SMS 201: BUSINESS STATISTICS I

COURSE GUIDE

NATIONAL OPEN UNIVERSITY OF NIGERIA

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INTRODUCTION:

SMS 201: Business Statistics I is a one semester, 2 credit units first year level course. It will be available to all first degree of the school of Business and Human resources Management at the National Open University, Nigeria. It will also be useful for those seeking introductory knowledge in business statistics.

The course consists of eighteen units that involved basic concepts and principles of statistics and decision making process, forms of data, methods of data collection, summarizing data, graphical presentation of data, measures of both central tendency and dispersion, set theory, permutations and combinations, some elements of probability concepts, probability distributions of both discrete and continuous random variables.

There are no compulsory prerequisites for this course, although a credit pass level in Mathematics at Senior Secondary School Certificate examination, General Certificate Examination. Ordinary Level, National examination Council (NECO) is a must for success in the course.

The course requires you to study the course materials carefully, supplement the materials with other resources from Statistics Textbooks both to be prescribed and those not prescribed that may treat the contents of the course.

This Course Guide tells you what the course is about, what course materials you will be using and how you can work your way through these materials. It suggests some general guidelines for the amount of time you are likely to spend on each unit of the course in order to complete it successfully. It also gives you some guidance on your tutor--marked assignments. Detailed information on tutor-marked assignment is found in the separate file.

There are likely going to be regular tutorial classes that are linked to the course. It is advised that you should attend these sessions. Details of the time and locations of tutorials will be communicated to you by National Open University of Nigeria (NOUN).

What You Will Learn In The Course

The overall aim of STA 101 Business Statistics I is to introduce you to the nature of Statistical Information, Collection, Summarizing and Presentation and analyzing the statistical information in such a way that the reality contained in the information may be revealed for decision-making. During the course, you will be exposed to nature of statistical information, collection and processing of statistical data. You will also be exposed to elementary concepts in probability and nature, characteristics and uses of some important probability distributions of both discrete random variable and continuous random variable.

Course Aims

The course aims to give you an understanding of statistical information and presentation for decision-making. It exposes you to measures that are computed and used for processing materials for decision-making. It also gives the basic knowledge of some concepts used for making decisions and carefully summarizes some Probability Distributions.

This will be achieved by:

1. Introducing you to nature and form of statistical data

2.   Showing how the statistical data can be collected and presented
3.   Showing you how to compute measurement of dispersion in a sample or population
4.   Showing you how to compute value of permutations and combinations
5.   Introducing you to the basic concepts of elementary probability
6.   Give the basic principles for the application of some important probability distributions

Course Objectives

To achieve the aims set above the course sets overall objectives; in addition, each unit also has specific objectives. The unit objectives are included at the beginning of a unit, you should read them before you start working through the unit. You may want to refer to them during your study of the unit to check on your progress. You should always look at the unit objectives after completing a unit. In this way you can be sure you have done what was required of you by the unit.

We set out wider objectives of the course as a whole below. By meeting these objectives, you should have achieved the aims of the course.

On successful completion of the course, you should be able to:

1.   Explain nature of Statistical information.
2.   Explain types of Statistical information
3.   Collect Statistical information
4.   Summarize Statistical information
5.   Present Statistical information
6.   Compute measures of Central Tendency and Dispersion for Statistical information
7.   Explain basic concepts in set theory
8.   Perform operations in set theory
9.   Compute values of Permutations and combinations for arrangement of objects
10.   Define Probability of an event
11.   Explain Properties of Probability
12.   Calculate Probability events
13.   Explain concepts used in probability
14.   Explain the principles underlying the application of various probability distributions
15.   Compute measures for Probability Distributions.

Working Through This Course

To complete this course, you are required to read the study units, read set books and other materials on the course.

Each unit contains self-assessment exercises called Student Assessment Exercises, SAE. At some points in the course, you are required to submit assignments for assessment purposes. At the end of the course there is a final Examination. This course should take about 15 weeks to complete. Some listed components of the course, what you have to do and how you should allocate your time to each unit in order to complete the course successfully on time, are given below

Below you will find listed components of the course, what you have to do and how you should allocate your time to each unit in order to complete the course successfully on time.

Course Materials

Major components of the course are:

(1)   Course Guide
(2)   Study Units
(3)   Textbooks
(4)   Assignment File

(5)   Presentation Schedule.

Study Units

There are eighteen Study Units in this Course, as follows:

Unit 1:   Statistics and Decision Making Process
Unit 2:   Nature, Source and Method of Data Collection
Unit 3:   Summarizing Data
Unit 4:   Graphical Presentation of Data
Unit 5:   Measures of Central Tendency 1 - The Arithmetic Mean
Unit 6:   Measures of Central Tendency 2 - Geometric Mean and Harmonic Mean.

Unit 7:   Measures of Central Tendency 3 - Median and Mode
Unit 8:   Fractiles, Skewness and Kurtosis
Unit 9:   Measures of Dispersion
Unit 10:   Set theory
Unit 11:   Permutations and Combinations
Unit 12:   Some Elementary Probability Concepts
Unit 13:   Probability Rules, Events and Bayes' Theorem
Unit 14:   Probability Distribution of a Discrete Random Variable.
Unit 15:   Binomial Distribution
Unit 16:   Poisson Distribution
Unit 17:   Hypergeometric Distribution
Unit 18:   Normal Distribution.

The first four units concentrate on the nature, collection and presentation of statistical data. This constitutes Module 4.1. The next five units, module 2, concentrate on computation of measures of central tendency and dispersion from samples and populations. Units 10 to 13, Module3, deal with the basic concepts and principles in elementary probability. The last five units, module 4, teach the principles underlying the applications of some important probability distributions.

Each unit consists of on week direction for study, reading material, other resources and summaries of key issues and ideas. The units direct you to work on exercises related to the required readings

Each unit contains a number of self-tests. In general, these self-tests question you on the material you have just covered or required you to apply it in some way and thereby help you to assess your progress and to reinforce your understating of the material. Together with tutor-marked assignments, these exercises will assist you in achieving the stated learning objectives of the individual units and of the course.

Set Textbooks

It is advisable you have some of the following books

Daniel W.W. and Terrel J.C. (1979) Business Statistics: Basic Concepts and Methodology, 2"d ed. Houghton Mifflin co. Boston

Harper W.M. (1982) Statistics. 4`" ed, Macdonald and Evans Handbook Series

Levin R.L (1990) Statistics for Management. 4t' ed., Prentice Hall of India Privatize Limited, New Delhi.

Assignment File

In this tile, you will find the details of the work you must submit to your tutor for marking. The marks you obtain for these assignments will count toward the final mark you obtain for this course. Further information on assignments will be found in the Assignment File itself and later in this Course Guide in the section on Assessment.

There are four assignments in this course. The four course assignments will cover:

Assignment 1 - All question in Units 1 - 4
Assignment 2 - All TMAs' question in Units 5 - 9
Assignment 3 - All TMAs' question in Units 10 - 13
Assignment 4 - All TMAs' question in Unit 14 -18.

Presentation Schedule

The presentation schedule included in your course materials gives you the important dates for this year for the completion of tutor-marking assignments and attending tutorials. Remember, you are required to submit all your assignments by due date. You should guide against falling behind in your work.

Assessment

There are two types of the assessment of the course. First are the tutor-marked assignments; second, there is a written examination.

In tackling the assignments, you are expected to apply information, knowledge and techniques gathered during the course. The assignments must be submitted to your tutor for formal Assessment in accordance with the deadlines stated in the Presentation Schedule and the Assignments File. The work you submit to your tutor for assessment will count for 50 % of your total course mark.

At the end of the course, you will need to sit for a final written examination of three hours' duration. This examination will also count for 50% of your total course mark.

Tutor-Marked Assignments (TMAs)

There are four tutor-marked assignments in this course. You will submit all the assignments. You are encouraged to work all the questions thoroughly. Each assignment counts 12.5% toward your total course mark.

Assignment questions for the units in this course are contained in the Assignment File. You will be able to complete your assignments from the information and

materials contained in your set books, reading and study units. However it is desirable in all degree level education to demonstrate that you have read and researched more widely than the required minimum. You should use other references to have a broad viewpoint of the subject and also to give you a deeper understanding of the subject.

When you have completed each assignment, send it, together with a TMA form, to your tutor. Make sure that each assignment reaches your tutor on or before the deadline given in the Presentation File. If for any reason, you cannot complete your work on time, contact your tutor before the assignment is due to discuss the possibility of an extension. Extensions will not be granted after the due date unless there are exceptional circumstances.

