COURSE GUIDE

BHM 713
CAPITAL INVESTMENT AND FINANCIAL DECISIONS

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Published by
National Open University of Nigeria

Printed 2013

Reprinted 2016

ISBN: 978-058-771-3

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INTRODUCTION

BHM 713: Capital Investment and Financial Decisions is a semester course work of two credit hours. It will be taken by students offering the Post Graduate Diploma programme in the School of Business and Human Resources Management.

The course, Capital Investment and Financial Decisions, consists of 14 units involving conceptual issues in investment and financial decisions through viability tests. The course involves some applications of elementary Mathematics. You are expected to be familiar with the use of simple calculators.

The course guide tells you what the course BHM 713 is all about.

WHAT YOU WILL LEARN IN THIS COURSE

The course contents consist of conceptual issues in investments such as definition, types and characteristics of investments and types and features of decisions. Financial decisions predicated on viability tests shall also be discussed.

COURSE AIMS

The truth is that the future is very uncertain. Yet, decisions are future-oriented. This course, therefore, aims at exposing you to the various techniques that can be applied in financial decisions – thus leading to scientific decisions being taken.

The aims will be achieved by:

- explaining the basis for long-term decision making
- discussing the traditional technique for appraising investments which include the Accounting Rate of Return (ARR) and the Payback Period of Investments (PPI)
- discussing the discounted techniques, namely, the Net Present Value (NPV), Internal Rate of Return (IRR) and Profitability Index (PI)
- explaining the merits and demerits of the various techniques
- highlighting the impacts of inflation on investment appraisal
- discussing why uncertainty must be considered in investment appraisal
- explaining how probabilities can be used to influence the impact of risk and uncertainty in investment decisions
- explaining how to conduct a sensitivity analysis of an investment.

COURSE OBJECTIVES

At the end of this course, you should be able to:

- define capital investment – distinguishing between the various classes
- compute the Accounting Rate of Return (ARR) and the Payback Period of Investments (PPI)
- compute the Net Present Value (NPV), Internal Rate of Return (IRR) and Profitability Index (PI) of projects
- appraise capital investments based on both the traditional and discounted techniques
- apply probabilities to compute the (i) expected value (i) standard deviation and (ii) co-efficient of correlation of projects
- conduct sensitivity analysis of investments
- evaluate investments under inflationary condition and
- evaluate investments when capital is not adequate thus resulting to capital rationing.

COURSE MATERIALS

1. Course guide
2. Study units
3. Textbooks
4. Assignment file

STUDY UNITS

There are 14 units in this course. The units should be studied carefully.

The Modules

Module 1

Unit 1 Conceptual Issues in Capital Investment
Unit 2 Decisions: Types, Features and Tools for Decision-Making
Unit 3 The Payback Period
Unit 4 The Accounting Rate of Return
Unit 5 Compounding and Discounting

Module 2

Unit 1 The Net Present Value (NPV)
Unit 2 The Net Present Value (Annuity)

Unit 3 The Internal Rate of Return
Unit 4 The International Rate of Return
Unit 5 The Profitability Index

Module 3

Unit 1 The Impact of Inflation on Investment Proposals
Unit 2 Using Probability to Assess Impact of Risks on Capital Investments
Unit 3 Sensitivity Analysis
Unit 4 Capital Rationing

The first two units delved into the basic principles in capital investments. Module 1 unit 3 – Module 2 unit 5 focused on the basic capital investment appraisal techniques while Module 3 units 1 to 4 looked at the impact of inflation on investment appraisals, the use of probabilities in controlling the impact of uncertainty in investment, sensitivity analysis and capital rationing.

Each study unit is expected to take you at least two hours of concentrated studies. Every unit includes introductions, objectives, main content, self assessment exercises, conclusion, summary, tutor-marked assignment and references/further reading. You are required to study the materials religiously and thereafter try the exercises. This is what is called "practice." Being a quantitative course, these practices are very crucial and central to understanding the topics under consideration. You are advised to use some of the textbooks under references, for further reading and practices. They are meant to give additional information. All these efforts, put together, shall enable you to achieve the learning objectives which we stated earlier.

ASSIGNMENT FILE

All together, there are five assignment questions and you are expected to attempt all of them. You may wish to be guided by the following schedules.

- **Question One:** This is centred on the basic principles of investment and financial decisions (refer to units 2, 3, and 4).
- **Question Two**: Computation of the Accounting Rate of Return (ARR) and Payback Period (PBP) of capital investments (refer to units 5and 6).
- **Question Three*:*** Computation of Net Present Value (NPV) of projects (refer to Units 8 and 9).
- **Question Four**: Computation of the Internal Rate of Return (IRR) of some capital investments (refer to units 10, 11 and 12).

- **Question Five:** Computation of expected value, standard deviation and coefficient of variation of some projects (refer to unit 14).

TUTOR-MARKED ASSIGNMENT

In doing the tutor-marked assignments, you are expected to apply what you have learnt in the contents of the study units. These assignments, which are five in number, are expected to be turned to your tutor for grading. They constitute 30% of the total score.

FINAL EXAMINATIONS AND GRADING

At the end of the course, you will write the final examination. It will attract the remaining 70%. This makes the total final score to be 100%.

SUMMARY

This course, Capital Investment and Financial Decisions (BHM 713), exposes you to the basic principles of capital investment and investment appraisal techniques (viability tests). Having successfully completed the course, you are expected to be at home with the procedures and techniques that facilitate financial decisions, variously called, capital budgeting investment appraisal and project evaluation. Best of luck.

MAIN COURSE

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MODULE 1

UNIT 1 CONCEPTUAL ISSUES IN CAPITAL INVESTMENT

CONTENTS

1.0 INTRODUCTION

Any discussion on investment must begin with this simple truth: investment requires taking some risks. Your hope for investment success depends, in part, on your ability to control those risks without passing up reasonable rewards (Miller, 2003: 13). It therefore follows that investment involves some elements of sacrifice in anticipation of future returns.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define investment
- state types of investment
- identify some basic features of investment
- explain some conceptual issues in investment.

3.0 MAIN CONTENT

3.1 Definition of Investment

Investment, in its broad sense, means the sacrifice of current naira for future naira. Two different attributes are generally involved: time and risk. The sacrifice takes place in the present and is certain. The rewards come later, if at all and the magnitude is generally uncertain. In some cases, the element of time predominates (for example, government bonds). In other cases, risk is the dominant attribute (for example, call options on common stock). In yet others, both time and risk are important (for example, shares of common stock) (William, F Sharpe, Garden J. Alexandra and Jeffery V. Bairley, 1995: 1). Viewed in this sense, marriage is an investment.

3.2 Types of Investment

Broadly speaking, investment may be classified into:

a. direct
b. indirect
c. real assets (tangible)
d. paper assets (financial instrument)

3.3 Basic Features of Investment

Our discussions so far can be summarised by highlighting the essential features of investment.

a. Investments are undertaken in anticipation of benefits which are not expected to accrue concurrently with the investment outlay. As a result of this inevitable time lag between outlay and benefit, almost every investment involves some risks, the risk that anticipated benefits may not ultimately be realised.
b. Investments can be made in real or financial assets. Irrespective of the media, all investments can be measured in terms of the total outlay of funds.
c. Unlike capital, investment is a flow variable. Consequently, it ought to be measured as a time-rate of change in capital stock.
d. Since investment benefits accrue overtime, there is the expectation that the asset in which any investment is denominated shall be retained by the investor for some reasonable period. Hence the value of the asset should be carefully established at the time the investment is made.

e. Every investment involves some current capability for consumption. As a result of this feature, economists usually expect an identity between the level of savings and investment.

3.4 Investment and Speculation

According to Okafor (1983), the distinction between investment and speculation is not easy to make by simply observing the overt actions of the individuals involved. He went further to provide a beautiful summary comparing the two as follows.

Table 1: Investment and Speculation Compared

Possible considerations		**Investment**	**Speculation**
1.	Degree of risk assumed	Less	More, if not infinite.
2.	Level of income/profit expected	Moderate	High
3.	Income orientation	Income to accrue over time	Income to accrue quickly and in a lump sum
4.	Major consideration	Future value of assets and future earnings potential	Direction and extent of expected price movement
5.	Nature of income	Regular income and possible terminal capital gains.	Capital gains.

3.5 Basis for Classifying Investments

Broadly speaking, investments can be classified into two- investment in real assets and investment in financial assets. In the words of Okafor (1983) both types of investments can further be classified on the basis of a number of parameters.

a. **Magnitude of outlay**
Major investments could be distinguished from minor investments. In investment outlay, size is relative. An investment is major or minor depending on the relative proportion of the outlay to the total size of a firm. Thus, whereas an investment of ₦20, 000 could be considered a

minor investment by a firm capitalised at ₦20 million, it is a very major investment to a small firm with total assets valued at ₦40, 000.

b. **Risk environment of investment**

A distinction is made between investment under conditions of certainty, investments under conditions of risk and investments under conditions of uncertainty. The problem of risk and uncertainty will be discussed in the subsequent unit.

c. **Motivation for investment**

A distinction could be made among investments for asset replacement, capacity expansion or modernisation, and investments for strategic purposes.

d. **Sequencing of cash flows**

Conventional investments are distinguished from non-conventional investments on the basis of the timing and sequencing of cash flow arising from the investment. The nature of both types of investments, and the differences between them, are discussed subsequently, in this course.

e. **Nature of expected benefits**

A distinction exists between cost-saving and revenue-yielding, real asset investment. The former is illustrated by a firm that replaces old equipment in the hope of cutting operating costs over the life of the new equipment. In a revenue expansion programme, on the other hand, funds are invested in order to increase gross revenue either through additional sales volume or through increased price per unit of sales.

When evaluating a cost-saving investment, the value of total costs saved is compared with the additional investment made. In the latter situation, the investor would have to compare the increased costs with the additional sales revenue.

f. **Relationship to other investments**

The costs and benefits of a given investment may or may not be affected by alternative investments. In this regard, dependent investments are different from independent investments.

g. **Investment in real assets**

Investment in real assets takes one of three major forms, that is, investment in business fixed assets, investment in inventory and investment in residential construction.

h. **Investment in projects**

Real asset investment is either on single fixed assets or on a group of inter-related assets. Where the group of inter-related assets provides facilities capable of completing a production or a service process, the investment activity is described as a project. Investment projects are such that the facilities provided by the component assets can only be effective if operated as a unit.

Hence the component assets must necessarily be accepted or rejected as a set.

Contrary to popular expectation, the basic difference between projects and single asset investments does not lie in the value of the investment outlay. The cost of a single turbine in a hydro-electricity generating plant could be many times the total investment outlay in a corn grinding mill. In terms of our definition, the latter is a project because it can complete a processing cycle. Outlay on the hydro-electricity generating turbine is not by itself a project. The distinction must, however, be given a common sense interpretation. It is wrong, for example, to regard the purchase of a single taxi cab as a project, though such a cab can operate as a unit. A project necessarily involves the interplay of a number of single assets. (Okafor, 1983).

3.6 Further Classifications

1. **Conventional and non-conventional investments**
   According to Okafor (1983), investment activities in which periods of net cash outflows are expected to precede periods of net cash inflows are described as conventional investments. Non-conventional investments, on the other hand, are those in which there is no specific pattern in the sequencing of cash flows.
2. **Cash flows**
   The definition of net cash inflow or outflow used above is not identical with the accounting concept of income or expenditure. Net cash inflow from an investment for any period includes both the accounting income for the period and the non-cash expenses charged to operating revenue in determining such income such as depreciation.
3. **Dependent and independent investments**
   Two or more investments are economically independent if the expected cash flow from each would be unaffected whether or not the alternate investments are carried out concurrently.

Investment proposals are dependent if they are either technically dependent or economically dependent. (Okafor, 1983).

3.7 Degree of Dependence in Investments

There are degrees of dependence of investment opportunities. In one extreme case, one investment (A) is so dependent on another (B) that the net benefits of A would be virtually insignificant unless both themes are carried out simultaneously. Given that situation, investment B is a prerequisite for A. Where the degree of dependence is reciprocal, the alternatives are complementary.

The other extreme case of dependence occurs where the alternatives are so inter-related that the decision to carry out one implies *ipso facto* a rejection of the other. This is a case of *mutual exclusion* which occurs either because of technical dependence or because the alternative investments serve the same market which can only support one of the alternatives. Cases of mutual exclusion in investment alternatives abound in industry.

Note Well

The distinction between dependent and independent investments is important for one main reason. Whereas an independent investment is evaluated on the basis of its absolute cash flows, a dependent investment must be evaluated on the basis of its incremental cash flows.

4.0 CONCLUSION

In this unit, you have learnt that capital investments can contribute a lot towards national development. Accordingly, it is advisable that individual families, churches and states should embark on one form of investment or the other.

5.0 SUMMARY

In this unit, you have seen that capital investments involve making sacrifices today in anticipation of future benefits. You also learnt that investments could, broadly speaking, be divided into two namely, direct and indirect investments or real assets (tangible) and paper assets (Financial Instrument). You also looked at the features of investments and finally you were able to distinguish between investments and speculations.

6.0 TUTOR-MARKED ASSIGNMENT

1. "There is no basic difference between the behaviour of speculators and those who are interested in making as much income as possible from a given capital outlay." Discuss.
2. Discuss the major similarities and differences between investment in real assets and investment in financial assets.
3. Evaluate at least five government policies currently in force, which either induce or stifle private investment.

7.0 REFERENCES/FURTHER READING

Nweze, A.U. (2006). Investment Opportunities in the Nigerian Capital Market. Enugu: M'Cal Communications

Okafor, F.O. (1983). Investment Decision: Evaluation of Projects and Securities. London: Cassell Ltd.

UNIT 2 DECISIONS: TYPES, FEATURES AND TOOLS FOR DECISION-MAKING

CONTENTS

1.0 INTRODUCTION

In real life situations, one is often faced with more than one choice out of many possible courses of action. These possible courses of action are known as alternatives. Faced with these alternatives, the question then that naturally arises is: Which of the alternatives do l embark upon?

Collectively, all the steps and processes to be taken in order to arrive at the best possible course of action are known as decision making. This is the focus of this unit.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- use tools for decision making at various levels
- define some terms in "Decision Analysis".

3.0 MAIN CONTENT

3.1 Categories in Decision-Making

Decisions could be grouped into five.

a. **Routine planning decision -** decision here is often concerned with how to make the best use of scarce resources. Example of this is budgeting.

b. **Non-routine (short-run problem) -** this is a "one-off" special decision of a non-recurring nature, where cost-benefit analysis could quickly be carried out.

c. **Investment or disinvestment decision -** this decision involves large cash outflow and the potential benefits are expected to accrue over a reasonably long period.

d. **Long-range decision -** this is a decision that is not frequently made. In a way, therefore, it can be seen as a quantitative policy decision. (Shilling law, 1963). This type of decision tries to provide a continuing solution to a continuing or recurring problem.

e. **Control decision-** this decision involves evaluating performance provided a proper cost-benefit analysis is carried out before implementation. (MAYO/BPP.1988:314).

Alternatively, decisions could simply be categorised into two.

a. "Accept or Reject" decisions
b. Ranking decisions.

To facilitate our discussion, let's look at the various concepts in decision-making, starting with the concept of relevant costs.

3.2 Concepts in Decision-Making

Relevant costs

According to the official terminology of the Chartered Institute of Management Accountants (CIMA), relevant costs are defined as "cost appropriate to aiding the making of specific management decisions."

MAYO/BPP (1988: 316) is of the view that the costs which should be used for decision making are often referred to as relevant costs. That is a relevant cost is a future cash flow arising as a direct consequence of a decision.

Relevant costs are future costs. Past costs are only useful in so far as they provide information for predicting future costs.

Relevant costs are cash flows. The following should be ignored.

- Depreciation/amortisation
- Notional rent or interest
- All overhead costs absorbed

- A relevant cost is one which arises as a direct consequence of a decision.

Differential costs

These are the differences in costs between two alternative courses of action. Example: If going to Lagos by road from Enugu costs ₦2,000 and going by air costs ₦10,000, the differential cost is therefore ₦(10,000 – 2,000) = ₦8,000.

Incremental costs

These are relevant costs, which are simply the additional costs incurred as a consequence of a decision. Example: in a "decision, the cost of processing further is known as incremental cost and the associated benefit is known as incremental revenue. The difference between the incremental revenue and incremental cost forms a basis for the decision to be processed further.

How does incremental cost differ from differential cost?
Whereas differential costs compare the differences in costs between two alternative courses of action, incremental costs are ways of stating the relevant costs when three or more options are compared (MAYO/BPP, 1983: 318).

Avoidable costs

They are defined as "those costs that can be identified with an activity or sector of a business and which would be avoided if that activity or sector did not exist.

Opportunity costs

An opportunity cost is the benefit foregone by choosing one opportunity instead of the next best alternative.

Sunk costs

In the Holy Bible, the Gospel of St. Matthew, 22:14, Luke 14: 15 – 24, the parable of the 'Wedding Feast' was told. According to the passage, those invited failed to come and the king said, "now go to the main streets and invite to the feast as many people as you find (Matt. 22: 9).

If we hold constant the moral or spiritual lessons, we can then extract the accounting information contained therein which is as follows.

Realising that the cooked food and meat and wine were all perishable – possibly there was no refrigerator then – the king reasoned that all the food and meat were sunk costs (they had no viable alternative uses, if not consumed that day). Hence, he decided to invite other people to come and eat the food and meat, so that on the assessment day (Matt. 25: 31 – 46), he may be told, "I was hungry and you fed me, thirsty and you gave me a drink: I was a stranger and you received me in your home" (Matt. 25: 35).

