$

Square both sides

$\hat{y}_t^2 = \left(\hat{y}_t + \hat{U}_t\right)^2$

$\hat{}^2 = \hat{}^2 + \hat{u}^2 + 2$

$y_t \quad y_t \quad {}_1 \quad y_t \quad {}_{t\hat{}2}\hat{U}$

Σ

Multiply through with .

$\sum\left[\hat{y}_t^2 = \hat{y}_t^2 + \hat{U}\ {}_t^2 + 2\hat{y}_t + \quad \hat{U}\ {}_t\right]$

$\sum \hat{Y}_t^2 = \sum \hat{Y}_t^2 + \sum \hat{U}_t^2 + \quad 2 \sum$

$= \sum \hat{Y}_t^2 + \sum \hat{U}_t^2$

$= \hat{\beta}_2^2 \sum \hat{X}\ {}_t^2 + \sum U\hat{}_t^2$

Since $\sum \hat{y}_t + \hat{U} \quad = 0$ and $\hat{y}\ {}_t =$

$\hat{y}_t + \hat{U}_t$

$\hat{\beta}_2 \hat{X}_t \dots\dots$

$\dots\dots\dots\dots\dots (57)$

The various sums of squares appearing in (57) can be described as follows:

$\sum \hat{Y}_t^2 = \sum\left(\hat{y}_t + \hat{U}_t\right)^2$ = total variation of the actual Y values about their sample mean, which may be called the total sum of square (TSS). $\sum \hat{Y}_t^2 = \sum\left(\hat{Y}_t - \bar{Y}\right)^2 = \sum\left(\hat{Y}_t - \bar{Y}\right)^2 = \hat{\beta}_2^2 \sum X_t^2$ = variation of the estimated Y values about their mean $\left(\hat{Y}_t - \bar{Y}\right)$, which appropriately may be called the sum of squares due to regression (i.e. due to the explanatory variables) or explained by regression, or simply the explained sum of squares (ESS). $\sum \hat{U}_t^2$ = residual or unexplained variation of the Y values about the regression line, or simply the residual sum of square (RSS). Thus equation (57) is:

TSS = ESS + RSS _____(58)

and shows that the total variation in the observed Y values about their mean value can be partitioned into two parts, one attributable to the regression line and the other to random forces because not all actual Y observations lie on the fitted line.

Dividing equation (58) by TSS

$$1 = \frac{ESS}{TSS} + \frac{RSS}{TSS} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (59)$$

$$= \frac{\sum\left(\hat{Y}_t - \bar{Y}\right)^2}{\sum(Y_t - \bar{Y})^2} + \frac{\sum \hat{U}_t^2}{\sum(Y_t - \bar{Y})^2}$$

We now define r^2 as

$$r^2 = \frac{\sum\left(\hat{Y}_t - \bar{Y}\right)^2}{\sum(Y_t - \bar{Y})^2} = \frac{ESS}{TSS} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (60)$$

or, alternatively, as:

$$r^2 = 1 - \frac{\sum \hat{U}_t^2}{\sum(Y_t - \bar{Y})^2} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (60a)$$

$$= 1 - \frac{RSS}{TSS}.$$

The quantity r^2 thus defined is known as the (sample) coefficient of determination and is the most commonly used measure of the goodness of fit of a regression line. Verbally, r^2 measure the proportion or percentage of the total variation in Y explained the regression model.

Two properties of r^2 may be noted:

(1) It is a nonnegative quantity

(2) Its limit are $0 \leq r^2 \leq 1$. An r^2 of 1 means a perfect fit, that is, $\hat{Y}_t = Y_t$ from each t. On the other hand, an r^2 of zero means that there is no relationship between the regressand and the regressor whatsoever (i.e $\hat{\beta}_2 = 0$). In this case as $\hat{Y}_t = \hat{\beta}_t = \bar{Y}$, that is the best prediction of any Y value is simply its mean value. In this situation therefore the regression line will be horizontal to the X axis.

Although r^2 can be computed directly from its definition given in equation (60) it can be obtained more quickly from the following formula;

$$r^2 = \frac{ESS}{TSS} = \frac{\sum \hat{Y}_t^2}{\sum \hat{Y}_t^2} = \frac{\hat{\beta}_2^2 \sum X_t^2}{\sum Y_t^2}$$
$$= \hat{\beta}_2^2 \left(= \frac{\sum X_t^2}{\sum Y_t^2} \right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (61)$$

If we divide the numerator and the denominator of equation (61) by the sample size n(or $n\ Y$ if the sample size is small), we obtain:

$r^2 = \hat{\beta}_2^2 \left(= \frac{S_X^2}{S_Y^2} \right)$.

Where S_Y^2 and S_X^2 are the sample variables of Y and X respectively

Since $\hat{\beta}_2 = \sum X_t + Y_t \,/ \sum_2 X_t^2$, equation (61) can also be expressed as

$$r^2 = \frac{(\sum X_t + Y_t)}{\sum X_t^2 \sum Y_t^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (62)$$

an expression that may be computationally easy to obtain. Given the definition of r^2, we can express ESS and RSS discussed earlier as follows:

$$\text{ESS} = r^2 . TSS$$
$$= r^2 \sum Y$$
$$RSS = TSS - ESS$$
$$= TSS(1 -$$
$$= \sum Y_t^2 =$$
$$_t^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (63)$$

$$ESS/TSS) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (64$$
$$(1 - r^2)$$

Therefore, we can write: $TSS = ESS + RSS$

$$\sum Y_t^2 = r^2 \sum Y_t^2 + (1 - r^2) \sum Y_t^2 \ldots\ldots\ldots\ldots\ldots\ldots (65)$$ an expression that we will find useful later. A quantity closely related to but conceptually very much different from r^2 is the coefficient of correlation, which is a measure of the degree of association between two variables. It can be computed either from:

$$r = \pm\sqrt{r^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (65)$$

or from its definition

$$r = \frac{\sum X_t + Y_t}{\sqrt{(\sum X_t^2)(\sum Y_t^2)}}$$
$$= \frac{n \sum X_t + Y_t - (\sum X_t)(\sum Y_t)}{\sqrt{[n \sum X_t^2 - (\sum X_t)^2][n \sum Y_t^2 - (\sum Y_t)^2]}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (66)$$

Which is known as the sample correlation coefficient.

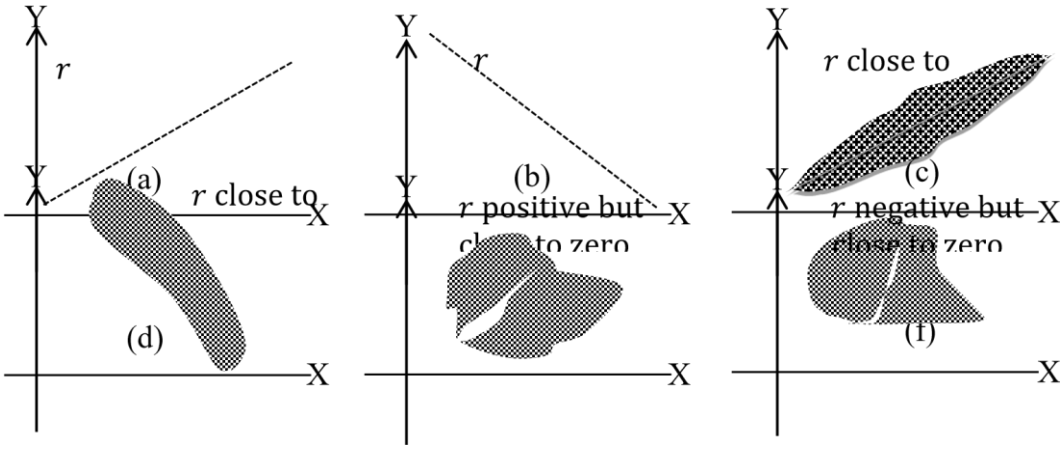


Figure 5.3 Showing the correlation patterns (adapted from Henri Theil, introduction to Econometrics, Prentice – Hall, Englewood Cliffs, N.J, 1978. P. 86)

Some of the properties of r are as follows:

(1) It can be positive or negative, the sign depending on the sign of the term in the numerator of (66) which measures the sample co variation of two variables.

(2) It lies between the limits of -1 and $+1$; that is, $-|\leq r \leq|$.

(3) It is symmetrical in nature; that is, the coefficient of correlation between Y and X (r_{xy}) is the same as that between Y and X (r_{yx}).

(4) It is independent of the origin and scale; that is/ if we define $X_t^* = aX_t + c$ and $Y_t^* = b\,Y_t + d$, where $a > 0, b > 0$ and c and d are constants, then r between X^* and Y^* is the same as that between the original variables X and Y.

(5) If X and Y are statistically independent, the correlation coefficient between them is zero, but if r = 0, it does not mean that to variables are independence.

(6) It is a measure of linear association or linear dependence only; it has no meaning for describing nonlinear relations.

(7) Although it is a measure of linear association between two variables, it does not necessarily imply any cause and effect relationship.

In the regression context, r^2 is a more meaningful measure than r, for the former tells us the proportion of variation in the dependent variable explained by the explanatory variable(s) and therefore provides on overall measure of the extent to which the variation in one variable determines the variation in the other.

The latter does not have such value. Moreover as we shall see, the interpretation of r (= R) is a multiple regression model is of dubious value. However, the student should note that r^2 defined previously can also be computed q the squared coefficient of correlation between actual Y_t and the estimated Y_t, namely $\hat{Y}_t$ that is using equation (66), we can write:

$$r^2 = \frac{\left[\sum(Y_t - \bar{Y}_t)(\hat{Y}_t - \bar{Y}_t)\right]^2}{\sum(Y_t - \bar{Y}_t)^2 \sum(\hat{Y}_t - \bar{Y}_t)^2} \text{ that is:}$$

$$r = \frac{\left(\sum Y_t \hat{Y}_t\right)^2}{\left(\sum Y_t^2\right)\left(\sum \hat{Y}_t^2\right)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (67)$$

where Y_t = actual Y, $\hat{Y}_t$= estimated Y and ¯ = ˆ = the mean of Y.

Let us take a look at the problem below:

Given $\hat{Y}_t = -184.0780 + 0.7064\mathrm{X_t}$

$\mathrm{Var}(\hat{\beta}_1) = 240.1707$ Se $(\hat{\beta}_1) = 46.2619$

$\mathrm{Var}(\hat{\beta}_2) = 0.000061$ Se $(\hat{\beta}_2) = 0.007827$

$r^2 = 0.998406$ $\hat{\sigma}^2 = 411.4913.$

The straight line equation above is a sample regression a analysis and it represent the aggregate (i.e. for the economy as a whole) Keynesian consumption function. As this equation shows the marginal propensity to consume (MPC) is about 0.71, suggesting that if income goes up by one naira, the average personal consumption expenditure (PCE) goes up by about 71 percents. From Keynesian theory, the MPC is less than 1. The intercept value of about – 184 billion naira. Of course, such, a mechanical interpretation of the intercept term does not make economic sense in the present instance because the zero income value is out of range of values we are working with and does not represent a likely outcome. As we will see on many occasion, very often the intercept term may not make much economic sense.

Therefore, in practice the intercept term may not be very meaningful although on occasions it can be very meaningful, in some analysis. The more meaningful value is the slope coefficient MPC in the present case.

The r^2 value of 0.9984 means approximately 99 percent of the variation in the PCE is explained by variation in the GDP. Since r^2at most can be 1, we can say that the regression line in the equation above fit our data extremely well; as you can see from that figure the actual data points are very tightly clustered around the estimated regression line.

5.4. Self-Assessment Exercises 1

1. What is meant by goodness of fit?
2. What is the meaning of R^2?

5.5. SUMMARY

A statistical test known as "goodness-of-fit" evaluates how well sample data fits a distribution from a population having a normal distribution. Simply put, it makes assumptions about whether a sample is biased or accurately reflects the facts that would be present in the wider population. The disparity between the actual values and those predicted by the model in the case of a normal distribution is established via goodness-of-fit. The chi-square is one of the techniques for figuring out goodness-of-fit. However, the coefficient of determination (R^2), which ranges from 0 to 1, expresses how accurately a statistical model forecasts a result. R^2 can be interpreted as the percentage of variation in the dependent variable that the statistical model predicts.

5.6. REFERENCES/Further Reading

Gujarat, D. N. (2007) Basic Econometrics, 4th Edition, tata Mcgraw – Hill publishing company limited, New Delhi.

Hall, S. G., & Asterion, D. (2011) Applied Econometrics, 2nd Edition, Palgrave Macmillian, New York city, USA.

5.7. Possible Answers to SAEs

These are the possible answers to the SAEs within the content.

Answers to SAEs 1

Goodness-of-fit is a statistical method for assessing how well a sample of data matches a given distribution as its population. It derives a comparison of observed values and expected values and explains whether the model developed fits the set of observations.

Answers to SAEs 2

The coefficient of determination, or R^2, is a measure that provides information about the goodness of fit of a model. In the context of regression it is a statistical measure of how well the regression line approximates the actual data.

Module 3: Normal Linear Regression Model (CNLRM)

This module introduces you to classical normal linear regression model, OLS Estimators under the normality Assumption, the method of maximum likelihood (ML), confidence intervals for regression coefficients $\boldsymbol{\beta_1}$ and $\boldsymbol{\beta_2}$ and hypotheses testing.

MODULE THREE: NORMAL LINEAR REGRESSION MODEL (CNLRM)

Unit One: Classical Normal Linear Regression Model
Unit Two: OLS Estimators under the Normality Assumption
Unit Three: The Method of Maximum Likelihood (ML)
Unit Four: Confidence Intervals for Regression Coefficients $\boldsymbol{\beta_1}$ AND $\boldsymbol{\beta_2}$
Unit Five: Hypotheses Testing

UNIT ONE: CLASSICAL NORMAL LINEAR REGRESSION MODEL

Unit Structure

1.1. Introduction
1.2. Learning Outcome
1.3. The Probability Distribution of Disturbances Ui
1.4. The Normality Assumption for Ui
1.5. Why the Normality Assumption
1.6. Summary
1.7. References/Further Readings/Web Resources
1.8. Possible Answers to Self-Assessment Exercises (SAEs)

1.1. INTRODUCTION

What is known as the classical theory of statistical inference consists of two branches, namely, estimation and hypothesis testing. We have thus far covered the topic of estimation of the parameters of the (two variable) linear regression model. Using the method of OLS we were able to estimate the parameters $\hat{\beta}_1$, $\hat{\beta}_2$ and σ^2. Under the assumptions of the *classical linear regression model (CLRM),* we were able to show that the estimators of these parameters, $\hat{\beta}_1$ $\hat{\beta}_2$, and $\hat{\sigma}^2$, satisfy several desirable statistical properties, such as unbiasedness, minimum variance, etc. (Recall the BLUE property.) Note that, since these are estimators, their values will change from sample to sample.

Therefore, these estimators are *random variables.*

But estimation is half the battle. Hypothesis testing is the other half. Recall that in regression analysis our objective is not only to estimate the sample regression function (SRF), but also to use it to draw inferences about the population regression function (PRF), as emphasized in Chapter 2. Thus, we would like to find out how

close flu is to the true flu or how close $\hat{\sigma}^2$ is to the true σ^2. For instance, in Example 3.2, we estimated the SRF as shown in Eq. (3.7.2). But since this regression is based on a sample of 55 families, how do we know that the estimated MPC of 0.4368 represents the (true) MPC in the population as a whole?

Therefore, since $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\sigma}^2$ are random variables, we need to find out their probability distributions, for without that knowledge we will not be able to relate them to their true values.

1.2. Learning Outcome

At the end of this unit, you should be able to:

i. know how to calculate the probability normality assumption for Ui
ii. understand the normality assumption for U_i
iii. understand why we have to conduct the normality assumption.

1.3. THE PROBABILITY DISTRIBUTION OF DISTURBANCES u_1

To find out the probability distributions of the OLS estimators, we proceed as follows. Specifically, consider$\hat{\beta}_2$. As we showed in Appendix 3A.2,

$$\hat{\beta}_2 = \sum K_i Y_1 \qquad (\mathbf{4.1.1}) \text{ where}$$

$k_i = x_i / \sum x_i^2$. But since the X's are assumed fixed, or non stochastic, because ours is conditional regression analysis, conditional on the fixed values of *X,,* Eq. (4.1.1) shows that /32 is a *linear* function of Y_i, which is random by assumption. But since $Y_i = \sum K_i(\beta_1 + \beta_2 X_i + u_i$, we can write (4.1.1) as

$$\hat{\beta}_2 = \sum K_i(\beta_1 + \beta_2 X_i + u_i) \qquad (\mathbf{4.1.2})$$

Because k_i, the betas, and X_i are all fixed,$\hat{\beta}_2$ is ultimately a *linear* function of the random variable *tit,* which is random by assumption. Therefore, the probability distribution of 42 (and also of it,) will depend on the assumption made about the probability distribution of u_i. And since knowledge of the probability distributions of OLS estimators is necessary to

draw inferences about their population values, the nature of the probability distribution of u_i assumes an extremely important role in hypothesis testing. Since the method of OLS does not make any assumption about the probabilistic nature of u_i, it is of little help for the purpose of drawing inferences about the PRF from the SRF, the Gauss-Markov theorem notwithstanding. This void can be filled if we are willing to assume that the u's follow some probability distribution. For reasons to be

explained shortly, in the regression context it is usually assumed that the u's follow the normal distribution. Adding the normality assumption for *it* to the assumptions of the classical linear regression model (CLRM) discussed in Chapter 3, we obtain what is known as the classical normal linear regression model (CNLRM).

Self-Assessment Exercises 1

What do you mean by probability distribution?

1.4. THE NORMALITY ASSUMPTION FOR u_1

Under the normality assumption, the ML and OLS estimators of the intercept and slope parameters of the regression model are identical. How- ever, the OLS and ML estimators of the variance of ui are different. In large samples, however, these two estimators converge.

1. Assumption 1: Linear Relationship.
2. Assumption 2: Independence.
3. Assumption 3: Homoscedasticity.
4. Assumption 4: Normality.

The classical *normal* linear regression model assumes that each u_i is distributed *normally* with

Mean: $E(u_i) = 0$ **(4.2.1)** Variance: $E[u_i - E(u_i)]^2 = E(u_i^2) = \sigma^2$ **(4.2.2)** cov (u_i, u_j): $E\{[(u_i - E(u_i)][u_j - E(u_j)]\} = (u_i\, u_j) = 0\ i \neq j$ **(4.2.3)**

The assumptions given above can be more compactly stated as

$$u_i \sim N(0, \sigma^2)$$ **(4.2.4)** where the symbol — means *distributed as* and *N* stands for the *normal distribution*, the terms in the parentheses representing the two parameters of the normal distribution, namely, the mean and the variance.

As noted in Appendix A, for two normally distributed variables, zero covariance or correlation means independence of the two variables. Therefore, with the normality assumption, (4.2.4) means that u_i, and u_j are not only uncorrelated but are also independently distributed. Therefore, we can write (4.2.4) as

$$u_i \sim NID(0, \sigma^2)$$

(4.2.5) where NID stands for *normally and independently distributed.*

1.4.1. Why the Normality Assumption?

The normalcy assumption, which is necessary for parametric assumptions, states that the obtained data has a normal distribution. The normality test is essentially supported by the majority of statistical tools, although the results only provide P values and not the normality test's power. The reason for the widespread belief in a Normality assumption is easy to see. If outcomes are indeed normally distributed then several different mathematical criteria identify the t-test and ordinary least squares regression as optimal analyses.