Final Examination and Grading

The final examination will be of three hours' duration and have a value of 50% of the total course grade. The examination will consist of questions which reflect the types of selftesting, practice exercises and tutor-marked problems you have previously encountered. All areas of the course will be assessed

Use the time between finishing the last unit and sitting the examination to revise the entire course. You might find it useful to review your self-tests, tutor-marked assignments and comments on them before the examination. The final examination covers information from all parts of the course.

COURSE OVERVIEW

This table brings the units, the number of weeks you should take to complete them and the assignment as follows.

UNIT	Title of work	Weeks Activity	Assessment (end of Unit)
	Course Guide	1	
1	Statistics and Decision Making Process	1	
2	Nature, Source and Method of Data Collection		
3	Summarizing Data	1	
4	Graphical Presentation of Data		Assignment 1
5	Measures of Central Tendency 1 - The Arithmetic Mean	1	
6	Measures of Central Tendency 2 - Geometric Mean and Harmonic Mean.		
7	Measures of Central Tendency 3 - Median and Mode	1	
8	Fractiles, Skewness and Kurtosis	1	
9	Measures of Dispersion	1	Assignment 2
10	Set theory	1	
11	Permutations and Combinations		
12	Some Elementary Probability Concepts	1	
13	Probability Rules, Events and Bayes' Theorem	1	
14	Probability Distribution of A Discrete Random Variable.	1	Assignment 3
15	Binomial Distribution	1	
16	Poisson Distribution	1	
17	Hypergeometric Distribution	1	
18	Normal Distribution.	1	
	Total	15wks	Assignments 4

Table 2 Course Organizer

How To Get The Most From This Course

In distance learning the study units replace the university lecturer. This is one of the great advantages of distance learning; you can read and work through specially designed study materials at your own pace and at a time and place that suit you best. Think of it as reading the lecture instead of listening to a lecturer. In the same way that a lecturer might set you some reading to do, the study units tell you when to read your books or other material, and when to undertake computing practical work. Just as a lecturer might give you an in-class exercise, your study units provides exercises for you to do at appropriate points.

Each of the study units follows a common format. The first item is an introduction to the subject matter of the unit and how a particular unit is integrated with the other units and the course as a whole. Next is a set of learning objectives. These objectives let you know what you should be able to do by the time you have completed the unit. You should use these objectives to guide your study. When you have finished the unit you must go back and check whether you have achieved the objectives. If you make a habit of doing this you will significantly improve your chances of passing the course.

The main body of the unit guides you through the required reading from other sources. This will usually be either from your set books or from a Readings section. Some units require you to undertake practical work or a computer. You will be directed when you need to use a computer and guided through the tasks you must do. The purpose of the computing work is twofold. First, it will enhance your understanding of the material in the unit. Second, it will give you practical experience of using programs, which you could well encounter in your work outside your studies. In any event, most of the techniques you will study are applicable on computers in normal working practice, so it is important that you encounter them during your studies.

Self-tests are interspersed throughout the units, and answers are given at the ends of the units. Working through these tests will help you to achieve the objectives of the unit and prepare you for the assignments and the examination. You should do each self-test as you come to it in the study unit. There will also be numerous examples given in the study units; work through these when you come to them, too.

The following is a practical strategy for working through the course. If you run into any trouble, contact your tutor. Remember that your tutor's job is to help you. When you need help, don't hesitate to call and ask your tutor to provide it.

1.   Read this Course Guide thoroughly.

2.   Organize a study schedule. Refer to the `Course overview' for more details. Note the time you are expected to spend on each unit and how the assignments relate to the units. Important information, e.g. details of your tutorials, and the date of the first day of the semester is available from study centre. You need to gather together all this information in one place, such as your dairy or a wall calendar. Whatever method you choose to use, you should decide on and write in your own dates for working breach unit.

3.   Once you have created your own study schedule, do everything you can to stick to it. The major reason that students fail is that they get behind with their course work. If you get into difficulties with your schedule, please let your tutor know before it is too late for help.

4.   Turn to Unit 1 and read the introduction and the objectives for the unit.

5.   Assemble the study materials. Information about what you need for a unit is given in the `Overview' at the beginning of each unit. You will also need both the study unit you are working on and one of your set books on your desk at the same time.

6.   Work through the unit. The content of the unit itself has been arranged to provide a sequence for you to follow. As you work through the unit you will be instructed to read sections from your set books or other articles. Use the unit to guide your reading.

7.   Up-to-date course information will be continuously delivered to you at the study centre.

8.   Well before the relevant due date (about 4 weeks before due dates), get the Assignment File for the next required assignment. Keep in mind that you will learn a lot by doing the assignments carefully. They have been designed to help you meet the objectives of the course and, therefore, will help you pass the exam. Submit all assignments no later than the due date.

9.   Review the objectives for each study unit to confirm that you have achieved them. If you feel unsure about any of the objectives, review the study material or consult your tutor.

10.   When you are confident that you have achieved a unit's objectives, you can then start on the next unit. Proceed unit by unit through the course and try to pace your study so that you keep yourself on schedule.

11.   When you have submitted an assignment to your tutor for marking do not wait for it return `before starting on the next units. Keep to your schedule. When the assignment is returned, pay particular attention to your tutor's comments, both on the tutor-marked assignment form and also written on the assignment. Consult your tutor as soon as possible if you have any questions or problems.

12.   After completing the last unit, review the course and prepare yourself for the final examination. Check that you have achieved the unit objectives (listed at the beginning of each unit) and the course objectives (listed in this Course Guide).

Tutors And Tutorials

There are some hours of tutorials (ten 2-hour sessions) provided in support of this course. You will be notified of the dates, times and location of these tutorials. Together with the name and phone number of your tutor, as soon as you are allocated a tutorial group.

Your tutor will mark and comment on your assignments, keep a close watch on your progress and on any difficulties you might encounter, and provide assistance to you during the course. You must mail your tutor-marked assignments to your tutor well before the due date (at least two working days are required). They will be marked by your tutor and returned to you as soon as possible.

Do not hesitate to contact your tutor by telephone, e-mail, or discussion board if you need help. The following might be circumstances in which you would find help necessary. Contact your tutor if.

• You do not understand any part of the study units or the assigned readings
• You have difficulty with the self-tests or exercises
• You have a question or problem with an assignment, with your tutor's comments on an assignment or with the grading of an assignment.

You should try your best to attend the tutorials. This is the only chance to have face to face contact with your tutor and to ask questions which are answered instantly. You can raise any problem encountered in the course of your study. To gain the maximum benefit from course tutorials, prepare a question list before attending them. You will learn a lot from participating in discussions actively.

Summary

Stat 101 Business Statistics I intend to introduce you to the nature and types of statistical Data, Collection, Presentation and analysis of the data to bring out the reality

of the information contained in the data for decision making. Upon completion of this course, you will be equipped with the basic knowledge in data and analyzing data to obtain the reality of the information contained in them. You will be able to answer these kind of question:

- •:•   What are the natures of statistical data?
- •:•   What are the methods of collecting Statistical data?
- •:•   How can statistical data be presented?
- •:•   The important measures computed from statistical data?
- •••   How are these measures computed?
- •:•   What are the concepts in elementary Probability?
- •:•   What are the characteristics of probability?
- •:•   How can you compute the probability of an event?
- •:•   What are the unique characteristics of probability distributions?
- •:•   How can you use the probability distribution to solve problems

There are more and more questions you can ask. To gain a lot from the course, please try to apply anything you learn in the course to real life situations. We wish you success with the course and hope that you will find it both interesting and useful.

SMS 201:   BUSINESS STATISTICS I

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UNIT 1

STATISTICS AND DECISION MAKING PROCESS

1.0   Introduction

You will realize that the activities of man and those of the various organizations, that will often be referred to as firms, continue to increase. This brings an increase in the need for man and the firms to make decisions on all these activities. The need for the quality and the quantity of the information required to make the decisions increases also. The management of any firm requires scientific methods to collect and analyze the mass of information it collects to make decisions on a number of issues. Such issues include the sales over a period of time, the production cost and the expected net profit. In this regard, statistics plays an important role as a management tool for making decisions.

2.0   Objectives

By the end of this unit, you should be able to:

•   Understand the various definitions of statistics
•   Describe the uses of statistics
•   Define the basic concepts in statistics.

3.1   Definitions of Statistics

Statistics can be defined as a management tool for making decision. It is also a scientific approach to presentation of numerical information in such a way that one will have a maximum understanding of the reality represented by such

information. Statistics is also defined as the presentation of facts in numerical forms. A more comprehensive definition of statistics shows statistics as a scientific method which is used for collecting, summarizing, classifying, analyzing and presenting information in such a way that we can have thorough understanding of the reality the information represents.