The above vividly illustrates the concept of sunk cost defined as "the cost of an asset which has no significant alternative use. Examples include:

- dedicated fixed assets
- development costs already incurred.

Committed costs

A cost is said to be a committed cost if the cash outflow must of necessity take place regardless of whatever decision is taken now about alternative opportunities.

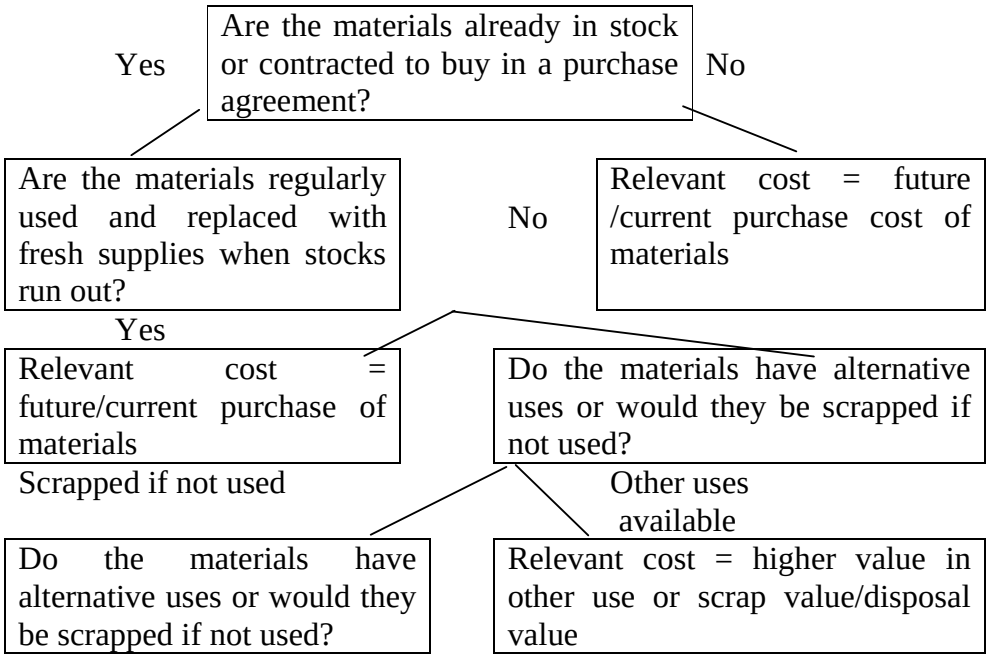
Notional (imputed) costs

This is in line with the matching concept in accounting. It is a hypothetical account cost which reflects the use of a benefit for which no actual cash expense is incurred. Examples are the followings.

- Rent charged on a building owned by an organisation.
- Interest charged on own capital employed.

The relevant cost of materials

In the word of MAYO/BPP (1988: 322), the decision tree below shows how the relevant costs of materials can be identified, provided that the materials are not supplies and so do not have an internal opportunity cost.



Relevant costs in non-routine decisions

Types of non-routine decisions

When performing the manufacturing and selling functions, management is constantly faced with the problem of choosing between alternative courses of action. Typical questions include: what to make? How to make it? Where to sell the product? And what price should be charged? In the short run, management is typically faced with the following non-routine, non-recurring types of decision.

- Acceptance or rejection of special order
- Pricing standard products
- Make or buy
- Sell or process further
- Add or drop a certain product line
- Utilisation of scarce resources

Relevant costs defined

In each of the above situations, the ultimate management decision rests on cost data analysis. However, not all costs are of equal importance in decision-making, and managers must identify the costs that are expected future costs (and revenues) which differ between the decision alternatives.

3.3 The Management Accountant's Role in Short Term Decision

Management accountant has an important role in the problem-solving process, not decision-making but as a collector and a reporter of relevant data. His reports must provide valid data in numbers that measure the quantities pertinent to the decision at hand. Many managers want the management accountant to offer recommendations about the proper decisions even though the final choice always rests with the operating executives. Problem-solving is essential in decision-making i.e. choosing among several courses of action.

The management accountant's role in problem solving is primarily that of technical expert on cost analysis. His responsibility is to be certain that the manager is guided by relevant data/information that would lead the manager to the best decision.

Under the concept of relevant costs, which may be appropriately titled the incremental, differential, or relevant cost approach, or relevant cost approach, the decision involves the following steps:

- gather all costs associated with each alternative
- drop the sunk costs
- drop those costs (not in absorption costing), that do not differ between alternatives
- select the best alternatives based on the cost data.

3.4 Feasibility and Viability Studies

Before embarking on any capital investment, it is always advisable to conduct both feasibility and viability studies. Whereas feasibility study is aimed at establishing the practicability or workability of an investment, viability study tries to evaluate the degree of profitability.

Feasibility study

This starts with environmental assessment (since certain investments cannot take place in some environments). Other issues to be considered include the followings.

- Management/personnel
- Availability of raw materials
- Market share assessment

Viability tests

These tests are normally conducted using either the traditional techniques or the discounted techniques or both.

4.0 CONCLUSION

As soon as you are faced with many alternatives, decision-making comes in. The truth, therefore, is that decision making is future-oriented. Therefore, everything humanly possible must be done to ensure that only economically viable, socially desirable and technically feasible investment decisions are taken.

5.0 SUMMARY

In this unit, you have learnt about the fundamental concepts in decision-making. The various types of decisions were also discussed. Feasibility and viability studies were equally considered.

6.0 TUTOR-MARKED ASSIGNMENT

1. State and discuss the basic types of decisions.
2. Distinguish between the following decision terms:
   a. Differential and incremental costs
   b. avoidable and unavoidable costs
   c. opportunity and sunk costs
   d. relevant and irrelevant costs
   e. routine and non-routine decisions

7.0 REFERENCES/FURTHER READING

Nweze, A. U. (2004). *Profit Planning*: *A Quantitative Approach*. Enugu: M'Cal Communications.

Osisioma, B. C. (1976). *Studies in Accountancy: Text and Readings*. Enugu: Acena Publishers.

Pandey, I. M. (1988). *Financial Management*. New Delhi: Vikas Publishing House PVT Ltd.

UNIT 3 THE PAYBACK PERIOD

CONTENTS

1.0 INTRODUCTION

For an average rational investor, one question that must be considered before embarking on a capital investment is: how long shall it take to recover or recoup the amount to be invested? This explains why a young man that wants to go into motorcycle (okada) business would first of all ask: how long shall it take to get back the cost of the motorcycle? Potential landlords also ask similar questions.

This is where the payback period readily comes in.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define, apply and compute payback period
- outline the merits and the demerits of the payback period.

3.0 MAIN CONTENT

3.1 Payback Period

Payback period is defined as the period, usually expressed in years, which it takes for the projects net cash inflows to recoup the original investment.

Illustrations

i. When the cash flows form an annuity, the payback period is simply "cash outflow divided by cash inflow." Example, Alhaji Haruna spent ₦10, 000,000 to build a house and receives ₦1, 000,000 annually as rent. The payback period is ₦ 10m ÷ ₦1m = 10 years.

ii. When the cash inflows do not form an annuity.
Ebele Nig. Ltd. has the option of investing in any of the following three projects which associated cash flows are presented thus:

Year	**Project I Cash Flow ₦'000**	**Project II Cash Flow ₦'000**	**Project III Cash Flow ₦'000**
0	(15,000)	(15,000)	(15,000)
1	6,000	4,000	3,000
2	5,000	5,000	5,000
3	4,000	6,000	4,000
4	-	-	3,000
5	-	-	2,000
6	-	-	1,000

Required: Advise the company on which of the three projects to invest in (base your advice on the payback approach).

Project I

This project generated ₦15, 000,000 in exactly three years. Therefore, the payback period is three (3) years.

N. B: There was no other cash inflow after the third year.

Project II

Again, the payback period is three (3) years since it took exactly three years to recoup ₦15, 000,000.

Also, no further cash inflow was recorded after the third year.

Project III

Even though the payback period for this project is 4 years, it recorded additional ₦2, 000,000 and ₦1, 000,000 in years 5 and 6 respectively.

Advice: Choose either project (I or II).

Solution

Ebele Nig. Ltd.

Net Cash Flows

Year	Project I		Project II		Project III	
	Cashflow	**Cumulative Cashflow**	**Cashflow**	**Cumulative Cashflow**	**Cashflow**	**Cumulative Cashflow**
	₦'000	**₦'000**	**₦'000**	**₦'000**	**₦'000**	**₦'000**
0	(15,000)	(15,000)	(15,000)	(15,000)	(15,000)	(15,000)
1	6,000	(9,000)	4,000	(11,000)	3,000	(12,000)
2	5,000	(4,000)	5,000	(6,000)	5,000	(7,000)
3	4,000	NIL	6,000	NIL	4,000	(3,000)
4	-	-	-	-	3,000	NIL
5	-	-	-	-	2,000	2,000
6	-	-	-	-	1,000	3,000

N.B: By usual notations:

Year 0 = Now (the date of investment)
Year 1 = the end of the1st year
Year 2 = the end of the 2nd year
and so on
Any figure in a bracket means cash outflow
A positive figure means cash inflow

3.2 Merits and Demerits of Payback Period

Merits

a. Simple to calculate
b. Easy to understand
c. It does not recognise depreciation as an expense
d. It favours projects that have quick return potentials

Demerits

a. It completely ignores any other cash inflow once the payback period has been arrived at.
b. It ignores the timing of cash inflows. For example, in the above illustration, Projects I and II ranked equally even when it is obvious that the cash inflows for project I are better than that for project II.

N.B: Because of its simplicity, the Payback Period Approach is undoubtedly the most commonly applied in practice.

Illustration

Bola Plc is considering the following three projects for which associated cash flows are given thus:

Year	Project A ₦'000	Project B ₦'000	Project C ₦'000
0	(500,000)	(500,000)	(500,000)
1	100,000	150,000	200,000
2	150,000	250,000	250,000
3	250,000	300,000	300,000
4	500,000	300,000	450,000

Required: Calculate the payback periods for each of the projects and advise Bola Plc. accordingly.

Solution

Bola Plc.

Project A

The sum of the cash inflows for the first 3 years is equal to ₦500,000,000.

∴ The payback period is 3 years.

Project B

The sum of the cash inflows for the first 2 years is ₦400, 000,000 and for the third year, the cash inflow is ₦300, 000,000 even when only ₦100, 000,000 was required to recoup the cost of the investment conclusively.

Therefore, the payback period is

$$\frac{500,000,000 - 400,000,000}{300,000,000}$$

= $2^1/_3$ years

Project C

The sum of the cash inflow for the first 2 years is ₦450, 000,000 and for the third year, the cash inflow is ₦3, 000,000,000.

Therefore, the payback period is

$$\frac{5{,}000{,}000{,}000 - 450{,}000{,}000}{300{,}000{,}000}$$

= $2^1/_6$ years

Comments

Since project C has the least payback period of ($2^1/_6$ years) compared with period B ($2^1/_3$ years), or project A (3 years), the management of Bola Plc. is advised to embark on project C.

Illustration

Modern Tech Services Ltd. is considering two alternative projects for a business expansion programme in the Northern part of the country. The projects have the following naira cash flow profiles according to the data supplied by the company's accountant:

Year	Project I	Project II
0	-1 million	-3 million
1	-2 million	.20 million
2	-95 million	-50 million
3	.85 million	.65 million
4	.78 million	.75 million
5	.62 million	.80 million
6	.40 million	1.90 million
7	.10 million	.20 million

SELF-ASSESSMENT EXERCISE

i. Calculate the payback period for each project (10 marks)
ii. Based on payback periods, advise on which of the two projects should be chosen (1 mark)
iii. State the advantages and disadvantages of the payback period criterion on investment appraisal (5 marks).
(Total 16 marks)
ICAN (May 1994) Mgt. Acc. Q7.

Solution

Project I		
Period	**Cash flow ₦'000**	**∑CF (₦)**
0	(1,000)	(1,000)
1	(2,000)	(3,000)
2	950	(2,050)
3	850	(1,200)
4	780	(420)
5	620	200
6	400	600
7	100	700
PBP = 4 years + 420,000 ÷ 620,000 = $4^{6}/_{7}$ years		

Project 2		
Period	**Cash flow ₦'000**	**∑CF (₦)**
0	(3,000)	(3,000)
1	(200)	(2,800)
2	500	(2,300)
3	650	(1,650)
4	750	(900)
5	800	(100)
6	1,900	1,800
7	200	2,000
PBP = 5 years + 100,000 ÷ 1,900,000 = 5 years + 0.522631578 PBP = 5 years		

Project 1 = $4^{6}/_{7}$ years - 1^{st} position
Project 2 = 5 years - 2^{nd} position

Advice

From this, I advise modern technical service to accept project 1 because it has a shorter payback period (PBP) than project 2.

Merits of PBP

a. It is not very costly to adopt
b. It is simple to understand and easy to calculate
c. It doesn't recognise depreciation as an expense
d. It favours projects that have quick return potentials
e. It is virtually suitable for many categories of management

f. It gives an insight into the liquidity of the project.

Demerits of PBP

a. It ignores the cash inflows earned after the payback period
b. It does not take the time value into consideration
c. Payback periods are arbitrarily set by management, hence they are usually subjective.
d. It is not consistent with the objective of maximising the market value of the firm's shares, as share prices are not dependent on the firm's payback period.

4.0 CONCLUSION

Predicated on the foregoing discussions, you can now conclude that the payback period is a very simple method for evaluating capital investments – the weaknesses notwithstanding.

5.0 SUMMARY

In this unit, you learnt the definition of the payback period as a method for appraising capital investment. You also looked at the merits and demerits of the method and had some illustrations of both when the cash inflows form an annuity and when the cash inflows fail to form an annuity.

6.0 TUTOR-MARKED ASSIGNMENT

AKPEBOR OTUOKENA BEAUTY (AOB)
PROJECTS

		A	B	C
Initial costs (₦)		400,000	460,000	360,000
Expected life		5 years	5 years	4 years
Scrap value expected		₦20,000	₦30,000	₦16,000
Expected Cash Inflow		₦'000	₦'000	₦'000
End of year	1	160	200	110
	2	140	140	130
	3	130	100	190
	4	120	100	200
	5	110	100	0

The company estimates its cost of capital to be 18% and discount factors are:

Year	1	0.8475
		0.7182
		0.6086
		0.5158
		0.4371

Required

i. Calculate the following:
   a. the payback period for each project
   b. the Internal Rate of Return(IRR) for each project
   c. the Net Present Value(NPV) of each project (10 marks)

ii. Which project should be accepted? Give reasons (2 marks) (Total 15 marks)
(ICAN, November 2002) Mgt. Acc. Q3.

7.0 REFERENCES/FURTHER READING

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UNIT 4 THE ACCOUNTING RATE OF RETURN

CONTENTS

1.0 INTRODUCTION

Oftentimes, accountants are interested in profits maximisation or more appropriately wealth maximisation. These profits are normally expressed as percentages of the amount invested. This way, a benchmark could be set. This is where the Accounting Rate of Return becomes relevant. It has its origin from the concepts of Return of Investment (ROI) or Return on Capital Employed (ROCE).

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define Accounting Rate of Return (ARR)
- state the formulae for ARR
- identify the investment criteria for ARR
- state the advantages and disadvantages of ARR.

3.0 MAIN CONTENT

3.1 Accounting Rate of Return (ARR)

This is defined as the ratio of average annual profits after depreciation to capital invested. This is the basic definition. Other variants exist. For example:

- profits may be before or after tax
- capital may or may not include working capital
- capital invested may mean the initial capital investment or the average of the capital invested over the life of the project.

From the above variations, one can state at a very high confidence level that the ARR (Accounting Rate of Return) and (Return on Capital Employed) could be used interchangeably.

Illustration

Modesta Nig. Ltd. is considering three projects each with an initial capital of ₦1, 000,000 and a life of 10 years. The profits generated by the projects are estimated to be as follows.

After Tax and Depreciation Profits

Year	**Project A ₦'000**	**Project B ₦'000**	**Project C ₦'000**
1	100	175	75
2	100	175	75
3	100	100	75
4	100	100	75
5	100	75	75
6	100	75	75
7	100	75	100
8	100	75	100
9	100	75	175
10	100	75	175
Total	1000	1000	1000

Required: Calculate the Accounting Rate of Return (ARR) using:

- initial capital
- average capital

Solution

Modesta Nig. Ltd.

i. Accounting Rate of Return on Initial Capital

	Project A	**Project B**	**Project C**
Average Profits	₦1,000,000	₦1,000,000	₦1,000,000
	10	10	10
	= 100,000	=100,000	= 100,000
∴ ARR	100,000	100,000	100,000
	1,000,000	1,000,000	1,000,000
	= 10%	= 10%	=10%

ii. Accounting Rate of Return on Average Capital

	Project A	**Project B**	**Project C**
Average Profits	₦1,000,000 / 2 = 500,000	₦1,000,000 / 2 = 500,000	1,000,000 / 2 = 500,000
∴ ARR	100,000 / 500,000 = 20%	100,000 / 500,000 = 20%	100,000 / 500,000 = 20%

N.B: Calculation of average capital.