The normalcy assumption, which is necessary for parametric assumptions, states that the obtained data has a normal distribution. The normality test is essentially supported by the majority of statistical tools, although the results only provide P values and not the normality test's power.

Why do we employ the normality assumption? The several reasons why we employ normality assumption are the following:

1. u_i represent the combined influence (on the dependent variable) of a large number of independent variables that are not explicitly introduced in the regression model. As noted, we hope that the influence of these omitted or neglected variables is small and at best random. Now by the celebrated central limit theorem (CLT) of statistics (see Appendix A for details), it can be shown that if there are a large number of independent and identically distributed random variables, then, with a few exceptions, the distribution of their sum tends to a normal distribution as the number of such variables increase indefinitely.' It is the CLT that provides a theoretical justification for the assumption of normality of u_i.
2. A variant of the CLT states that, even if the number of variables is not very large or if these variables are not strictly independent, their sum may still be normally distributed.[2]
3. With the normality assumption, the probability distributions of OLS estimators can be easily derived because, as noted in Appendix A, one property of the normal distribution is that any linear function of normally distributed variables is itself normally distributed. OLS estimators $\hat{\beta}_1$, and $\hat{\beta}_2$ are linear functions of u_i. Therefore, if u_i are normally distributed, so are $\hat{\beta}_1$, and $\hat{\beta}_2$, which makes our task of hypothesis testing very straightforward.
4. The normal distribution is a comparatively simple distribution involving only two parameters (mean and variance); it is very well known and its theoretical properties have been extensively studied in mathematical *statistics.* Besides, many phenomena seem to follow the normal distribution.

5. Finally, if we are dealing with a small, or finite, sample size, say data of less than 100 observations, the normality assumption assumes a critical role. It not only helps us to derive the exact probability distributions of OLS estimators but also enables us to use the *t*, *F*, and X^2 *statistical* tests for regression models. The *statistical* properties of *t, F,* and χ^2probability distributions are discussed in Appendix A. As we will show subsequently, if the sample size is reasonably large, we may be able to relax the normality assumption.

A *cautionary note:* Since we are "imposing" the normality assumption, it behooves us to find out in practical applications involving small sample size data whether the normality assumption is appropriate. Later, we will develop some tests to do just that. Also, later we will come across situations where the normality assumption may be inappropriate. But until then we will continue with the normality assumption for the reasons discussed previously.

1.5. Self-Assessment Exercises 2

Why do we need normality assumption?

1.6. SUMMARY

We come to the conclusion in this unit that one of the most frequently misunderstood statistical concepts is the normalcy assumption. As opposed to what is frequently believed, the assumption needing a normal distribution in multiple regressions only applies to the disturbance factor. Perhaps the difficulty in comprehending what this assumption refers to the random error in the link between the independent variables and the dependent variable in a regression model is what causes confusion regarding this assumption. In reality, each case in the sample has a unique random variable that includes all the noise that explains discrepancies between observed and predicted values produced by a regression equation. The distribution of this disturbance term or noise for all cases in the sample should be normally distributed.

1.7. REFERENCES/Further Reading

Gujarat, D. N. (2007) Basic Econometrics, 4th Edition, tata Mcgraw – Hill publishing company limited, New Delhi.

Hall, S. G., & Asterion, D. (2011) Applied Econometrics, 2nd Edition, Palgrave Macmillian, New York city, USA.

1.8. Possible Answers to SAEs

These are the possible answers to the SAEs within the content.

Answers to SAEs 1

A probability distribution is a mathematical function that describes the probability of different possible values of a variable. Probability distributions are often depicted using graphs or probability tables.

Answers to SAEs 2

The explanation for the widespread acceptance in the Normality assumption is simple. If the outcomes are normally distributed, various mathematical criteria identify the t-test and ordinary least squares regression as suitable analyses.

UNIT TWO: OLS ESTIMATORS UNDER THE NORMALITY ASSUMPTION

Unit Structure

2.1. Introduction
2.2. Learning Outcome
2.3. Properties of OLS Estimators under the Normality Assumption
2.4. Summary
2.5. References/Further Readings/Web Resources
2.6. Possible Answers to Self-Assessment Exercises (SAEs)

2.1. INTRODUCTION

The OLS estimators that we create through linear regression give us a relationship between the variables. However, performing a regression does not automatically give us a reliable relationship between the variables. In order to create reliable relationships, we must know the properties of the estimators and show that some basic assumptions about the data are true under the normality assumption. One must understand that having a good dataset is of enormous importance for applied economic research. Therefore, in this unit we will vividly discuss the OLS estimators under the normality assumption.

2.2. Learning Outcome

At the end of this unit, you should be able to:

i. Understand the properties of OLS estimators under the normality assumption
ii. Understand the meaning of probability distribution

2.3. PROPERTIES OF OLS ESTIMATORS UNDER THE NORMALITY ASSUMPTION

The OLS estimates thus obtained possess certain desirable properties such as unbiasdness and minimum variance. These properties are enough to ensure a good estimateof the unknown population parameter value.

With the assumption that u, follow the normal distribution as in (4.2.5), the OLS estimators have the following properties; Appendix A provides a general discussion of the desirable statistical properties of estimators.

1. They are unbiased.
2. They have minimum variance. Combined with 1, this means that they are minimum-variance unbiased, or efficient estimators.
3. They have consistency; that is, as the sample size increases indefinitely, the estimators converge to their true population values.
4 $\hat{\beta}_1$ (being a linear function of u,) is *normally distributed* with

Mean: $E(\hat{\beta}_1) = \beta_1$ **(4.3.1)**

$\text{Var}(\hat{\beta}_1) = \sigma^2_{\hat{\beta}_1} = \frac{\sum X_i^2}{n\sum x_i^2}\sigma^2$ **= (3.3.3)** **(4.3.2)**

Or more compactly,

$$\hat{\beta}_1 \sim N(\hat{\beta}_1, \sigma^2_{\hat{\beta}_1})$$

Then by the properties of the normal distribution the variable *Z*, which is defined as

$$Z = \frac{\hat{\beta}_1 - \beta_1}{\sigma_{\hat{\beta}_1}} \quad \textbf{(4.3.3)}$$

follows the standard normal distribution, that is, a normal distribution with zero mean and unit $(= 1)$ variance, or

$Z \sim N(0, 1)$

5. $\hat{\beta}_2$(being a linear function of u_i) is *normally* distributed with

Mean: $E(\hat{\beta}_1) = \hat{\beta}_2$ **(4.3.4)**

$\text{Var}(\hat{\beta}_1)$: $\sigma^2_{\hat{\beta}_1} = \frac{\sigma^2}{\sum X_i^2}$

$= (\mathbf{3.3.1})$ $(\mathbf{4.3.5})$

Or, more compactly,

$$\hat{\beta}_2 \sim N\left(\hat{\beta}_2, \sigma^2_{\hat{\beta}_2}\right)$$

Then, as in (4.3.3),

$$Z = \frac{\beta_2 - \beta_2}{\sigma_{\hat{\beta}_1}} \quad \textbf{(4.3.6)}$$

also follows the standard normal distribution.

Geometrically, the probability distributions of $\hat{\beta}_1$and $\hat{\beta}_2$ are shown in Figure 4.1.

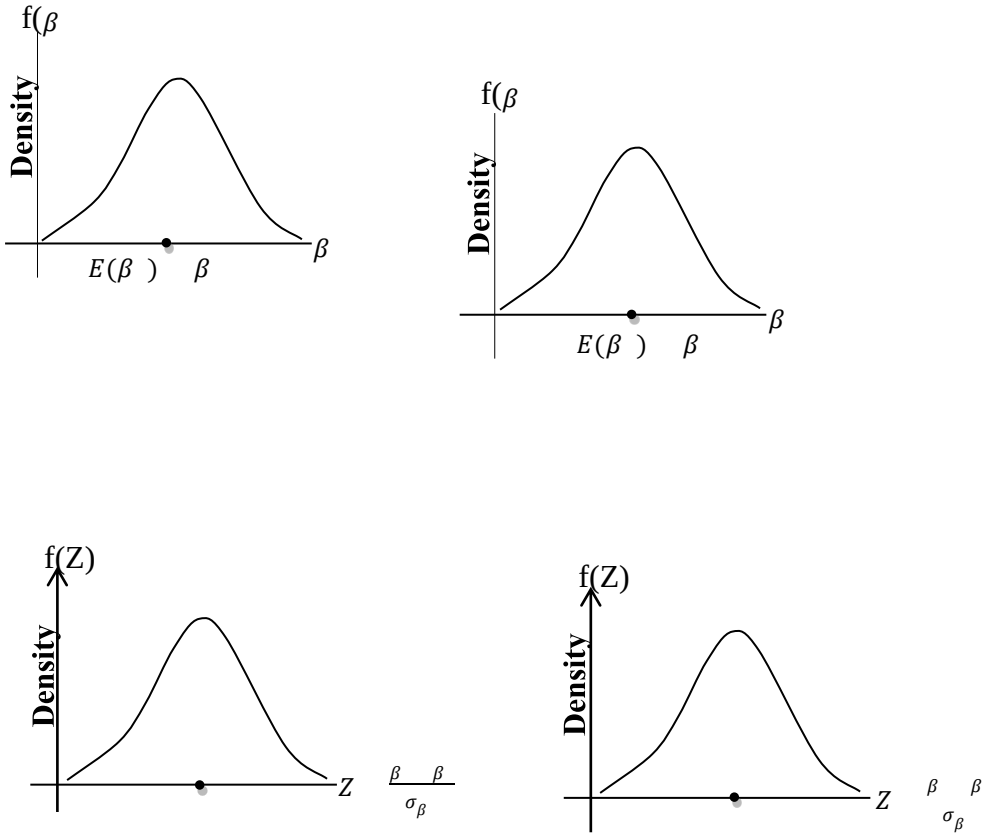


Figure 4.1 Probability distribution of ˆ ˆ

6. *(n* – 2)($\hat{\sigma}^2/\sigma^2$) is distributed as the X^2 (chi-square) distribution with (n – 2)df. [3] This knowledge will help us to draw inferences about the true $\sigma^2$from the estimated σ^2.
7. ($\hat{\beta}_1, \hat{\beta}_2$) are distributed independently of $\hat{\sigma}^2$. The importance of this will be explained in the next chapter.
8. $\hat{\beta}_1$ and $\hat{\beta}_2$ *have minimum variance in the entire class of unbiased estimators, whether linear or not.* This result, due to Rao, is very powerful because, unlike the Gauss-Markov theorem, it is not restricted to the class of linear estimators only. 4 Therefore, we can say that the least-squares estimators are **best unbiased estimators (BUE);** that is, they have minimum variance in the entire class of unbiased estimators.

In passing, note that, with the assumption that $u_i \sim$ N(0, σ^2), Y_i, being a linear function of u_i, is itself normally distributed with the mean and variance given by

$$E(Y_i) = \hat{\beta}_1 + \hat{\beta}_2 X_i \quad \textbf{(4.3.7)}$$

$$Var(Y_i) = \sigma^2 \quad \textbf{(4.3.8)}$$

More neatly, we can write

$$Y_i \sim N(\beta_1 + \beta_2 X_i, \sigma^2) \quad \textbf{(4.3.9)}$$

2.4. Self-Assessment Exercises 1

What are the properties of the OLS estimators?

2.5. SUMMARY

To summarize, the normality assumption allows us to derive the probability, or sampling, distributions of and (both normal) and (connected to the chi square). This simplifies the work of establishing confidence intervals and testing (statistical) hypotheses, as we will see in the following chapter.

2.6. REFERENCES/Further Reading

Gujarat, D. N. (2007) Basic Econometrics, 4th Edition, tata Mcgraw – Hill publishing company limited, New Delhi.

Faraday, M.N., (2014) Applied Econometrics, 1st Edition, Pentagon Publication limited.

2.7. Possible Answers to SAEs

These are the possible answers to the SAEs within the content.

Answers to SAEs 1

Three properties of the OLS estimators are that they are linear (running in a straight line rather than curved), they are unbiased (they average out the same as the data they purport to represent), and they have less variance than alternative models.

UNIT 3 THE METHOD OF MAXIMUM LIKELIHOOD (ML)

Unit Structure

3.1. Introduction
3.2. Learning Outcome
3.3. Maximum Likelihood Estimation of two variable regression Model.
2.4. Summary
3.5. References/Further Readings/Web Resources
3.6. Possible Answers to Self-Assessment Exercises (SAEs)

3.1. INTRODUCTION

A method of point estimation with some stronger theoretical properties than the method of OLS is the method of maximum likelihood (ML). Since this method is slightly involved, it is discussed in the appendix to this chapter. For the general reader, it will suffice to note that if u_i are assumed to be normally distributed, as we have done for reasons already discussed, the ML and OLS estimators of the regression coefficients, the
β'_s, are identical, and this is true of simple as well as multiple regressions. The ML estimator of σ^2 is $\sum \hat{u}_1^2 / n$. This estimator is biased, whereas the OLS estimator of $\sigma^2 \sum \hat{u}_1^2 / (n-2)$, as we have seen, is unbiased. But comparing these two estimators of σ^2, we see that as the sample size n gets larger the two estimators of σ^2 tend to be equal. Thus, asymptotically (i.e., as *n* increases indefinitely), the ML estimator of σ^2 is also unbiased.
Since the method of least squares with the added assumption of normality of u, provides us with all the tools necessary for both estimation and hypothesis testing of the linear regression models, there is no loss for readers who may not want to pursue the maximum likelihood method because of its slight mathematical complexity.

3.2. Learning Outcome

At the end of this unit, you should be able to:

i. understand the meaning of Maximum Likelihood Estimation of two variable regression Model.
ii. to understand how to derive the Maximum Likelihood Estimation model

3.3. MAXIMUM LIKELIHOOD ESTIMATION OF TWO VARIABLE REGRESSION MODEL

The parameters of a linear regression model can be estimated using a least squares procedure or by a maximum likelihood estimation procedure. Maximum likelihood estimation is a probabilistic framework for automatically finding the probability distribution and parameters that best describe the observed data.

Given some observed data, maximum likelihood estimation (MLE) is a method for estimating the parameters of an assumed probability distribution. This is accomplished by maximizing a probability function so that the observed data is most likely under the assumed statistical model. The maximum likelihood estimate is the point in the parameter space that maximizes the likelihood function. Because the logic of maximum likelihood is both intuitive and versatile, it has become a dominating method of statistical inference.

If the probability function is differentiable, the derivative test for locating maxima can be used. In some circumstances, the first-order requirements of the likelihood function can be solved analytically; for example, the ordinary least squares estimator for a linear regression model maximizes the likelihood when the random errors are assumed to have normal distributions with the same variance. MLE is generally comparable to maximum a posteriori (MAP) estimation with uniform prior distributions (or a normal prior distribution with an infinity standard deviation) in Bayesian inference. MLE is a specific example of an extremum estimator in frequentist inference, with the likelihood as the objective function.

Assume that in the two-variable model $Y_i + \beta_1 + \beta_2 X_i + u_i$ the Y_i are normally and independently distributed with mean = $\beta_1 + \beta_2 X_i$ and variance = σ^2. [See Eq. (4.3.9).] As a result, the joint probability density function of $Y_1, Y_2, \ldots\ldots Y_n$, given the preceding mean and variance, can be written as

$$f(Y_1, Y_2, \ldots\ldots Y_n | \beta_1 + \beta_2 X_i, \sigma^2)$$

But in view of the independence of the Y's, this joint probability density function can be written as a product of n individual density functions as

$$f(Y_1, Y_2, \ldots\ldots Y_n | \beta_1 + \beta_2 X_i, \sigma^2) = f(Y_1 | \beta_1 + \beta_2 X_i, \sigma^2) f f(Y_2 | \beta_1 + \beta_2 X_i, \sigma^2) \ldots f(Y_n | \beta_1 + \beta_2 X_i, \sigma^2) \quad \textbf{(1)}$$

Where

$$f(Y_i) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\frac{(Y_i - \beta_1 - \beta_2 X_i)^2}{\sigma^2}\right\} \quad (2)$$

which is the density function of a normally distributed variable with the given mean and variance.

(Note: exp means *e* to the power of the expression indicated by [].)

Substituting (2) for each Y, into (1) gives

$$f(Y_1, Y_2, \ldots\ldots\ Y_n|\beta_1 + \beta_2 X_i, \sigma^2) = \frac{1}{\sigma^n\left(\sqrt{2\pi}\right)^n} \exp\left\{-\frac{1}{2}\sum \frac{(Y_i - \beta_1 - \beta_2 X_i)^2}{\sigma^2}\right\} \quad (3)$$

If $Y_1, Y_2, \ldots\ldots\ Y_n$ are known or given, but fit, h, and σ^2 are not known, the function in (3) is called a likelihood function, denoted by $\mathrm{LF}(\beta_1, \beta_2, \sigma^2)$, and written as [1].

$$\mathrm{LF}(\beta_1, \beta_2, \sigma^2) = \frac{1}{\sigma^n\ \sqrt{2\pi}^{\ n}} \exp\left\{-\frac{1}{2}\sum \frac{(Y_i - \beta_1 - \beta_2 X_i)^2}{\sigma^2}\right\} \quad (4)$$

Maximum Likelihood Estimation of Two-Variable Regression Model

Assume that in the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i$ the Y_i are normally and independently distributed with mean $= \beta_1 + \beta_2 X_i$ and variance$= \sigma^2$. [See Eq. (4.3.9).] As a result, the joint probability density functions of $Y_1, Y_2, \ldots\ldots, Y_n$,given the preceding mean and variance, can be written as

$$f(Y_1, Y_2, \ldots\ldots\ Y_n|\beta_1 + \beta_2 X_i, \sigma^2)$$

But in view of the independence of the Ys, this joint probability density function can be written as a product of n individual density functions as

$$f(Y_1, Y_2, \ldots\ldots\ Y_n|\beta_1 + \beta_2 X_i, \sigma^2) = f(Y_1|\beta_1 + \beta_2 X_i, \sigma^2) f(Y_2|\beta_1 + \beta_2 X_i, \sigma^2) \ldots f(Y_n|\beta_1 + \beta_2 X_i, \sigma^2) \quad (1)$$

Where

$$f(Y_1) = \frac{1}{\sigma\left(\sqrt{2\pi}\right)^n} \exp\left\{-\frac{1}{2}\frac{(Y_i - \beta_1 - \beta_2 X_i)^2}{\sigma^2}\right\} \quad (2)$$

which is the density function of a normally distributed variable with the given mean and variance.