From all these definitions, you will realize that statistics are concerned with numerical data.. Examples of such numerical data are the heights and weights of pupils in a primary school when evaluating the nutritional well being of the pupils and the accident fatalities on a particular road for a period of time.

You should also know that when there are numerical data, there must be non-numerical data such as the taste of brands of biscuits, the greenness of some vegetables and the texture of some joints of a wholesale cut of meat. Non-numerical data cannot be subjected to statistical analysis except they are transformed to numerical data. To transform greenness of vegetables to numerical data, a five point scale for measuring the colour can be developed with 1 indicating very dull and 5 indicating very green.

3.2   The Roles of Statistics

You will realize that statistics is useful in all spheres of human life. A woman with a given amount of money, going to the market to purchase foodstuff for the family, takes decision on the types of food items to purchase, the quantity and the quality of the items to maximize the satisfaction she will derive from the purchase. For all these decisions, the woman makes use of statistics

Government uses statistics as a tool for collecting data on economic aggregates such as national income, savings, consumption and gross national product. Government also uses statistics to measure the effects of external factors on its policies and to assess the trends in the economy so that it can plan future policies.

Government uses statistics during census. The various forms sent by the government to individuals and firms on annual income, tax returns, prices, costs, output and wage rates generate a lot of statistical data for the use of the government

Business uses statistics to monitor the various changes in the national economy for the various budget decisions. Business makes use of statistics in production, marketing, administration and in personnel management.

Statistics is also used extensively to control and analyze stock level such as minimum, maximum and reorder levels. It is used by business in market

research to determine the acceptability of a product that will be demanded at various prices by a given population in a geographical area. Management also uses statistics to make forecast about the sales and labour cost of a firm. Management uses statistics to establish mathematical relationship between two or more variables for the purpose of predicting a variable in terms of others. For the conduct and analyses of biological, physical, medical and social researches, we use statistics extensively.

3.3   Basic Concepts In Statistics

Let us quickly define some of the basic concepts you will continue to come across in this course.

- **Entity**: This may be person, place, and thing on which we make observations. In studying the nutritional well being of pupils in a primary school, the entity is a pupil in the school.

- **Variable**: This is a characteristic that assumes different values for different entities. The weights of pupils in the primary school constitute a variable.

- **Random Variable:** If we can specify, for a given variable, a mathematical expression called a function, which gives the relative frequency of occurrence of the values that the variable can assume, the function is called a probability function and the variable a random variable.

- **Quantitative Variable:** This is a variable whose values are given as numerical quantities.

    Examples of this is the hourly patronage of a restaurant

- **Qualitative Variable:** This is a variable that is not measurable in numerical form or that cannot be counted. Examples of this are colours of fiuits, taste of some brands of a biscuit.

- **Discrete Variable:** This is the variable that can only assume whole numbers. Examples of these are the number of Local Government Council Areas of the States in Nigeria, number of female students in the various programmes in the National Open University.

    A discrete variable has "interruptions" between the values it can assume. For instance between 1 and 2, there are infinite number of

values such as 1.1, 1.11, 1.111, 1. I I l land so on. These are called interruptions.

- **Continuous Variable:** This is a variable that can assume both decimal and nondecimal values. There is always a continuum of values that the continuous variable can assume. The interruptions that characterize the discrete variable are absent in the continuous variable. The weight can be both whole values or decimal values such as 20 kilograms and 220.1752 kilograms.

- **Population:** This is the largest number of entities in a study. In the study of how workers in Nigeria spend their leisure hours, the number of workers in Nigeria constitutes the population of the study.

- **Sample:** This is the part of the population that is selected for a study. In studying the income distribution of students in the National Open University, the incomes of 1000 students selected for the study, from the population of all the students in the Open University will constitute the sample of the study.

- **Random Sample:** This is a sample drawn from a population in such a way that the results of its analysis may be used to generalize about the population from which it was drawn.

Exercise 1.1

What is the importance of Statistics to human activities? Your answer can be obtained in section 3.2 of this unit.

4.0   Conclusion

In this unit you have learned a number of important issues that relate to the meaning and roles of statistics. The various definitions and examples of concepts given in this unit will assist tremendously in the studying of the units to follow.

5.0   Summary

What you have learned in this unit concerns the meaning and roles of statistics, and the various concepts that are important to the study of statistics.

6.0   Tutor Marked Assignment

What is Statistics? Of what importance is statistics?

7.0   References, Further Reading and Other Sources

Daniel, W.W. and Terrel J.C. (1979) Business Statistics: Basic Concepts And Methodology 2"ᵃ ed. Houghton Mifflin Company Boston.

Hannapan, T.J. (1982) Mastering Statistics. The Macmillan Press Ltd.

UNIT 2

DATA: NATURE, SOURCE AND METHODS OF COLLECTION

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3.3.4	Questionnaire
4.0	Conclusion
5.0	Summary
6.0	Tutor Marked Assignment
7.0	References, further Reading and Other Resources

1.0   Introduction

This is the second unit in the course. The first unit has shown that statistics involves the scientific method of collecting, summarizing, classifying, analyzing and presenting information, usually numerical information, in a way that we can have a thorough understanding of the reality the information represents. You will realize that it is important to know the nature, the sources and methods of collecting the data. This unit takes you through these aspects of data. There are self-assessment exercises designed to make you pause and reflect on what you are reading.

At the end of the unit, there are again self-assessment questions which are designed to know the extent of your learning of the contents in the unit.

2.0   Objectives

After studying unit and going through the exercises, you should be able to

- Know the nature of primary and secondary data
- Identify various sources of micro-and macro-statistical information
- Describe the various methods of data collection

3.1   Nature of Statistical Data

3.1.1 Primary Data

You are already aware that statistics uses numerical data. The Numerical data can be divided into:

a)   Primary and
b)   Secondary Data

Primary Data are data collected by or on behalf of the person or people who are going to make use of the data. It is the data collected specifically for a purpose and used for the purpose for which they are collected.

Examples of Primary data are:

(i)   Heights and weights of students collected to determine their nutritional well being.
(ii)   The population of Primary school pupils in the states of the country to allow the Federal Government plan for the primary education.
(ii)   The academic performance of students in secondary schools under various types of leaders to know the leadership style suitable for the secondary schools

The various methods of collecting primary data include surveys, interview, observation, questionnaire and experiments. These will be further discussed in this unit.

Primary data could be very expensive to collect when the elements of the study sample are widely scattered and when the items of equipment for data collection, as in many experiments, are capital intensive. However errors can be minimized when collecting primary data since the researcher can always take adequate precaution in collecting primary data.

3.1.2   Secondary Data

This is the data that is used by a person or people other than the person or people by whom or for whom the data was collected. These are the data collected for some other purpose, frequently for administrative reasons, and used for the purpose for which they were not collected.

Secondary data are always collected from published sources, like textbooks, journals, Newspapers, magazines, and gazette.

Examples of secondary data include:

(i)   Accident fatalities on a particular road over a period of time collected from the Police or Road safety corps.
(ii)   Dietary requirements of various age groups collected from a nutrition textbook.
(iii)   Age distribution in Nigeria collected from the publications of the National Population Census.

From the discussion so far, you will know that secondary data are second-hand data. This shows the need to know as much as possible about the data. In trying

to know much about the secondary data, we need to consider the following points:

(i)   How the data was collected
(ii)   How the data has been processed
(iii)   The accuracy of the data
(iv)   How far the data has been summarized
(v)   How comparable the data is with other tabulations.
(vi)   How to interpret the data, especially when figures collected for one purpose are used for another.

With secondary data, we usually strike a compromise between what we want and what we are able to find.

Secondary data have the following advantages:

(i)   They are very inexpensive to collect since they are readily and abundantly available in published sources.
(ii)   There is a great variety of secondary data on a wide range of subjects.
(iii)   Many of these data have been collected for many years hence we can use them to establish trends for forecasting.

With all these advantages of secondary data, we must use secondary data with great care since such data may not give the exact kind of information required and the data may not be in the most suitable form they are required.

Student Assessment Exercise 2.1

For each of the Primary and Secondary data list:

(i)   The characteristics
(ii)   The advantages
(iii)   The disadvantages

3.2   SOURCES OF DATA

3.2.1   Micro-Statistical Information

The firms and private organizations, when monitoring the activities of their businesses, produce a lot of information that are specific to the organization and firm and used for their decision-making processes. The information produced by these form and organization are called micro-statistical Information.

In micro-economics, the concern is for the household and firm. The information produced or generated at these levels are micro-statistical information. For firm, such information include those generated from production, marketing and personnel.

3.2.2 Macro-Statistical Information

These are produced by the Public sector of the economy. They are related to the whole country as a whole. Such information include population, education, rate of inflation, level of unemployment and so on.