Given a scrap value of zero, then the average capital investment is calculated thus:

$$\frac{\text{Sum of the terms}}{\text{No of Terms}}$$

Sum = 1,000,000 + 0

= 1,000,000

No of terms = 2 (that is, 1,000,000 and 0)

$$\therefore \text{Average} = \frac{1,000,000}{2}$$

= ₦500, 000

By way of formula therefore,

$$\text{Average capital} = \frac{\text{Initial Capital}}{2}$$

Where the scrap value is not zero, then the average capital becomes:

$$\frac{\text{Initial Capital + Scrap Value}}{2}$$

3.2 ARR: Merits and Demerits

Merits

a. Simple to calculate
b. Easy to understand

Demerits

a. It ignores the time value of money. For example, the above three projects are ranked equally even when there are obvious differences in the timing of cash flows.
b. It recognises depreciation as an expense.
c. There are many variations of accounting rate of return.
d. Accordingly, it lacks objectivity.

4.0 CONCLUSION

Even though the Accounting Rate of Return (ARR) does not recognise the time value of money, it remains a very valuable method for evaluating capital projects. This is simply because it is very much associated with the concept of Return on Investment or Return on Capital Employed – which is used to evaluate divisional performance.

5.0 SUMMARY

In this unit, you have learnt the basic definition of Accounting Rate of Return (ARR) and also looked at the other variants of the definition. Such variants include the followings.

- The definition of profits: is it Profit Before Tax (PBT) or Profit after Tax (PAT)?
- The definition of capital: is the working capital included or not? Is it the initial capital or the average capital?

6.0 TUTOR-MARKED ASSIGNMENT

Duro Plastic Plc. is considering investment in two mutually exclusive projects – A and B, each having a life span of 5 years and no residual value.

	Project A ₦	**Project B** ₦
Initial Investment		
Net Cash Inflows	250,000	320,000
Year 1	80,000	70,000
Year 2	60,000	75,000
Year 3	65,000	80,000
Year 4	70,000	85,000
Year 5	75,000	85,000

The criteria for accepting or rejecting a project are:

i. accounting rate of return on average capital employed should not be less than 15%.
ii. payback period should not be more than four years, and
iii. profitability index based on net present value should not be less than 3% using a discounting rate of 12%.

You are required to present necessary calculations to show whether the above projects A and B are acceptable to the company or not.

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UNIT 5 COMPOUNDING AND DISCOUNTING

CONTENTS

1.0 INTRODUCTION

In investment decisions, it is generally believed that money has time value. Hence, we often talk of future value (when we are compounding) or present value (when we are discounting). This is so because ₦1.00 received today is worth more than ₦1.00 receivable tomorrow. Yes, even if we hold constant the application of time value of money, the saying that "a bird at hand, is worth more than one million in the bush", becomes relevant. It is for this reason that we now talk about compounding and discounting.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define compounding and discounting in investment decisions
- compute the formulae for compounding and discounting
- compute simple discounting factors
- compute annuity factors (present values)
- solve some practical problems involving compounding and discounting.

3.0 MAIN CONTENT

3.1 The Components of Cash Flows?

The following components according to Lucey (1988) are typical cash flow items.

Cash Inflows

i. The project revenue
ii. Government grants
iii. Resale or scrap value of assets
iv. Tax receipts
v. Other cash inflows caused by accepting the project.

Cash Outflows

i. Initial investment in acquiring the assets
   a. Project cost (labour materials sets)
   b. Working capital investment
   c. Tax payments
   d. Any other cash outflow caused by accepting the project

3.2 Time Value of Money

Suppose you are given the option of receiving either ₦100, 000 (one hundred thousand naira) today or ₦120, 000 (one hundred and twenty thousand naira) at the end of the year, which option would you choose given that the prevailing interest rate is at 10% per annum and that the risk of default is zero?

Given that redeeming the ₦120, 000 in a year's time has a probability that it must necessarily be redeemed, then the question of a "bird at hand is worth more than ten in the bush" becomes inapplicable. In that case, we are left with only rational reasoning.

To reason rationally, one has the option of asking the following questions.

At 10% per annum, how much is ₦100, 000 worth in a year's time? and
At 10% per annum, how much is ₦120, 000 receivable in a year's time worth today?

Answers

While the first question addresses a concept known as "COMPOUNDING," the second centres on "DISCOUNTING."

Option 1: In a year's time, ₦100, 000 at 10% per annum will worth ₦100, 000 x 1.1 = ₦110,000.

Option 2: The present value (today's value) of ₦120, 000 receivable in a year's time at 10% per annum is ₦120, 000 $(1.1)^{-1}$ = ₦109, 091.

Analysis

Either way, the option of receiving ₦120, 000 in a year's time has a higher value than receiving ₦100, 000 today.

What if the option was between receiving equal amounts say ₦120, 000 either today or in a year's time?

Obviously, the question becomes easy since any rational being would opt for receiving the money today.

Hence, ₦10, 000 received today is worth more than ₦10,000 received tomorrow. This is the concept of time value of money.

Comment

For investment appraisal, discounting is preferred to compounding. Put differently, for investment appraisal, our focal date should be today and not tomorrow!

3.3 Compound Interest

According to the New Webster's Dictionary (1995: BD 25), compound interest is interest upon principal plus accrued interest.

Put differently, compound interest is generally calculated for long term loans and the interest payable in one period forms a part of the principal in the coming or subsequent periods; which itself earns interests for other periods. The ultimate effect is that at the end of each period, the principal keeps increasing following the addition of interest earned in the preceding period. Interests are usually calculated and added to the principal at the end of each regular interval usually called the conversion intervals or periods.

The conversion period/intervals may be one year (annually), six months (semi-annually), four months (term), three months (quarterly), monthly, weekly, daily or even hourly.

Usual Notations

Initial principal	=	P
Rate of interest	=	Jm pr (J, M)

That is, nominal interest rate = J per year
Number of conversion intervals per year = M
Rate of interest per conversion period = I
Period for which principal is invested = conversion interval
Accumulated amount after k conversion, periods = Sk
Accumulated amount at the end of the term = S

From the above notation, interest rate per conversion interval is equal to nominal interest rate divided by number of conversions per year. There 1 = j/m.

Also, the accumulated amount at the end of the first conversion period is equal to the principal at the beginning of the first conversion period plus interest earned at the first conversion period that is:

$$S_1 = P + pi = P(1 + i)$$

Similarly, the accumulated amount at the end of the second conversion period is given as:

$$S_2 = S_1 + S,I = S_1 (1 = i)$$
$$= P(1 + i)(1 + i) = P(1 + i)$$

Similarly, $S_3 = S_2 S_2 i$
$$= S_2 (1 + i)$$
$$= P(1 + i)^2 (1 + i)$$
$$= P(1 + i)^3$$

It can be observed from the expressions of S_1, S_2 and S_3 that they form a sequence in which the power of (1 + i) is equal to the number of conversion intervals. Therefore, the accumulated amount at the end of k conversion period gives us:

$$S_k = P(1 + i)^k$$

Similarly, the accumulated amount at the end of the conversion period N where n is the number of terms

$$S_n = P(1 + i)^n$$

N.B: If there are “M” conversion intervals per year then the number of conversions in the period of “T” years is “TM”.

Illustration

Find the accumulated amount of the following:

a. ₦250, 000 after a period of 3 years at J = 6% and M = 2
b. ₦325, 000 at the end of 6 years and 3 months at J4 = 0.10
c. ₦275, 000 after the period of $3^1/_2$ years at J4 = 8%
d. ₦250, 000 after the period of 10 years at 6% converted quarterly.

Solution

a. P = ₦250, 000 n = TM 3 x 2 = 6
I = JM = 3% = 0.03
But S = $P (1 +i)^n$
Substituting,
S = $250,000 (1.03)^6$
= 250,000 x 1.194052297
= ₦298,513.07

b. P = ₦325, 000; t = $6^1/_4$; J4 = 10%
∴n = $6^1/_4$ x 4 = 25 and 1 = 10/4 = 2.50%
S = $P (1 + i)^n$
Substituting,
P = $325,000 (1.025)^{25}$
= 325,000 x 1.853944098
= ₦602, 531.82

c. P = ₦275, 000; t = $3^1/_2$ years, J_{12} = 8%
∴ n = 41 and i = 8/12
S = $P (1 + i)^n$
∴ S = $275,000 (1.006666666)^{42}$
= 275,000 x 1.321900923
= ₦363, 522.75

d. P = 250,000; t = 10 years, j = 6%
M = 4
∴ n = 40, i = 6/4= 1.5%
∴S = $250,000 (1.015)^{40}$
= ₦453, 504.60

3.4 Annuity

According to the New Webster's Dictionary (1995: BD5), annuity is a series of payment for a fixed future period or for life, payable monthly, semi-annually, annually or at any other specified intervals. It is frequently used to describe a part of the retirement allowance derived

from the accumulated contributions made by the members, contributions which are called "pension."

Put differently, an annuity is a sequence of equal payments made at equal time intervals. For example:

- the payment of a sum of money at the end of each month as rent
- mortgage payments
- installment payments (hire purchase, lease).

Payment intervals
This is the time interval between two consecutive payments.

Term of an annuity
This is the time period from the payment interval preceding the first payment to the date of the last payment.

Amount of an annuity (A)
This is the total equivalent value of all payments on the day of the last payment at a given rate of interest.

Present value of an annuity (A)
This is the total equivalent value of all payments at the beginning of the term given a rate of interest.

Simple annuities
These are annuities in which the payment intervals and conversion COINCIDE. Under the simple annuity, payments are made at the end of the payment intervals.

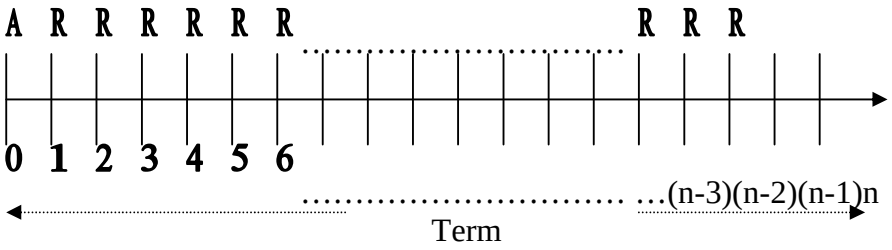
Formulae
Let's recall that since annuity is a sequence of equal payment made at equal time intervals, it therefore becomes a geometric series in which:

- the first term, $a = 1$
- the common rate, $r = 1 + i$
- the last term, $1 = (1 + i)^{n-1}$

Usual notations:

R	=	regular payments
N	=	number for regular payment
A	=	present value of an annuity
S	=	amount of the annuity
I	=	interest rate per payment interval

Time Diagram



Let's assume that our focal date is the date of the last payment (n). By the concept of equivalent values, let's identify the two sets of obligation.

First set of obligation

The accumulated sum (S) on the date of the last payment (n) is:

$$S(1 + i)^0 = S$$

Second set of obligation

This is the future value of all the regular payments. That is:

$$R(1 + i)^{n-1} + R(1 + i)^2 + R(1 + i)^{n-3} + R(1 + i)^{n-4} + \ldots R(1 + i) + R(1 + i)^0$$
$$= R[1 + i)^{n-1} + (1 + i)^{n-2} + (1 + i)^{n-3} + (1 + i)^{n-4} + (1 + i) + 1]$$

Rewrite the terms in the reverse order.

$$R[1 + (1 + i) + \ldots + (1 + i)^{n-3} + (1 + i)^{n-2} + (1 + i)^{n-1}]$$

The above, has therefore metamorphosed into a geometric series in which:

- The first term, a = 1
- The common ratio, r = 1 + i
- The last term, $I = (1 + 1)^{n-1}$

The last term has (n – 1) and not n since no payment was made in the nth period. Therefore, the sum of the series, S is the same as the sum of the geometric series given as:

$$S_n = \frac{a - rI}{1 - r} = \frac{a(r^n - 1)}{r - 1}$$

for r > 1

By substituting

$$\frac{a - rI}{1 - r}$$

In the above equation

$$\frac{1 - (1 + I)(1 + I)^{n-1}}{1 - (1 - I)} = \frac{1 - (1 + 1)^n}{-i}$$

On multiplying both sides by –1, we have:

$$\frac{(1 + i)^n - 1}{i}$$

$$\therefore S_n = RI \frac{(1 + I)^n - 1I}{i}$$

For given values of I and n

$$\frac{(1 + I)^n - 1}{I}$$

(As in annuity table) – This is a constant and therefore denoted as S_nI (usually referred as "S angle n at i". Therefore, instead of writing

$$S = R \left[\frac{(1 + i)^n - 1}{I}\right.$$

$$S = RS_n\, I.$$

Present Value, A

Assume that t = 0, value of all regular payment made to repay, for instance, a loan at the beginning of the term = $R (1 + i)^{-1} + R (1 + i)^2 + \ldots + R (1 + I)^{-n}$

$$A = R [(1 + i)^{-1} + (1 + i)^{-2} + \ldots + (1 + i)^{-n}].$$

Again, the above series becomes a geometric progression which

first term (a) = $(1 + i)^{-1}$
Common ratio (r) = $(1 + i)^{-1}$
But $S_n = \frac{a(1 - r^n)}{1 - r}$
for r < 1
substituting

$$A = \frac{(1 + i)^{-1} [1 - (1 + i)^{-1(n)}]}{1 - (1 + i)^{-I}}$$

$$= \frac{(1+i)^{-1}[1-(1+i)^{-n}]}{1-{}^{1}/_{1+i}}$$

$$= \frac{(1+i)-1[1-(1+i)^{n}]}{\frac{1+i-1}{1+i}}$$

$$= \frac{(1+i)^{-1}[1-(1+i)^{-n}]}{\frac{i}{1+i}}$$

$$= \frac{(1+i)^{-1}[1-(1+i)^{-n}] \times 1+i}{i}$$

$$= \frac{1-(1+i)^{-n}}{i}$$

$$\therefore A = R\frac{[1-(1+i)^{-n}]}{i}$$

$\therefore A = Ran\ i$

Tutorial Notes

a. Ordinary annuity certain: This is an annuity in which the payments are made at the end of the payment intervals.
b. Students are urged to display each annuity on a time line with the interest period (IP) as the unit of measure marking at least the beginning of the term (0 on the scale) and the end of the term (n on the scale) and a few other periods.
c. Difficulties with annuities arise from a failure to keep two facts in mind when using the formula.
d. The formula S = Rsn i, gives the amount of an annuity just after payment has been made.
e. The formula A = Ran i give the value of annuity one period before the first payment was made.
f. If an unknown is associated with the amount of an annuity, take a focal date of t = n in which case the amount is known (given or inferred).
g. If an unknown is associated with the present value of an annuity, take the focal date of t = 0 in which case the present value is known (given or inferred).

SELF-ASSESSMENT EXERCISE

For the past ten years, Austin has been depositing ₦50, 000 at the end of each year in a savings account, which pays 15 percent per annum. How much was to his credit just after the 10^{th} deposit?

Solution

R = 50,000
i = 0.15
n = 10 (by usual notation)

$$S = \frac{R[(1 + I)^n - 1]}{i}$$

Substituting

$$S = ₦50,000 \frac{[(1.15)^{10} - 1]}{0.15}$$
$$= ₦50,000 \frac{[4.0455577 - 1]}{0.15}$$
$$= ₦50,000 \times 20.3037184$$
$$= ₦1,015,185.91$$

SELF-ASSESSMENT EXERCISE

Today, Uche purchased an annuity of ₦250, 000 per year for 15 years from an insurance company, which uses 3% compound annually. If the first payment is due in one year, what did the annuity cost him?

Solution

R = ₦250, 000
i = 0.03
n = 15

$$A = \frac{R[1 - (1 + i)^{-n}]}{I}$$

Solution

$$₦250,000 \frac{[1 - (1.03)^{-15}]}{0.03}$$
$$= ₦250,000 \times 11.93793509$$
$$= ₦2,984,483.77$$

4.0 CONCLUSION

In this unit, you have learnt that in financial mathematics in general, and in capital investment appraisal in particular, the twin concepts of compounding and discounting are so fundamental that they can not be done away with. This is because of the all-important concept of "time value of money." Accordingly, You are advised to understand this "foundation" aspect of capital investment appraisal technique.

5.0 SUMMARY

In this unit, you have learnt a wide range of concepts in financial mathematics. These concepts include:

- cash flows
- time-value of money
- compound interest and annuity

You are now conversant with the various formulae for computation.

6.0 TUTOR-MARKED ASSIGNMENT

1. Assuming that Prof. Bello decides to deposit in his savings account ₦10,000.00 at the end of every 6 months for 5 years, followed by deposits of ₦15,000 at the end of every 6 months for 12 years and ₦30,000 at the end of every 6 months for 8 years. If the bank pays interest at 7 per cent per annum, compounded semi annually, how much will he collect at the end of 25 years?
2. Ifebuche Local Government built a bridge that will need no repairs until the end of the next 5 years when ₦30, 000 will be required for repairing. After that, it is estimated that ₦30, 000 will be needed at the end of each year for the next 20 years. You are required to find the present value of the upkeep of the bridge, if money is worth 10 percent per annum.

7.0 REFERENCES/FURTHER READING

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MODULE 2

UNIT 1 THE NET PRESENT VALUE (NPV)

CONTENTS

1.0 INTRODUCTION

Discounted Cash Flow (DCF I)

Against the backdrop that the traditional techniques ignore the timing of cash flow, a new approach known as the discounted cash flow has been developed. This approach uses cash flows rather than accounting profits. According to Lucey, (1988), accounting profits are invariably calculated for stewardship purposes and are period-oriented (usually monthly, quarterly or annually) thus necessitating accrual accounting with its attendant conventions and assumptions. Therefore, for investment appraisal purposes, a project-oriented approach using cash flow is to be preferred since it disallows depreciation as an expense and also recognises the timing of cash flows.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define Net Present Value (NPV)
- apply the formula for simple NPV
- describe the investment criteria under NPV.