(Note: exp means e to the power of the expression indicated by [].) Substituting (2) for each Y_i into (1) gives

$$f(Y_1, Y_2, \ldots\ldots\ Y_n|\beta_1 + \beta_2 X_i, \sigma^2) = \frac{1}{\sigma^n\ \sqrt{2\pi}^{\ n}} \exp\left\{-\frac{1}{2}\sum \frac{(Y_i - \beta_1 - \beta_2 X_i)^2}{\sigma^2}\right\} \quad (3)$$

(

If $Y_1, Y_2, \ldots\ldots\ Y_n$ are known or given, but fib β_1, β_2 and σ^2 are not known, the function in (3) is called a likelihood function, denoted by LF (β_1, β_2 and σ^2), and written as[1].

$$\mathrm{LF}(\beta_1, \beta_2\ \sigma^2\ ) = \frac{1}{\sigma^n\left(\sqrt{2\pi}\right)^n}\exp\left\{-\frac{1}{2}\sum\frac{(Y_i-\beta_1-\ \beta_2X_i)^2}{\sigma^2}\right\} \qquad (\mathbf{4})$$

The method of maximum likelihood, as the name indicates, consists in estimating the unknown parameters in such a manner that the probability of observing the given Y's is as high (or maximum) as possible. Therefore, we have to find the maximum of the function (4). This is a straightforward exercise in differential calculus. For differentiation it is easier to express (4) in the log term as follows.[2] (Note: $\mathrm{I_n}$ = natural log.).

$$\mathrm{I_n LF} = -n\ln\sigma - \frac{n}{2}\ln(2\pi) - \frac{1}{2}\sum\frac{(Y_i-\beta_1-\ \beta_2X_i)^2}{\sigma^2}$$

$$= -\frac{n}{2}\ln\sigma^2$$

$$-\frac{1}{-}\ln(2\pi)\sum\frac{(Y_i-\beta_1-\ \beta_2X_i)^2}{2} \qquad (\mathbf{5})$$

Differentiating (5) partially with respect to σ^2 β_1, β_2 and σ^2, we obtain

$$\frac{\partial \mathrm{In}LF}{\partial\beta_1} = -\frac{1}{\sigma^2}\sum(Y_i-\beta_1 - \beta_2X_i)(-1) \qquad (\mathbf{6})$$

$$\frac{\partial \mathrm{In}LF}{\partial\beta_2} = -\frac{1}{\sigma^2}\sum(Y_i-\beta_1 - \beta_2X_i)(-X_i) \qquad (\mathbf{7})$$

$$\frac{\partial \mathrm{In}LF}{\partial\sigma^2} = -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4}\sum(Y_i-\beta_1 - \beta\ X\ )^2 \qquad (\mathbf{8})_{2\ \ i}$$

Setting these equations equal to zero (the first-order condition for optimization) and letting $\bar{\beta}_1, \bar{\beta}_2$ and $\bar{\sigma}^2$ denote the ML estimators, we obtain[3].

$$\frac{1}{\bar{\sigma}^2}\sum(Y_i - \bar{\beta}_1 - \bar{\beta}_2 X_i) = 0 \quad (9)$$

$$\frac{1}{\bar{\sigma}^2}\sum(Y_i - \bar{\beta}_1 - \bar{\beta}_2 X_i)X_i = 0 \quad (10)$$

$$-\frac{n}{2\bar{\sigma}^2} + \frac{1}{2\bar{\sigma}^4}\sum\left(Y_i - \bar{\beta}_1 - \bar{\beta}_2 X_i\right)^2 = 0 \quad (11)$$

After simplifying, Esq. (9) and (10) yield

$$\sum Y_i = n\bar{\beta}_1 + \bar{\beta}_2 \sum X_i \quad (12)$$

$$\sum Y_i X_i = \bar{\beta}_1 \sum X_i + \bar{\beta}_2 \sum X_i^2 \quad (13)$$

Which are precisely the normal equations of the least-squares theory obtained in (3.1.4) and (3.1.5). Therefore, the ML estimators, the $\bar{\beta}_s$, given in (3.1.6) and (3.1.7). This equality is not accidental. Examining the likelihood (5), we see that the last term enters with a negative sign. Therefore, maximizing (5) amounts to minimizing this term, which is precisely the least-squares approach, as can be seen from (3.1.2). Substituting the ML, (= OLS) estimators into (11) and simplifying, we obtain the ML estimator of $\bar{\sigma}^2$ as

$$\bar{\sigma}^2 = \frac{1}{n}\sum\left(Y_i - \bar{\beta}_1 - \bar{\beta}_2 X_i\right)^2 = \frac{1}{n}\sum\left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i\right)^2 = \frac{1}{n}\sum \hat{u}_i^2 \quad (14)$$

From (14) it is obvious that the ML estimators $\bar{\sigma}^2$ differs from the OLS estimator $\bar{\sigma}^2 = [1/(n-2)]\sum \hat{u}_i^2$, which was shown to be an unbiased estimator of σ^2 in Appendix 3A, Section 3A.5. Thus, the ML estimator of σ^2 is biased. The magnitude of this bias can be easily determined as follows. Taking the mathematical expectation of (14) on both sides, we obtain.

$$\mathrm{E}(\bar{\sigma}^2) = \frac{1}{n}E\left(\sum \hat{u}_i^2\right)$$
$$= \left(\sum \frac{n-2}{n}\right)\sigma^2 \qquad \text{using Eq. (16) of Appendix 3A, Section 3A. 5} \qquad (\mathbf{15})$$
$$= \sigma^2 - \frac{1}{n}\sigma^2$$

Which shows that $\bar{\sigma}^2$ is biased downward (i.e., it underestimates the true σ^2) in small samples. But notice that as n, the sample size, increase indefinitely, the second term in (15), the bias factor, tends to be zero. Therefore, asymptotically (i.e., in a very large sample),$\bar{\sigma}^2$ is unbiased too, that is, lim E($\bar{\sigma}^2$) = σ^2 as $n \to \infty$. It can further be proved that $\bar{\sigma}^2$ is also a consistent estimator.[4]; that is, as n increase indefinitely $\bar{\sigma}^2$ converges to its true value σ^2.

3.4. Self-Assessment Exercises 1

Which model can be estimated by maximum likelihood estimator?

3.5. SUMMARY

The maximum likelihood (ML) technique is an alternative to the least-squares method. However, in order to employ this strategy, one needs make an assumption regarding the probability distribution of the disturbance term. In the context of regression, the most commonly used assumption is that follows the normal distribution. However, under the normality assumption, the ML and OLS estimators of the regression model's intercept and slope parameters are equal. The OLS and ML estimators of the variance of, on the other hand, disagree. However, in large samples, these two estimators converge. As a result, the ML approach is commonly referred to as a large-sample method. The ML approach has a broader use in that it may be used on regression models with nonlinear parameters.

3.6. REFERENCES/Further Reading

Gujarat, D. N. (2007) Basic Econometrics, 4th Edition, tata Mcgraw – Hill publishing company limited, New Delhi.

Hall, S. G., & Asterion, D. (2011) Applied Econometrics, 2nd Edition, Palgrave Macmillian, New York city, USA.

3.7. Possible Answers to SAEs

These are the possible answers to the SAEs within the content.

Answers to SAEs 1

The parameters of a logistic regression model can be estimated by the probabilistic framework called maximum likelihood estimation.

UNIT 4 CONFIDENCE INTERVALS FOR REGRESSION COEFFICIENTS β_1 AND β_2

Unit Structure

4.1. Introduction
4.2. Learning Outcome
4.3. Confidence intervals
4.3.1. Confidence Interval for $\boldsymbol{\beta_1}$
4.3.2. Confidence interval for σ^2
4.4. Summary
4.5. References/Further Readings/Web Resources
4.6. Possible Answers to Self-Assessment Exercises (SAEs)

4.1. INTRODUCTION

Interval estimates can be contrasted with point estimates. A point estimate is a single value given as the estimate of a population parameter that is of interest, for example the mean of some quantity. An interval estimate specifies instead a range within which the parameter is estimated to lie. Confidence intervals are commonly reported in tables or graphs along with point estimates of the same parameters, to show the reliability of the estimates.

For example, a confidence interval can be used to describe how reliable survey results are. In a poll of election voting-intentions, the result might be that 40% of respondents intend to vote for a certain party. A 99% confidence interval for the proportion in the whole population having the same intention on the survey might be 30% to 50%. From the same data one may calculate a 90% confidence interval, which in this case might be 37% to 43%. A major factor determining the length of a confidence interval is the size of the sample used in the estimation procedure, for example the number of people taking part in a survey.

In statistics, a confidence interval (CI) is a type of interval estimate of a population parameter. It is an observed interval (i.e., it is calculated from the observations), in principle different from sample to sample, that frequently includes the value of an unobservable parameter of interest if the experiment is repeated. How frequently the observed interval contains the parameter is determined by the confidence level or confidence coefficient. More specifically, the meaning of the term "confidence level" is that, if CI are constructed across many separate data analyses of replicated (and possibly different) experiments, the proportion of such intervals that contain the true value of the parameter will match the given confidence level. Whereas two-sided confidence limits form a confidence interval, their one-sided counterparts are referred to as lower/upper confidence bounds (or limits). The confidence interval contains the parameter values that, when tested, should not be rejected with the same sample. Greater levels of variance yield larger confidence intervals, and hence less precise estimates of the parameter. Confidence intervals of difference parameters not containing 0 imply that there is a statistically significant difference between the populations.

In applied practice, confidence intervals are typically stated at the 95% confidence level. However, when presented graphically, confidence intervals can be shown at several confidence levels, for example 90%, 95% and 99%.

Certain factors may affect the confidence interval size including size of sample, level of confidence, and population variability. A larger sample size normally will lead to a better estimate of the population parameter. Confidence intervals were introduced to statistics by Jerzy Neyman in a paper published in 1937

4.2. Learning Outcome

At the end of this unit, you should be able to:

i. Understand confidence interval for β_1 and σ^2

ii. Understand how to interpret the confidence interval

4.3. Confidence Interval

4.3.1. Confidence Interval for β_1

In statistics, a confidence interval denotes the likelihood that a population parameter will fall between a set of values a certain percentage of the time. Analysts frequently utilize confidence ranges that include 95% or 99% of the expected observations.

With the normality assumption for *u,,* the OLS estimators β_1 and β_2 are themselves normally distributed with means and variances given therein. Therefore, for example, the variable.

$$Z = \frac{\hat{\beta}_2 - \beta_2}{Se(\hat{\beta}_2)} = \frac{(\hat{\beta}_2 - \beta_2)\sqrt{\sum x_i^2}}{\sigma}\hat{\beta}_1 \qquad (\mathbf{4.3.1})$$

as noted in (4.3.6), is a standardized normal variable. It therefore seems that we can use the normal distribution to make probabilistic statements about $\beta_2$provided the true population variance σ^2 is known. If σ^2 is known, an important property of a normally distributed variable with mean μ and variance σ^2 is that the area under the normal curve between *it* ± σ is about 68 percent, that between the limits μ ± 2σ is about 95 percent, and that between μ ± 3σ is about 99.7 percent.

But σ^2 is rarely known, and in practice it is determined by the unbiased estimator $\hat{\sigma}^2$. If we replace σ by ˆ, (5.3.1) may be written as

$$Z = \frac{\hat{\beta}_2 - \beta_2}{Se(\hat{\beta}_2)} = \frac{\text{estimator} - \text{parameter}}{\text{estimated standard errors of estimator}} = \frac{(\hat{\beta}_2 - \beta_2)\sqrt{\sum x_i^2}}{\hat{\sigma}}\hat{\beta}_1 \qquad (\mathbf{4.3.2})$$

where the se ($\hat{\beta}_2$) now refers to the estimated standard error. It can be shown that the *t* variable thus defined follows the *t* distribution with n – 2 df. [Note the difference between (4.3.1) and (4.3.2).] Therefore, instead of using the normal distribution, we can use the *t* distribution to establish a confidence interval for $\hat{\beta}_2$ as follows:

Pr (—$t_{\alpha/2}$ ≤ t ≤ $t_{\alpha/2}$) = 1 —α **(4.3.3)** where the *t* value in the middle of this double inequality is the *t* value given by (4.3.2) and where $t_{\alpha/2}$ is the value of the *t* variable obtained from the *t* distribution for α/2 level of significance and n — 2 df; it is often called the **critical** *t* value at α/2 level of significance. Substitution of (4.3.2) into (4.3.3) yields

$$Pr\left[-t_{\alpha/2} \le \frac{\hat{\beta}_2 - \beta_2}{Se(\hat{\beta}_2)} \le t_{\alpha/2}\right] = \mathbf{1} - \alpha \qquad (\mathbf{4.3.4})$$

Rearranging (5.3.4), we obtain

$$\boldsymbol{Pr}\left[\widehat{\boldsymbol{\beta}}_{\mathbf{2}} - \boldsymbol{t}_{\boldsymbol{\alpha}/\mathbf{2}}\boldsymbol{Se}\left(\widehat{\boldsymbol{\beta}}_{\mathbf{2}}\right) \le \boldsymbol{\beta}_{\mathbf{2}} \le \widehat{\boldsymbol{\beta}}_{\mathbf{2}} + \boldsymbol{t}_{\boldsymbol{\alpha}/\mathbf{2}}\,\boldsymbol{Se}\left(\widehat{\boldsymbol{\beta}}_{\mathbf{2}}\right)\right] = \mathbf{1} - \boldsymbol{\alpha} \qquad (\mathbf{4.3.5})^{\,3}$$

Equation (4.3.5) provides a 100(1 - a) percent **confidence interval** for β_2, which can be written more compactly as

100(1 – α)% confidence interval for β_2:

$$\hat{\beta}_2 \pm t_{\alpha/2} Se(\hat{\beta}_1) \qquad (4.3.6)$$

Arguing analogously, and using (4.3.1) and **(4** 3 2), we can then write:

$$Pr\ \beta_1 - t_{\alpha/2}\ Se\ \beta_1 \leq \beta_2 \leq \beta_1 + t_{\alpha/2}\ Se\ \beta_1 = 1 - \alpha \qquad (4.3.7)$$

or, more compactly

100(1 – α)% confidence interval for β_1:

$$\hat{\beta}_1 \pm t_{\alpha/2} Se(\hat{\beta}_1) \qquad (4.3.8)$$

Notice an important feature of the confidence intervals given in (4.3.6) and (4.3.8): In both cases *the width of the confidence interval is proportional to the standard* error *of the estimator.* That is, the larger the standard error, the larger is the width of the confidence interval. Put differently, the larger the standard error of the estimator, the greater is the uncertainty of estimating the true value of the unknown parameter. Thus, the standard error of an estimator is often described as a measure of the precision of the estimator, i.e., how precisely the estimator measures the true population value.

Returning to our illustrative consumption-income example, we found that $\hat{\beta}_2$= 0.5091, se ($\hat{\beta}_2$) = 0.0357, and df = 8. If we assume α = 5%, that is, 95% confidence coefficient, then the *t* table shows that for 8 df the critical $t_{\alpha/2} = t_{0.025}$= 2.306. Substituting these values in (5.3.5), the reader should verify that the 95% confidence interval for β_2 is as follows:

$$0.4268 \leq \beta_2 \leq .5914 \qquad (4.3.9)$$

Or, using (4.3.6), it is

$$0.5091 \pm 2.306(0.0357) \text{ that is,}$$

$$0.5091 \pm .0823 \qquad (4.3.10)$$

The Interpretation of this confidence interval is: Given the confidence coefficient of 95%, in the long run, in 95 out of 100 cases intervals like (0.4268, 0.5914) will contain the true β_2. But, as warned earlier, we cannot say that the probability is 95

percent that the specific interval (0.4268 to 0.5914) contains the true β_2 because this interval is now fixed and no longer random; therefore, β_2 either lies in it or does not: The probability that the specified fixed interval includes the true β_2 is therefore 1 or 0.

Confidence Interval for $\boldsymbol{\beta_1}$

Following (4.3.7), the reader can easily verify that the 95% confidence interval for flu of our consumption-income example is

$$9.6643 \leq \beta_2 \leq 39.2448 \qquad \mathbf{4.3.11}(\quad)$$

Or, using (4.3.8), we find it is

$$24.4545 \pm 2.306(6.4138)$$

This is,

$$24.4545 \pm .7902 \qquad (\mathbf{4.3.12})$$

14 that is,

Again you should be careful in interpreting this confidence interval. In the long run, in 95 out of 100 cases intervals like (4.3.11) will contain the true β_1; the probability that this particular fixed interval includes the true β_1 is either 1 or 0.

Confidence Interval for $\boldsymbol{\beta_1}$ and $\boldsymbol{\beta_2}$ Simultaneously

There are occasions when one needs to construct a joint confidence interval for β_1 and β_2 such that with a confidence coefficient $(1-\alpha)$say, 95%, that interval includes β_1 and β_2 simultaneously.

4.3.2. CONFIDENCE INTERVAL FOR $\boldsymbol{\sigma^2}$

As pointed out in our previous discussion, under the normality assumption, the variable
Figure 5.1 The 95% confidence interval for x^2 (8 df).

follows the X^2 distribution with n – 2 df.[5] Therefore, we can use the X^2 distribution to establish a confidence interval for σ^2.

$$\Pr\left(x^2_{1-\alpha/2} \leq X^2 \leq X^2_{\alpha/2}\right) = 1 - \alpha \qquad (\mathbf{4.4.2})$$

where the X^2 value in the middle of this double inequality is as given by (5.4.1) and where $x^2_{1-\alpha/2}$ and $X^2_{\alpha/2}$ are two values of X^2 (the critical X^2 values) obtained from the chi-square table for n – 2 df in such a manner that they cut off 100(α/2) percent tail areas of the X^2 distribution, as shown in Figure 5.1. Substituting X^2 from (5.4.1) into (5.4.2) and rearranging the terms, we obtain

$$\Pr\left[(n-2)\frac{\hat{\sigma}^2}{X^2_{\alpha/2}} \leq \sigma^2 \leq (n-2)\frac{\hat{\sigma}^2}{x^2_{1-\alpha/2}}\right] = 1 - \alpha \qquad (\mathbf{4.4.3})$$

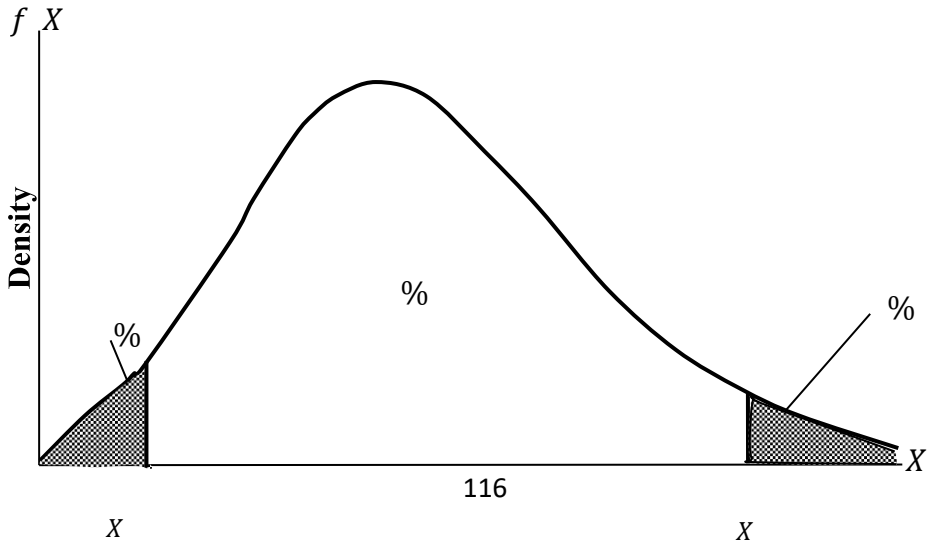
which gives the 100(1 - a)% confidence interval for a 2 .

To illustrate, consider this example, when $\hat{\sigma}^2$ = 42.1591 and df = 8. If α is chosen at 5 percent, the chi-square table for 8 df gives the following critical values: $X^2_{0.025}$ = 17.5346, and $X^2_{0.975}$= 2.1797. These values show that the probability of a chi-square value exceeding 17.5346 is 2.5 percent and that of 2.1797 is 97.5 percent. Therefore, the interval between these two values is the 95% confidence interval for X^2.

Substituting the data of our example into (4.4.3), the students should verify that the 95% confidence interval for 5 2 is as follows:

$$19.2347 \leq \sigma^2 \leq 154.7336 \qquad \mathbf{(4.4.4)}$$

The interpretation of this interval is: If we establish 95% confidence limits on *a2* and if we maintain a priori that these limits will include true a 2 , we shall be right in the long run 95 percent of the time.