Macro-economics is concerned with such aggregates as national income, gross national products, saving, consumption, gross domestic product and so on. These are macro-statistics information.

3.2.3 Statistical Information in Business

In The performance and monitoring of various activities in the firms, a lot of data is produced. The quality and the quantity of the information generated in the firm depend on the size of the firm and the resources available to the firm

The firm is interested in what is happening to the national economy and what is happening in the industry it belongs to. You should know that a combination of firms makes the industry. As regards national income, the firm wishes to know the interest rates and unemployment. As regards the industry, the firm will always wish to know wage rates, prices, level of output so that it can compare its performance with the competitors in the industry.

The firm also generates data in the following areas.

(i)   Production:

        The firm collects data on the results of quality control, prices of raw materials, defectiveness of consignment of raw materials, wage rates, stock of materials, accident rates, absenteeism rates, unit cost.

(ii)   Marketing:
        The firm will to want know the budgeted sales, the advertising cost to sustain the sales, the distribution costs etc.

        Other information could be generated from personnel and Accounts departments to aid decision-making process of the firm.

3.2.4 Government Statistics

As you were informed in unit 1 section 3.2, the governments produce statistics to be able to measure the effects of their policies, to monitor the effects of external factors on their policies and to be able to assess trends so that they can plan future policies. The macrostatistical information is generated by the governments as you have learned in this unit. The various governments rely in the firms and the individuals to generate these macro statistical data

Student Assessment Exercises 2.2

a)   Look at a firm around you and list ten pieces of information the firm generates from the activities of the firm.

c)   Look at your Local Government Council Area list ten pieces of Information the council generates for the governance of the council.

3.3   Methods Of Data Collection

3.3.1   Surveys

In this unit, you have learned that secondary data are already available; hence they must be collected when there is the need to use them. The collection of primary data involves survey or inquiry of one type or the other.
Some surveys can be limited in the sense that they can be carried out with a few minutes of observation. Others can be detailed. When surveys are detailed, information from the surveys are more acceptable and valued than when they are limited.

Examples of surveys are:

(i)   Government Survey

(ii)   Market research surveys - carried out for one particular client and not published in any form.

(iii)   Research surveys - carried out by academicians and published in journals

(iv)   Firms commission ad hoc surveys on a wide variety of subjects.

Survey methods consist of the following stages:

(i)   The survey design - This depends on the objectives of the survey, the available methods, the amount of money, and time that can be allocated for collecting the information

(ii)   The Pilot survey: This is the preliminary survey carried out on a very small scale to make sure that the design and methodology of the survey are likely to produce the information required

(iii)   Collection of Information - This involves the use of observation, interview and questionnaire.

(iv)   Coding - We may need to pre-code the questions to facilitate classification and tabulation.

(v)   Tabulation - There is the need to tabulate the data, to give a summary of the data.

(vi)   Secondary statistics - We will need to calculate secondary statistics such as means and percentages.

(vii) Reports - Reports must be written on the results and the results must be illustrated with graphs and diagrams.

3.3.2   Observations

This is one of the methods of collecting primary data. It can be used to know the use a particular facility is put. It can be used to study the behaviours of people in a work place.

There can be the following types of observation:

- Participant observation
- Systematic observation
- Mechanical observation

In participant observation, the observer is involved in the activities he is trying to observe. Examples of these are the vice-chancellor who participates in the eating at the cafeteria to observe the performance of the cooks and the acceptability of the food by the students; the lecturer who sits for his own paper with the students to observe the difficulty encountered by the students in the paper. This method can have serious influence in the entities the observer is observing. The method may also consume a lot of time.

In systematic observation, the observer does not take part in the activity. The method is used when events can be investigated without the participants knowing that somebody is observing them. Though the method is objective, it does not question the motives of people observed.

In mechanical observation, mechanical devices do the observation. For instance, the number of vehicles passing a particular point on the road can be recorded mechanically. Sophisticated mechanical means such as television, film and tape recorders are used to provide more complex information. Mechanical observations can be more effective than those observation made by individuals observer who can be subject to bias.

Generally, observation has the following problems:

- Objectivity- to remain objective, the observer cannot ask the question that will help him to understand the events he is observing.
- Selectivity- an observer can unintentionally become selective in perception, recording and reporting.
- Interpretation- the observer may impute meanings to the behaviour of people that the people do not intend.
- Chance- a chance event may be mistaken for a recurrent one.
- Participation- the participation of the observer can influence the behaviour of people being observed.

3.3.3 Interviewing

This is a conversation with a purpose. There can be formal and informal interviews. Informally everybody uses interviews to obtain information.
The formal interview is also initiated by the interviewer who approaches the person he is interested in interviewing. The interviewer therefore arranges the venue, and the time, and prepares the questions to be asked. The interviewer also secures the means of recording the responses.

Interviews are used in a number of situations that include:

(i)   Obtaining opinion polls for the success of a candidate in an election.
(ii)   Studying why people behave in a way
(iii)   Selecting applicants for some jobs
(iii)   Reporting especially in the radio and television

Before the interview is conducted, the interviewer must have knowledge of his respondents. The interviewer must also secure an initial rapport with his respondents. The interviewer must explain clearly and briefly the purpose of the interview to his respondents.

The interviewer should be very objective. He must not express his own opinions and must not influence the answers of the respondents. The language of the question must be at the level of the respondents. If the questions are written in a language different from that of the respondents, the question must be translated to that of the respondent but the answers must be written by the interviewer in the language of the questions. This was the case during the last census in Nigeria. The questionnaire used was written in English language. That is not the language many Nigerians understand. For the purpose of getting the response of people in this group; the questions were translated to the language they understand.

Interviewing method has a number of advantages:

(i)   It allows the interviewer to have personal contact with the respondents therefore allowing more questions to be asked which improves the quality of the information.
(ii)   It allows the interviewer to persuade unwilling respondents.
(iv)   It allows experienced interviewer to know when to make calls and recalls

Interviewing method also has a number of disadvantages.

(i)   Biased interviewer may influence the responses of the respondents to suit his own opinions.
(ii)   A biased respondent, in a matter-affecting ego, may give false responses.
(iii)   It is very expensive and time consuming especially when respondents are widely scattered.
(v)   It may be difficult to interview some top people in government and business

3.3.4   Questionnaire

This is a list of questions drawn in such a way that the questions are related to the objectives of the study being conducted, and the responses to the question will be analyzed to provide solutions to the problems we attempt to solve in the study.

There are two types of questionnaire namely:

(i)   Structured or fixed-response questionnaire:
(ii)   Unstructured or open ended questionnaire

The structured questionnaire consists of a list of questions drawn on the study being conducted. Each question is accompanied by alternative answers from which the respondent picks appropriate answer or answers. An example of a structured question is this:

What is your monthly salary

•   Below N7500
•   N7500 - N10,000 0

- N10, 000 - N12, 500 0
- Above N12,500

Structured questionnaire has a number of advantages:

(i)   It is very easy to complete and analyze.
(u)   Most of the questions are answered
(iii)   The responses are always related to the objectives of the study.

A major disadvantage of the structured questionnaire is the fact that it does not allow the views of the respondents, which may enhance the quality of the information collected.

Unstructured questionnaire is a list of questions drawn on the study on which information is required. The questions are not accompanied by alternative answers as in the structured questionnaire. The respondents are free to provide their own responses.
Example of a question in an unstructured questionnaire is "What is your monthly salary?" Unstructured questionnaire are not difficult to construct since no question is accompanied by alternative answers. It has the following advantages.

(i)   It allows for the views of the respondents
(ii)   The questionnaire are not difficult to construct since no question is accompanied by alternative answers.

However, the unstructured questionnaire has the following disadvantages.

(i)   It is not easy to complete and analyze.
(ii)   Many of the responses supplied by the respondents may not be related to the objectives of the study.
(iii)   The respondents may not answer all questions, especially those considered difficult by them. This will definitely reduce the quality of the information obtained.

In drawing a standard questionnaire, the following must be considered.

(1)   List all the questions you want to ask.
(ii)   Decide whether to use direct or indirect questions or both.
(iii)   A question must be limited to an idea.
(v)   Ask simple and interesting questions before the difficult and uninteresting ones.
(v)   State the questions clearly.

(vi)   A question must mean the same thing to all the respondents.
(vii)   The language must be at the level of the respondents.
(viii)   Do not ask questions that will hurt your respondent
(ix)   The questionnaire should be pre-tested on a mock-audience before it is administered on the real sample to detect the difficulty in completing and analyzing the questionnaire so as to review the questionnaire before it finally gets to the real audience it is meant for.