3.0 MAIN CONTENT

3.1 Assumptions Underlying the Basic Discounted Cash Flow Appraisal

According to Lucey (1988), certain assumptions are made initially so that the underlying principles can be more easily understood.

These are as follows:

- uncertainty does not exist
- inflation does not exist
- the appropriate discount rate to use is known
- a perfect capital market exists, that is unlimited funds can be raised at the market rate of interest.

Later, each of the above assumptions will be isolated and handled accordingly.

3.2 Net Present Value (NPV)

Net Present Value (NPV) is defined as the difference between the present value of cash inflows and those of the cash outflows all discounted at the cost of capital.

According to Okafor (1983: 222), the net present worth of a project is the present value of the discounted net proceeds anticipated throughout the economic life of the project. The cash outflows and inflows are discounted using the same rate of discount. The algebraic sum of the discounted stream of cash flows is the Net Present Value (NPV).

That is

$$NPV = \sum_{t=0}^{n} \frac{FC_t}{(1+K)^t}$$

where

NPV = net present value
CF_t = net cash flow at time t
K = discount rate

For most conventional investments, the net cash outflow would occur at the initial period, that is, at t = 0. In such cases, the equation becomes:

$$\sum_{\text{NPV}}^{n} \frac{CF_i}{(1+K)^i} CF_0$$
$$t = 0$$

The present value of one ratio today, is of course ₦1. Therefore, CF_0, would be equal to the initial cost of the project.

Decision Rule

The general criteria under the NPV appraisal techniques are the followings.

- INVEST: if NPV > 0. That is, invest if the NPV is positive.
- DON'T INVEST: if NPV < 0. That is, do not invest if the NPV is negative.
- Remain indifferent: if NPV = 0. That is, you may or may not invest if the NPV = 0.

According to Okafor (1983: 223), choosing among alternatives and mutually exclusive projects, the decision rule is to rank them according to their relative net present worth. The project with the highest NPV is presumed to be the most preferable.

Illustration

Ayodele Engineering Co. is trying to decide which type of machine tool to buy, of the two types available. Type A costs ₦10, 000,000 and the net annual income from the first three years of its life will be ₦3, 000,000, ₦4, 000,000 and ₦5, 000,000 respectively. At the end of this period, it will be worthless except for scrap value of ₦1, 000,000. To buy a type A tool, the company would need to borrow from a Finance Group at 9%. Type B will last for three years too, but will give a constant net annual cash flow of ₦3, 000,000. It costs ₦6, 000,000 but credit can be obtained from its manufacturer at 6% interest. It has no ultimate scrap value. Which investment would be the more profitable? Give reason for your answer.

Solution

Ayodele Engineering Company

Type A Year	Cash flow ₦'000	Discount Factor (9%) ₦'000	Net Present Value ₦'000
0	(10,000)	1.000	(10,000)
1	3,000	0.917	2,751
2	4,000	0.842	3,368
3	6,000	0.772	4,632
	NPV		₦751

N.B: 6,000 = 5,000 cash flow + 1,000 scrap value.

Type B	**Cash flow**	**Discount Factor (6%)**	**Net Present Value**
Year	₦'000	₦'000	₦'000
0	(6,000)	1.000	(6,000)
1	3,000	0.943	2,829
2	3,000	0.890	2,670
3	3,000	0.840	2,520
	NPV		₦2,019

Alternatively, for project B, since the cash inflows form an annuity, we then use annuity factor. For n = 3, r = 6%, the annuity factor is:

$$\frac{1 - (1.06)^{-3}}{0.06} = 2.673$$

The NPV = 3000 x 2.673 – 6000 = 2019.

Thus, we can see that type B has a far higher NPV and this will be the better investment.

Illustration

Femi Nig. Ltd is proposing to purchase a new machine for ₦20, 000,000 which will have a life span of 6 years. The cash inflows estimated to be generated by the machine are as follows: Year 1 = ₦12,400,000; Year 2 = ₦6,000,000; Year 3 = ₦7,100,000; Year 4 = ₦2,203,000 and Year 5 = ₦2,774,000 and removed in year 6 an estimated net cash outflow of ₦1,477,000.

The company's cost of capital is 15%. Should investment be proceeded with?

Solution

Femi Nig. Ltd

Year	Cash flow ₦'000	15% Discount ₦'000	Net PV at 15% ₦'000
0	(20,000)	1.000	(20,000)
1	12,400	0.870	10,788
2	6,000	0.756	4,536
3	7,100	0.658	4,672
4	2,203	0.572	1,260
5	2,774	0.497	1,379
6	-1,477	0.432	-638
		Net Profit Value (NPV) =	+ 1,997

The NPV is positive, hence 'go' for the project.

4.0 CONCLUSION

In this unit, you have learnt about the most fundamental methods for appraisal capital projects – the Net Present Value (NPV). This approach must be understood and applied most religiously.

5.0 SUMMARY

In this unit, you are acquainted with the Net Present Value (NPV) method of capital investment approach. You are now familiar with the basic definition and formula. You have also learnt about the computational technique and the investment criteria.

6.0 TUTOR-MARKED ASSIGNMENT

Because of public credit policy binding in the current year, Omoloju Ltd. is unable to raise all the funds it requires for investments, which must be made in the current year. There are investments opportunities open to it this year, but it can only finance one to them. The projects' cash flows are as follows:

Year	**A**	**B**	**C**
0	(500,000)	(350,000)	(532,360)
1	300,000	100,000	350,000
2	300,000	260,000	300,000
3	300,000	200,000	200,000
4	(100,000)	50,000	10,000

Required: Assuming that the marginal cost of capital of Omoloju Ltd. is 28%, advises the company on which of the three investment opportunities to choose from.

i. Write short notes on the following.

   a. Net Present Value
   b. Payback Model
   c. The Internal Rate of Return (IRR)

ii. The following pieces of information relate to three possible capital projects: because of capital rationing, only one project can be accepted by the management of Akpebor Otuokena Beauty (AOB).

Projects

		A	**B**	**C**
Initial costs (₦)		400,000	460,000	360,000
Expected life		5 years	5 years	4 years
Scrap value expected		₦20,000	₦30,000	₦16,000
Expected cash Inflow		₦'000	₦'000	₦'000
End of year	1	160	200	110
	2	140	140	130
	3	130	100	190
	4	120	100	200
	5	110	100	0

The company estimates its cost of capital to be 18% and discount factors are:

Year	1.	0.8475
	2.	0.7182
	3.	0.6086
	4.	0.5158
	5.	0.4371

Required

i. Calculate the following.
   a. The payback period for each project
   b. The Internal Rate of Return for each project

The Net Present Value of each project (10 marks)

ii. Which project should be accepted? Give reasons (2 marks) (Total 15 marks)
(ICAN, November 2002) Mgt. Acc. Q3.

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UNIT 2 THE NET PRESENT VALUE (ANNUITY)

CONTENTS

1.0 INTRODUCTION

In the previous section, we looked at investment appraisal involving the computation of Net Present Value (NPV) in situations where the cash inflows were not regular or constant. In that case, we used ordinary present value discounting factors. But, in situations where the cash inflows are constant, the time and effort shall be considerably minimised if we use the annuity factor present value instead. For periods less than 5, the beauty of this short cut may not be appreciated. However, with long periods, this short cut shall then become indispensable.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the concept of annuity
- apply the annuity factor to net present value
- solve practical questions involving annuity (lease rentals, hire purchase).

3.0 MAIN CONTENT

3.1 Illustration of Net Present Value

Obumneme Group of Companies leases land and erects building on it, financing the construction from term loans. The buildings are rented out by the company which can borrow and invest money at 15 percent per annum.

As the company's financial controller, you have been approached to advise it on how best to use a site it leased 25 years ago for 80 years, from Obumnenye Local Government for an initial premium of ₦50, 000,000 and annual ground rent of ₦6, 000,000. When the lease expires, the building will revert to the local government. The following options are available to the company on the use of the site in question.

a. The site could be out-leased for the remaining years at an annual rent of ₦40, 000,000.
b. A house could be constructed quickly on the site with the following estimated costs and income.

Building and other capital expenditure	₦500, 000,000
Annual management and maintenance fee	₦150, 000,000
Annual rental income (till the lease expires)	₦250, 000,000

c. Blocks of flats could be constructed on the site. However, this would entail a long development period and rents would not be collected till after 5 years. The estimated costs and income for this option are given as follows.

   Building and other capital expenses = ₦250, 000,000 per year (amounting to N1, 250,000.00) Annual management and maintenance costs of N200, 000,000.00 and annual rental income of N550, 000,000.00 (for the 50 years after completion)

Solution

This is an interesting question that brings out some cost/management concepts clearly. The concepts are the followings.

a. The initial premium of ₦50, 000.00. This cost is already incurred hence it is both sunk and irrelevant. Accordingly, we shall disregard it in our analysis.
b. The annual ground rent of ₦6, 000.00. This cost is yet to be incurred. Hence, it is a relevant cost. However, since it must necessarily be incurred regardless of the option embarked upon, it becomes a common cost. Accordingly, including or excluding it in our analysis shall not affect our decision. We shall exclude it.
c. The net cash inflows in each case form an annuity. Hence, we shall use annuity table (present value) instead of ordinary present value table.
d. Relevant period.

The land was rented 25 years ago for 80 years. The relevant period therefore is from today (the 25th year) to the 80th year. That is, 55 years.

Therefore, the annuity factor at 15 percent for 55 years is calculated thus.

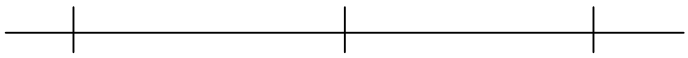
The formula is = $\frac{1 - (I + r)^{-n}}{r}$

where n = the number of periods
r = the interest rate

Substituting

$\frac{1 - (1.15)^{-55}}{0.15}$ = 6.6636

Option C however takes the form of a deferred annuity since the building would take 5 years to complete and cash inflows can only take place as from the 6th year. This we can represent on a number line thus:



<----3.3522 ---------->5<-------3.3114------>55

- a5 0.15 = 3.3522 and
- a55 0.15 = 6.6636
- difference = 3.3114

Therefore, the annuity factor for the deferred annuity is 3.3114. Anchored on foregoing comments, we then proffer our solution thus:

Option A: Out- lease the site for remaining years
Since the yearly income is ₦40, 000,000 for 55 years, the present value is ₦40, 000,000 x 6.6636 = ₦266, 544,000.00.

Option B: Quick construction of a house at the site.
This is purely theoretical, as house cannot be so quickly built. Yes, even at the "transfiguration of our Lord Jesus Christ," (Matt. 17: 1 – 8; Mark 9: 2 – 8 and Luke 9: 28 – 36), Peter said, "We will make three tents, one for you, one for Moses and one for Elijah" (Luke 9: 33).

Assuming that it is possible to quickly construct a house, then the cost of the house ₦500, 000,000 took place in year zero.

Also, since the annual management and maintenance fee is ₦150,000,000 and the annual rental income is ₦250,000,000, the net annual cash inflow is ₦100,000,000 (i.e. ₦250,000,000 – ₦150,000,000). Therefore, the NPV is ₦100, 000,000 x 6.6636 – ₦500, 000,000 = ₦166, 360,000.

Option C: Construction of a block of flats
Since the construction would last for 5 years @ ₦250, 000,000 per annum, the present value of the cost of the block of flats is:

₦250, 000,000 x 3.3522 = ₦838,050,000

Also, given that annual management and maintenance costs ₦200,000,000 and annual rental income of ₦550,000,000 shall commence after 5 years, the annual net cash inflows of ₦350,000,000 (₦550,000,000 – ₦200,000,000) form a deferred annuity whose present value is

₦350, 000,000 x 3.3114 = ₦1, 158,990,000

This leaves us with an NPV of ₦320, 940,000 (That is ₦1, 158,990,000 – ₦838, 050,000).

Summary

Option A: NPV = ₦266, 544,000
Option B: NPV = ₦166, 360,000
Option C: NPV = ₦320, 940,000

Therefore, since option C has the highest NPV, that option is the most preferable and hence recommended.

N.B: We most logically assumed the 25th year as our focal date.

4.0 CONCLUSION

In this unit, you have learnt that "multiplication, is the summary of addition," and with this knowledge, you can now develop a formula for present value – when the cash inflow is constant and hence forms an annuity. This method provides a very beautiful short-cut, particularly when you are thinking of longer periods. This method should also be understood in "hire purchase" and "finance lease" computations.

5.0 SUMMARY

In this unit, you have learnt about the application of annuity factor (present value) in capital investments.

6.0 TUTOR-MARKED ASSIGNMENT

Ola Nig. Ltd. invested ₦10m in a project that gives it ₦1m per annum for 40 years. If the cost of capital is 10 per cent per annum, compute the Net Present Value.

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UNIT 3 THE INTERNAL RATE OF RETURN I

CONTENTS

1.0 INTRODUCTION

In our previous sections, you are conversant with the various investments appraisal techniques. While the payback period tried to answer the question of how long it would take for the cost of the investment to be recovered, the Net Present Value (NPV) on the other hand centred on wealth maximisation.

Yet, there is another method that sets a hurdle rate, internally, before investment can take place. This is called the Internal Rate of Return (IRR). This is the focus of this unit.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define Internal Rate of Return (IRR)
- state IRR formula and how to derive unknown values within a range
- outline the investment criteria under the IRR
- state the merits and the demerits of IRR.

3.0 MAIN CONTENT

3.1 Internal Rate of Return (IRR)

According to Okafor (1983: 224), the IRR criterion follows the basic principles of the NPV method. Unlike the NPV method, the IRR does not use an exogenously determined (exogenously to the project being considered) discount rate. Rather, the principle is to find a rate of discount that will match the discounted value of cash inflows and

outflows. The rate of discount, which achieves that equality, is the internal rate of return.

Put differently, the internal rate of return is the rate at which NPV is zero; the rate, at which the present value of the cash inflows is equal to those of the outflows, and the hurdle rate or the break-even rate.

According to Lucey (1988: 414), alternative names for the IRR include DCF yield, marginal efficiency of capital, trial and error method, discounted yield and the actuarial rate of return.

According to Okafor (1983: 224) and Van Home (1986: 130), the IRR is derived mathematically by solving the following equation for r:

$$\sum_{t=0}^{n} \frac{CF_t}{(I=r)^t} = 0$$

For conventional projects, the equation becomes:

$$\sum_{t=0}^{n} \frac{CF_t}{(I=r)^t} = CF_0$$

3.2 Investment Criteria under the IRR Approach

Under the IRR, the investment criteria are as follows.

- Invest if IRR > cost of capital. That is invest if the internal rate of return is more than the cost of capital.
- Do not invest if the IRR < cost of capital. That is, do not invest if the internal rate of return is less than the cost of capital.
- Remain indifferent if IRR = cost of capital.

Illustration

Refer to illustration 2 under Module 2 unit 6. Compute the Internal Rate of Return (IRR).

Solution

Trial and Error: Let us try 20% since 15% gives NPV of ₦1, 997,000

Year	**Cash flow ₦'000**	**20% Discount ₦'000**	**Net Present ₦'000**
0	-20,000	1,000	-20,000
1	12,400	0.833	10,329
2	6,000	0.694	4.164
3	7,100	0.579	4,111
4	2,203	0.402	1,061
5	2,774	0.402	1,061
6	-1,477	.0335	-495
			285

Let us try 22%

Year	**Cash flow ₦'000**	**20% Discount ₦'000**	**Net Present ₦'000**
0	-20,000	1,000	-20,000
1	12,400	0.820	10,168
2	6,000	0.672	4.032
3	7,100	0.551	3,912
4	2,203	0.451	994
5	2,774	0.370	1,026
6	-1,477	0.303	-447
			-315

Since IRR lies between positive and negative numbers, it should lie between + 286 and –315.

Hence, using the formula to calculate the IRR, we have:

$$IRR = x + \left|\frac{a}{a+b}\right|(y - x)$$

where x = the lower rate of interest used

- Y = the higher rate of interest used
- a = the absolute NPV at X%
- b = the absolute NPV at Y%
- II = modulus i.e. assume every figure to be positive.
- IRR = Internal Rate of Return

Using the above formula, we have:

20% + [285/ (285 + 315)] x (22 – 20)

= 20 + (285 x 2)/600

= 20 + 0.95

∴ IRR = 20.95%

This is the highest cost of capital, which could be used on the project. As a check, calculate the NPV with 20.95% as your cost of capital.

Proof: **Femi Nig. Ltd.**

Year	Cash flows ₦	DF @ 20.95%	PV ₦
0	(20,000)	1.000	(20,000)
1	12,400	0.827	10,255
2	6,000	0.684	4,104
3	7,100	0.565	4,012
4	2,203	0.467	1,029
5	2,774	0.386	1,071
6	(1,477)	0.319	(471)
			0

Illustration

Haruna Nigeria Ltd.

An investment is being considered for which the net cash flows have been estimated as follows:

Year 0	Year 1	Year 2	Year 3	Year 4
₦	₦	₦	₦	₦
-9,500	3,000	4,700	4,800	3,200

What is the NPV if the discount rate is 20%? Is the project acceptable? Calculate the IRR.

Solution

From the table, at r = 20%

The discount factors are 0.833, 0.694, 0.579 and 0.482

∴ NPV = -9500 + (0.833 x 3000) + (0.694 x 4700) + (0.597 x 4800) + (0.482 x 3,200) = + ₦582.

Since, the NPV is positive, the project is acceptable. To calculate the IRR, we try higher rate say 25%. The NPV if r = 25% is calculated thus:

Year	**Cashflow ₦'000**	**20% Discount ₦'000**	**Net Present ₦'000**
0	-9500	1,000	-9,500
1	3,000	0.8000	2,400
2	4,700	0.6400	3,008
3	4,800	0.5120	2,458
4	3,200	0.4096	1,311

That gives NPV = -323.