Self-Assessment Exercises 1

What can you interpret from a confidence interval?

4.4. SUMMARY

Confidence intervals are a range of values (intervals) that serve as good estimates of an unknown population parameter; however, the interval generated from a specific sample does not always include the true value of the parameter. When we state, "we are 99% confident that the true value of the parameter is in our confidence interval," we mean that 99% of the hypothetically observed confidence intervals will contain the parameter's true value. The population parameter is either in the interval, realized or not after any single sample is obtained; it is not a question of chance. If a comparable hypothesis test is done, the confidence level is the complement of the respective level of significance, i.e., a 95% confidence interval reflects a significance level of 0.05.

4.5. REFERENCES/Further Reading

Gujarat, D. N. (2007) Basic Econometrics, 4th Edition, tata Mcgraw – Hill publishing company limited, New Delhi.

Hall, S. G., & Asterion, D. (2011) Applied Econometrics, 2nd Edition, Palgrave Macmillian, New York city, USA.

4.6. Possible Answers to SAEs

These are the possible answers to the SAEs within the content.

Answers to SAEs 1

A confidence interval for a mean gives us a range of plausible values for the population mean. If a confidence interval does not include a particular value, we can say that it is not likely that the particular value is the true population mean.

UNIT FIVE: HYPOTHESIS TESTING

Unit Structure

5.1. Introduction
5.2. Learning Outcome
5.3. Analysis of Hypothesis Testing
5.4. Hypothesis testing: the confidence interval Approach
5.5. Hypothesis testing: the test-of-significance Approach
5.6. Testing the significance of σ^2: the Chi Square (X^2)Test
5.6.1. Errors in Hypothesis Testing
5.6.2. Types of Statistical Tests
5.6.3. P-Value Testing
5.6.4. Further Explanation on Hypothesis Testing
5.6.5. The Test Statistic
5.6.6. The Rejection Regions

5.7. Summary
5.8. References/Further Readings/Web Resources
5.9. Possible Answers to Self-Assessment Exercises (SAEs)

5.1. INTRODUCTION

A hypothesis test is a statistical test that is used to determine whether there is enough evidence in a sample of data to infer that a certain condition is true for the entire population. A hypothesis test examines two opposing hypotheses about a population: the null hypothesis and the alternative hypothesis. The null hypothesis is the statement being tested. Usually the null hypothesis is a statement of "no effect" or "no difference". The alternative hypothesis is the statement you want to be able to conclude is true.
Based on the sample data, the test determines whether to reject the null hypothesis. You use a p-value, to make the determination. If the p-value is less than or equal to the level of significance, which is a cut-off point that you define, then you can reject the null hypothesis.

5.2. Learning Outcome

At the end of this unit, you should be able to:

i. Understand the meaning of Hypothesis
ii. Understand the hypothesis testing using the confidence interval Approach and test-of-significance Approach
iii. Understand the testing of significance of σ^2 and the Chi Square (X^2)Test

5.3. ANALYSIS OF HYPOTHESIS TESTING

The use of sample data in hypothesis testing determines the plausibility of a theory. Given the facts, the test gives evidence of the hypothesis's plausibility. Statistical analysts test hypotheses by measuring and analysing a random sample of the population being studied.
Having discussed the problem of point and interval estimation, we shall now consider the topic of hypothesis testing. In this unit we will discuss briefly some general aspects of this topic.
The problem of statistical hypothesis testing may be stated simply as follows: *Is* a *given observation or finding compatible with some stated hypothesis or not?* The

word "compatible," as used here, means "sufficiently" close to the hypothesized value so that we do not reject the stated hypothesis. Thus, if some theory or prior experience leads us to believe that the true slope coefficient β_2 of the consumption-income example is unity, is the observed $\hat{\beta}_2$ = 0.5091 from the statistical table consistent with the stated hypothesis? If it is, we do not reject the hypothesis; otherwise, we may reject it.

In the language of statistics, the stated hypothesis is known as the null hypothesis and is denoted by the symbol H_0. The null hypothesis is usually tested against an alternative hypothesis (also known as maintained hypothesis) denoted by H_1, which may state, for example, that true β_2 is different from unity. The alternative hypothesis may be simple or composite.[6] For example, $H_1: \beta_2$ = 1.5 is a simple hypothesis, but $H_1: \beta_2 \pm 0$ 1.5 is a composite hypothesis.

The theory of hypothesis testing is concerned with developing rules or procedures for deciding whether to reject or not reject the null hypothesis. There are two *mutually complementary* approaches for devising such rules, namely, confidence interval and test of significance. Both these approaches predicate that the variable (statistic or estimator) under consideration has some probability distribution and that hypothesis testing involves making statements or assertions about the value(s) of the parameter(s) of such distribution. For example, we know that with the normality assumption $\hat{\beta}_2$ is normally distributed with mean equal to β_2 and variance. If we hypothesize that β_2 = 1, we are making an assertion about one of the parameters of the normal distribution, namely, the mean. Most of the statistical hypotheses encountered in this text will be of this typemaking assertions about one or more values of the parameters of some assumed probability distribution such as the normal, F, t, or X^2.

Self-Assessment Exercises 1

What is hypothesis testing and types?

5.4 HYPOTHESIS TESTING: THE CONFIDENCE-INTERVAL APPROACH

Both confidence intervals and hypothesis tests are inferential methods that rely on an approximated sample distribution. Confidence intervals estimate a population parameter using data from a sample. To test a hypothesis, hypothesis tests employ data from a sample. Because both procedures rely on the same fundamental methodology, confidence intervals and hypothesis testing are closely related. Furthermore, there is a strong relationship

between significance and confidence levels. Indeed, they are so closely related that hypothesis tests and confidence intervals always agree on statistical significance. A confidence interval is derived from a sample and provides a range of values that most likely include the unknown value of a population parameter. Read Understanding Confidence Intervals to learn more about confidence intervals in general, how to interpret them, and how to calculate them.

(i) Two-Sided or Two-Tall Test

To illustrate the confidence-interval approach, once again we revert to the consumption income example. As we know, the estimated marginal propensity to consume (MPC), $\hat{\beta}_2$,is 0.5091. Suppose we postulate that

$$H_0: 02 = 0.3 \qquad H_1: 02 \neq 0.3$$

that is, the true MPC is 0.3 under the null hypothesis but it is less than or greater than 0.3 under the alternative hypothesis. The null hypothesis is a simple hypothesis, whereas the alternative hypothesis is composite; actually it is what is known as a two-sided hypothesis. Very often such a two-sided alternative hypothesis reflects the fact that we do not have a strong a priori or theoretical expectation about the direction in which the alternative hypothesis should move from the null hypothesis.

Is the observed $\hat{\beta}_2$ compatible with H_0? To answer this question, let us refer to the confidence interval (5.3.9). We know that in the long run intervals like (0.4268, 03914) will contain the true β_2 with 95 percent probability. Consequently, in the long run (i.e., repeated sampling) such intervals provide a range or limits within which the true β_2 may lie with a confidence coefficient of, say, 95%. Thus, the confidence interval provides a set of plausible null hypotheses. Therefore, if $\beta_2$under H_0 falls within the $100(1 - \alpha)$% confidence interval, we do not reject the null hypothesis; if it lies outside the interval, we may reject it.

Decision Rule: Construct a $100(1 - \alpha)$ % confidence interval for β_2. If the β_2 under H_0 falls within this confidence interval, do not rejectH_0, but if it falls outside this interval, rejectH_0.

Following this rule, for our hypothetical example, H_0: β_2 = 0.3 clearly lies outside the

95% confidence interval given in (4.3.9). Therefore, we can reject

Figure 4.2 A $100(1 - \alpha)$% confidence interval for β_2.

the hypothesis that the true MPC is 0.3, with 95% confidence. If the null hypothesis were true, the probability of our obtaining a value of MPC of as much as 0.5091 by sheer chance or fluke is at the most about 5 percent, a small probability.

In statistics, when we reject the null hypothesis, we say that our finding is statistically significant. On the other hand, when we do not reject the null hypothesis, we say that our finding is not statistically significant.

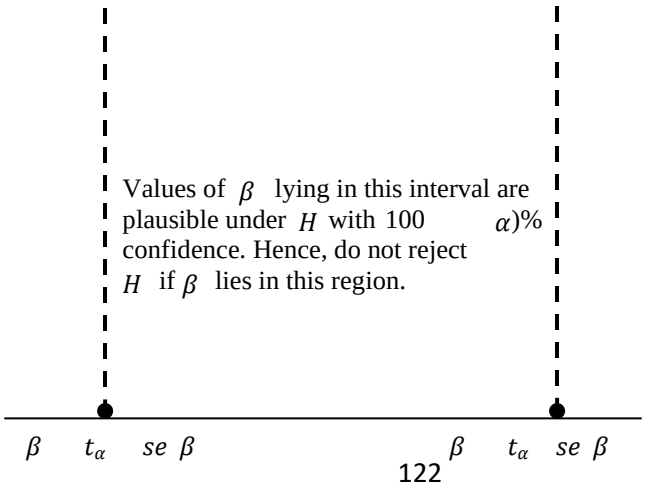
Some authors use a phrase such as "highly statistically significant." By this they usually mean that when they reject the null hypothesis, the probability of committing a Type I error (i.e., a) is a small number, usually 1 percent. it is better to leave it to the researcher to decide whether a statistical finding is "significant," "moderately significant," or "highly significant."

(ii) One-sided or One-Tall Test

Sometimes we have a strong a priori or theoretical expectation (or expectations based on some previous empirical work) that the alternative hypothesis is one-sided or unidirectional rather than two-sided, as just discussed. Thus, for our consumption-income example, one could postulate that

$$H_0: \beta_2 \leq 0.3 \text{ and } H_1: \beta_2 < 0.3$$

Perhaps economic theory or prior empirical work suggests that the marginal propensity to consume is greater than 0.3. Although the procedure to test this hypothesis can be easily derived from (4.3.5), the actual mechanics are better explained in terms of the test-ofsignificance approach discussed next.



Self-Assessment Exercises 2

What are the approaches to hypothesis testing?

5.5. HYPOTHESIS TESTING: THE TEST-OF-SIGNIFICANCE APPROACH

A test of significance is a formal procedure for comparing observed data with a claim (also called a hypothesis), the truth of which is being assessed. The claim is a statement about a parameter, like the population proportion p or the population mean μ.

(i) Testing the Significance of Regression Coefficients: The t Test

An *alternative but complementary approach* to the confidence-interval method of testing statistical hypotheses is the test-of-significance approach developed along independent lines by R. A. Fisher and jointly by Neyman and Pearson.' Broadly speaking, a test of significance is a procedure by which sample results are used to verify the truth or falsity of a null hypothesis. The key idea behind tests of significance is that of a test statistic (estimator) and the sampling distribution of such a statistic under the null hypothesis. The decision to accept or reject Ho is made on the basis of the value of the test statistic obtained from the data at hand.

As an illustration, recall that under the normality assumption the variable

$$t = \frac{\hat{\beta}_2 - \beta_2}{se(\hat{\beta}_2)} = \frac{(\hat{\beta}_2 - \beta_2)\sqrt{\sum X_i^2}}{\hat{\sigma}} \qquad (\mathbf{4.3.2})$$

follows the *t* distribution with n – 2 df. If the value of true β_2 is specified under the null hypothesis, the *t* value of (5.3.2) can readily be computed from the available sample, and therefore it can serve as a test statistic. And since this test statistic follows the *t* distribution, confidence-interval statements such as the following can be made:

$$\Pr\left[t_{\alpha/2} \leq \frac{\hat{\beta}_2 - \beta_2}{se(\hat{\beta}_2)} \leq t_{\alpha/2}\right] = 1 - \alpha \qquad (\mathbf{4.7.1})$$

where β_2^* is the value of β_2 under H_0 and where $-t_{\alpha/2}$ and $t_{\alpha/2}$ are the values of *t* (the critical *t* values) obtained from the *t* table for $(\alpha/2)$ level of significance and n – 2 df [cf. (4.3.4)].

Rearranging (5.7.1), we obtain

$$\Pr\left[\beta_2^* - t_{\alpha/2}\, se(\hat{\beta}_2) \leq \hat{\beta}_2 \leq \beta_2^* + \left(\hat{\beta}_2\right)\right] = 1 - \alpha \qquad \textbf{(4.7.2)}$$

which gives the interval in which 42 will fall with $1 - \alpha$ probability, given $\beta_2 = \beta_2^*$. In the language of hypothesis testing, the $100(1 - \alpha)\%$ confidence interval established in (4.7.2) is known as the region of acceptance (of the null hypothesis) and the *region(s)* outside the confidence interval is (are) called the region(s) of rejection (of H_0) or the critical region(s). As noted previously, the confidence limits, the endpoints of the confidence interval, are also called critical values.

The intimate connection between the confidence-interval and test-of significance approaches to hypothesis testing can now be seen by comparing (4.3.5) with (4.7.2). In the confidence-interval procedure we try to establish a range or an interval that has a certain probability of including the true but unknown β_2, whereas in the test-ofsignificance approach we hypothesize some value for β_2 and try to see whether the computed $\hat{\beta}_2$ lies within reasonable (confidence) limits around the hypothesized value. Once again let us revert to our consumption-income example. We know that $\hat{\beta}_2$= 0.5091, se ($\hat{\beta}_2$) = 0.0357, and df = 8. If we assume α = 5 percent, $t_{\alpha/2}$ = 2.306. If we let $H_0: \beta_2 =$

$\beta_2^* = 0.3$ and $H_1: \beta_2 \neq 0.3$, (4.7.2) becomes

Pr (0.2177 ≤ $\hat{\beta}_2$0.3823) = 0.95 **(4.7.3)**

as shown diagrammatically in Figure 5.3. Since the observed 42 lies in the critical region, we reject the null hypothesis that true β_2 = 0.3.

In practice, there is no need to estimate (4.7.2) explicitly. One can compute the *t* value in the middle of the double inequality given by (4.7.1) and see whether it lies between the critical *t* values or outside them. For our example,

$$t = \frac{0.5091 - 0.3}{0.0357}$$

$$= 5.86 \qquad (\mathbf{4.7.4})$$

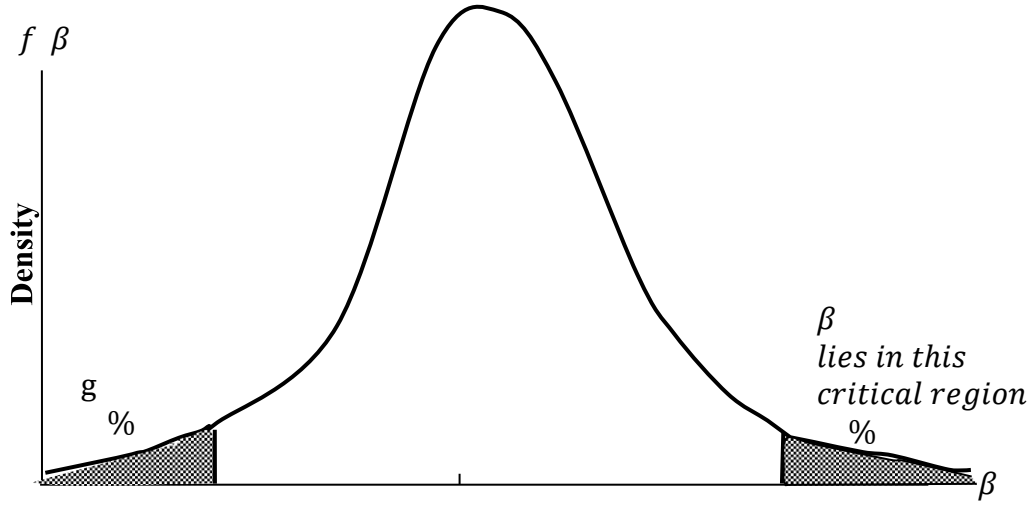


FIGURE 5.3 The 95% confidence interval for under the hypothesis that = 0.3.

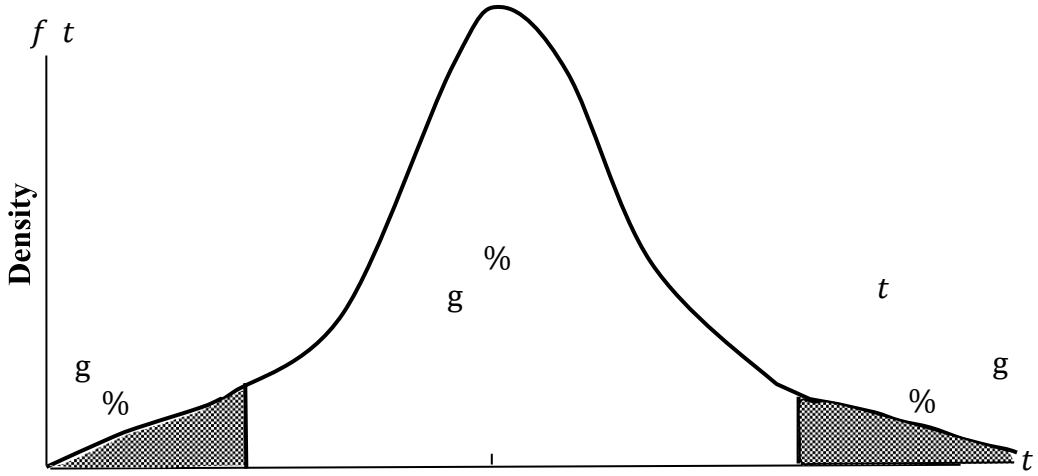


FIGURE 5.4 The 95% confidence interval for t(8 df). $\hat{\beta}_2$

β_2

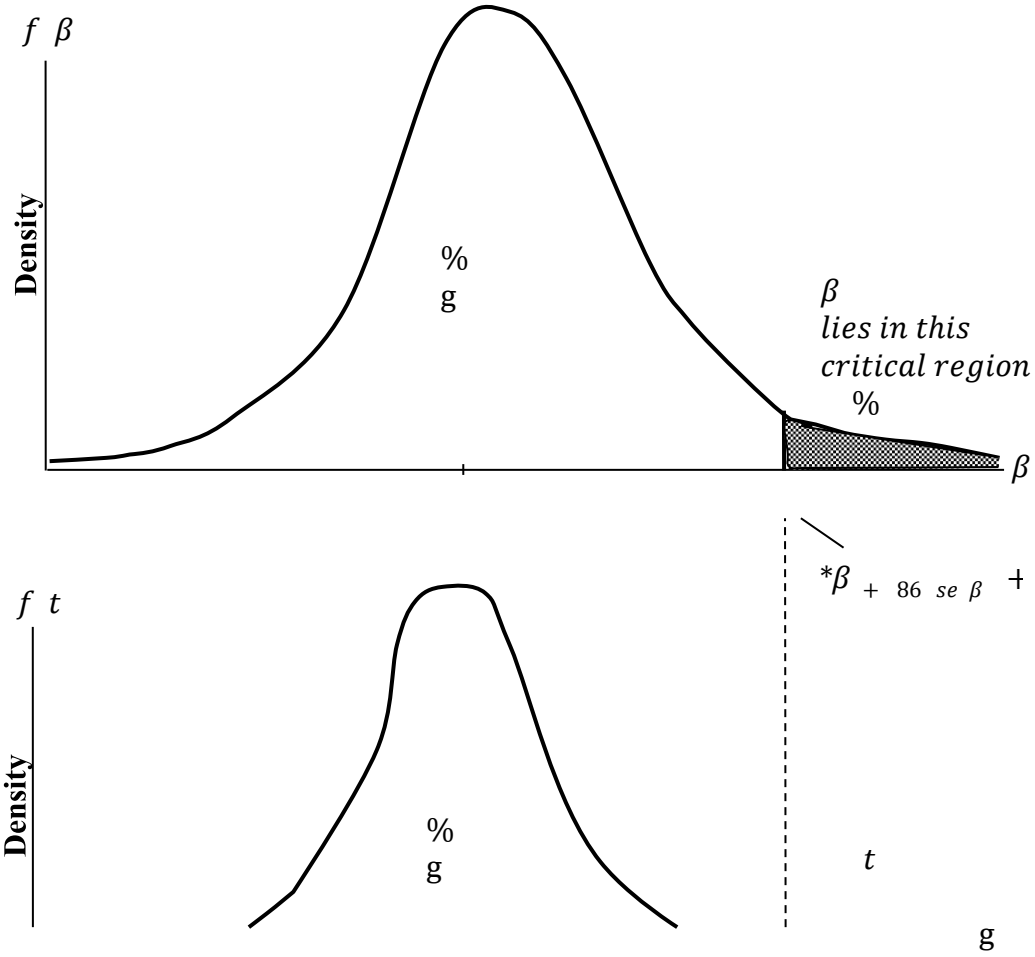
which clearly lies in the critical region of Figure 5.4. The conclusion remains the same; namely, we reject H_0.