Student Assessment Exercise 2.3

You may wish to put off your course material, pause for a while and list the various methods of data collection, their descriptions, their advantages and disadvantages

4.0   Conclusion

In this unit, you have learned primary data and secondary data; their nature, examples, sources, advantages and disadvantages. You should also be aware of the various methods of collecting data, their advantages and disadvantages. In the unit to follow, you will learn how to summarize data to facilitate the necessary analysis that will bring out the reality contained in the information.

5.0   Summary

What you have learnt in this unit consists of the nature of statistical data, the sources of the data and the methods of collecting the data. The unit to follow will build upon this.

6.0   Tutor Marked Assignment

What are the distinguishing features between interviews and observation?

7.0   References, Further Reading And Other Resources

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UNIT 3

SUMMARISING DATA

Table of Contents

1.0   Introduction

You learned the methods of collecting data in the previous unit. In the first unit, you have learned that statistics is a scientific method of collecting, summarizing, classifying, analyzing and presenting information in such a way that we can have a thorough understanding of the reality the information represents. From this definition of statistics, summarizing the data is very important to the analysis of the data. You will learn in this unit how to arrange the mass of unordered array of mass of data in such a way that detailed analysis on the data can be performed.

2.0   Objectives

By the end of this unit, you should be able to:

• Arrange an unordered array of data in ordered form.
• Prepare frequency distributions for both grouped and ungrouped data.
• Construct relative frequency, cummulative relative frequency for both ungrouped and grouped data

3.1   Ordered Array

When data are collected, they are collected in such a way that there is no particular arrangement of the values. This unordered array of values does not facilitate the process of analyzing the data. The data must therefore be arranged either in ascending or descending order of magnitude so as to facilitate the analysis.

Example 3.1: Suppose the ages of twenty pupils in a primary school are as follows (age to the nearest years)

6, 8, 10, 11, 7, 5, 8, 9, 11, 12. 11, 9, 8, 12, 5, 6, 10, 8, 7, 9.

A look through the values shows no order of arrangement. An ordered form of the values shows the following:

5, 5, 6, 6, 7, 7, 8, 8, 8, 8. 9,9,9,10,10,11,11,11,12,12.

The data arranged in this form has a number of advantages over the raw data.

(i)   We can quickly know the lowest and highest values in the data.
(ii)   We can easily divide the data into sections.
(iii)   We can see whether any value appears more than once in the array.
(iv)   We can observe the distance between succeeding values in the data.

Student Assessments Exercise 3.1

Go through the values listed below and arrange them in ascending order of magnitude

The daily births in a maternity home in a month are given below

 5, 3, 6, 7, 4, 1, 2, 4, 0, 3
3, 5, 2, 6, 2, 5, 0, 7, 2, 5
2, 3, 5, 4, 6, 7, 5, 1, 1, 3

3.2   Frequency Distribution

3.2.1   Frequency Distribution for Ungrouped Data

An ordered array of data does not sufficiently summarize the data.   The   data can be further summarized by preparing the frequency distribution for the data.

A frequency distribution is a table showing the values of the data and the number of occurrence of each of the values.

Example 3.2

For the data in exercise 3.1 above, present the frequency distribution for the values. The values will be represented by Xi and the frequency is represented by Fi

Frequency Distribution

Xi	Fi
0	2
1	3
2	5
3	5
4	3
5	6
6	3
7	3

This table shows the values of the variable and their respective frequencies.

3.2.2   Frequency Distribution of Grouped Data

It is possible to have the frequency in example 3.2 without grouping the data because the values are not many. There are only seven values. In situation where the values are in thousands or millions, it may be difficult to analyze the values if they are not grouped.
For many of sophisticated analyses, we need to group the data before analysis commences. We therefore group the data into class intervals

Class intervals: are defined as contiguous, non-overlapping intervals selected in such a way that they are mutually exclusive and collectively exhaustive. They are mutually exclusive in the sense that a value is Placed in one and only one class interval.
The class intervals could be 5-9, 10-14, 15-19, 20-24..................... It   could also be 11-20, 21-30, 31-40,............................
It may even take either form. In this unit there will be more examples of class intervals. The class should not be too few and should not be too many. Too few

class intervals can result in a loss of much detail while too many class intervals may not condense the

Example 3.2

The table below shows the scores of 50 students in mathematics in a Senior Secondary Examination

19	50	57	25	61	42	26	33	46	45
63	31	80	36	78	56	38	69	83	40
52	17	35	65	13	63	72	29	56	57
22	45	53	44	76	47	86	55	66	48
41	64	38	43	23	58	55	32	52	46

For this we need to prepare the tallies from the tallies we obtain the frequency of each of the class intervals.

Class Intervals	Tallies	Frequency
11-20	III	3
21-30	JHT	5
31-40	JHT III	8
41-50	JHT JHT I	11
51-60	JHT JHT	10
61-70	JHT II	7
71-80	IIII	4
81-90	II	2

What you have above is the frequency distribution of a grouped data. To further summarize grouped data, some basic concepts are important. These concepts will be defined and computed now.

(i)   **Class Limit:** For any class interval, there are two class limits, the lower and upper class limits. For the example 3.2, the lower class limits

are 11, 21, 31, 41, 51, 61, 71, 81. The upper class limits are 20, 30, 40, 50, 60, 70, 80 and 90.

(ii)   **Class Boundaries**:   For any class interval, there are two class boundaries. The lower class boundary of a class interval is the mean of the lower class limit of the interval and the upper limit of the preceeding interval. Let us compute the lower class boundary of the interval 11-20. The lower class limit of the class interval is 11 and the upper class limit of the preceeding class interval is suppose to be 10. The lower class boundary of the class interval is therefore:

$$\frac{10+11}{2} = 10.5$$

For the example 3.2, the lower class boundaries of the class intervals are 10.5, 20.5, 30.5, 40.5, 50.5, 60.5, 70.5, and 80.5.

The upper class boundary of a class interval is the mean of the upper class limit of the class interval and the lower class limit of the succeeding class interval. For example 3.2, the upper class boundary of interval 11-20 is 20.5. The upper class limit of the class interval is 20 while the lower class limit of the succeeding class interval is 21. The upper class boundary of the interval is therefore equal to:

$$\frac{20+21}{2} = 20.5$$

For the example, the upper class boundaries of the class intervals are respectively 20.5, 30.5, 40.5, 50.5, 60.5, 70.5, 80.5, and 90.5. From these values, you will see that the upper class boundary of a class interval is the lower class boundary of the succeeding class interval. The classes can also be given in terms of class boundaries rather than class limits. When this is done, there is overlapping of the class intervals. For example 3.2, if the class boundaries are used, we will have the following as our frequency distribution.

(iii) **Class Width:** This is the difference between the upper and lower

Class Intervals	Frequency
10.5-20.5	3
20.5-30.5	5
30.5-40.5	8
40.5-50.5	11
50.5-60.5	10
60.5-70.5	7
70.5-80.5	4
80.5-90.5	2

class boundaries (not class limits). For our example 3.2, the class width for all the class intervals is 10. For the first interval, the class width is 20.5-10.5 = 10. It is not 20-11.

(iv)   Class Mark: This is the mean of the upper and the lower class

boundaries. It can also be the mean of the lower and the upper class

limits. For example 3.2, the class mark for the first interval is:

$$\frac{10.5+20.5}{2} = 15.5. \text{ It can also be } \frac{11+20}{2} = 15.5$$

For the classes in the example 3.2, we have the following as class width for the respective class intervals 15.5, 25.5, 35.5 , 45.5, 55.5, 65.5, 75.5, and 85.5. There is need to summarize what we have done so far into class limits, class boundaries, class marks and class width.

Class Limits	Class Boundaries	Class Marks	Class Width
11-20	10.5-20.5	15.5	10
21-30	20.5-30.5	25.5	10
31-40	30.5-40.5	35.5	10
41-50	40.5-50.5	45.5	10
51-60	50.5-60.5	55.5	10
61-70	60.5-70.5	65.5	10
71-80	70.5-80.5	75.5	10

| 81-90 | 80.5-90.5 | 85.5 | 10 |

3.3   Relative Frequency

3.31   Ungrouped Data

The relative frequency of a value is defined by the total frequencies of all the values contained in the set of values.