The IRR can be calculated as follows:

$$IRR = 20\% + 5\% \left(\frac{582}{905}\right) = 23.22\%$$

a b c d

where:

- is a discount rate, which gives a positive NPV? In this example, 20% gives ₦-582.
- is the difference between (a) and the rate, which gives a negative NPV? In this example, 25% - 20% = 5%.
- is the positive NPV at the discount rate chosen in (a)? In this example, it is 582?
- is the total range of NPV at the rates chosen? In this example, + 582 to – 323 = 905?
  (Lucey 1988: 415)

Illustration

MPC (Megini Prince Chima) has been looking for a suitable investment which will give a target internal rate of return of 17 to 20%. An investment adviser has offered the company a project, the details of which are given below.

Pineapple Squash Bottling Project

Initial investment involves purchase of machinery for ₦1, 800,000 and installation expenses of ₦310, 000. The plant can produce ₦100,000 cartons of pineapple squash per annum, during the first two years, rising to125,000 cartons per annum, for the next three years. Cost of

production of each carton, excluding depreciation costs is ₦21 and the selling price will be ₦27. The plant will be scrapped at the end of the 5th year and is expected to have negligible scrap value.

You are required to calculate the actual internal rate of return of the above project. You may ignore the effect of taxation.

Solution

Present Value factor = $(1 + r)^n$

where r is the rate; and
n is the number of year.

Solution

i. Pineapple Squash Bottling Project

Year	**Cash flow ₦**	**PV Factor 17%**	**Present Value**	**PV Factor 20%**	**Present Value**
0	(211,000)	1.000	(211,000)	1.000	(211,000)
1	60,000	0.855	51,300	0.833	49,980
2	60,000	0.731	43,860	0.694	41,640
3	75,000	0.624	46,800	0.579	36,150
4	75,000	0.534	40,050	0.482	36,150
5	75,000	0.456	34,200	0.402	30,150
Net present value			5,210		(9,655)

Cash flow – years 1 and 2 (₦27 – 21) x ₦10, 000 = ₦60, 000

Cash flow – years 3, 4, and 5 (₦27 – 21) x ₦12, 5000 = ₦75,000

N/B: This is a neat way of determining the cash inflows.

Actual Rate of Return = $a + \frac{c}{c + d} (b - a)$

Where a = the low discount rate
b = the high discount rate
c = the low rate of present value
d = the high rate of net present value

$$17 + \frac{5210}{5210 + 9655} (20 - 17)$$

$$= \quad 17 + \frac{5210 \times 3}{14865}$$

$$= \quad 17 + 1.05 \quad = \quad 18\%$$

4.0 CONCLUSION

This method called Internal Rate of Return is also very important. Even with the possibility of multiple rates, it is still very important.

5.0 SUMMARY

In this unit, you have learnt the various definitions of Internal Rate of Return (IRR). You are now aware of computational techniques and the investment criteria.

6.0 TUTOR-MARKED ASSIGNMENT

International Oil Ltd., a refinery consortium, which specialises in the running of refinery plants in Africa, is considering setting up a refinery plant in Port Harcourt.

Information available shows that only two types of refineries can thrive in the Nigerian environment. These include the followings.

i. FCC based refinery
ii. Hydro-cracking type

The following investment information is also available in respect of the two projects.

	Fluid Catalytic Cracking (FCC) Refinery	**Hydro Cracking Refinery (HC)**
Cost of installation	₦2,600 million	₦1,500 million
annual productivity (Average)	100 million metric tons (MT)	60 million metric tons (MT)
Plant life (years)	15	16
Processing fees ₦/Mt (by the refinery	5.00	5.00
Commencement of production	Year 3	Year 3
Annual operating cost other information	₦30 million	₦20 million

Cost of capital is assumed to be 15%

Cash outflows for the refineries are set out below:

Year	**FCC Million**	**HC Million**
Immediately	500	400
Year 1	1,000	900
Year 2	600	200
Year 3	500	----
	2,600 million	1,500 million

Ignore taxation.

You are required to:

- determine the approximate IRR of the two refinery projects (8 marks)
- determine the more viable of the two refineries (4 marks)
- determine other factors that need to be considered in the construction and management of the refinery plant (4 marks)

Total (16 marks)

Rate of Discount

- 0.870
- .756
- .658
- .572
- .497
- .432
- .376
- .324
- .284
- .247
- .215
- .187
- .163
- .141
- .123
- .107
- .093
- .081

(ICAN, May, 1997, Q5).

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UNIT 4 INTERNAL RATE OF RETURN II

CONTENTS

1.0 INTRODUCTION

In the previous unit, you learnt that the computation of IRR involved a lot of trial and error except when y are using computers. Therefore, any discussion that could considerably reduce the quantum of trial and error shall be a welcome development.

Accordingly, this unit concerns itself with developing short cuts to trial and error approach.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- outline the steps to be adopted in order to find an alternative to pure "trial and error" in applying the IRR method
- read the annuity table
- apply the annuity table in solving IRR problems
- explain that at times IRR could yield multiple rates.

3.0 MAIN CONTENT

3.1 Short Cut to IRR Computation?

Since the calculation of IRR is based on trial and error, any technique to minimise the extent of the trial and error would be highly appreciated. The following steps would be helpful.

Step 1: Sum up the cash inflows
Step 2: Find the average of the cash inflows. Let this be x

Step 3: Given that the cash outflow occurred in year zero and taking year zero as the focal date, we then establish an equation of values,

Thus x and i = CFO

Where x = the average of cash inflows

And i = annuity factor for a given value of n and i

CFO = cash outflow in year 0.

Step 4: From the annuity table (present value) read up the nearest (most approximate) rate in which annuity factor at the given value of n is very close to the quotient

Step 5: The rate obtained in Step 4 above becomes the base rate.

Step 6: Compute the NPV using the rate as the discount rate.

Step 7: If the NPV derived from above is positive, a higher rate of discount is tried and if negative, a lower rate is tried.

Step 8: Upon arriving at two rates, one having a positive NPV and the other a negative NPV, resort to interpolation viz:

$$IRR = x + \left|\frac{a}{a+b}\right|(y-x)$$

where IRR = internal rate of return

x = the lower rate
a = NPV at x
y = the higher rate
b = NPV at y
II = modulus sign (meaning assume every figure to be positive).

Illustration

Anulika Nig. Ltd. is considering investing in a project which cash flows were as follows:

Year 0 ₦**'000**	**Year 1** ₦**'000**	**Year 2** ₦**'000**	**Year 3** ₦**'000**	**Year 4** ₦**'000**	**Year 5** ₦**'000**
-144	+15	+25	+35	+45	+ 60

Given that the cost of capital is at 10% per annum, should Anulika Nig. Ltd. invest in it or not – using the IRR approach?

Solution

To minimise the extent of the trial and error, the above eight steps are then sequentially followed in the following way.

Step 1: Sum up the cash inflows ₦ (15,000 + 25,000 + 3,500 + 45,000 + 60,000) = ₦180, 000

Step 2: Find the average of the cash inflows: the average is ₦180, 000 ÷ 5 = ₦36, 000

Step 3: Given that the outflow occurred in year zero and taking year zero as the focal date, we then establish an equation of values, thus:

X an i = CFO

Substituting

36,000 as 5 i = 144,000
∴ a 5 i = 4.00

Step 4: From the annuity table (present value) read up the nearest (most approximate) rate which annuity factor at the given value of n is very close to the quotient, CFO: X

Substituting

144,000: 36,000 = 4.00

From the annuity table (present value) given that n = 5 and a5 i = 4.00
The nearest values of i are 7% (4.100) and 8% (3.993).

Step 5: The rate obtained in Step 4 above becomes the base rate. In this case 8%

Step 6: Compute the NPV using the base rate at the discount rate.

Year	**Cash flow ₦'000**	**DCF @ 8% ₦'000**	**Present Value ₦'000**
0	-144	1,000	-144.00
1	15	0.926	13.89
2	25	0.857	21.43
3	35	0.794	27.79
4	45	0.735	33.08
5	60	0.681	40.86
		NPV =	-6.95

Step 7: If the NPV derived in Step 6 above is positive, a higher rate of discount is tried and if negative, a lower rate is tried.

Accordingly, let's try lower rate say 6%

Year	**Cash flow ₦'000**	**DCF @ 8% ₦'000**	**Present Value ₦'000**
0	-144	1,000	-144.00
1	15	0.943	14.15
2	25	0.890	22.25
3	35	0.839	29.37
4	45	0.792	35.64
5	60	0.747	44.82
		NPV =	-2.23

Step 8: Upon arriving at the two rates, one having a positive NPV and the other a negative NPV, resort to interpolation viz:

$$IRR = x + \left|\frac{a}{a+b}\right|(y-x)$$

where IRR = internal rate of return
x = the lower rate
a = NPV at x
y = the higher rate
b = NPV at y
II = modulus

Substituting:

$$IRR = 6 + \frac{2.23}{2.23 + 6.95}(8-6)$$

$$= 6 + \frac{2.23 \times 2}{9.18}$$

$$= 6 + 0.4858$$

$$= 6.486$$

As a check, let's now compute the NPV given that the discount rate = 6.486%.

Year	Cash flow ₦'000	DCF @ 6.486% ₦'000	Present Value ₦'000
0	-144	1,000	-144.00
1	15	0.9391	14.0865
2	25	0.8819	22.0475
3	35	0.8282	28.987
4	45	0.7777	34.999
5	60	0.7304	43.824
		NPV =	-0.056*

- For all practical purposes, the NPV at IRR should be zero. However, occasionally, one could record a negligible negative or positive NPV (-0.056 in this case) due to rounding up of error.

4.0 CONCLUSION

In this unit, you have learnt that the computation of Internal Rate of Return (IRR) is essentially anchored on "trial and error method". An attempt was made to develop a short cut that is reliable.

5.0 SUMMARY

In this unit, you have learnt about a short-cut to the "trial and error" approach to IRR computations.

6.0 TUTOR-MARKED ASSIGNMENT

Recompute illustrations 2 and 3 (Module 2, unit 4.), by applying the short cut.

7.0 REFERENCES/FURTHER READING

Horngren, C. T., Datar, S. & Foster, (1997). *Cost Accounting: A Managerial Emphasis*. New Delhi: Prentice Hall.

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UNIT 5 THE PROFITABILITY INDEX

CONTENTS

1.0 INTRODUCTION

As scientific as the Net Present Value (NPV) approach to investment appraisal may appear to be, it has one major limitation– it fails to consider the quantum of capital that generated the NPV. This is a major weakness since ordinarily; a higher capital base will generate a higher NPV. Logically, therefore, a relative NPV or better still, an NPV per unit of capital base would give a better evaluation results. This is where the Profitability Index (PI) comes from.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the meaning of profitability index (PI)
- state the formulae for PI
- apply the formulae for PI
- outline the merits and the demerits of PI
- compare the IRR method with the NPV method.

3.0 MAIN CONTENT

3.1 Profitability Index or Excess Present Value Index (EPV I)

There are two possible formulae to calculate this index.

a. According to Okafor (1983: 229), the profitability index (PI) of a project is the ratio of the sum of the present values of all its cash inflows to the sum of the present values of its cash outflows, i.e.

$$PIi = \frac{P\ v\ i}{Ci}$$

Where

PIi = profitability index of project i
Pvi = sum of present value of cash inflows from project I
Ci = sum of present value of cash outflows of project i.

b. According to Lucey (1988: 419), the EPVI is merely a variant of the basic NPV method and is the ratio of the NPV of a project to the initial investment.

i.e. $$EPVI = \frac{NPV}{\text{Initial Investment}}$$

Thus, the index is a measure of relative and not absolute profitability. Because of this, it suffers from the same general criticisms when used for ranking purposes as the IRR.

Decision rule

The decision rules for the profitability index are as follows.

- Accept only projects that have profitability index of more than 1 (one)
- Reject projects that have profitability index of less than one
- Remain indifferent if the index is zero.

For the excess present value index, the decision rules are as follows.

- Accept only projects which EPVI is positive
- Reject projects which EPVI is negative
- Remain indifferent if the EPVI is zero.

SELF-ASSESSMENT EXERCISE

Akachukwu Company is considering five different investment opportunities. The company's cost of capital is 12 percent. Data on these opportunities under consideration are given below.

Project	**Investment ₦'000**	**PV at 12% ₦'000**	**NPV ₦'000**	**IRR ₦'000**	**Profitability Index ₦'000**
a.	35,000	39,325	4,325	16	1.12
b.	20,000	22,930	2930	15	1.15
c.	25,000	27,453	2,453	14	1.10
d.	10,000	10,854	854	18	1.09
e.	9,000	8,749	(251)	11	0.97

i. Rank the five projects in descending order of preference, according to:

- NPV (Net Present Value)
- IRR (Internal Rate of Return)
- Profitability Index.

Which ranking would you prefer?
Based on your answer in part 2, which projects would you select if ₦55, 000,000 is the limit to be spent?

Solution

i. Akachukwu Company

Order of Preference	**NPV**	**IRR**	**Profitability Index**
a.	1	2	2
b.	2	3	1
c.	3	4	3
d.	4	1	4
e.	5	5	5

ii. The profitability index approach is generally considered the most dependable method of ranking projects competing for limited funds. It is an index of relative attractiveness, measured in terms of how much you get out for each naira invested.

iii. Based on the answer in part 2, projects (a) should be selected, where combine NPV would be ₦7, 255 (₦2,930 + ₦4,325) with the limited budget of ₦55,000,000.

4.0 CONCLUSION

In this unit, you have learnt that Net Present Value (NPV) has one major weakness when one is faced with two or more projects – it fails to take into consideration the quantum of capital outlay that generated the NPV. This is a weakness because huge capital outlays are likely to have huge NPV relative to small capital outlay. This is where the Profitability Index (PI) comes in hence; PI is defined as NPV per unit of capital.

5.0 SUMMARY

In this unit, you have looked at the basic definition of profitability index. You also looked at the computational techniques and the investment criteria.

6.0 TUTOR-MARKED ASSIGNMENT

United Development Corporation has ₦2.5million naira available for investment in projects. The following projects are under consideration.

Project No	**Initial ₦**	**Annual ₦**	**Life**
1.	800,000	230,000	6 years
2.	600,000	190,000	6 years
3.	700,000	210,000	6 years
4.	900,000	240,000	6 years
5.	300,000	92,000	6 years
6.	950,000	300,000	6 years

The corporation expects a minimum rate of return of 18%. Projects Nos. 2 and 5 are complementary to each other. They have to be accepted together or rejected together. Projects Nos. 2 and 5 are mutually exclusive due to their nature.

You are required to:

i. calculate profitability of all the six projects (6 marks)
ii. advise the corporation on selection of projects to maximise profitability, bearing in mind that only ₦2.5 million capital is available (6 marks)

Note: Present value of annuity of ₦1 for the next 6 years at 18% is ₦3.497 (ICAN, Nov. 1999, Q3).

7.0 REFERENCES/FURTHER READING

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MODULE 3

UNIT 1 THE IMPACT OF INFLATION ON INVESTMENT PROPOSALS

CONTENTS

1.0 INTRODUCTION

Inflation can be simply defined as an increase in the average price of goods and services. The accepted measure of general inflation in Nigeria is the Retail Price Index (RPI) which is based on the assumed expenditure patterns of an average family. General inflation is a factor in investment appraisal but of more direct concern is what may be termed **specific inflation** i.e. the changes in prices of the various factors which make up the project being investigated, e.g. wage rates, sales prices, material costs, energy costs, transportation charges and so on.

Every attempt should be made to estimate specific inflation for each element of the project in as detailed a manner as feasible. Generally, overall estimates, based on the RPI, are likely to be inaccurate and misleading.

Synchronised and Differential Inflation

Differential inflation is where costs and revenues change at differing rates of inflation or where the various items of cost and revenue move at different rates. This is normal but the concept of synchronised inflation - where costs and revenues rise at the same rate - although unlikely to be

encountered in practice, is useful for illustrating various facets of projects appraisals involving inflation.

Money cash flows and real cash flows

Money cash flows are the actual amounts of money changing hands whereas real cash flows are the purchasing power equivalents of the actual cash flows. In a world of zero inflation, there would be no need to distinguish between money and real cash flows as they would be identical. Where inflation does exist, then a difference arises between money cash flows and their real value and this difference is the basis of the treatment of inflation in project appraisal.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define inflation
- explain the meaning of money and real cash flows
- explain the effects of inflation on investment appraisal.

3.0 MAIN CONTENT

3.1 Dealing with Inflation

The following example will be used to illustrate the way that inflation occurs in an investment appraisal.

Example One

A labour saving machine costs ₦24, 000 p.a. at current wage rates. The machine is expected to have a 3 year life and nil scrap value. The firm's cost of capital is 10%.

Calculate the project's NPV:

a. with no inflation
b. with general inflation of 15% which wage rates are expected to follow (i.e synchronised inflation)
c. with general inflation of 15% and wages rising at 20% p.a. (i.e differential inflation)

Solution

a. NPV - no inflation

$$-60{,}000 + 24{,}000\ A_{3]10\%}$$

$$= \quad -60{,}000 + 24{,}000 \times 2.487 \quad = \quad \textbf{₦312}$$

Project unacceptable as it has a negative NPV at company's cost of capital

b. General inflation 15%, wages increasing at 15%

Wage Savings p.a. with no inflation	**Wage Savings p.a. with 15% inflation**
24,000	27,600
24,000	31,740
24,000	36,501

With no inflation, the appropriate discounting rate was 10%. With inflation at 15%, the 10% discounting rate is sufficient to bring cash sums arising at different periods into equivalent purchasing power terms. Without inflation ₦1 now was deemed equivalent to ₦1.10 (1.15) = ₦1.265, thus the discount rate to be used is 26½%.