Notice that if the estimated β_2 (= $\hat{\beta}_2$) is equal to the hypothesized β_2 , the *t* value in (4.7.4) will be zero. However, as the estimated β_2 value departs from the hypothesized β_2 value, It I (that is, the absolute *t* value; *note: t* can be positive as well as negative) will be increasingly large. *Therefore, a "large" It value will be evidence against the null hypothesis.* Of course, we can always use the *t* table to determine whether a particular *t* value is large or small; the answer, as we know, depends on the degrees of freedom as well as on the probability of Type I error that we are willing to accept. If you take a look at the *t statistical* table you will observe

that for any given value of df the probability of obtaining an increasingly large | | value becomes progressively smaller. Thus, for 20 df the probability of obtaining a | | value of 1.725 or greater is 0.10 or 10 percent, but for the same df the probability of obtaining a | | value of 3.552 or greater is only 0.002 or 0.2 percent.

Since we use the t distribution, the preceding testing procedure is called appropriately the t test. In the language of significance tests, a statistic is said to be statistically significant if the value of the test statistic lies in the critical region. In this case the null hypothesis is rejected. By the same token, a test is said to be statistically insignificant if the value of the test statistic lies in the acceptance region. In this situation, the null hypothesis is not rejected. In our example, the t test is significant and hence we reject the null hypothesis. Before concluding our discussion of hypothesis testing, note that the testing procedure just outlined is known as a two-sided, or two-tail, test of-significance procedure in that we consider the two extreme tails of the relevant probability distribution, the rejection regions, and reject the null hypothesis if it lies in either tail. But this happens because our H_1 was a two-sided composite hypothesis; $\beta_2 \neq 0.3$ means β_2 is either greater than or less than 0.3. But suppose prior experience suggests to us that the MPC is expected to be greater than 0.3. In this case we have: $H_0: \beta_2 \leq 0.3$ and $H_1: \beta_2 > 0.3$. Although H_1 is still a composite hypothesis, it is now one-sided. To test this hypothesis, we use the one-tail test (the right tail), as shown in Figure 5.5. (See also the discussion in Section 5.6.).

The test procedure is the same as before except that the upper confidence limit or critical value now corresponds to $t_\alpha = t_{.05}$, that is, the 5 percent level. As Figure 5.5 shows, we need not consider the lower tail of the *t* distribution in this case. Whether one uses a two- or one-tail test of significance will depend upon how the alternative hypothesis is formulated, which, in turn, may depend upon some a priori considerations or prior empirical experience.



Self-Assessment Exercises 1

What is the concept on the test of significance?

5.6. Testing the significance of σ^2 :The Chi Square (X^2)Test

In statistics, tests of significance are used to obtain a judgment on whether to reject or support statements based on sample data. Statistics is a subfield of mathematics that deals with the collection and calculation of numerical data. This subject is well recognized for statistical survey research. The phrase "significance" appears frequently and is significant during a statistical procedure.

In statistics, it is critical to understand whether or not the outcome of an experiment is statistically significant. There are certain preset tests that could be used to determine the relevance. These tests are known as significance tests or simply significance tests.

There is a degree of error in this statistical testing. In advance definition of the probability of sampling error is necessary for particular studies. The sampling error does exist in any test that does not take the complete population into account. In statistical study, testing for significance is crucial.

The threshold at which a given event's statistical significance can be recognized is known as the significance level. P-value is another name for this. Since larger samples are seen to be less susceptible to chance, sample size is a key factor in determining the statistical significance. For significance testing, one should only use representative and random samples.

The likelihood that a relationship exists is, in essence, what the importance is. Tests of significance provide information on the likelihood of whether and to what extent a relationship is the result of random chance. This reveals the error that would be committed by us if the discovered relationship were taken for granted.

Technically stated, the statistical significance relates to the likelihood that a finding from a statistical test or piece of study will come about by chance. Finding the truth is essentially the major goal of statistical study. The researcher must take a number of actions to ensure the sample quality, accuracy, and good measures in this process. The researcher must decide if the experiment's results are the result of a thorough analysis or merely a lucky coincidence.

The significance is a probability value showing that the outcome of a study happened entirely by chance. Both mild and strong statistical significance are possible. It does not necessarily mean that it has any application. The relevance of an experiment may occasionally be misunderstood if a researcher does not use language in the report of their experiment with care.

The statisticians and psychologists seek for a probability of 5% or less, which indicates that 5% of the outcomes are the result of chance. Additionally, this suggests that there is a 95% chance that the outcomes will not be accidental. When the outcome of our experiment is determined to be statistically significant, it means we can be 95% certain the results are not the result of chance.

In the process of testing for statistical significance, there are the following steps:

1. Stating a Hypothesis for Research
2. Stating a Null Hypothesis
3. Selecting a Probability of Error Level

4. Selecting and Computing a Statistical Significance Test
5. Interpreting the results

To determine whether the frequency distribution of a categorical variable deviates from your expectations, the chi-square goodness of fit test is utilized. In order to determine if two category variables are connected to one another, the chi-square test of independence is utilized.

When the sample sizes are large, a chi-squared test (also known as a chi-square or 2 test) is a statistical hypothesis test used in the study of contingency tables. Simply said, the main purpose of this test is to determine if two categorical variables (two dimensions of the contingency table) have independent effects on the test statistic (values in the table). The test, specifically Pearson's chi-squared test and its variations, is valid if the test statistic is chi-squared distributed under the null hypothesis. If there is a statistically significant difference between the expected frequencies and the observed frequencies in one or more categories of a contingency table, it can be determined using Pearson's chi-squared test. A Fisher's exact test is used as an alternative for contingency tables with smaller sample sizes.

The observations are categorized into classes that are mutually exclusive in the conventional applications of this exam. The test statistic generated from the observations follows a 2 frequency distribution if the null hypothesis, that there are no differences between the classes in the population, is true. The goal of the test is to determine how likely it would be for the observed frequencies to occur under the null hypothesis.

When the observations are unrelated, test statistics with a 2 distribution are produced. Additionally, depending on observations of the pairings, there are 2 tests to examine the independence of a pair of random variables. Chi-squared tests are frequently tests for which the distribution of the test statistic approaches the 2 distribution asymptotically, i.e., the sampling distribution of the test statistic (if the null hypothesis is true) increasingly resembles a chi-squared distribution as sample sizes rise.

Therefore, let us illustrate the test-of-significance methodology, consider the following variables

$$X^2 = (n - 2)\frac{\hat{\sigma}^2}{\sigma^2} \qquad (\mathbf{4.4.1})$$

which, as noted previously, follows the x 2 distribution with n – 2 df. For the hypothetical example, $\hat{\sigma}^2$ = 42.1591 and df = 8. If we postulate that $H_0: \sigma^2 = 85$ vs. $H_1: \sigma^2 \neq 85$, Eq.
(4.4.1) provides the test statistic for H_0. Substituting the appropriate values in (4.4.1), it can be found that under $H_0, X^2 = 3.97$. If we assume α = 5%, the critical χ^2 values are 2.1797 and 17.5346. Since the computed χ^2 lies between these limits, the data

support the null hypothesis and we do not reject it. This test procedure is called the chi-square test of significance.

1. The Logic of Hypothesis Testing

As just stated, the logic of hypothesis testing in statistics involves four steps. We expand on those steps in this section:

First Step: State the hypothesis

Stating the hypothesis actually involves stating **two opposing hypotheses** about the value of a population parameter.

Example: Suppose we have are interested in the effect of prenatal exposure of alcohol on the birth weight of rats. Also, suppose that we know that the mean birth weight of the population of untreated lab rats is 18 grams.

Here are the two opposing hypotheses:

- **The Null Hypothesis (Ho).** This hypothesis states that **the treatment has no effect**. For our example, we formally state:
- The null hypothesis (Ho) is that prenatal exposure to alcohol **has no effect** on the birth weight for the population of lab rats. The birth weight will be equal to 18 grams. This is denoted

$$H_0: \mu = 18$$

- **The Alternative Hypothesis (H1).** This hypothesis states that **the treatment does have an effect**. For our example, we formally state:

The alternative hypothesis (H1) is that prenatal exposure to alcohol **has an effect** on the birth weight for the population of lab rats. The birthweight will be different than 18 grams. This is denoted

$$H_1: \mu \neq 18$$

2. **Second Step: Set the Criteria for a decision.**

The researcher will be gathering data from a sample taken from the population to evaluate the credibility of the null hypothesis.

A criterion must be set to decide whether the kind of data we get is different from what we would expect under the null hypothesis.

Specifically, we must set a criterion about wether the sample mean is different from the hypothesized population mean. The criterion will let us conclude whether (reject null hypothesis) or not (accept null hypothesis) the treatment (prenatal alcohol) has an effect (on birth weight).

3. Third Step: Collect Sample Data.
Now we gather data. We do this by obtaining a random sample from the population.

16 Obs \ 1 Vars	BirthWeight
	Numeric
Obs0	10.00
Obs1	11.00
Obs2	12.00
Obs3	13.00
Obs4	12.00
Obs5	15.00
Obs6	14.00
Obs7	13.00
Obs8	12.00
Obs9	13.00
Obs10	14.00
Obs11	15.00
Obs12	11.00
Obs13	14.00
Obs14	16.00
Obs15	13.00

Example: A random sample of rats receives daily doses of alcohol during pregnancy. At birth, we measure the weight of the sample of newborn rats. We calculate the mean birth weight.

4. Fourth Step: Evaluate the Null Hypothesis We compare the sample mean with the hypothesis about the population mean.

- If the data are consistent with the hypothesis we conclude that the hypothesis is reasonable.
- If there is a big discrepency between the data and the hypothesis we conclude that the hypothesis was wrong.

Example: We compare the observed mean birth weight with the hypothesized values of 18 grams.

- If a sample of rat pups which were exposed to prenatal alcohol has a birth weight very near 18 grams we conclude that the treatement does not have an effect. Formally we do not reject the null hypothesis.
- If our sample of rat pups has a birth weight very different from 18 grams we conclude that the treatement does have an effect. Formally we reject the null hypothesis.

5.6.1. Errors in Hypothesis Testing

The central reason we do hypothesis testing is to decide whether or not the sample data are consistent with the null hypothesis.

In the second step of the procedure we identify the kind of data that is expected if the null hypothesis is true. Specifically, we identify the mean we expect if the null hypothesis is true.

If the outcome of the experiment is consistent with the null hypothesis, we believe it is true (we "accept the null hypothesis"). And, if the outcome is inconsistent with the null hypothesis, we decide it is not true (we "reject the null hypothesis").

We can be wrong in either decision we reach. Since there are two decisions, there are two ways to be wrong.

(i). Type I Error: A type I error consists of rejecting the null hypothesis when it is actually true. This is a very serious error that we want to seldomly make. We don't want to be very likely to conclude the experiment had an effect when it didn't.

The experimental results look really different than we expect according to the null hypothesis. But it could come out the way it did just because by chance we have a wierd sample.

Example: We observe that the rat pups are really heavy and conclude that prenatal exposure to alcohol has an effect even though it doesn't really. (We conclude, erroneously, that the alcohol causes heavier pups!) There could be for another reason.
Perhaps the mother has unusual genes.
(ii). Type II Error: A type II error consists of failing to reject the null hypothesis when it is actually false. This error has less grevious implications, so we are will to err in this direction (of not concluding the experiment had an effect when it, in fact, did).

The experimental results don't look different than we expect according to the null hypothesis, but they are, perhaps because the effect isn't very big.

Example: The rat pups weigh 17.9 grams and we conclude there is no effect. But "really" (if we only knew!) alcohol does reduce weight, we just don't have a big enough effect to see it.

5.6.2. Types of Statistical Tests

The statistical significance of a certain parameter in a given set of data is computed using one-tailed and two-tailed statistical tests, respectively. These tests can either be one-sided or two-sided.

When an estimated parameter deviates in one way from a benchmark value that has been established, the one-tailed test can be employed in research. On the other hand, the two-tailed test should be used when deviations from the benchmark value in both directions are thought to be theoretically feasible.

The term "tail" is employed in the names of these tests because, like the bell curve or normal distribution, the extreme points of the distributions where observations tend to reject the null

hypothesis are relatively small and "tail off" to zero. The research hypothesis determines whether a one-tailed or two-tailed significance test should be used.

Example

1. The one-tailed test can be used to test the null hypothesis, which states that in the 10th standard, boys won't score considerably higher than girls. In this case, the direction of the difference is indirectly assumed by the null hypothesis.

2. The null hypotheses could be tested using the two-tailed test: Boys' and girls' 10 Standard test scores do not significantly differ.

5.6.3. P-Value Testing

The p-value is a crucial concept for hypothesis testing when discussing the statistical significance of a set of data. The p-value, which is used to test statistical hypotheses, is stated to be a function of observed sample findings. Prior to running the test, a threshold value must be chosen. The significance level, which is often 1% or 5%, is this value. It is indicated by.

The data is considered to be inconclusive if our null hypothesis is true and the p-value is less than or equal to the significance level. As a result, the null hypothesis should be disproved and a different hypothesis should be presumed to be true.

Keep in mind that a lesser p-value implies that the study hypothesis does not sufficiently explain the data, and a larger significance should be assumed. Such a test manages type I error rate not to be more than the significance level (), provided the p-value is determined correctly. In a wide range of disciplines, including psychology, sociology, science, economics, social science, biology, criminal justice, etc., the use of p-values in statistical hypothesis testing is highly widespread.

5.6.4. Further Explanation on Hypothesis Testing

The information about a parameter from the sample data is used in hypothesis testing to provide answers to such yes-or-no queries, though not always with a high degree of certainty.

Hypothesis Tests consist of:

a. Specification of the null hypothesis, H0:0;

b. Specification of the alternative hypothesis, H1:1;

c. Specification of the test statistic and its distribution under the null hypothesis;

d. Selection of the significance level α in order to determine the rejection region;

e. Calculation of the test statistic from the data sample;

f. Conclusions, which are based on the test statistic and the rejection region;

Let's assume that our linear regression model has the following form, and we will examine each point separately.

$Y=X\beta+\varepsilon$,

where $\beta=[\beta 0/\beta 1]$, and assume that (UR.1)-(UR.4) hold true.

1. **The Null Hypothesis**

For the univariate regression, the null hypothesis is written as H0:i, where c is the constant value in which we are interested. We may either succeed in rejecting or fail to succeed in rejecting the null hypothesis when testing it. Until sufficient facts are presented to show otherwise, the null hypothesis is assumed to be accurate.

If we are unable to rule out the null hypothesis, it does not always follow that it is accurate. A hypothesis test merely evaluates whether there is sufficient evidence to reject the null hypothesis, not whether hypothesis is correct or which is most likely to be true.

2. The Alternative Hypothesis

After stating the null hypothesis, we must compare it to the alternative hypothesis, H1.

We can describe the alternative hypothesis for the null hypothesis H0:i=c0 in one of three ways:

We "accept" the conclusion that i>c since we reject H1:i>c. Economic theory typically offers details on the variable parameter signals. For instance, we would compare H0: INCOME = 0 versus H1: INCOME > 0 because economic theory strongly implies that food expenditures will increase as income increases.

H1: $\beta i < c <$ - rejecting H0, leads us to “accept” the conclusion that $\beta i < c$.
H1: $\beta i \neq c$: $\neq$ - rejecting H0, leads us to “accept” the conclusion that βi is either greater *or* smaller than c. The null, which we either reject or fail to reject—we never accept the null—is the standard by which we discuss hypothesis testing. As a result, if we reject the null, we must accept (or deal with) the alternative.

5.6.5. The Test Statistic

The test statistic is calculated based on the null hypothesis, or with the assumption that it is accurate. The statistic's distribution is known under the null hypothesis. We determine whether or not to reject the null based on the test statistic's value. In our univariate regression model, we can compute the following t-statistic under the null hypothesis H0: i= c.
If the null hypothesis is incorrect, the t-statistic has some other distribution rather than a t-distribution with N-2 degrees of freedom.

5.6.6. The Rejection Regions

Values with a low likelihood of appearing when the null hypothesis is true make up the rejection zone. The alternative hypothesis's specification determines the rejection region. It is doubtful that the null hypothesis is true if the estimated test statistic value is inside the rejection zone (i.e., an unlikely event would occur under the null).

Selecting a threshold of significance a likelihood of the rare event, often 0.010.01, 0.050.05, or 0.10.1 determines the size of the rejection zones. We will compare the calculated t-statistic, ti, to the critical value, tc to decide whether to reject the null hypothesis or not.

Self-Assessment Exercises 2

1. What is the importance of hypothesis testing in econometrics?
2. What are the different types of statistical tests?
3. What is the rejection region of a sample?

5.7. SUMMARY

In econometrics, hypothesis testing is the process by which a population parameter assumption is put to the test. Depending on the type of data used and the goal of the

research, the analyst will choose a certain approach. Using sample data from a wider population, hypothesis testing is used to deduce the outcome of a hypothesis. How to decide whether to accept a null hypothesis or to reject it in favor of a competing hypothesis. A statistic is calculated from the results of a survey or test, and then it is examined to see if it fits within a predetermined acceptable range. If so, the null hypothesis is accepted; otherwise, it is disproven.

5.8. REFERENCES/Further Reading

Gujarat, D. N. (2007) Basic Econometrics, 4th Edition, tata Mcgraw – Hill publishing company limited, New Delhi.

Hall, S. G., & Asterion, D. (2011) Applied Econometrics, 2nd Edition, Palgrave Macmillian, New York city, USA.

5.9. Possible Answers to SAEs

These are the possible answers to the SAEs within the content.

Answers to SAEs 1

Hypothesis testing allows the researcher to determine whether the data from the sample is statistically significant. Hypothesis testing is one of the most important processes for measuring the validity and reliability of outcomes in any systematic investigation.

Answers to SAEs 2

In conclusion, there are two main types of statistical tests: parametric and non-parametric. Parametric tests make certain assumptions about the data, while non-parametric tests do not make any assumptions about the data. Both types of tests are used to make inferences about a population based on a sample

Answers to SAEs 3

The rejection region (also called the critical region) is the range of values of a sample statistic that will lead to rejection of the null hypothesis.