Example 3.3. Suppose the frequency distribution of the scores of the twenty students in a test is as presented below

Scores	Frequency
2	1
3	2
4	2
5	4
6	5
7	3
8	2
9	1
TOTAL	20

The total frequency is 20. The relative frequency of the first score is given as 1/20 = 0.05

The relative frequency distribution of the scores is therefore given as:

Scores	Frequency	Relative Frequency
2	1	0.05
3	2	0.10
4	2	0.10
5	4	0.20
6	5	0.25
7	3	0.15
8	2	0.10
9	1	0.05

3.3.2   Grouped Data

For the grouped data, the relative frequency of a class interval is the frequency of the interval divided by the total frequencies of all class intervals. For example 3.2, the relative frequency distribution of the class intervals is as presented

Relative Frequency Distribution of Grouped Data

Class Intervals	Frequency	Relative Frequency
11-20	3	0.06
21-30	5	0.10
31-40	8	0.16
41-50	11	0.22
51-60	10	0.20
61-70	7	0.14
71-80	4	0.08
81-90	2	0.04

Going through the relative frequencies table presented in the unit, you will realize that the sum of the relative frequencies for a set of values is 1

3.4   Cumulative Relative Frequency Distribution

3.4.1   Ungrouped Data

In this unit, you have learned construction of relative frequency for ungrouped data. A further summary of data can be in form of cumulative relative frequency. To construct the cumulative relative frequency, we need to construct the cumulative frequency for the data. The cumulative relative frequency of a value is the cumulative frequency of the values divided by the total frequency of the values contained in the array of data.

For the example 3.3, the cumulative frequency and the cumulative relative frequency for the set of data is as presented below

Score	Frequency	Cumulative Frequency	Cumulative Relative Frequency
2	1	1	1/20 = 0.05
3	2	3 = 1+2	3/20 = 0.15
4	2	5 = 3+2	5/20 = 0.25
5	4	9= =5+4	9/20 = 0.45
6	5	14=9+5	14/20=0.70
7	3	17 = 14+3	17/20 =.85
8	2	19 = 17+2	19/20 =.095
9	1	20 = 19+1	20/20 = 1.00

3.4.2   GROUPED DATA

The cumulative frequency of grouped data is constructed the same way as that of ungrouped data.

Example 3.4

The table below shows the frequency distribution of the ages of employees in a firm. Prepare the cumulative frequency and the cumulative relative frequency for the ages of the employees.

Age (In Years)	Frequency
18-20	2
21-23	5
24-26	13
27-29	25
30-32	22
33-35	8
36-38	6
39-41	3
42-44	5
45-47	3
48-50	3
51-53	2
54-56	2
57-59	1

The cumulative frequency and the cumulative relative frequency distributions are as follows.

Ages (Years)	Frequency	Cumulative Frequency	Cumulative Relative Frequency
18-20	2	2 =2	2/100 = 0.02
21-23	5	2+5= 7	7/100 = 0.07
24-26	13	7+13 = 20	20/100 = 0.20
27-29	25	20+25= 45	45/100 = 0.45
30-32	22	45+22= 67	67/100 = 0.67
33-35	8	67+8= 75	75/100 = 0.75

36-38	6	75+6 = 81	81/100 = 0.81
39-41	3	81+3 =84	84/100 = 0.84
42-44	5	84+5 = 89	89/100 = 0.89
45-47	3	89+3= 92	92/100 = 0.92
48-50	3	92+3= 95	95/100 = 0.95
51-53	2	95+2= 97	97/100 = 0.97
54-56	2	97+2= 99	99/100 = 0.99
57-59	1	99+1= 100	100/100 = 1.00

4.0   Conclusion

In this unit you have learned how to arrange data in ordered form. You have also learned how to summarize data, both grouped and ungrouped data, into frequency distribution, relative and cumulative frequency distributions. You have also learned some basic concepts that will aid the analysis of grouped data latter in subsequent units.

What you have learned in this unit will facilitate further presentation and analysis of the data in the subsequent units.

5.0   Summary

In this unit, we have summarized data using frequency distribution, relative and cumulative frequency distributions. Some concepts that will aid the graphical presentation of the unit that will follow this unit were also defined.

6.0   Tutor Marked Assignment

For the table presented below prepare

(i)   The frequency distribution for the class interval
         11-20, 21-30, 31-40........
(ii)   Cummulative Frequency distribution.
(iii)   Relative Frequency.
(v)   Cummulative Relative Frequency Distribution

Ages of 100 Employees

58	37	21	28	27	24	27	39	38	30
60	20	30	50	44	33	23	26	31	32
18	23	41	32	27	42	40	29	28	34
56	19	28	29	23	47	29	24	31	34
30	19	22	53	49	26	16	38	42	36
41	32	33	20	31	36	32	32	31	32
21	24	55	21	24	34	37	29	33	32
32	49	38	48	33	43	26	38	30	28
24	46	15	43	43	23	23	28	34	28
41	23	25	19	51	23	36	31	35	31

7.0   References, Further Readings and other Resources

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UNIT 4

GRAPHICAL PRESENTATION OF DATA

1.0   Introduction

You learned in Unit 3 the summarizing of data using frequency distribution, relative and cumulative relative frequency distributions. In this unit, you will learn how to represent these data graphically. Graphical representation has a way of showing the salient features of data without having to interpret the column of numbers. Some other data that are not summarized into frequency distributions will be represented with charts in this unit.

2.0   Objectives

At the end of this unit, you should be able to:

•   Plot histogram, frequency polygon and Ogive for a set of values. • Construct simple, component and multiple bar charts for a set of data.
•   Draw pie chart

3.1   Histogram:

3.1.1   One of the ways of representing a frequency distribution is by means of a histogram. In constructing a histogram we plot the frequencies of the class intervals against the class boundaries [not the class limit]. The vertical axis is used for the frequencies and the horizontal axis for the class boundaries.

Example 4.1

Suppose the table below shows the distribution of 50 spectators in a secondary school sports competition. You are required to represent the data with a histogram

Table 4.1: Age Distribution of spectators in school sports.

Age (Years)	Frequency
10-14	2
15-19	3
20-24	5
25-29	7
30-34	11
35-39	8
40-44	6
45-49	5
50-54	3

In solving the exercise, you require a graph sheet; you also need to prepare the class boundaries for the class intervals. You will then plot the frequency against the respective class boundaries. You still need to recall how the class boundaries are computed in unit 3 we will not discuss it here.

Age (Years)	Class Boundaries	Frequency
Class Interval		
10-14	9.5-14.5	2
15-19	14.5-19.5	3
20-24	19.5-24.5	5
25-29	24.5-29.5	7
30-34	29.5-34.5	11
35-39	34.5-39.5	8
40-44	39.5-44.5	6
45-49	44.5-49.5	5
50-54	49.5-54.5	3

FIGURE 4.1 Histogram Showing The Data In Example 4.1
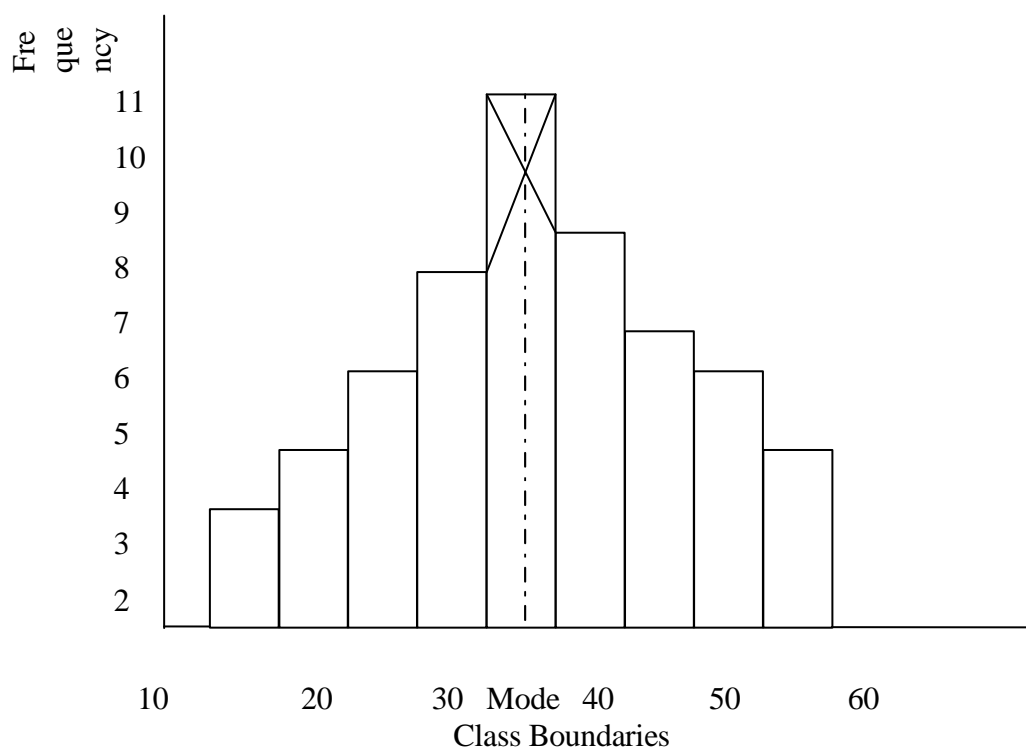
The histogram can be used to estimate the mode of the distribution. To do this you have to locate the highest cell in the histogram, join the upper class boundary of the cell with the upper boundary of the preceding cell, join the lower class boundary of the highest cell with the lower class boundary of the succeeding cell, locate the intersection, draw a vertical line from the intersection to the horizontal. The value of the vertical line on the horizontal axis is the mode. You need to see the construction on the histogram in fig. 4.1. The mode read from figure 4.1 is 32.5.