Project NPV with 15% Synchronised Inflation

Year	**Cash Flow**	**26½% Discount**	**Present**
0	-60,000	1,000	-60,000
1	+27,600	0.792	21,859
2	+31,740	0.624	19,806
3	+35,501	0.494	18,031
		₦PV =	**₦304**

∴ Project unacceptable

It will be seen that the answers with no inflation and with 15% synchronised inflation are virtually the same, (the difference being due to rounding up in three figure tables). This equivalence is to be expected, as with synchronised inflation, the firm, in real terms, is no better or no worse off.

c. Project with 15% general inflation and wages rising at 20% p.a. (differential inflation.)

	Wage per annum	
Year 1	24,000(1.20)	= 27,600
2	$24,000(1.20)^2$	= 31,740
3	$24,000(1.20)^3$	= 36,501

Project NPV with Differential Inflation

Year	**Cash Flow**	**26½% Discount**	**Present**
0	-60,000	1.000	-60,000
1	+28,800	0.792	22,810
2	+34,560	0.624	21,565
3	+41,472	0.494	20,487
		NPV =	**₦4,862**

∴ Project acceptable

Thus it will be seen that with differential inflation, the project is acceptable. In this case, this is to be expected because it was a labour saving project so that in real terms, the firm is better off if the rate of wage inflation is greater than the general rate of inflation.

Frequently, differential inflation works to the disadvantage of the firm, for example, when costs are rising faster than prices. Each case is different and detailed, individual analysis is required - not generalised assumptions.

3.2 Money and Real Discount Rates

The 26½% discount rate used in Example 1 was a money discount factor and was used to discount the money cash flows of the project. The relationship between real and money discount factors is as follows.

$$\text{Real discount factors} = \frac{1 + \text{Money discount factor}}{1 + \text{Inflation Rate}} - 1$$

Using the data from example 1 the real discount factor can be calculated.

$$\textbf{Real Discount Factor} = \frac{1 + 0.265}{1 + 0.15} - 1 = 0.1 \text{ i.e. } 10\%$$

In this case, of course, the real discount factor was already known and the above calculation was for illustrative purpose only.

The real discount factor can be used for project appraisal provided that the money cash flows are first converted into real cash flows by discounting at the general inflation rate as follows.

Example Two

Re-work part (c) of example 1 using real cash flows and the real discount factor.

Real Cash Flow Evaluation					
Year	**Money Cash Flow**	**General Inflation 15% discount**		**26½% Discount**	**Present**
0	-60,000	1.000	-60,000	1.000	-60,000
1	+28,800	0.870	25,056	0.909	22,776
2	+34,560	0.756	26,127	0.826	21,581
3	+41,472	0.658	27,289	0.751	20,494
				NPV =	**N4,851**

It will be seen that the two methods give identical results.

Thus it will be seen that there are two approaches to investment appraisal where inflation is present.

Single Discounting: Money cash flows discounted by money discount factor.

Two Stage Discounting: Money cash flows discounted by general inflation rate and then the real cash flows produced discounting by real discount factor.

The two approaches produce the same answer because the money discount factor includes the inflation allowance. However, because of this and because money cash flows are the most natural medium in which estimates will be made, it is recommended that money cash flows should be discounted at an appropriate money discount factor. Take great care never to discount money cash flows by a real discount factor or real cash flows by a money discount factor. If real cash flows are directly provided in a question, take care to discount once only using a real discount factor.

Illustration

A firm is considering a project with a cash outlay of ₦1, 000,000 now and 5 yearly cash flows of ₦500, 000.

i. What is the NPV at 10%?
ii. What is the NPV assuming a general inflation rate of 8% and an increase in cash flows to ₦510, 000 per annum?

Solution

i. Given that the cash inflow is constant, we now apply the annuity factor (present value). At 10% per annum for 5 years, the annuity factor present value is arrived at by evaluating

$$\frac{1 - (1.10)^{-5}}{0.10} = 3.79$$

$$\therefore \quad NPV = (1{,}000{,}000) + 3.79 \times 500{,}000$$
$$= (1{,}000{,}000) + 1{,}895{,}393$$
$$= + ₦895, 393$$

ii. Discount rate with 8%
Inflation = 1.10(1.08) = 1.188
= 19%
With cash outlay at ₦1m and cash inflow at ₦510, 000 for 5 years discount rate at 19%
Annuity factor (present value)

$$\frac{1 - (1.19)^{-5}}{0.19} = 3.058$$

$$\therefore \quad \text{The NPV} = (1{,}000{,}000) \times 3.058 \times 510{,}000$$
$$= (1{,}000{,}000) \times 1{,}559{,}393$$
$$= 559{,}393$$

4.0 CONCLUSION

In this unit, you have learnt that since price levels always change continuously, there is the need to evaluate investments under inflationary trends.

5.0 SUMMARY

In this unit, you have learnt the basic definition of inflation and other associated terms namely.

- Retail Price Index (RPI)
- Specific inflation
- Synchronised and differential inflation
- Money cash flows and real cash flows.

You also had some illustrations on how to evaluate investments under an inflationary trend.

6.0 TUTOR-MARKED ASSIGNMENT

1. Chief Ugwoke is considering a project with a cash outlay of ₦5, 000,000.00 now and 5 yearly cash inflows of ₦2, 500,000.00.
2. What is the APV assuming a general inflation rate of 8% and an increase in cash flow are up to ₦2, 550,000.00

7.0 REFERENCES/FURTHER READING

Lucey, T. (2002). *Quantitative Techniques*, London: MPG Books Ltd.

Nweze, A.U. (2004). *Quantitative Approach to Management Accounting*. Enugu: Amazing Grace Publishers.

UNIT 2 THE USE OF PROBABILITIES TO ASSESS THE IMPACT OF RISK ON CAPITAL INVESTMENTS

CONTENTS

1.0 INTRODUCTION

In this unit, you shall be looking at the probability estimate of cash flows.

Environmental Conditions

Investment decisions are carried out under one of the following three possible environmental conditions:

i. condition of certainty
ii. condition of risks and
iii. condition of uncertainty

i. **Condition of certainty**
Condition of certainty can be said to prevail where a potential investor has full knowledge of the ultimate outcome of an investment opportunity. This implies that:

a. perfect knowledge, from the onset, of the exact nature and timing of the stream of cash flow to be expected from the investment opportunity.
b. the expectation (belief) that the anticipated (ultimate) outcome would not change.

A situation of single value expectation is however rare in the investment world (Okafor, 1983).

ii. **Condition of risks**

Where an investor knows exactly the range of possible outcomes to expect from an investment opportunity, as well as the likelihood (probability) of each outcome, the investor is exposed to a condition of risk

Since conditions of certainty are rare in the investment world, this part of the text would focus mainly on conditions of risk and uncertainty. After all, freedom from uncertainty is a luxury rarely enjoyed by the contemporary management.

Methods of incorporating uncertainty and risk when appraising projects

The major business risk faced by business organisations in the appraisal of long-term capital project is the possibility that actual outcome of such investments will deviate from forecasts used in the appraisal overtime.

The methods of analysing risk and the uncertainty of capital expenditure projects include the followings.

- Risk premium method
- Adjusted payback period
- Simulation modelling
- Probability estimates of cash flows
- Sensitivity analysis
- Certainty equivalent
- Standard deviation of the expected Net Present Value [NPV]
- Worst possible and best possible outcomes
- Portfolio theory
- Decision tree
- Finite horizon method

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define probability
- explain the following terms, namely, expected value, standard deviation and co-efficient of variation as applied in investment appraisal
- evaluate investments by applying probability estimates.

3.0 MAIN CONTENT

3.1 Illustration of Probability

This section is best discussed with an illustration.

The General Manager of a company is confronted with the option of embarking on one project out of the two projects which are as follows.

Project A		Project B	
Project ₦'000	**Probability**	**Profit ₦'000**	**Probability**
20	0.40	10	0.20
22	0.30	27	0.70
30	0.30	35	0.10

Prepare a report advising the General Manager on which of the two projects to embark upon. Consider the use of coefficient of variation in choosing among the two projects.

Source: Management Accounting, ACA, Nov. 1988 Q7

Comment

The above problem is based on the probability estimates of cash flows. Accordingly, we shall use the problem to illustrate probability estimates in greater details.

3.2 Probability Estimate of Cash Flows

Probability has been defined as the qualification of uncertainty. Probabilities have values ranging from zero (0) to one (1).

A probability of zero means that the event can never take place. For example, flying to the moon unaided cannot be contemplated. This we can express as p (flying to the moon unaided) = 0.

A probability of one means that the event must surely take place. For example, every mortal must die. This we can express as:

P (dying) = 1

Going down the memory lane, let’s now use our knowledge of statistics to establish the formula for probability approach to uncertainty.

3.3 Frequency Distribution

Since the value of probabilities can only lie between 0 and 1, it follows that the probabilities of any given event must necessarily sum up to 1.

That means, given a set of cash flows with their associated probabilities, the sum of probabilities and hence the sum of the frequencies must necessarily be 1.

Hence,

$$\Sigma f = \Sigma p = 1$$

It will be recalled that for a grouped data, $n = \Sigma f$.
Hence, for our probability estimates of cash flows

$$n = \Sigma f = \Sigma p = 1.$$

Average (Expected Value)

For a grouped data, the average (x), is given as:

$$X = \frac{\Sigma X}{\Sigma f} = \frac{\Sigma px}{\Sigma p} = \Sigma px \text{ (Since } \Sigma p = 1).$$

In probability estimates of cash flows, the average is known as the expected value. Hence our expected value, EV is given as

$$EV = \Sigma px = \bar{X}$$

Standard Deviation

For grouped data, the standard deviation, d, is given as:

$$d = \frac{\sqrt{\Sigma f (X - X)^2}}{\Sigma f}$$

For probability estimates of cash flows, our formula for standard deviation becomes:

$$d = \frac{\sqrt{\Sigma f (X - X)^2}}{\Sigma f}$$

$$= \frac{\sqrt{\Sigma f (X - EV)^2}}{\Sigma p}$$

$$= \sqrt{\Sigma f (X - EV)^2}$$
(Since $\Sigma p = 1$).

3.4 Interpretation of the Value of Standard Deviation

Since the residuals (the extent to which the various observed values of Y deviate from the regression line) are assumed to have a constant variance, the higher this constant variance, the more widely the various observed values of Y are scattered and hence the higher the standard deviation.

It therefore follows that the higher the standard deviation the higher the risk.

This can further be illustrated using our experience in soccer.

When a team concedes an indirect kick (very close to the 18) many of the players would come together and form a wall of defence. This way, they are not widely scattered and the standard deviation is low and as a corollary, the risk of conceding a goal is equally reduced.

Posers. What do you think is the relationship between distant marriage and the chances of extra-marital affairs?

Which one is easier – breaking a stick of broom or breaking a bunch of brooms?

Let's now solve the problem at hand, bearing the above discussion in mind.

Solution

Expected Value (EV) of Profits

$EV = \Sigma px$

Where p = probability

Project A			**Project B**		
Project ₦'000	**Probability**	**Expected Value ₦**	**Profit ₦'000**	**Probability**	**Expected Value**
20,000	0.40	8,000	10,000	0.20	2,000
22,000	0.30	6,600	27,000	0.70	18,900
30,000	0.30	9,000	35,000	0.10	3,500
		23,600			**24,400**

Comment

Based on the expected value of profits, it would appear that project B should be favoured to project A because project B has a higher expected value of profits. This measure however, does not take into account the risk in the projects, as measured by the dispersion of the outcome, so that a better measure, such as standard deviation of the outcome, which takes the risks in project into accounts, is considered in selecting the most viable of the projects.

b. Standard deviation of profits

Standard deviation d is calculated by using the formula:

d = $\sqrt{\Sigma p (X - X)^2}$
d = $\sqrt{\Sigma p (X - EV)^2}$
d = Standard figures
x = profit figures
p = probability

Project A					
X **₦**	**Prob.** **₦**	**PX** **₦**	**X – X** **₦**	**$(X - X)^2$** **₦**	**$P(X - X)^2$** **₦**
20,000	0.40	8,000	- 36000	12,960,000	5,184,000
22,000	0.30	6,600	- 1600	2,560,000	768,000
30,000	0.30	9,000	+ 6400	40,960,000	12,288,000
	0	**23,600**	Variance =		**18,240,000**

d = √Variance = √18240000
= ₦427.838301

Y **₦**	**Prob.** **₦**	**PY**	**Y – Y**	**$(Y - Y)^2$**	**$P (Y - Y)^2$**
10000	0.20	2000	- 4400	270,360,000	41,472,000
27000	0.70	18900	- 2600	6,760,000	47,320,000
35000	0.10	35000	10600	112,360,000	11,236,000
	0	**24400**	**Variance**		**57,440,000**

d = √Variance = √57440000
= ₦7578.918

Comment

The above shows that project A has a lower degree of risk in absolute terms than project B. The degree of risk measured by the standard

deviation shows the degree of dispersion of the profits of the projects around the means profit or the expected value of profits of the projects. Since project A shows a lower degree of dispersion in absolute terms, the project is considered less risky than project B, which shows a higher standard deviation of a project.

This measure, however, is not a relative measure as it does not take the size of the projects or the expected value of the projects into consideration. A more reliable relative measure is the coefficient of variation, which relates the standard deviation of the size of the project and its expected value.

3.5 Co-efficient of Variation

The formula for calculating the co-efficient of variation =

$$\frac{d}{X} \times \frac{100}{1}$$

Where

d = standard deviation
X = mean or expected value of profits.

Co-efficient of Variation

PROJECT A	PROJECT B
₦4,270.83130	₦7,578.91813
₦23,600	₦24,400
= 0.18096	= 0.31061

Comment

From the above, it may be concluded that project B is considered riskier than project A, and should therefore be foregone in preference to project A. (MAYO).

Illustration

Usman Nig. Ltd is considering a major investment in a new productive process. The total cost of the investment has been estimated to be ₦2, 000,000 but if this is increased to ₦3, 000,000, the productive capacity could be substantially increased. Due to the nature of the process, once the basic plant has been established, to increase its capacity at a future date is exceptionally costly. One of the problems facing management is that the demand for process output is very uncertain. However, the

market research and finance departments have been able to produce the following first estimates.

Investment A (₦3m)		**Investment B (₦2m)**	
Demand Probability	**Annual Net Cash Inflow (₦M)**	**Demand Probability**	**Net Cash Inflow (₦M)**
0.3 yrs 1 – 4	1.0	0.4 yrs 1 – 4	0.6
5 – 10	0.7	5 – 10	0.2
0.5 yrs 1 – 4	0.8	0.4 yrs 1 – 4	0.6
5 – 10	0.4	5 – 10	0.2
0.2 yrs 1 – 10	0.1	0.2 yrs 1 – 4	0.2
Cost of Capital	15%	Cost of Capital	15%

You are required to prepare a statement, which clearly indicates the financial implications of each of the projects. Select the better investment.

Solution to Illustration

Usman Nig. Ltd.

Investment A				**Investment B**			
Year	**Cash Flow ₦m**	**Annuity Factor**	**Present Value ₦m**	**Year**	**Cash Flow ₦m**	**Annuity Factor**	**Present Value ₦m**
1 – 4	1.0	2.8550	2.855	1 – 4	0.60	2.855	1.713
5 – 10	0.7	2.164	1.515	5– 10	0.50	2.164	1.082
			4.370				2.795

Probability 2

Investment A				**Investment B**			
Year	**Cash Flow ₦m**	**Annuity Factor**	**Present Value ₦m**	**Year**	**Cash Flow ₦m**	**Annuity Factor**	**Present Value ₦m**
1 – 4	0.80	2.855	2.855	1 – 4	0.60	2.855	1.713
5 – 10	0.40	2.164	0.860	5– 10	0.50	2.164	0.433
			3.150				2.146

Probability 3

Investment A: Years 1 – 10. ₦0.10m x 5.019 = ₦50.19m
Investment B: Years 1 – 1

Expected NPV (Possible Outcome x Probability).

	Investment A	**Investment B**
1.	₦4.370 x 0.30m = ₦1.311	₦2.795 x 0.040m = ₦1.118m
2.	₦3.1496 x 0.50m= ₦1.5748	₦2.1458 x 0.40m = ₦0.85832
3.	₦0.5019 x 0.20m= ₦0.10038	₦1.0038 x 0.20m = ₦0.20076
Total present value =	₦3.0000m	₦21.1708m
Less initial outlay =	(₦0.01382m)	(₦0.177708m)

Decision: Accept Investment B with a positive NPV.

4.0 CONCLUSION

In this unit, you learnt the definition of probability. You are also exposed to the following terms: expected value, standard deviation and coefficient of variation as applicable to investment appraisal. Finally, you can now evaluate investment by applying probability estimates.

5.0 SUMMARY

This unit has exposed you to the conditions of certainty, uncertainty and risks in the probability estimates of cash flows. You are now aware of the methods for incorporating certainty, uncertainty and risks when appraising projects. Many examples had been cited to drive home the points discussed.