Module 4: Calculation of Some Econometrics Test

This module introduces you to Calculation of some econometrics test. The module consists of 4 units which include: accepting and rejecting an hypothesis, the level of significance, Regression analysis and analysis of variance and normality tests

Unit One: Accepting & Rejecting an Hypothesis
Unit Two: The Level of Significance
Unit Three: Regression Analysis and Analysis of Variance
Unit Four: Normality tests

UNIT ONE: ACCEPTING AND REJECTING AN HYPOTHESIS

Unit Structure

1.1. Introduction
1.2. Learning Outcome
1.3. The meaning of accepting or rejecting an hypothesis
1.4. The zero null hypothesis and the —2-t rule of thumb
1.5. Forming the null and Alternative hypothesis
1.6. Summary
1.7. References/Further Readings/Web Resources
1.8. Possible Answers to Self-Assessment Exercises (SAEs)

1.1. INTRODUCTION

In order to undertake hypothesis testing you need to express your research hypothesis as a null and alternative hypothesis. The null hypothesis and alternative hypothesis are statements regarding the differences or effects that occur in the population. You will use your sample to test which statement (i.e., the null hypothesis or alternative hypothesis) is most likely (although technically, you test the evidence against the null hypothesis). So, with respect to our teaching example, the null and alternative hypothesis will reflect statements about all statistics students on graduate management courses.

The null hypothesis is essentially the "devil's advocate" position. That is, it assumes that whatever you are trying to prove did not happen (*hint:* it usually states that something equals zero). For example, the two different teaching methods did not result in different exam performances (i.e., zero difference). Another example might be that there is no relationship between anxiety and athletic performance (i.e., the slope is zero). The alternative hypothesis states the opposite and is usually the hypothesis you are trying to prove (e.g., the two different teaching methods did result in different exam performances). Initially, you can state these hypotheses in more general terms (e.g., using terms like "effect", "relationship", etc.).

1.2. Learning Outcome

At the end of this unit, you should be able to:

i. uunderstand the meaning of accepting and rejecting an hypothesis
ii. Identify a null and alternative hypothesis.

1.3. The meaning of "Accepting" or "Rejecting" an Hypothesis

If a study or collection of facts supports a hypothesis as being true, you can state "I accept the (research) hypothesis." When the data contradicts your theory, you state, "I don't have enough support to accept the hypothesis." You have formally accepted the alternative hypothesis when there is insufficient evidence to support it.
However, if on the basis of a test of significance, say, the *t* test, we decide to "accept" the null hypothesis, all we are saying is that on the basis of the sample evidence we

have no reason to reject it; we are not saying that the null hypothesis is true beyond any doubt.

Why? To answer this, let us revert to our consumption-income example and assume that $H_0: \beta_2$ (MPC) = 0.50. Now the estimated value of the MPC is $\hat{\beta}_2$= 0.5091 with a $se\ (\hat{\beta}_2) = 0.0357$. Then on the basis of the *t* test we find that $t = (0.5091 - 0.50)/0.0357 = 0.25$, which is insignificant, say, at α = 5%. Therefore, we say "accept" H_0. But now let us assume $H_0: \beta_2 = 0.48$. Applying the *t* test, we obtain $t = (0.5091 - 0.48)/0.0357 = 0.82$, which too is statistically insignificant. So now we say "accept" this H_0. Which of these two null hypotheses is the "truth"? We do not know. Therefore, in "accepting" a null hypothesis we should always be aware that another null hypothesis may be equally compatible with the data. It is therefore preferable to say that we *may* accept the null hypothesis rather than we (do) accept it. Better still, just as a court pronounces a verdict as "not guilty" rather than "innocent," so the conclusion of a statistical test is "do not reject" rather than "accept."

Self-Assessment Exercises 1

What is accepting and rejecting a hypothesis?

1.4. The "Zero" Null Hypothesis and the "2-t" Rule of Thumb

The "2-t" Rule of Thumb and the "Zero" Null Hypothesis • H0: 2 = 0, which states that the slope coefficient is zero, is a null hypothesis that is frequently tested in empirical research. The goal of this "zero" null hypothesis is to determine whether or not Y is related to X, the explanatory variable, in any way.

A null hypothesis that is commonly tested in empirical work is $H_0: \beta_2 = 0$, that is, the slope coefficient is zero. This "zero" null hypothesis is a kind of straw man, the objective being to find out whether Y is related at all to *X*, the explanatory variable. If there is no relationship between Y and X to begin with, then testing a hypothesis such as $\beta_2 = 0.3$ or any other value is meaningless.

This null hypothesis can be easily tested by the confidence interval or the t-test approach discussed in the preceding sections. But very often such formal testing can be shortcut by adopting the "2-t" rule of significance, which may be stated as "2-t" Rule of Thumb. If the number of degrees of freedom is 20 or more and if α, the level

of significance, is set at 0.05, then the null hypothesis $\beta_2 = 0$ can be rejected if the *t* value $\left[= \hat{\beta}_2 / se\left(\hat{\beta}_2\right)\right]$ computed from (4.3.2) exceeds 2 in absolute value.

The rationale for this rule is not too difficult to grasp. From (4.7.1) we know that we will reject $H_0: \beta_2 = 0$ if

$$t = \hat{\beta}_2 / se\left(\hat{\beta}_2\right) > t_{\alpha/2} \qquad \text{when}$$

$\hat{\beta}_2 > 0$ or

$$t = \hat{\beta}_2 / se\left(\hat{\beta}_2\right) < -t_{\alpha/2} \quad \text{when } \hat{\beta}_2 < 0$$

or when

$$|t| = \left|\frac{\hat{\beta}_2}{se\left(\hat{\beta}_2\right)}\right| > t_{\alpha\ 2} \qquad (\mathbf{4.8.1})_{/}$$

for the appropriate degrees of freedom.

Now if we examine the *t Statistical* table, we see that for df of about 20 or more a computed *t* value in excess of 2 (in absolute terms), say, 2.1, is statistically significant at the 5 percent level, implying rejection of the null hypothesis. Therefore, if we find that for 20 or more df the computed *t* value is, say, 2.5 or 3, we do not even have to refer to the *t* table to assess the significance of the estimated slope coefficient. Of course, one can always refer to the *t* table to obtain the precise level of significance, and one should always do so when the df are fewer than, say, 20.

In passing, note that if we are testing the one-sided hypothesis $\beta_2 = 0$ versus $\beta_2 > 0$ or $\beta_2 < 0$, then we should reject the null hypothesis if

$$|t| = \left|\frac{\hat{\beta}_2}{se\left(\hat{\beta}_2\right)}\right| > t_{\alpha} \qquad (\mathbf{4.8.2})$$

If we fix α at 0.05, then from the *t* table we observe that for 20 or more df a *t* value in excess of 1.73 is statistically significant at the 5 percent level of significance (one-tail). Hence, whenever a *t* value exceeds, say, 1.8 (in absolute terms) and the df are 20 or more, one need not consult the *t* table for the statistical significance of the observed coefficient. Of course, if we choose α at 0.01 or any other level, we will have to decide on the appropriate *t* value as the benchmark value. But by now the reader should be able to do that.

Self-Assessment Exercises 2

What is the two T rule of thumb?

1.5. Forming the Null and Alternative Hypotheses

Statistical hypothesis testing employs null and alternate hypotheses. The alternative hypothesis of a test states your research's forecast of an effect or relationship while the null hypothesis always anticipates no effect or relationship between variables. There are opposing explanations for your research question provided by the null and alternative hypotheses. "Does the independent variable affect the dependent variable?" is the research question's first part.

• "No, there is no effect in the population," is the response given by the null hypothesis (H0).

• The contrary hypothesis (Ha) provides the affirmative response, "Yes, there is an effect in the population."

The alternative and null are both population-based assertions. The reason for this is that the purpose of hypothesis testing is to draw conclusions about a population from a sample. By examining disparities between groups or correlations between variables in the sample, we can frequently determine whether there is an effect in the population. Strong hypothese must be written for your research. To determine whether the evidence supports the null or alternative hypothesis, you might employ a statistical test. The null and alternative hypotheses must be stated in a precise form for each type of statistical test. The hypotheses can, however, also be stated in a more general manner that is applicable to any test.

Given the null and the alternative hypotheses, testing them for statistical significance should no longer be a mystery. But how does one formulate these hypotheses? There are no hard-and-fast rules. Very often the phenomenon under study will suggest the nature of the null and alternative hypotheses. For example, consider the capital market line (CML) of portfolio theory, which postulates that $E_i = \beta_1 + \beta_2 \sigma_i$, where E = expected return on portfolio and a = the standard deviation of return, a measure of risk. Since return and risk are expected to be positively related–the higher the risk, the higher the return–the natural alternative hypothesis to the null hypothesis that $\beta_2 = 0$ would be $\beta_2 > 0$. That is, one would not choose to consider values of β_2 less than zero.

> But consider the case of the demand for money. As we shall show later, one of the important determinants of the demand for money is income. studies of the money demand functions have shown that the income elasticity of demand for money (the

percent change in the demand for money for a 1 percent change in income) has typically ranged between 0.7 r, 1.3. Therefore, in a new study of demand for money, if one postulates t the incomeelasticity coefficient β_2 is 1, the alternative hypothesis could he that $\beta_2 \neq 1$, a two-sided alternative hypothesis.

Thus, theoretical expectations or prior empirical work or both can be relied upon to formulate hypotheses. But no matter how the hypotheses are formed, *it is extremely important that the researcher establish these hypotheses before carrying out the empirical investigation.* Otherwise, he or she will be guilty of circular reasoning or selffulfilling prophesies. That is, if one were to formulate hypotheses after examining the empirical results, there may be the temptation to form hypotheses that justify one's results. Such a practice should be avoided at all costs, at least for the sake of scientific objectivity. Keep in mind the Stigler quotation given at the beginning of this chapter!

Self-Assessment Exercises 2

How to formulate null and alternate hypothesis?

3.5. SUMMARY

We create a hypothesis known as the null hypothesis in the first stage of the hypothesis-testing procedure. This has a few unique qualities. It offers the foundation for generating what is known as a p-value and is a precise statement regarding population parameters. A common abbreviation for the null hypothesis is H. Your current opinion is represented by the null hypothesis. You do not reject the null hypothesis (Ho) if the data support it. However, if the data gives sufficient evidence to the contrary, you must reject Ho. If the null hypothesis is rejected or not, that is the outcome of the hypothesis test. You could be asking yourself, "Why not just accept H instead of 'not rejecting' it?" This is thus because the goal of statistical hypothesis testing is to disprove H rather than to prove it.

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3.7. Possible Answers to SAEs

These are the possible answers to the SAEs within the content.

Answers to SAEs 1

If our test statistic is both positive and over the cutoff point, we have enough data to reject the null hypothesis and accept the alternative one. The null hypothesis must be accepted if it is positive and less than or equal to the critical value.

Answers to SAEs 2

A good general rule of thumb is to reject the null hypothesis if the t statistic's absolute value is greater than 2 and accept it otherwise. Using 2 as a rough approximation to the t value of 1.960, this would apply when n is more than or equal to roughly 40. Thus, it is simple to quickly determine which t statistics in a column are significant.

Answers to SAEs 3

1. The assertion that must be taken into account becomes the alternative hypothesis. These assertions may be wholly untrue or an attack on reality as it currently stands.

2. The null hypothesis is the opposite of the alternate hypothesis.

UNIT TWO: The Level of Significance

Unit Structure

2.1. Introduction
2.2. Learning Outcome
2.3. Analysis of Level of significance
2.4. The exact level of significance: the *P* value
2.5. Significance Analysis
2.6. The Choice between confidence-interval and test-of-significance Approaches to hypothesis testing
2.7. Summary
2.8. References/Further Readings/Web Resources
2.8. Possible Answers to Self-Assessment Exercises (SAEs)

2.1. INTRODUCTION

The significance level, also denoted as alpha or α, is the probability of rejecting the null hypothesis when it is true. For example, a significance level of 0.05 indicates a 5% risk of concluding that a difference exists when there is no actual difference. However, a type I error occurs when the researcher rejects a null hypothesis when it is true. The probability of committing a Type I error is called the significance level, and is often denoted by α.

2.2. Learning Outcome

At the end of this unit, you should be able to:

i. understand the meaning of Level of significance

ii. Understand the Choice between confidence-interval and test-of-significance Approaches to hypothesis testing

2.3. LEVEL OF SIGNIFICANCE

It should be clear from the discussion so far that whether we reject or do not reject the null hypothesis depends critically on a, the level of significance or the *probability of committing* a Type I error–the probability of rejecting the true hypothesis. But even then, why is a commonly fixed at the 1, 5, or at the most 10 percent levels? As a matter of fact, there is nothing sacrosanct about these values; any other values will do just as well. In an introductory book like this it is not possible to discuss in depth why the chooses the 1, 5, or 10 percent levels of significance, for that will take us into the field of statistical decision making, a discipline unto itself. A brief summary, for a given sample size, if we try to reduce a *Type I error*, a *Type II error* increases, and vice versa. That is, given the sample size, if we try to reduce the probability of rejecting the true hypothesis, we at the same time increase the probability of accepting the false hypothesis. So there is a tradeoff involved between these two types of errors, given the sample size. Now the only way we can decide about the tradeoff is to find out the relative costs of the two types of errors. Then,

If the error of rejecting the null hypothesis which is in fact true (Error Type I) is costly relative to the error of not rejecting the null hypothesis which is in fact false (Error Type II), it will be rational to set the probability of the first kind of error low. If, on the other hand, the cost of making Error Type I is low

relative to the cost of making Error Type II, it will pay to make the probability of the first kind of error high (thus making the probability of the second type of error low).[13]

Of course, the rub is that we rarely know the costs of making the two types of errors. Thus, applied econometricians generally follow the practice of setting the value of α at a 1 or a 5 or at most a 10 percent level and choose a test statistic that would make the probability of committing a Type II error as small as possible. Since one minus the probability of committing a Type II error is known as the **power of the test,** this procedure amounts to maximizing the power of the test.

But this entire problem with choosing the appropriate value of *a* can be avoided if we use what is known as the ***p value*** of the test statistic, which is discussed next.

Self-Assessment Exercises 1

What is meant by 0.05 level of significance?

2.4. The Exact Level of Significance: The *p* Value

As just noted, the Achilles heel of the classical approach to hypothesis testing is its arbitrariness in selecting α. Once a test statistic (e.g., the *t* statistic) is obtained in a given example, why not simply go to the appropriate statistical table and find out the actual probability of obtaining a value of the test statistic as much as or greater than that obtained in the example? This probability is called the *p* value (i.e., probability value), also known as the observed or exact level of significance or the exact probability of committing a Type I error. More technically, the p value is defined as the lowest significance level at which a null hypothesis can be rejected.

To illustrate, let us return to our consumption–income example. Given the null hypothesis that the true MPC is 0.3, we obtained a *t* value of 4.86 in (4.7.4). What is the p value of obtaining a *t* value of as much as or greater than 5.86? Looking up the *t* table**,** we observe that for 8 df the probability of obtaining such a *t* value must be much smaller than 0.001 (one-tail) or 0.002 (two-tail). By using the computer, it can be shown that the probability of obtaining a *t* value of 5.86 or greater (for 8 df) about 0.000189." This is the p value of the observed *t* statistic. This observed, or exact, level of significance of the *t* statistic is much smaller than the conventionally, and arbitrarily, fixed level of significance, such as 1, 5, or 10 percent. As a matter of fact,

if we were to use the p value just computed, and reject the null hypothesis that the true MPC is 0.3, the probability of our committing a Type I error is only about 0.02 percent, that is, only about 2 in 10,000.

As we noted earlier, if the data do not support the null hypothesis, $|t|$obtained under the null hypothesis will be "large" and therefore the p value of obtaining such a | |value will be "small." In other words, for a given sample size, as | | increases, the p value decreases, and one can therefore reject the null hypothesis with increasing confidence.

What is the relationship of the p value to the level of significance a? If we make the habit of fixing a equal to the p value of a test statistic (e.g., the *t* statistic), then there is no conflict between the two values. To put it differently, it is better to give up fixing fi arbitrarily at some level and simply choose the *p* value of the test statistic. It is preferable to leave it to the reader to decide whether to reject the null hypothesis at the given p value. If in an application the p value of a test statistic happens to be, say, 0.145, or r 14.5 percent, and if the reader wants to reject the null hypothesis at this (exact) level of significance, so be it. Nothing is wrong with taking a chance of being wrong 14.5 percent of the time if you reject the true null hypothesis. Similarly, as in our consumption-income example, there is nothing wrong if the researcher wants to choose a p value of about 0.02 percent and not take a chance of being wrong more than 2 out of 10,000 times. After all, some investigators may be risk-lovers and some risk-averters.

Self-Assessment Exercises 1

What is the exact significance level?

2.5. SIGNIFICANCE ANALYSIS

Let us revert to our consumption-income example and now hypothesize t the true MPC is 0.61 (H_0:β_2 = 0.61). On the basis of our sample result $\hat{\beta}_2$ 0.5091, we obtained the interval (0.4268, 0.5914) with 95 percent confidence. Since this interval does not include 0.61, we can, with 95 percent confidence, say that our estimate is statistically significant, that is, significantly different from 0.61.

But what is the practical or substantive significance of our finding? That is, what difference does it make if we take the MPC to be 0.61 rather than 0.5091? Is the 0.1009 difference between the two MPCs that important practically?

The answer to this question depends on what we really do with these estimates. For example, from macroeconomics we know that the income multiplier is 1/(1 – MPC). Thus, if MPC is 0.5091, the multiplier is 2.04, but it is 2.56 if MPC is equal to 0.61. That is, if the government were to increase its expenditure by N1 to lift the economy out of a recession, income will eventually increase by N2.04 if the MPC is 0.5091 but by N2.56 if the MPC is 0.61. And that difference could very well be crucial to resuscitating the economy.

The point of all this discussion is that one should not confuse statistic significance with practical, or economic, significance. As Goldberger notes:

When a null, say, $\beta_j = 1$, is specified, the likely intent is that β_j is close to 1, so close that for all practical purposes it may be treated as if it were 1. But whether 1.1 is "practically the same as" 1.0 is a matter of economics, not of statistics. One cannot resolve the matter by relying on a hypothesis test, because the test statistic [t =]$(b_j - 1)/\hat{\sigma}_{bj}$ measures the estimated coefficient in standard error units, which are not meaningful units in which to measure the economic parameter $\beta_j - 1$. It may be a good idea to reserve the term

"significance" for the statistical concept, adopting "substantial" for the economic concept. The point made by Goldberger is important. As sample size becomes very large, issues of statistical significance become much less important but issues of economic significance become critical. Indeed, since with very large samples almost any null hypothesis will be rejected, there may be studies which the magnitude of the point estimates may be the only issue.

Self-Assessment Exercises 1

What is significance analysis?

2.6. The Choice between Confidence-Interval and Test-of-Significance Approaches to Hypothesis Testing

We may get a range of potential values as well as an estimate of the precision for our parameter value from the confidence intervals. How confident we are in inferring

information about the population parameter from our sample depends on the results of hypothesis tests. Both confidence intervals and hypothesis testing are inferential procedures that employ samples to either estimate a population parameter or assess the veracity and strength of a claim. If the value associated with the null hypothesis is discovered within our confidence interval, this would indicate that our confidence interval is poor and that we have a high p-value. Our null hypothesis will typically be the value 0 (point of no difference), and if we see 0 in our confidence interval, it means that there is a good likelihood that we will really find NO DIFFERENCE, which is typically the reverse of what we desire.