3.1.2   Frequency Polygon.

Another way of representing frequency distribution graphically is by the means of a frequency polygon.

In constructing a frequency polygon, we plot the frequency against the class marks. You learned in unit 3 that class mark of a class interval is the mean of the lower and the upper class boundaries or limits of the class interval.

Example 4.2

Construct a frequency polygon for the data in example 4.1

To construct the frequency polygon, you need to compute the class mark for the class intervals, you need to make the polygon touch the horizontal at both ends. To do this, you have to compute the class mark for an imaginary class interval at the beginning and another imaginary class interval at the end of the distribution.

If you look at table 4.1 that is of interest here, there is no class interval 5-9 at the beginning and there is no class interval 55-59 at the end of the distribution. We need to bring these intervals in and assign a frequency of 0 to each of them.

Let us now compute the class mark for the class intervals.

Class Intervals	Class Marks	Frequency
5-9	7	0
10-14	12	2
15-19	17	3
20-24	22	5
25-29	27	7
30-34	32	11
35-39	37	8
40-44	42	6
45-49	47	5
50-54	52	3
55-59	57	0

The next activity is to plot the frequency against the class marks. The frequency is on the vertical axis and the class mark on the horizontal axis. Frequency polygon must be plotted on a graph sheet

FIGURE 4.2: Frequency Polygon for Example 4.2

3.1.3 **Ogive**

This is another way of representing frequency distribution graphically. The other name for ogive is cumulative frequency distribution curve. This curve is very important in the determination of median, quartriles, percentiles, semi-interquartile range, that will be discussed in some subsequent units.

In plotting ogive for a distribution, you will do the following

(a) Compute the upper class boundaries of all the classes including that of an imaginary class at the beginning of the distribution.
b)   Prepare a cumulative frequency distribution for the data
(c)   Plot the cumulative frequency against the upper class boundary.

Example 4.3.

For table 4. l, construct an ogive for that distribution.

Class Interval	Frequency	Less than	Cumulative Frequency
5-9	0	9.5	0
10-14	2	14.5	2
15-19	3	19.5	5
20-24	5	24.5	10
25-29	7	29.5	17
30-34	11	34.5	28
35-39	8	39.5	36
40-44	6	44.5	42
45-49	5	49.5	47
50-54	3	59.5	50

You will realize that the class interval 5-9 was introduced. A frequency of 0 was also assigned to the class interval since the original table did not show the class. This is done so that the ogive can take its origin from the horizontal

line. From the table you will see that all the values that are less than 24.5 are contained jn class interval 5-9, 10-14, 15-19 and 20-24. The sum of the values which is equal to the cumulative frequency of the interval is 0+2+3+5 = 10.

You should know that ogive can only be plotted on a graph sheet.

FIGURE 4.3: Ogive Showing the Frequency Distribution in Table 4.1

Cummulative Frequency

Exercise 4.1

The frequency distribution of salaries (monthly) of workers in a firm is as follows

Salaries	Number of Workers
2,000-7,000	5
7,000-12,000	8
12,000-17,000	24
17,000-22,000	5
22,000-27,000	3
27,000-32,000	2
32,000-37,000	1

For the distribution, prepare

(i)   a histogram and
(ii)   an ogive

Using the histogram, estimate the mode of the distribution

3.2   Bar Chart

3.2.1   Simple Bar Chart

Another way of representing data graphically is by means of bar chart. A bar chart shows vertical bars with equal width to represent the values of a variable in some intervals of time. The area of the bar is proportional to the magnitude of the quantity it represents. The bars must be drawn on graph sheet and they must have equal width. There are simple components and multiple bar charts. These will be discussed in this unit.

In a simple bar chart, we use the bars to represent the value of a variable in a period of time.

Example 4.4

Suppose the monthly sales of a firm for three consecutive months are given as follows

Months	Sales (In million of Naira)
January	5.2
February	7.4
March	10.6

You are required to represent the data on a simple bar chart.

Figure 4.4 Simple Bar Chart for Example 4.4

Months
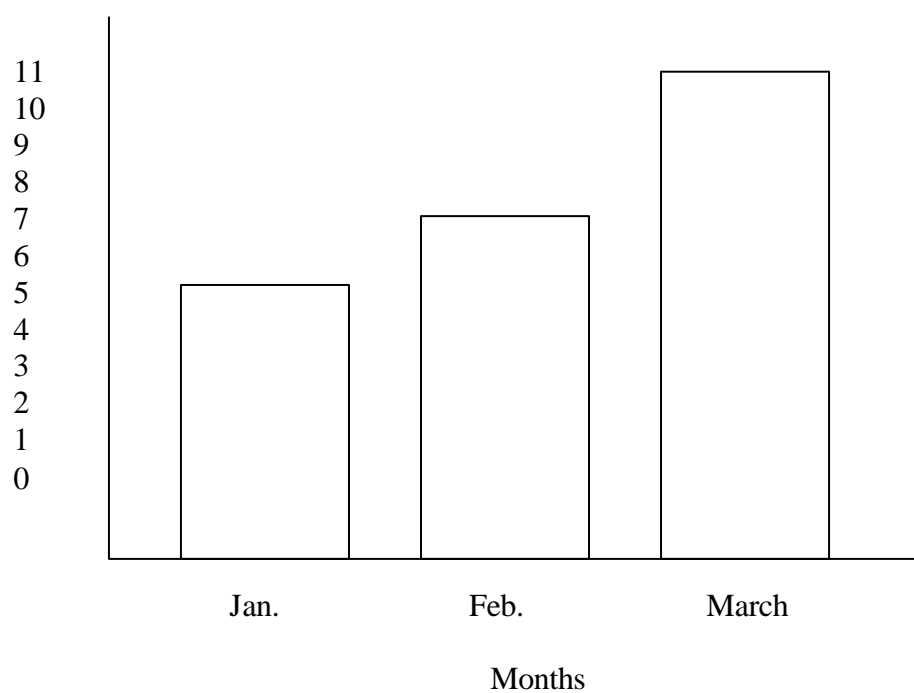
3.2.2 Component Bar Charts

Another bar chart that shows the total value for a time period and the values of the components that makeup the total is the component bar chart. In this case, the bar for the total value for a period is divided into the values for the components that make up the total.

Example 4.5

Suppose a hotel has three departments A, B, C from where sales are made and the annual records of the net profit of the departments for three consecutive years are as presented below.

Years	Net Profits in the Departments (Millions of Naira)		
	A	B	C
1999	3.2	3.4	2.8
2000	2.8	3.0	2.6
2001	4.0	3.2	3.6

You are to represent the values of the net profits with the aid of a component bar chart.

You will need to plot the values of the components A, B, and C for three years. The first year will show a bar of length 9.4cm divided into 3.2cm for A, 3.4cm for B, and 2.8cm for C. You will then repeat the exercise for the values of A, B and C in years 2000 and 2001. There must be a legend to show the shading of the component.