6.0 TUTOR-MARKED ASSIGNMENT

1. Two components are assembled to make a finished product. There is 0.2 probability that the first component will cost ₦50 and 0.8 probability it will cost ₦60. While there is 0.3 probability that the second component will cost ₦90 and 0.7 probability it will cost ₦120. The assembly cost is ₦21.

    a. Compute the expected cost of the finished product. (6 marks)

    b. Company ABC has predicted its costs and sales in respect of a product selling for ₦10 as follows.

i.	Marginal cost per unit	Probability
		0.6
		0.4
ii.	Fixed cost	Probability
	₦15,000	0.8
	₦20,000	0.2
iii	Sales unit	Probability
	₦4,000	0.5
	₦6,000	0.5

Compute the profit expectation (12 marks)
(Total 18 marks)
ICAN, Nov., 1998. Mgt Acc Q6.

Baro Pharmaceutical Nig. Ltd. can purchase the patents and the manufacturing rights of any one of the following three drugs. The cost of the rights are as follows.

Drug X = ₦260, 000
Drug Y = ₦380, 000
Drug Z = ₦400, 000

At the company's board meeting, the management accountant declared that it was general knowledge that the venture is a very short-term project. The fixed manufacturing and advertising cost of each venture will be.

	X ₦'000	**Y ₦'000**	**Z ₦'000**
Fixed manufacturing costs	240	40	40
Advertising costs	100	60	40

Sales and production will, once known, dovetail and therefore, there will be no stock build up. The sales price and variable costs per unit are:

	X ₦'000	**Y ₦'000**	**Z ₦'000**
Sales price per unit	680	630	260
Variable cost per unit	280	220	140

However, the sales volume is the key factor. The company does not know what the sales level will be but it knows the various probabilities of what the sale level could be. Drug X could be a complete flop, it could sell well, or it might sell very well.

Drug Y is also quite variable whereas with Drug Z, the range of outcome is very small. The various possible sales volumes and their associated probabilities are as follows.

Drugs					
X		**Y**		**2**	
Sales volume in unit	Probability	Sales volume in Unit	Probability	Sales volume in Unit	Probability
0	0.1	3,000	0.1	7,000	0.8
2,500	0.4	4,000	0.3	8,000	0.1
4,000	0.5	6,000	0.3	9,000	0.1
		8,000	0.3		

The company now wishes to decide on which of the drugs it should manufacture and sell.

Required

Calculate the expected money value of each drug and on the basis of this, advise the board on which drug to produce.

(16 marks)

(ICAN, Nov. 2003, Mgt. Acc. Q2)

7.0 REFERENCES/FURTHER READING

Nweze, A.U. (2006). "Investment Opportunities in the Nigerian Capital Market". Enugu: M'Cal Communications.

Okafor, F.O. (1983). Investment Decision: Evaluation of Projects and Securities. London: Cassell Ltd.

UNIT 3 SENSITIVITY ANALYSIS

CONTENTS

1.0 INTRODUCTION

Managers try to identify the critical variables in their capital budgeting decisions. The variables are normally - the cash flows, cost of capital and the life of the product. The objective is to find out how sensitive the project is in order to change any or all these variables.

Two ways can be identified for the treatment of sensitivity analysis.

a. To alter the value of the variables arbitrarily and find out if the decision on the project will change.
b. Alternatively, the analyst can calculate the percentage change in a variable that will result in a change of his decision on the project.

Shortcomings in sensitivity analysis (Multi Co-linearity)

There can be an inherent weakness in the sensitivity analysis.
In the sensitivity analysis, each element is varied individually. It is however, likely that interrelationships exist between many of the elements, and two or more elements will in reality vary simultaneously. "This is the problem of simultaneous relationships, or multi co-linearity, among the independent variables. It means simply that the independent variables are not really independent of one another, but rather have values that are jointly or simultaneously determined".

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define sensitivity analysis
- identify the key factors in any investment appraisal
- evaluate investments by varying the key factors in turn, and assessing their effects on the investment

- identify the advantages and disadvantages of sensitivity analysis.

3.0 MAIN CONTENT

3.1 Sensitivity Analysis

This is a practical way of showing the effects of uncertainty by varying the values of the key factors (e.g. sales volume, price, rates of inflation, and cost per unit) and showing the resulting effect on the project. The objective is to establish which of the factors affects the project most. When this is done, it is the management's task to decide whether the project is worthwhile, given the sensitivity of one or more of the key factors. It will be seen that this method does not ask for subjective probability estimates or likely outcomes, but attempts to provide the data upon which judgements may be made. The method is illustrated by the following example.

Example 5

Assume that a project (using single valued estimates) has a positive NPV of £25,000 at a 10% discounting rate. This value would be calculated by the normal methods using particular values for sales volume, sales price, cost per unit, inflation rate, length of life, etc.

Once the basic value (i.e. the NPV of £25,000) has been obtained, the sensitivity analysis is carried out by flexing, both upwards and downwards, each of the factors in turn.

An abstract of the results of a sensitivity analysis for the project above might be as follows.

Table 3

Sensitivity Analysis Abstract

Original NPV = £25,000

A **Element to be varied**	**B** **Alteration from basic**	**C** **Revised NPV £**	**D** **Increase + Decrease £**	**E** **Percentage Change**	**F** **Sensitivity Factor, i.e E/B**
Sales Volume (Basic value 8,000 units in Period 1 8,500 in Period 2, etc)	+15% +10% -10% -15% -20%	46,000 33,000 17,000 14,000 9,000	+21,000 +8,000 -8,000 -11,000 -16,000	84 32 32 44 64	5.6 3.2 3.2 2.9 3.2
Sales Price (Basic value £6 units in Period 1 £6.25 in Period 2, etc)	+20% +10% -10% -15% -20%	42,000 31,000 17,000 11,000 2,000	+17,000 +6,000 -8,000 -14,000 -23,000	68 24 32 56 92	3.4 2.4 3.2 3.73 4.6
Cost/unit (Basic value £2.50 in Period 1 £2.60 in Period 2, etc.	+25% +10% -5% -10%	-12,000 6,000 34,000 47,000	-37,000 -19,000 +9,000 +22,000	148 76 36 88	5.9 7.6 7.2 8.8

From such an analysis, the more sensitive elements can be identified. Once identified, further analysis and study can take place on these factors to try to establish the likelihood of variability and the range of values that might be expected so as to be able to make a more reasoned decision whether or not to proceed with the project.

Advantages of sensitivity analysis

a. shows the effect on project outcome by varying the value of the elements which make up the project (e.g. sales, costs, etc).
b. simple in principle.
c. enables the identification of the most sensitive variables.

Disadvantages of sensitivity analysis

a. gives no indication of the likelihood of a variation occurring.
b. considerable amount of computation involved.
c. only considers the effect of a single change at a time which may be unrealistic.

Illustration Two

Alhaji Shehu Buraimah is contemplating investing in a project and the following tentative estimates have been made.

Cash outlay	₦100,000 (in year 0)
Sales price/unit	₦30
Unit cost	₦20
Discount rate	10% p.a.
Life span of project	3 yrs.
Year	Sales volume
1	4,000 units
2	6,000 units
3	3,000 units

Required

a. Calculate the maximum tolerable unfavourable change in each of the areas (as a percentage of the original estimated value) in:

   i. sales unit
   ii. unit cost
   iii. sales volume
   iv. initial outlay
   v. project file

   Comment on the results. Could sales volume be treated separately in this analysis?

b. Now, assume that the government anti-inflationary policy allows sales prices to rise by 10% per annum compound, and unit costs are expected to rise at annual rate of 20% compound (both starting in year 0). What initial cash subsidy would be necessary to retain the viability of the project?

Discounting Factor at 10%

Year	PV
0	1.000
1	0.909
2	0.826
3	0.751

Source: Financial Management, ACA, Nov. 1988m Q2

Solution to Illustration

a. Computation of the Net Present Value (NPV)

Year Value	Cash Flow	Discount Factor	Present
	₦	**(DF) at 10**	**(PV)**
0	(100,000)	1,000	(100,000)
1	40,000	0.909	36,360
2	60,000	0.826	49,560
3	30,000	0.751	22,530
			₦8,450

Cash flows as computed viz:

Sales Volume x (Sales Price - Unit Cost)

Year	D.F	Sales	PV of Sales	Costs (₦)	PV of Costs
1	0.909	120,000	109,080	80,000	72,720
2	0.826	180,000	148,680	120,000	99,120
3	0.751	90,000	67,590	60,000	45,060
			325,350		216,900

i. Sensitivity to price

$$= \frac{\text{NPV}}{\text{PV of sales}} \times \frac{100}{1}$$

Substituting

$$\frac{8,450}{325,350} \times \frac{100}{1}$$

Comment

Sales price should not be reduced by more than 2.60% otherwise the project becomes unacceptable.

ii. Sensitivity to unit cost

$$= \frac{\text{NPV}}{\text{PV of sales}} \times \frac{100}{1}$$

Substituting

$$\frac{8,450}{216,900} \times \frac{100}{1}$$

Comment

The unit cost should not be increased by more than 3.8% otherwise the project becomes unacceptable.

iii. Sensitivity to sales volume

$$= \frac{\text{NPV}}{\text{PV of sales}} \times \frac{100}{1}$$

Where PV of contribution
= 325,350 - 216,900 = 108,450

Substituting

$$\frac{8,450}{108,450} \times \frac{100}{1}$$

$$= 7.8\%$$

Comment

A fall in contribution beyond 7.8% level would make the project unacceptable.

iv. Sensitivity of initial outlay

$$= \frac{\text{NPV}}{\text{PV of initial outlay}} \times \frac{100}{1}$$

$$= \frac{8,450}{100,000} \times \frac{100}{1}$$

Comment

The initial outlay should not increase beyond 8.45%.

v. Sensitivity of project life

Suppose the analyst assumes the project lasts for 2 years.

Year	Cash Flow (N)	DF @ 10%	PV (N)
0	(100,000)	1.000	(100,000)
1	40,000	0.909	36,360
2	60,000	0.826	49,560
			(14,080)

By Interpretation

$$2 + \frac{14{,}080 \quad (3 - 2)}{14{,}080 \quad + 8{,}450} = 2.63 \text{ years}$$

Sensitivity to product life

$$\frac{3 - 2.63}{3} = 12.30\%$$

Comment

The maximum tolerable reduction in a project life to retain viability is 12.3%

vi. Alternative

Method of interpreting the life span of a project

A reduction of one year in the life of a project will lead to a rejection of the project since the NPV arrived at is negative i.e. ₦14, 808.

Unless the analyst assumes that the net cash inflows accrue evenly over the period, it is not advisable to interpolate.

The analyst must assume that inflows after the initial outflow occur at the end of the years to which they relate. In this case, a reduction of the project life from three years to two years will be an unfavourable change and will alter our decision.

The percentage change will be:

$$\frac{3 - 2}{3} \times \frac{100}{1} = 33.33\%$$

The maximum tolerable change = 33.33%

General comment

The project is sensitive to all the factors tested above. The selling price and the unit cost should be watched, as the project is particularly sensitive to these. The other factors are relatively sensitive. In general, a slight change in the factors should have grave consequences on the viability of the project.

The sales volume could be treated separately in the analysis. The sensitivity of sales volume could be calculated for each of the respective years.

$$\text{Year 1} \quad \frac{\text{NPV}}{\text{PV contributed (year 1)}} \times \frac{100}{1}$$

$$= \frac{8{,}450}{36{,}360} \times \frac{100}{1}$$

$$= 23.24\%$$

$$\text{Year 2} \quad \frac{\text{NPV}}{\text{PV contributed (year 2)}} \times \frac{100}{1}$$

$$= \frac{8{,}450}{49{,}560} \times \frac{100}{1}$$

$$= 17.05\%$$

$$\text{Year 3} \quad \frac{\text{NPV}}{\text{PV contributed (year 3)}} \times \frac{100}{1}$$

$$= \frac{8{,}450}{22{,}530} \times \frac{100}{1}$$

$$= 37.50\%$$

b. Incorporating inflation effects

Year	Sales (₦)	Cost (₦)
1	4000 (1.1) x 30 = 132,000	4000 (1.2) x 20 = 96,000
2	6000 $(1.1)^2$ x 30 = 217,800	6000 $(1.2)^2$ x 20 = 172,800
3	3000 $(1.1)^3$ x 30 = 119,790	3000 $(1.2)^3$ x 20 = 103,680

Year	DF 10%	Cash Flow	PV (₦)
0	1.000	(100,000)	(100,000)
1	0.909	36,000	32,724
2	0.826	45,000	37,170
3	0.751	16,110	12,099
			(18,007)

Initial subsidy needed to retain the viability of the project is ₦18, 007.
(MAYO ASSOCIATES).

4.0 CONCLUSION

In this unit, you are exposed to the definition of sensitivity analysis. You are now able to identify the key factors in any investment appraisal. You also learnt how to evaluate investments by varying some key factors and assessing their effects on investments. Finally, you can now outline the advantages and the disadvantages of sensitivity analysis.

5.0 SUMMARY

In this unit, various factors in sensitivity analysis were treated. The factors identified were cash flows, cost of capital and the life of the product. You are also exposed to the shortcomings in sensitivity analysis. You are now aware of the fact that the independent variables are not really independent of one another, but rather, they have values that are jointly or simultaneously determined.

6.0 TUTOR-MARKED ASSIGNMENT

A company is considering the production of a new product. The relevant data about the product as revealed by research carried out by the company's marketing manager are as follows.

Contribution per unit ..... ₦4

Annual fixed costs ..... ₦10, 000

Break even sales volume ..... 2,500 units

Probable Sales Schedule	
Units	**%**
5,000	10
4,000	20
3,000	40
2,000	20
1,000	10

The management, before accepting to produce this product, is very much concerned about the possibility of risk of losses associated with the production taking into consideration the probable sale forecast as above.

You are required

i. To advise the management whether to go ahead with the production of this product. Assume that the company will not mind 5% chance of the product turning out to make losses.
ii. To calculate the coefficient of variation of the probable sales to buttress our decision in (1) above. (10 marks).

ICAN, Nov. 1994 Mgt Acc. Q7

7.0 REFERENCES/FURTHER READING

Okafor, F. O. (1983). Investment Decisions: Evaluation of Projects and Securities. London: Cassell.

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UNIT 4 CAPITAL RATIONING

CONTENTS

1.0 INTRODUCTION

Capital rationing is where the firm is unable to initiate all projects which are apparently profitable because insufficient funds are available. Under the assumptions given for the basic DCF model, a perfect capital market was presumed, that is as much finance as required could be raised at the market rate of interest. In imperfect capital market conditions, capital may be raised, but at increasing rates of interest; but there will be some point where there is an absolute limit to the amount that could be raised. This is known as **external capital rationing**. Alternatively, the effects of capital rationing may develop for internal purposes, for example, it may be decided that investment should be limited to the amount that can be financed solely from retained earnings or kept within a given capital expenditure budget. The external and internal factors, which impose quantitative limits, have led to two opposing view-points, known as the 'hand' and 'soft' views of capital rationing.

The 'hard' view is that there is an absolute limit on the amount of money a firm may borrow or raise externally whereas the `soft' view is that rationing by a quantitative limit such as an arbitrary capital expenditure budget should only be seen as temporary and administratively expedient because such a limit is not determined by the market (the assumption being that any amount of fund is available, at a

price) and such a limit would not be imposed by a profit maximising firm.

Whatever the causes of the limited capital supply available for investment purpose, it means that, not only must each project cover the cost of capital, but that the project or batch of projects selected must maximise the return from the limited funds available, i.e. some forms of ranking becomes necessary.

Before considering solution methods, some definitions need to be considered.

a. Single period capital rationing - the term is used where there is a limit on the fund available now but where it is anticipated that fund will be freely available in subsequent periods.
b. Multi-period capital rationing - where the limitation of fund extends over a number of periods or possibly indefinitely.
c. Divisible projects - projects where the whole project or any fraction may be undertaken. If a fractional part is undertaken, then it is assumed that the initial outlay and subsequent cash inflows and outflows are reduced pro rata. Although for most industrial projects, this seems somewhat hypothetical, the assumption of divisibility is frequently made in solving capital rationing problems, particularly in examination questions.
d. Indivisible projects - where the whole project must be undertaken or not at all.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define capital rationing
- evaluate projects and investments under inflationary trends.

3.0 MAIN CONTENT

3.1 Project Selection under Capital Rationing

Where capital rationing exists, the normal DCF decision rule, which is to accept all projects which have a positive NPV at the cost of capital, is insufficient to make the appropriate project selection.

The objective where capital rationing exists is to maximise the return from the batch of projects selected having regards to the capital limitation. This means that the investment decision changes from simply being accept or 'reject' to what is in effect a ranking problem. Ways of achieving this objective are shown below for the following rationing possibilities (single or multi period capital with divisible projects), where some are mutually exclusive.

3.2 Single Period Capital Rationing - Divisible Projects

This is the simplest situation and the solution method is to rank the projects in order of their EVPI (i.e. NPV per ₦ of outlay as described before). We are to choose projects, or fraction of a project, until the supply of capital for investment is exhausted.

Example One

Onwe Ltd. has a cost of capital of 15% and has a limit of ₦100, 000 available for investment in the current period. It is expected that capital will be freely available in the future. The investment required, the NPV at 15% and the EVPI for each of the 6 projects currently being considered are shown below.

What projects should be initiated?

Project	Outlay ₦	NPV @ 15% ₦	EPVI (NPV) Outlay
A	20,000	8,000	0.4
B	40,000	28,000	0.7
C	35,000	37,500	1.07
D	50,000	31,500	0.63
E	15,000	3,500	0.23
F	45,000	-5,000	-0.11

Solution

Ranking by EVPI is C, B, D, A and E. Project F cannot be considered because it fails the initial hurdle of achieving a positive NPV.