In most applied economic analyses, the null hypothesis is set up as a straw man and the objective of the empirical work is to knock it down, that is, reject the null hypothesis. Thus, in our consumption–income example, the null hypothesis that the MPC $\beta_2 = 0$ is patently absurd, but we often use it to dramatize the empirical results. Apparently editors of reputed journals do not find it exciting to publish an empirical piece that does not reject the null hypothesis. Somehow the finding that the MPC is statistically different from zero is more newsworthy than the finding that it is equal to, say, 0.7.
Thus, J. Bradford De Long and Kevin Lang argue that it is better for economists.

Self-Assessment Exercises 1

Why are confidence intervals recommended rather than significance tests?

2.7. SUMMARY

When the p-value is less than the specified significance level, a hypothesis test yields a statistically significant result (or one with statistical significance). The significance level or alpha tells a researcher how extreme results must be in order to reject the null hypothesis, whereas the p-value is the likelihood of getting a test statistic or sample result as extreme as or more extreme than the one seen in the study.

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2.9. Possible Answers to SAEs

These are the possible answers to the SAEs within the content.

Answers to SAEs 1

Your result would be considered statistically significant if your p-value was less than or equal to 0.05 (the significance level). Accordingly, the data is sufficient to reject the null hypothesis and support the alternative hypothesis.

Answers to SAEs 2

The likelihood of the observed outcome or a more extreme consequence is precisely computed. An association between the row and column variables is often assumed to exist when the significance threshold is less than 0.05.

Answers to SAEs 3

The term "significance testing" refers to the application of statistical techniques to ascertain if a sample taken from a population is genuinely representative of the community or if it was simply chosen at random. The set alpha level, which is typically set at 0.05, determines statistical significance most of the time.

UNIT 3 REGRESSION ANALYSIS AND ANALYSIS OF VARIANCE

Unit Structure

3.1. Introduction
3.2. Learning Outcome
3.3. Regression Analysis
3.4. Application of Regression Analysis The problem of prediction
3.4.1. Mean Prediction
3.4.2. Reporting the results of regression analysis
3.4.3. Individual Prediction
3.4.4. Evaluating the results of regression analysis
3.4.5. Evaluating the results of regression of regression analysis
3.5. Summary
3.6. References/Further Readings/Web Resources
3.7. Possible Answers to Self-Assessment Exercises (SAEs)

3.1. INTRODUCTION

In statistical modeling, regression analysis is a statistical process for estimating the relationships among variables. It includes many techniques for modeling and analyzing several variables, when the focus is on the relationship between a dependent variable and one or more independent variables (or 'predictors'). More specifically, regression analysis helps one understand how the typical value of the dependent variable (or 'criterion variable') changes when any one of the independent variables is varied, while the other independent variables are held fixed. Most commonly, regression analysis estimates the conditional expectation of the dependent variable given the independent variables – that is, the average value of the dependent variable when the independent variables are fixed. Less commonly, the focus is on a quantile, or other location parameter of the conditional distribution of the dependent variable given the independent variables. In all cases, the estimation target is a function of the independent variables called the regression function. In regression analysis, it is also of interest to characterize the variation of the dependent variable around the regression function which can be described by a probability distribution. Regression analysis is widely used for prediction and forecasting, where its use has substantial overlap with the field of machine learning. Regression analysis is also used to understand which among the independent variables are related to the dependent variable, and to explore the forms of these relationships. In restricted circumstances, regression analysis can be used to infer causal relationships between the independent and dependent variables. However this can lead to illusions or false relationships, so caution is advisable; for example, correlation does not imply causation.

Many techniques for carrying out regression analysis have been developed. Familiar methods such as linear regression and ordinary least squares regression are

parametric, in that the regression function is defined in terms of a finite number of unknown parameters that are estimated from the data. Nonparametric regression refers to techniques that allow the regression function to lie in a specified set of functions, which may be infinite dimensional.

The performance of regression analysis methods in practice depends on the form of the data generating process, and how it relates to the regression approach being used. Since the true form of the data-generating process is generally not known, regression analysis often depends to some extent on making assumptions about this process. These assumptions are sometimes testable if a sufficient quantity of data is available. Regression models for prediction are often useful even when the assumptions are moderately violated, although they may not perform optimally. However, in many applications, especially with small effects or questions of causality based on observational data, regression methods can give misleading results.

variance is the expectation of the squared deviation of a random variable from its mean, and it informally measures how far a set of (random) numbers are spread out from their mean. The variance has a central role in statistics. It is used in descriptive statistics, statistical inference, hypothesis testing, goodness of fit, Monte Carlo sampling, amongst many others. This makes it a central quantity in numerous fields such as physics, biology, chemistry, economics, and finance. The variance is the square of the standard deviation, the second central moment of distribution, and the covariance of the random variable with itself.

3.2. Learning Outcome

At the end of this unit, you should be able to:

i. understand the meaning of regression analysis and variance
ii. Know how to calculate the regression analysis and analysis of variance

3.3. Regression Analysis

In this unit we will study regression analysis from the point of view of the analysis of variance and introduce the reader to an illuminating and complementary way of looking at the statistical inference problem.

Regression analysis is a group of statistical techniques used in statistical modeling to determine the relationships between a dependent variable (often referred to as the "outcome" or "response" variable, or a "label" in machine learning jargon), and one or more independent variables (often referred to as "predictors," "covariates," "explanatory variables," or "features"). In the most prevalent type of regression analysis, called linear regression, the line (or a more complicated linear combination) that most closely matches the data in terms of a certain mathematical criterion is found.

Ordinary least squares, for instance, determines the specific line (or hyperplane) that minimizes the sum of squared differences between the genuine data and that line (or hyperplane). This enables the researcher to estimate the conditional expectation (or population average value) of the dependent variable when the independent variables take on a specified set of values for precise mathematical reasons (see linear regression). Regression techniques that are less prevalent, such as quantile regression and necessary condition analysis, utilize somewhat different methods to estimate alternative location parameters or to estimate the conditional expectation for a larger group of non-linear models (such as nonparametric regression).

Two conceptually separate uses of regression analysis predominate. First, there is a significant overlap between the usage of regression analysis and machine learning in the areas of prediction and forecasting. Second, regression analysis can be used to infer causal links between the independent and dependent variables in specific circumstances. Regressions by themselves, it should be noted, only illuminate connections between a dependent variable and a group of independent variables in a given dataset. Researchers must carefully explain why existing correlations have predictive value in a new context or why a link between two variables has a causal meaning before using regressions for prediction or to infer causal relationships, respectively. When attempting to estimate causal linkages using observational data, the latter is particularly crucial.

However, in previous discussion, we developed the following identity:

$$\sum y_i^2 = \sum \hat{y}_i^2 + \sum \hat{u}_i^2$$
$$= \hat{\beta}_2^2 \sum x_i^2 + \sum \hat{u}_i^2$$

that is, TSS = ESS + RSS, which decomposed the total sum of squares (TSS) into two components: explained sum of squares (ESS) and residual sum of squares (RSS). A study of these components of TSS is known as the analysis of variance (ANOVA) from the regression viewpoint.

Associated with any sum of squares is its df, the number of independent observations on which it is based. TSS has $n - 1$ df because we lose 1 df in computing the sample mean ¯. RSS has n – 2 df. (Why?) (Note: This is true only for the two-variable regression model with the intercept β_1 present.) ESS has 1 df (again true of the two-

variable case only), which follows from the fact that ESS $= \hat{\beta}_2^2 \sum x_i^2$ is a function of $\hat{\beta}_2$ only, since $\sum x_i^2$ is known.
However, we can also state that;

$$F = \frac{\text{MSS of ESS}}{\text{MSS of RSS}} = \frac{\hat{\beta}_2^2 \sum x_i^2}{\sum \hat{u}_i^2/(n-2)} = \frac{\hat{\beta}_2^2 \sum x_i^2}{\hat{\sigma}^2} \qquad (\mathbf{4.9.1})$$

If we assume that the disturbances u, are normally distributed, which we do under the CNLRM, and if the null hypothesis (H_0) is that β_2= 0, then it can be shown that the F variable of (4.9.1) follows the F distribution with 1 df in the numerator and (n – 2) df in the denominator.
What use can be made of the preceding F ratio? It can be shown that

$$E\left(\hat{\beta}_2^2 \sum x_i^2\right) = \sigma^2 + \beta_2^2 \sum x_i^2 \qquad (\mathbf{4.9.2})$$

and

$$E\frac{\sum \hat{u}_i^2}{n-2} = E(\hat{\sigma}^2) = \sigma^2 \qquad (\mathbf{4.9.3})$$

(Note that β_2 and σ^2appearing on the right sides of these equations are the true parameters.) Therefore, if β_2 is in fact zero, Eqs. (4.9.2) and (4.9.3) both provide us with identical estimates of true σ^2. In this situation, the explanatory variable X has no linear influence on Y whatsoever and the entire variation in Y is explained by the random disturbances u_i. If, on the other hand, β_2 is not zero, (4.9.2) and (4.9.3) will be different and part of the variation in Y will be ascribable to X. Therefore, the F ratio of (4.9.1) provides a test of the null hypothesis $H_0: \beta_2 = 0$. Since all the quantities entering into this equation can be obtained from the available sample, this F ratio provides test statistic to test the null hypothesis that true β_2 is zero. All that needs to be done is to compute the F ratio and compare it with the critical F value obtained from the F tables at the chosen level of significance, or obtain the p value of the computed F statistic.

To illustrate, let us continue with our consumption-income example.
Looking at the ANOVA value and the F statistics; therefore, the computed F value is seen to be 202.87. The p value of this F statistic corresponding to 1 and 8 degree of freedom (df) cannot be obtained from the F table, but by using electronic statistical tables it can be shown that the p value 0.0000001, an extremely small

probability indeed. If you decide to choose the level-of-significance approach to hypothesis testing and fix α at 0.01, or a 1 percent level, you can see that the computed F of 202.87 is obviously significant at this level. Therefore, if we reject the null hypothesis that $\beta_2 = 0$, the probability of committing a Type I error is very small. For all practical purposes, our sample could not have come from a population with zero $\beta_2$value and we can conclude with great confidence that *X*, income, does affect Y, consumption expenditure.

Thus, the *t* and the *F* tests provide us with two alternative but complementary ways of testing the null hypothesis that $\beta_2 = 0$. If this is the case, why not just rely on the *t* test and not worry about the *F* test and the accompanying analysis of variance? For the twovariable model there really is no need to resort to the *F* test. But when we consider the topic of multiple regressions we will see that the *F* test has several interesting applications that make it a very useful and powerful method of testing statistical hypotheses.

Self-Assessment Exercises 1

What is a regression analysis used for?

3.4. APPLICATION OF REGRESSION ANALYSIS: THE PROBLEM OF PREDICTION

For example let say we have a sample regression result:

$$\hat{Y}_i = 24.4545 + 0.5091X_i$$

where $\hat{Y}_i$ is the estimator of true E(Y,) corresponding to given *X*. What use can be made of this historical regression? One use is to "predict" or "forecast" the future consumption expenditure Y corresponding to some given level of income *X*. Now there are two kinds of predictions: (1) prediction of the conditional mean value of Y corresponding to a chosen *X*, say, X_0 , that is the point on the population regression line itself and (2) prediction of an individual Y value corresponding to X_0. We shall call these two predictions the mean prediction and individual prediction.

3.4.1. Mean Prediction

To fix the ideas, assume that X_0 = 100 and we want to predict E($Y|X_0$ = 100). Now it can be shown that the historical regression (3.6.2) provides the point estimate of this mean prediction as follows:

$$\begin{aligned}\hat{Y}_0 &= \hat{\beta}_1 + \hat{\beta}_2 X_0 \\ &= \\ &24.4545 + \\ &0.5091(100) \\ &= 75.3645\end{aligned} \qquad (\mathbf{4.10.1})$$

where $\hat{Y}_0$= estimator of E($Y|X_0$). It can be proved that this point predictor is a best linear unbiased estimator (BLUE).

Since $\hat{Y}_0$ is an estimator, it is likely to be different from its true value. The difference between the two values will give some idea about the prediction or forecast error. To assess this error, we need to find out the sampling distribution of $\hat{Y}_0$. It is shown in

Appendix 5A, Section 5A.4, that $\hat{Y}_0$ in Eq. (5.10.1) is normally distributed with mean $(\beta_1 + \beta_2 X_0)$ and the variance is given by the following formula:

$$\begin{aligned}&\text{var}(\hat{Y}_0) \\ &= \sigma^2 \left[\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum x_i^2}\right]\end{aligned} \qquad (\mathbf{4.10.2})$$

By replacing the unknown σ^2 by its unbiased estimator $\hat{\sigma}^2$, we see that the variable

$$\begin{aligned}&t \\ &= \frac{\hat{Y}_0 - (\beta_1 + \beta_2 X_0)}{se(\hat{Y}_0)}\end{aligned} \qquad (4.10.3)$$

follows the t distribution with n – 2 df. The t distribution can therefore be used to derive confidence intervals for the true E($Y_0|X_0$) and test hypotheses about it in the usual manner, namely,

$$\begin{aligned}&\Pr\left[\hat{\beta}_1 + \hat{\beta}_2 X_0 - t_{\alpha/2} se(\hat{Y}_0) \leq \beta_1 + \beta_2 X\right. \\ &\qquad\qquad {}_0 \leq \hat{\beta}_1 + \hat{\beta}_2 X_0 + t_{\alpha/2} se(\hat{Y}_0)\Big] \\ &= 1 - \alpha \qquad (4.10.4)\end{aligned}$$

where se (4) is obtained from (5.10.2). For our data (see Table 3.3),

$$\begin{aligned}\text{var}(\hat{Y}_0) &= 42.159\left[\frac{1}{10} + \frac{(100 - 170)^2}{33{,}000}\right] \\ &= 10.4759\end{aligned}$$

and

$$se\left(\hat{Y}_0\right) = 3.2366$$

Therefore, the 95% confidence interval for true $\mathrm{E}(Y|X_0) = \beta_1 + \beta_2 X_0$ is given by

$75.3645 - 2.306(3.2366) \leq \mathrm{E}(Y_0|X = 100) \leq 75.3645 + 2.306(3.2366)$ that is,

$$67.9010 \leq \mathrm{E}(Y|X = 100) \leq 82.8381 \qquad (\mathbf{4.10.5})$$

FIGURE 5.6 Confidence interval (bands) for mean Y and individual Y values.

Thus, given X_0 = 100, in repeated sampling, 95 out of 100 intervals like (4.10.5) will include the true mean value; the single best estimate of the true mean value is of course the point estimate 75.3645.

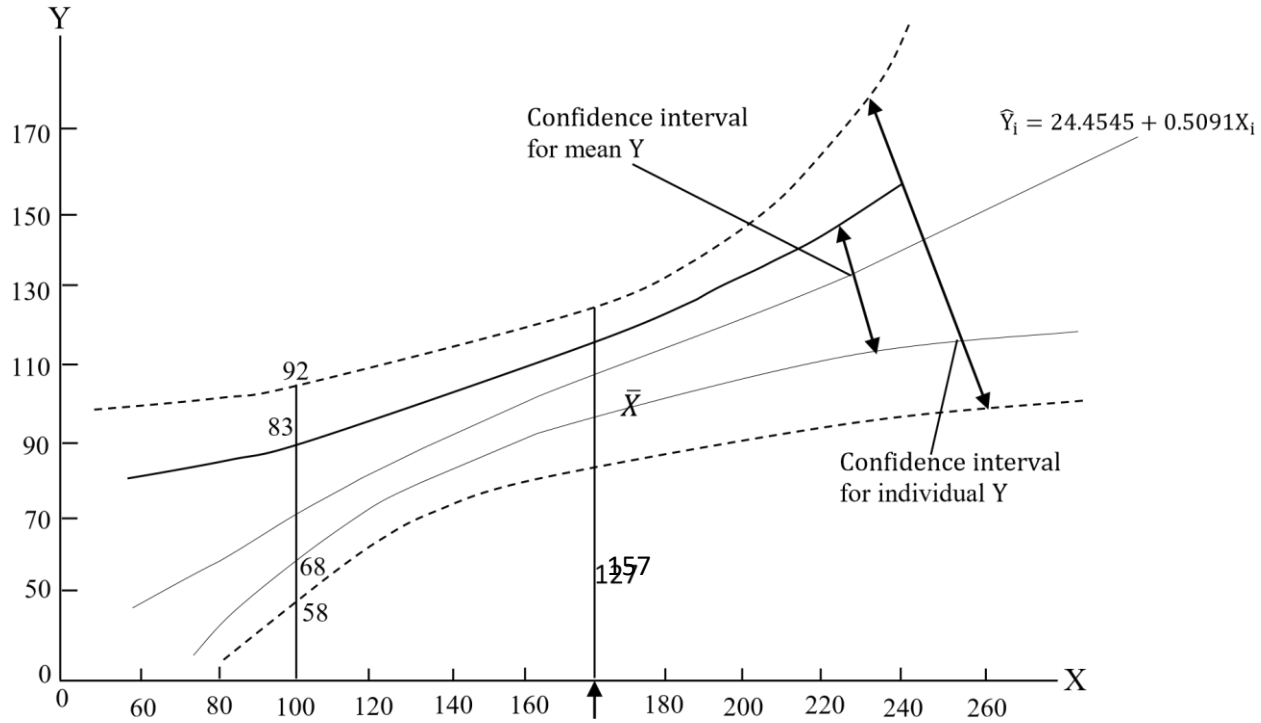
If we obtain 95% confidence intervals like (4.10.5) for each of the *X* values, we obtain what is known as the confidence interval, or confidence band, for the population regression function, which is shown in Figure 5.6.

3.4.2. Individual Prediction

If our interest lies in predicting an individual Y value, Y_0, corresponding to a given X value, say, Xo,, a best linear unbiased estimator of Y_0 is also given by (4.10.1), but its variance is as is follows:

$$\begin{aligned}\mathrm{var}\left(Y_0 - \hat{Y}_0\right) &= E\left[Y_0 - \hat{Y}_0\right]^2 \\ &= \sigma^2\left[1 + \frac{1}{n} + \frac{(X_0 - \bar{X}_0)^2}{\sum_i x^2}\right] \qquad (\mathbf{4.10.6})\end{aligned}$$

It can be shown further that Y_0 also follows the normal distribution with mean and variance given by (4.10.1) and (4.10.6), respectively. Substituting $\hat{\sigma}^2$ for the unknown σ^2, it follows that



$$t = \frac{Y_0 - \hat{Y}_0}{se\ (Y_0 - \hat{Y}_0)}$$

also follows the *t* distribution. Therefore, the *t* distribution can be used to draw inferences about the true Y_0. Continuing with our consumption-income example, we see that the point prediction of Y_0 is 75.3645, the same as that of $\hat{Y}_0$, and its variance is 52.6349 (the reader should verify this calculation). Therefore, the 95% confidence interval for Y_0 corresponding to X_0 = 100 is seen to be

$$(58.6345 \leq Y_0 | X_0 = 100 \leq 92.0945) \qquad (\mathbf{4.10.7})$$

Comparing this interval with (4.10.5), we see that the confidence interval for individual Y_0 is wider than that for the mean value of Y_0. (Why?) Computing confidence intervals like (4.10.7) conditional upon the *X* values, we obtain the 95% confidence band for the individual Y values corresponding to these *X* values. This confidence band along with the confidence band for Y_0 associated with the same X's is shown in Figure 5.6.

Notice an important feature of the confidence bands shown in Figure 5.6. The width of these bands is smallest when $X_0 = X$. (Why?) However, the width widens sharply as X_0 moves away from *X*. (Why?) This change would suggest that the predictive ability of the *historical* sample regression line falls markedly as X_0 departs progressively from *X*. Therefore, one should exercise great caution in "extrapolating" the historical regression line to predict $E(Y|X_0)$ or Y_0 associated with a given X_0 that is far removed from the sample mean X.

3.4.3 REPORTING THE RESULTS OF REGRESSION ANALYSIS

There are various ways of reporting the results of regression analysis, but in this text we shall use the following format, employing the consumption income example of Chapter 3 as an illustration:

$$\begin{aligned} \hat{Y}_i &= 24.4545 + 0.5091X_i \\ se &= (6.4138) \quad (0.0357) \\ &= 0.9621 \\ t &= (3.8128) \quad (14.2605) \\ p &= (0.002571) \quad (0 \\ & r2 \\ & (\mathbf{4.11.1}) \\ & df \\ & .000000289) \\ & = 8 \\ & F_1 \\ & _{.8} = 202.87 \end{aligned}$$

In Eq. (4.11.1) the figures in the first set of parentheses are the estimated standard errors of the regression coefficients, the figures in the second set are estimated *t* values computed

from (4.3.2) under the null hypothesis that the true population value of each regression coefficient individually is zero (e.g., 3.8128 = 24.4545 ÷ 6.4138), and the figures in the third set are the estimated p values. Thus, for 8 df the probability of obtaining a t value of 3.8128 or greater is 0.0026 and the probability of obtaining a t value of 14.2605 or larger is about 0.0000003.

By presenting the p values of the estimated t coefficients, we can see at once the exact level of significance of each estimated t value. Thus, under the null hypothesis that the true population intercepts value is zero, the exact probability (i.e., the p value) of obtaining a t value of 3.8128 or greater is only about 0.0026. Therefore, if we reject this null hypothesis, the probability of our committing a Type I error is about 26 in 10,000, a very small probability indeed. For all practical purposes we can say that the true population intercept is different from zero. Likewise, the p value of the estimated slope coefficient is zero for all practical purposes. If the true MPC were in fact zero, our chances of obtaining an MPC of 0.5091 would be practically zero. Hence we can reject the null hypothesis that the true MPC is zero.

Earlier we showed the intimate connection between the F and t statistics, namely, $F_{1.k} = t_k^2$. Under the null hypothesis that the true $\beta_2 = 0$, (4.11.1) shows that the F value is 202.87 (for 1 numerator and 8 denominator df) and the t value is about 14.24 (8 df); as expected, the former value is the square of the latter value, except for the roundoff errors.

3.4.4. EVALUATING THE RESULTS OF REGRESSION ANALYSIS

Now that we have presented the results of regression analysis of our consumption-income example in (4.11.1), we would like to question the adequacy of the fitted model. How "good" is the fitted model? We need some criteria with which to answer this question.

First, are the signs of the estimated coefficients in accordance with theoretical or prior expectations? A priori, β_2, the marginal propensity to consume (MPC) in the consumption function, should be positive. In the present example it is. Second, if theory says that the relationship should be not only positive but also statistically significant, is this the case in the present application? The MPC is not only positive but also statistically significantly different from zero; the p value of the estimated t value is extremely small.

The same comments apply about the intercept coefficient. Third, how well does the regression model explain variation in the consumption expenditure? One can use r^2 to answer this question. In the present example r^2 is about 0.96, which is a very high value considering that r^2 can be at most 1.

Thus, the model we have chosen for explaining consumption expenditure behavior seems quite good. But before we sign off, we would like to find out whether our model satisfies the assumptions of CNLRM. We will not look at the various assumptions now because the model is patently so simple. But there is one assumption that we would like to check, namely, the normality of the disturbance term, u_i. Recall that the *t* and *F* tests used

before require that the error term follow the normal distribution. Otherwise, the testing procedure will not be valid in small, or finite, samples.

Self-Assessment Exercises 1

What is the best application of a regression model?

3.5. SUMMARY

Using a statistical method, one can predict changes in a dependent variable (like sales revenue, for instance) based on changes in one or more independent variables (like population and income, for instance). Because a regression analysis equation can be used to fit a curve or line to data points in a way that minimizes the variations in the distances of the data points from the curve or line, this process is also known as curve fitting or line fitting. However, the relationships shown in a regression analysis are just associative, any cause-and-effect (causal) inference is purely arbitrary, and variance is the discrepancy between an anticipated and actual outcome, such as between a budget and actual spending.

3.6. REFERENCES/Further Reading

Gujarat, D. N. (2007) Basic Econometrics, 4th Edition, tata Mcgraw – Hill publishing company limited, New Delhi.

Hall, S. G., & Asterion, D. (2011) Applied Econometrics, 2nd Edition, Palgrave Macmillian, New York city, USA.

3.7. Possible Answers to SAEs

These are the possible answers to the SAEs within the content.

Answers to SAEs 1

Regression analysis is a group of statistical techniques used to estimate relationships between a dependent variable and one or more independent variables. It can be used to determine the strength of the link between variables and to predict how they will interact in the future.

Answers to SAEs 2

Regression analysis is frequently used in business to predict potential opportunities and dangers. For instance, demand analysis predicts how many items a buyer is likely to purchase. Demand is not the only dependent variable in business, though.

UNIT FOUR: Normality Tests

Unit Structure

4.1. Introduction
4.2. Learning Outcome
4.3. Normality test analysis
4.3.1. Histogram of Residuals
4.3.2. Normal Probability Plot
4.3.3. Jarque-Bera (JB) test of Normality
4.5. Summary
4.6. References/Further Readings/Web Resources
4.7. Possible Answers to Self-Assessment Exercises (SAEs)

4.1. INTRODUCTION

In statistics, normality tests are used to determine if a data set is well-modeled by a normal distribution and to compute how likely it is for a random variable underlying the data set to be normally distributed.
Before applying statistical methods that assume normality, it is necessary to perform a normality test on the data (with some of the above methods we check residuals for normality). We hypothesize that our data follows a normal distribution, and only reject this hypothesis if we have strong evidence to the contrary.

While it may be tempting to judge the normality of the data by simply creating a histogram of the data, this is not an objective method to test for normality – especially with sample sizes that are not very large. With small sample sizes, discerning the shape of the histogram is difficult. Furthermore, the shape of the histogram can change significantly by simply changing the interval width of the histogram bars.

4.2. Learning Outcome

At the end of this unit, you should be able to:

i. understand the meaning of histogram residuals
ii. understand the analysis of normal probability plot and Jarque-Bera test of normality.

4.3. NORMALITY TEST ANALYSIS

If sample data were collected from a regularly distributed population (within a certain tolerance), it would pass the normality test. A regularly distributed sample population is needed for a number of statistical tests, including the Student's t-test and the one-way and two-way ANOVA.

A data set's normality can be checked using a normality test, which can also be used to estimate the likelihood that a random variable underlying the data set has a normal distribution.
The tests, which are actually a type of model selection, can be understood in a variety of ways depending on how one views probability:

1. In descriptive statistics, the goodness of fit of a normal model to the data is measured; if the fit is low, the data are not properly modeled by a normal distribution in that aspect, without passing any judgment on any underlying variable.
2. The null hypothesis that data are regularly distributed is examined using frequentist statistics statistical hypothesis testing.
3. In Bayesian statistics, one does not "test normality" per se but rather computes the likelihood that the data come from a normal distribution with given parameters, (for all,), and compares that with the likelihood that the data come from other distributions under consideration. The most straightforward method is to use a Bayes factor, which indicates the relative likelihood of seeing the data given various models, or more precisely, one can take a prior distribution on potential models and parameters.

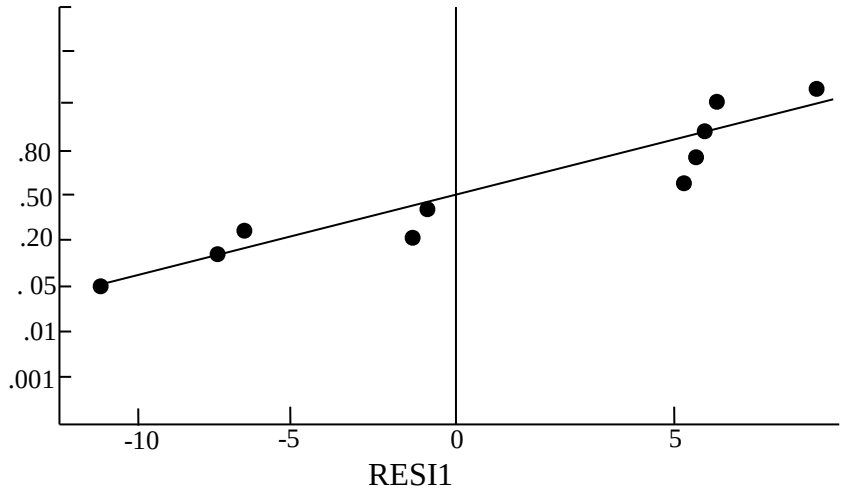
We will consider just three normality test: (1) histogram of residuals; (2) normal probability plot (NPP), a graphical device; and (3) the Jarque-Bera test.

4.3.1. Histogram of Residuals.

A histogram of residuals is a simple graphic device that is used to learn something about the shape of the PDF of a random variable. On the horizontal axis, we divide the values of the variable of interest (e.g., OLS residuals) into suitable intervals, and in each class interval we erect rectangles equal in height to the number of observation (i.e., frequency) in that class interval. If you mentally superimpose the bell-shaped normal distribution curve on the histogram, you will get some idea as to whether normal (PDF) approximation may be appropriate. It is always a good practice to plot the histogram of the residuals as a rough and ready method of testing for the normality assumption.

4.3.2. Normal Probability Plot.

A comparatively simple graphical device to study the shape of the probability density function (PDF) of a random variable is the normal probability plot (NPP) which makes use of normal probability paper, a specially designed graph paper. On the horizontal, or x, axis, we plot values of the variable of interest (say, OLS residuals, $\hat{u}_i$), and on the vertical, or y, axis, we show the expected value of this variable if it were normally distributed. Therefore, if the variable is in fact from the normal population, the NPP will be approximately a straight line. The NPP of the residuals from our consumption-income regression is shown in Figure 5.7 below, which is obtained from the MINITAB software package, version 13. As noted earlier, if the fitted line in the NPP is approximately a straight line, one can conclude that the variable of interest is normally distributed In Figure 5.7, we see that residuals from our illustrative example are op-proximately normally distributed, because a straight line seems to fit the data reasonably well. MINITAB also produces the Anderson-Darling normality test, known as the A^2 statistic. The underlying null hypothesis is that the variable under consideration is normally distributed. As Figure 5.7 shows, for our example, the computed A^2 statistic is 0.394. The p value of obtaining such a value of A^2 is 0.305, which is reasonably high. Therefore, we do not reject the



Average $= -0.0000000$ $A^2 = 0.394$
StDev $= 6.12166$ P $-$ value $= 0.305$
N $= 10$

FIGURE 5.7 Residuals from consumption-income regression

hypothesis that the residuals from our consumption-income example are normally distributed. Incidentally, Figure 5.7 shows the parameters of the (normal) distribution, the mean is approximately 0 and the standard deviation is about 6.12.

4.3.3. Jarque-Bera (JB) Test of Normality.

The JB test of normality is an *asymptotic,* or large-sample, test. It is also based on the OLS residuals. This test first computes the skewness and kurtosis measures of the OLS residuals and uses the following test statistic:

$$JB = n\left[\frac{S^2}{6} + \frac{(K-3)^2}{24}\right] \qquad (\mathbf{4.12.1})$$

where n = sample size, S = skewness coefficient, and K = kurtosis coefficient. For a normally distributed variable, $S = 0$ and $K = 3$. Therefore, the JB test of normality is a test of the joint hypothesis that S and K are 0 and 3, respectively. In that case the value of the TB statistic is expected to be 0.

Under the null hypothesis that the residuals are normally distributed, Jarque and Bera showed that *asymptotically (i.e., in large samples) the JB statistic given in (4.12.1) follows the chi-square distribution with 2 df.* If the computed *p* value of the JB statistic in an application is sufficiently low, which will happen if the value of the statistic is very different from 0, one can reject the hypothesis that the residuals are normally distributed. But if the p value is reasonably high, which will happen if the value of the statistic is close to zero, we do not reject the normality assumption.

The sample size in our consumption-income example is rather small. Hence, strictly speaking one should not use the JB statistic. If we mechanically apply the JB formula to our example, the JB statistic turns out to be 0.7769. The p value of obtaining such a value from the chi-square distribution with 2 df is about 0.68, which is quite high. In other words, we may not reject the normality assumption for our example. Of course, bear in mind the warning about the sample size.

EXAMPLE 5.1

Let us return to Example 3.2 about food expenditure in India. Using the data given in (3.7.2) and adopting the format of (5.11.1), we obtain the following expenditure equation:

$$\widehat{FoodExp}_i = 94.2087 + 0.4368\ \text{TotalExp},$$

se = (50.8563)	(0.0783)	
t = (1.8524)	(5.5770)	(5.12.2)
p = (0.0695)	(0.0000*	
r^2 = 0.3698;	df = 53	
$F_{1.53} = 31.1034$	(***p*** value = 0.0000)*	

where * denotes extremely small.

First, let us interpret this regression. As expected, there is a positive relationship between expenditure on food and total expenditure. If total expenditure went up by a rupee, on average, expenditure on food increased by about 44 paise. If total expenditure were zero, the average expenditure on food would be about 94 rupees. Of course, this mechanical Interpretation of the intercept may not make much economic sense. The r^2 value of about 0.37 means that 37 percent of the variation in food expenditure is explained by total expenditure, a proxy for income.

Suppose we want to test the null hypothesis that there is no relationship between food expenditure and total expenditure, that Is, the true slope coefficient $\beta_2 = 0$. The estimated value of β_2 is 0.4368. If the null hypothesis were true, what is the probability of obtaining a value of 0.4368? Under the null hypothesis, we observe from (5.12.2) that the *t* value is 5.5770 and the p value of obtaining such a *t* value is practically zero. In other words we can reject the null hypothesis resoundingly. But suppose the null hypothesis were that $\beta_2 = 0.5$. Now what?

Using the *t* test we obtain:

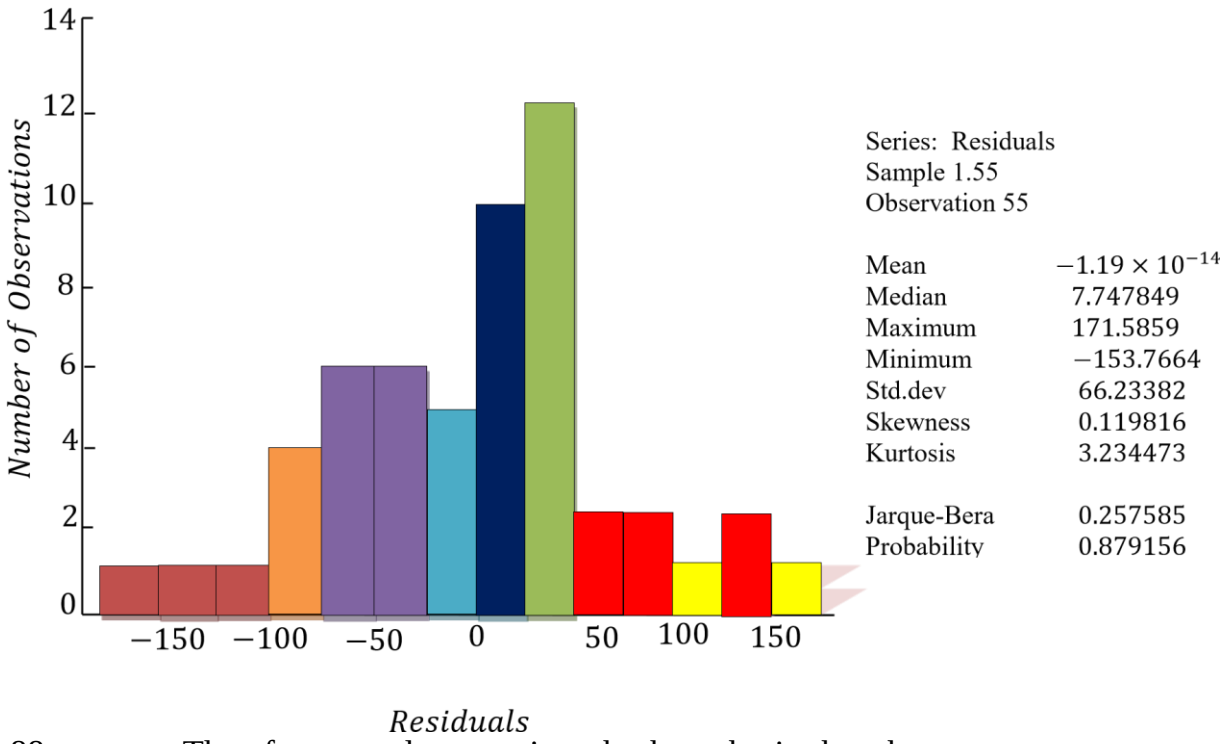
$$\frac{0.4368 - 0.5}{0.0783} = -0.8071$$

The probability of obtaining a | | of 0.8071 is greater than 20 percent. Hence we do not reject the hypothesis that the true β_2 is 0.5.

Notice that, under the null hypothesis, the true slope coefficient is zero, the *F* value is 31.1034, as shown in (5.12.2). Under the same null hypothesis, we obtained a *t* value of 5.5770. If we square this value, we obtain 31.1029, which is about the same as the *F* value, again showing the close relationship between the *t* and the *F*statistic. *(Note:* The numerator df for the *F* statistic must be 1, which is the case here.)

Using the estimated residuals from the regression, what can we say about the probability distribution of the error term? The information is given in Figure 5.8. As the figure shows, the residuals from the food expenditure regression seem to be symmetrically distributed.

Application of the Jarque-Bera test shows that the JB statistic is about 0.2576, and the probability of obtaining such a statistic under the normality assumption is about



88 percent. Therefore, we do not reject the hypothesis that the error terms are normally distributed. But keep in that the sample size of 55 observations may not be enough.

We leave it to the reader to establish confidence intervals for the two regression coefficients as well as to obtain the normal probability plot and do mean and individual predictions.

Self-Assessment Exercises 1

What is normality test used for?

4.4. SUMMARY

The outputs of processes that, under the null, produce random variables that are only asymptotically or nearly normal (with the 'asymptotically' component depending on some quantity that we cannot make huge); are typically subjected to normality tests; The null of the normal distribution should always be rejected by normality tests in the era of inexpensive memory, abundant data, and quick CPUs. As a result, normality tests should paradoxically only be applied to small samples because they are more likely to have poor power and little control on type I rate.

4.5. REFERENCES/Further Reading

Dimitrios, A & Stephen, G., (2011). Applied Econometrics, 2nd Edition, Edition 2006 and Revised Edition 2007.

4.6. Possible Answers to SAEs

These are the possible answers to the SAEs within the content.

Answers to SAEs 1

The outputs of processes that, under the null, produce random variables that are only asymptotically or nearly normal (with the 'asymptotically' component depending on some quantity that we cannot make huge); are typically subjected to normality tests; The null of the normal distribution should always be rejected by normality tests in the era of inexpensive memory, abundant data, and quick CPUs. As a result, normality tests should paradoxically only be applied to small samples because they are more likely to have poor power and little control on type I rate.