∴ Optimal Investment Plan

Project	Fraction Undertaken	Investment ₦	NPV
C	1.0	35,000	37,500
B	1.00	40,000	28,000
D	0.50	25,000	15,750
		₦100,000	₦81,250

It will be seen that this solution method uses the well known management accounting principle of maximising return per unit of the limiting factor - in this case NPV per ₦ of capital available for investment. It will be recalled that this principle is appropriate where there is a single constraint only - in this case investment finance for one period.

3.3 Single Period Capital Rationing with Mutually Exclusive Divisible Projects

Where two or more of the projects are mutually exclusive, the solution method of ranking of EVPI can still be used but the projects have to be divided into groups each containing one of the mutually exclusive projects. This is shown below.

Example Two

Assume the same data as example one except that projects B and D are mutually exclusive.

Which projects should be initiated?

Solution

It is necessary to divide the projects into two groups, rank by EVPI, and select projects up to the capital limit and compare the total NPV obtainable from each group.

Group I			Group II		
Project	Investment ₦	EVPI	Project	Investment ₦	EVPI
A	20,000	0.4	A	20,000	0.4
B	40,000	0.7	C	35,000	1.07
D	35,000	1.07	D	50,000	0.63
E	15,000	0.23	E	15,000	0.23

Ranking the groups and choosing the projects up to the investment limit produce the following:

Group I				Group II			
Project	Fraction	₦'000 Investment	NPV	Project	Fraction	₦ Investment	NPV
C	1.00	35.00	37.50	C	1.00	35.00	37.50
B	1.00	40.00	28.00	D	1.00	50.00	31.50
A	1.00	20.00	8.00	A	3/4	15.00	6.00
E	1/3	5.00	1,17				
		₦100.00	₦74,67			₦100.00	₦75.00

It will be seen that, by a narrow margin, Group II with the proportions indicated has the greater NPV and would be chosen.

3.4 Single Period Capital Rationing - Indivisible Projects

Where projects have to be accepted in their entirety or not at all, then EVPI ranking procedure does not necessarily produce the optimal solution. Provided that relatively few projects are involved, a trial and error approach can be used to find a solution. Where projects are indivisible, then it is likely that some of the capital available for investment may be unused and in such circumstances a full analysis should include the returns from external investment of under-utilised funds.

Example 9

Lloyds Ltd. has a cost of capital of 10% and has a limit of ₦100,000 available for investment in the current period. Capital is expected to be freely available in future periods. The following indivisible projects are being considered.

Initial Project	Investment ₦	NPV @ 10% ₦
A	35,000	17,500
B	40,000	22,500
C	65,000	38,000
D	48,000	31,500
E	23,000	9,000

It is required to calculate the optimal investment plan:

a. where there are no alternative investments available for any surplus funds

b. where surplus funds can be invested to produce 12% perpetuity.

Solution

a. Various combinations are tried to see which combination produces the maximum NPV.

Table 5 shows a few examples.

Table 5

Project Combinations ₦	Total Outlay for Combinations ₦	Surplus Funds ₦	Total NPV of Combinations ₦
AC	100,000	-	55,500
ABE	98,000	2,000	49,000
AD	83,000	17,000	49,000
BD	88,000	12,000	54,000
BE	63,000	37,000	31,500
CE	88,000	12,000	47,000
DE	71,000	29,000	40,500

It will be seen from Table 5 that the best investment plan is A and C which utilise all the funds available and produce a combined NPV of ₦55, 500.

When surplus funds can be invested externally, each of the combinations in Table 5 which has surplus funds must be examined to see if the project NPV and the return on external investment are greater than ₦55, 500.

Each ₦1, 000 invested at 12% in perpetuity yields ₦200 NPV, i.e.

$$\left[\frac{1{,}000 \times .12}{.1}\right] - 1{,}000 = ₦200$$

The project combinations and total NPV (Projects + External Investment) are shown in Table 6.

Combination	Total Project outlay ₦	Funds externally invested ₦	External investment NPV ₦	+	Project NPV ₦	= Total NPV ₦
ABE	98,000	2,000	400	+	49,000	49,400
AD	83,000	17,000	3,400	+	49,000	52,400
BD	88,000	12,000	2,400	+	54,000	56,400*
BE	63,000	37,000	7,400	+	31,500	38,900
CE	88,000	12,000	2,400	+	47,000	49,400
DE	71,000	29,000	5,800	+	40,500	46,300

Table 6

* When external investment is considered, then projects BD should be initiated and ₦12, 000 invested externally to produce a total NPV of ₦54, 500. It will be seen that this is slightly better than the AC combination shown in Table 5.

Note: Although ranking by EVPI in conditions of single-period capital rationing with indivisible projects does not necessarily produce the correct ranking, it usually provides an excellent guide to the best group of projects.

3.5 Multi-Period Capital Rationing

This has been previously defined as where investment funds are expected to be limited over several periods. In such circumstances, it becomes difficult to choose the batch of projects (some starting immediately, someone period hence, two periods hence, etc.) which yields the maximum return and yet which remains with the capital limits. The problem becomes one of optimising a factor (e.g. NPV) where resources are limited, i.e. the funds available over the periods being considered. This will be recognised as a situation where Linear Programming (LP) can be used and LP has been used successfully in solving Multi-period Capital Rationing problems.

SELF-ASSESSMENT EXERCISE 1

Examine the balance sheet of two manufacturing companies, two commercial firms and two financial institutions. Make a list of all capital items in the balance sheet. In each case, determine the ratio of financial to real assets. Explain the differences, if any, in the ratios as between the various industrial groupings studied.

SELF-ASSESSMENT EXERCISE 2

Udaku Plastics Products Limited is evaluating the proposal to purchase new machinery for the current year. The relevant data are as follows.

i. The cost of the machinery would be ₦200, 000. It would be depreciated on a straight-line basis over five years with no salvage.
ii. The pre-tax annual cash inflow from this investment is ₦100, 000 and the income tax rate is 45% paid the same year as incurred. All cash flows occur at year end.
iii. The company's investment policy is to embark on investments only if two of the following three conditions are satisfied.
    a. The after-tax accounting rate of return is at least 20%
    b. The payback period is less than four years
    c. The net present value of the new machine is favourable.

iv. The desired rate of return is 15% and the annuity table is as follows.

Year	Present value of an annuity in arrears of 1 at 15%
1	₦870
2	1.626
3	2.284
4	2.856
5	3.353
6	3.785

Required:

i. Would you, as the Chief Accountant of Udaku Plastics Products Limited, advise that the investment be undertaken. Why?
ii. How much would the company have had to invest four years ago at 15% compounded annually to have ₦300, 000 now *(16 marks).*

ICAN, (May 1986). Mgt Acc Q2.

SELF-ASSESSMENT EXERCISE 3

i. Write short notes on the followings.
    a. Net Present Value Model
    b. Payback Model
    c. The Internal Rate of Return *(3 marks).*

ii. The following pieces of information relate to three possible capital projects: because of capital rationing, only one project can

be accepted by the management of Akpebor Otuokena Beauty (AOB) Nigeria Limited.

	PROJECTS		
	A	B	C
Initial costs (₦)	400,000	460,000	360,000
Expected life	5 years	5 years	4 years
Scrap value expected	N20,000	₦30,000	₦16,000
Expected cash inflow	₦'000	₦'000	₦'000
End of year 1	160	200	110
2	140	140	130
3	130	100	190
4	120	100	200
5	110	100	0

The company estimates its cost of capital to be 18% and discount factors are as follows.

Year	1	0.8475
	2	0.7182
	3	0.6088
	4	0.5158
	5	0.4371

Required

i. Calculate the following.
   a. The payback period for each project
   b. The Internal Rate of Return for each project
   c. The Net Present Value of each project.
   (*10 marks)*

ii. Which project should be accepted? Give reasons (2 marks) (Total 15 marks).

ICAN, (November 2002). Mgt. Acc. Q3.

SELF-ASSESSMENT EXERCISE 4

FCE (Fredrick C. Egwuonwu) Plc. has ₦500, 000 available for investment

The following projects are under consideration.

Project	Initial Outlay	Net Cash Inflow During 6 Years Life	Residual
A	140,000	₦40,000 annually	5,000
B	180,000	₦80,000 for each of the first 3 years and ₦60,000 for each of the remaining 3 years	Nil
C	220,000	₦60,000 for each of the first 3 years and ₦80,000 for each of the remaining 3 years	Nil
D	160,000	₦300,000 annual amount for the first 3 years being 25% more than the annual amount for the next 3 years	Nil
E	120,000	First year nil remaining years at ₦50,000 per annum	Nil

The expected rate of return on capital is 15%

With supporting calculation advise management on which of the projects should be selected for investment.

SELF-ASSESSMENT EXERCISE 5

Wolex Limited has commenced a review of the price being charged for a major product line. Over the past three years, the product has had sales averaging 48,000 units per year at a standard selling price of ₦5.35. Cost has also been rising steadily over the past year and the company is considering raising this price to ₦5.85 or ₦6.35. The Sales Manager has produced the following schedule to assist with the decision:

Price	₦5.85k	₦6.35k
Estimates of demand		
Pessimistic estimate (probability 0.25)	35,000	12,000
Most likely estimate (probability 0.55)	40,000	20,000
Optimistic estimate (probability 0.20)	50,000	40,000

Currently the unit cost is estimated at ₦5.00 analysed as follows:

	₦
Direct material	2.50
Direct labour	1.00
Variable overheads	1.00
Fixed overheads	0.50
	5.00

The management accountant is of the view that the most likely value for unit variable cost over the next year is ₦4.90 (subjective probability 0.75) but that it could be as high as ₦5.20 (probability 0.15) and it might even be as low as ₦4.75 (probability 0.10). Total fixed costs are currently estimated as ₦24, 000 p.a. but it is estimated that the corresponding total for the ensuring year will be as follows.

- ₦25, 000 with probability of 0.20
- ₦27, 000 with probability of 0.60
- ₦30, 000 with probability of 0.20

Demand quantities, unit costs and fixed costs can be assumed to be statistically independent.

i. Analyse the above information in a way that will assist management in addressing the problem.
ii. Give your views on the situation and advise on the new selling price.
iii. Calculate the expected level of profit that would result from selling price that you recommend. *(15 marks).*

ICAN,(Nov. 2002) Mgt.Acc.

3.6 Features of Investment

Our discussions so far can be summarised by highlighting the essential features of investment.

a. Investments are undertaken in anticipation of benefits which are not expected to accrue concurrently with the investment outlay. As a result of this inevitable time lag between outlay and benefit, almost every investment involves some risks, the risk that anticipated benefits may not ultimately be realised.
b. Investments can be made in real or financial assets. Irrespective of the media, all investments can be measured in terms of the total outlay of funds.

c. Unlike capital, investment is a flow variable. Consequently, it ought to be measured as a time-rate of change in capital stock.
d. Since investment benefits accrue overtime, there is the expectation that the asset in which any investment is denominated shall be retained by the investor for some reasonable period. Hence the value of the asset should be carefully established at the time the investment is made.
e. Every investment involves some current capability for consumption. As a result of this feature, economists usually expect an identity between the level of savings and investment.

3.7 Investment and Speculation

According to Okafor (1983), the distinction between investment and speculation is not easy to make by simply observing the overt actions of the individuals involved. He went further to provide a beautiful summary comparing the two as follows.

Table 1: *Investment and Speculation Compared*

Possible considerations		*Investment*	*Speculation*
1	Degree of risk assumed	Less	More, if not infinite.
2	Level of income/profit expected	Moderate	High
3	Income orientation	Income to accrue over time	Income to accrue quickly and in a lump sum
4	Major consideration	Future value of assets and future earnings potential	Direction and extent of expected price movement
5	Nature of income	Regular income and possible terminal capital gains.	Capital gains.

3.8 Basis for Classifying Investments

Broadly speaking, investments can be classified into two: investment in real assets and investment in financial assets. In the words of Okafor (1983), both types of investment can further be classified on the basis of a number of parameters.

a. **Magnitude of outlay**

Major investments could be distinguished from minor investments. In investment outlay, size is relative. An investment is major or minor depending on the relative proportion of the outlay to the total size of a firm. Thus, whereas an investment of ₦20, 000 could be considered a minor investment by a firm capitalised at ₦20 million, it is a very major investment to a small firm with total assets valued at ₦40, 000.

b. **Risk environment of investment**

A distinction is made between investment under conditions of certainty, investments under conditions of risk, and investments under conditions of uncertainty. The problem of risk and uncertainty will be discussed in the subsequent units.

c. **Motivation for investment**

A distinction could be made among investments for asset replacement, capacity expansion or modernisation, and investments for strategic purposes.

d. **Sequencing of cash flows**

Conventional investments are distinguished from non-conventional investments on the basis of the timing and sequencing of cash flow arising from the investment. The nature of both types of investment, and the differences between them, are discussed subsequently, in this course.

e. **Nature of expected benefits**

A distinction exists between cost-saving and revenue-yielding real asset investment. The former is illustrated by a firm that replaces old equipment in the hope of cutting operating costs over the life of the new equipment. In a revenue expansion programme, on the other hand, funds are invested in order to increase gross revenue either through additional sales volume or through increased price per unit of sales.

When evaluating a cost saving investment, the value of total costs saved is compared with the additional investment made. In the latter situation, the investor would have to compare the increased costs with the additional sales revenue realised.

f. **Relationship to other investments**

The costs and benefits of a given investment may or may not be affected by alternative investments. In this regard, dependent investments are different from independent investments.

Investment in real assets

Investment in real assets takes one of three major forms, that is, investment in business fixed assets, investment in inventory and investment in residential construction.

Investment in projects

Real asset investment is either on single fixed assets or on a group of inter-related assets. Where the group of inter-related assets provides facilities capable of completing a production or a service process, the investment activity is described as a project. Investment projects are such that the facilities provided by the component assets can only be effective if operated as a unit. Hence the component assets must necessarily be accepted or rejected as a set.

Contrary to popular expectation, the basic difference between projects and single asset investments does not lie in the value of the investment outlay. The cost of a single turbine in a hydro-electricity generating plant could be many times the total investment outlay in a corn grinding mill. In terms of our definition, the latter is a project because it can complete a processing cycle. Outlay on the hydro-electricity generating turbine is not by itself a project. The distinction must, however, be given a common sense interpretation. It is wrong, for example, to regard the purchase of a single taxi cab as a project, though such a cab can operate as a unit. A project necessarily involves the interplay of a number of single assets. (Okafor, 1983).

3.9 Further Classifications

1. **Conventional and non-conventional investments**
   According to Okafor (1983), investment activities in which periods of net cash outflow are expected to precede periods of net cash inflows are described as conventional investments. Non-conventional investments, on the other hand, are those in which there is no specific pattern in the sequencing of cash flows.
2. **Cash flows**
   The definition of net cash inflow or outflow used above is not identical with the accounting concept of income or expenditure. Net cash inflow from an investment for any period includes both the accounting income for the period and the non-cash expenses charged to operating revenue in determining such income as depreciation.

3. **Dependent and independent investments**
   Two or more investments are economically independent if the expected cash flow from each would be unaffected whether or not the alternate investments are carried out concurrently.

Investment proposals are dependent if they are either technically dependent or economically dependent. (Okafor, 1983).

Degree of dependence in investments

There are degrees of dependence of investment opportunities. In one extreme case, one investment (A) is so dependent on another (B) that the net benefits of A would be virtually insignificant unless both of them are carried out simultaneously. Given that situations, investment B is a prerequisite for A. Where the degree of dependence is reciprocal, the alternatives are complementary.

The other extreme case of dependence occurs where the alternatives are so inter-related that the decision to carry out one implies *ipso facto* a rejection of the other. This is a case of *mutual exclusion* which occurs either because of technical dependence or because the alternative investments serve the same market which can only support one of the alternatives. Cases of mutual exclusion in investment alternatives abound in industries.

N.B.

The distinction between dependent and independent investments is important for one main reason. Whereas an independent investment is evaluated on the basis of its absolute cash flows, a dependent investment must be evaluated on the basis of its incremental cash flows.

3.10 Feasibility and Viability Studies

Before embarking on any capital investment, it is always advisable to conduct both feasibility and viability studies. Whereas feasibility study is aimed at establishing the practicability or workability of an investment, viability study tries to evaluate the degree of profitability.

Feasibility Study

This starts with environmental assessment (since certain investments cannot take place in some environments). Other issues to be considered include the followings.

- Management/Personnel

- Availability of Raw Material
- Market Share Assessment

Viability Tests

These tests are normally conducted using either the traditional techniques or the discounted techniques or both.

4.0 CONCLUSION

In this unit, you have learnt that capital investments can contribute a lot towards national development. Accordingly, you should be able to advise individuals, families, churches and states to embark on one form of investment or the other.

5.0 SUMMARY

In this unit, you learnt that capital investments involve making sacrifices today in anticipation of future benefits. You also learnt that investments could be divided into two namely: direct and indirect investments or real assets (tangible) and paper assets (financial instruments). You are also conversant with the features of investments and can now distinguish between investments and speculations.

6.0 TUTOR-MARKED ASSIGNMENT

1. "There is no basic difference between the behaviours of speculators and those who are interested in making as much income as possible from a given capital outlay". Discuss.
2. Discuss the major similarities and differences between investment in real assets and investment in financial assets.
3. Evaluate at least five government policies currently in force, which either induce or stifle private investment.

7.0 REFERENCES/FURTHER READING

Nweze, A.U. (2006). "Investment Opportunities in the Nigerian Capital Market". Enugu: M'Cal Communications.

Okafor, F.O. (1983). Investment Decision: Evaluation of Projects and Securities. London: Cassell Ltd.