FIGURE 4.5: Component Bar Chart For Example 4.5
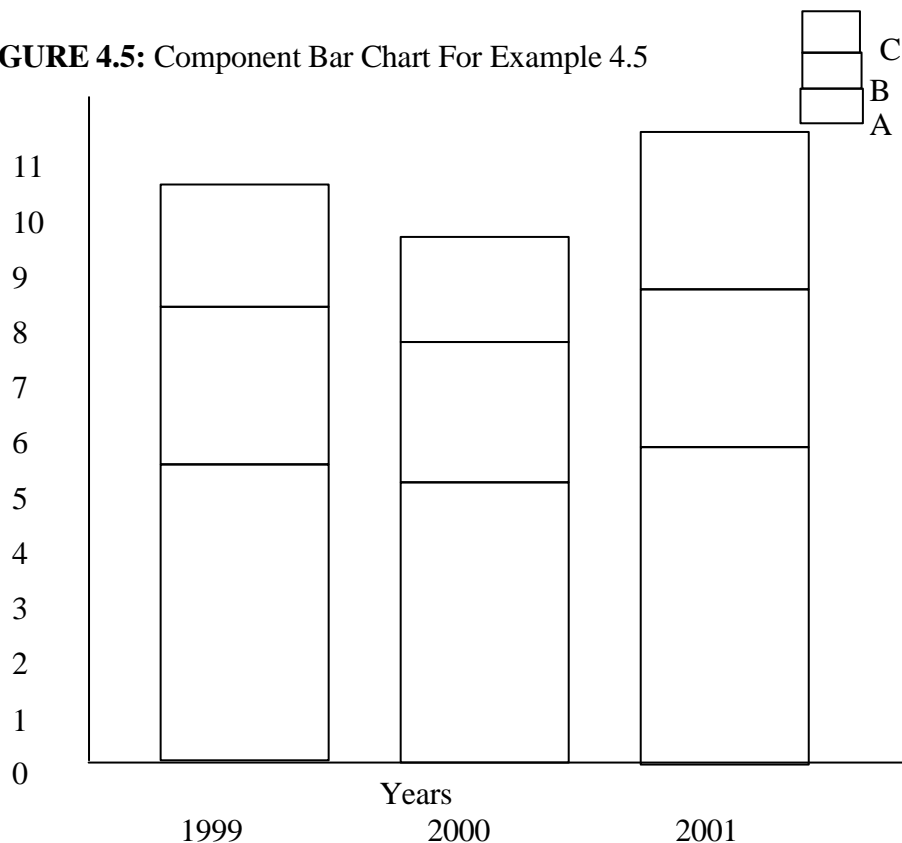
3.2.3   Multiple Bar Chart

For the multiple bar chart, each component of every year is presented by a bar whose length is corresponding to the value of the component.

Example 4.6

Using the values of net profit in example 4.5, construct a multiple bar chart for the hotel for the period three years.
In this exercise, you will draw single bar for each component of every year. The bars for a year will now look like histogram.

FIGURE 4.6: Multiple Bar Chart for Example 4.6
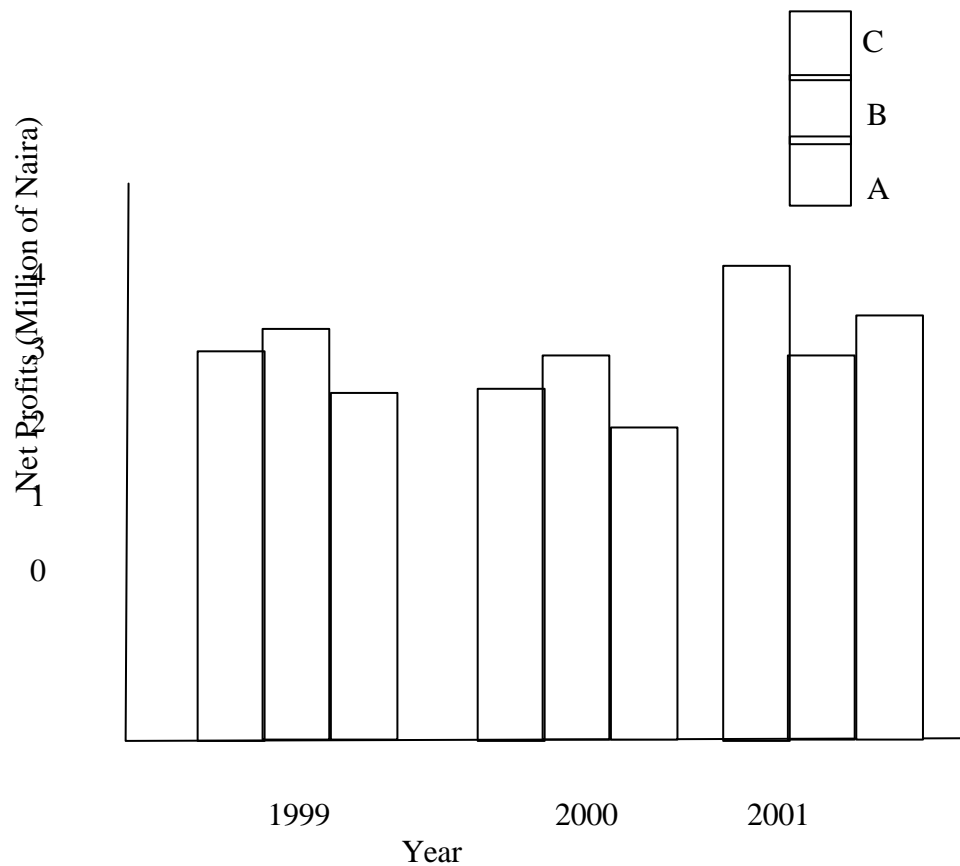
3.3   Pie Chart

This is another means of representing data graphically. The values of the Items represented with pie chart are proportional to the area of the sectors that represent them.

In the case of pie-chart, the sectorial angles are computed for the items based on their values and on the total values of the items.

Sectorial angle of an item = $\dfrac{\text{Value of Item}}{\text{Total Value of all items}}$ X 360°

After obtaining the sectorial angles for the items we use a pair of compasses, a pencil, protector and ruler to draw the angles of the sector.

Example 4.7

Suppose the monthly income of a worker is allocated as follows

Items	N
Feeding	9625
Rent	4125
Education	5500
Savings	6875
Others	1375
TOTAL	27,5000

You are required to represent the data on a pie char. We will therefore compute the sectoral angles

Items	Sectoral Angles
Feeding	126"
Rent	54"
Education	72°
Savings	90"
Others	18'
TOTAL	360

For instance the sectoral angle for feeding is calculated as

$$\frac{9625}{27500} \times 360^0 = 126^0$$

The total values of the items is N27, 500. This is to say that the monthly income of the worker is N27, 500. 360° is used in the calculation because the sum of angles at a point is 360°. If the sectoral angles are computed correctly, the sum of the angles must be equal to 360°.

FIGURE 4.7: Pie Chart showing Items in Example 4.7
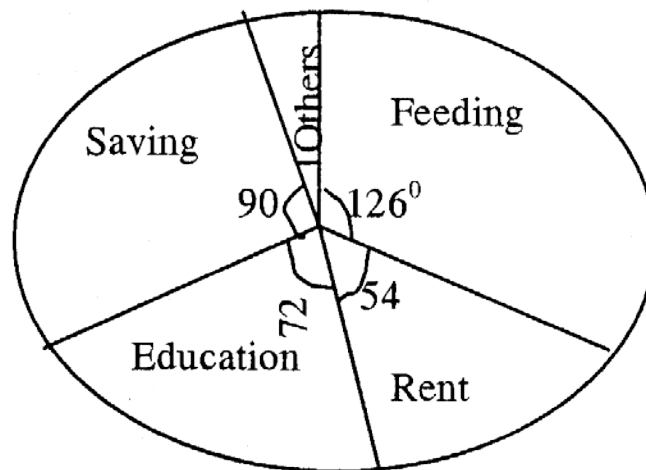
It is possible the values of the items are given as percentages of the total values of the items. To find the sectoral angles, we only need to multiply the respective percentage with 360°

Example 4.8

Suppose the basic elements of cost of a restaurant are expressed as percentages of sales as follows.

Elements	Cost as % Sales
Food Cost	40
Labour Cost	35
Overhead Cost	15
Net Profit	10
Sales	100

Obtain the sectoral angle for elements of cost.

Sectoral Angles

Food Cost = $\dfrac{40}{100}$ x $360^0 = 144^0$

Labour Cost = $\dfrac{35}{100}$ x $360^0 = 126$

Net Profit = $\dfrac{1}{1}$ x $36^0 = 36^0$

4.0   Conclusion

You learned in this unit the graphical representations that summarize the data further after the frequency distribution has been constructed. You learned that histogram is constructed by plotting frequency against the class boundaries [not class limits]; frequency polygon is constructed by plotting frequency against class mark and the ogive or cumulative frequency curve is constructed by plotting cumulative frequency against the upper class boundary.

You also learned how to represent other forms of data with bar charts and pie charts. Of all these graphs and charts only pie chart can be drawn on plan sheet of paper, all the others are to be constructed on graph sheets.

5.0   Summary

This unit taught graphic representation of frequency distributions in terms of histogram, frequency polygon and ogive. The unit also taught graphical representation of other forms of data with bar and pie charts. Subsequent units will teach the computation of measures of central tendency from ungrouped and grouped data.

6.0   Tutor Marked Assignment

6.1   The salaries of some staff in a department are given as follows: 4 clerical staff earn N1000 and more but less than N4000 each. 6 supervisors earn N4000 and more but less than N7000 each. 8 managers earn N7000 and more but less than N10, 000 each, 4 senior managers earn N10, 000 and more but less than N13, 000 each while 3 General managers earn N13, 000 and more but less than N16, 000 each. Using this data draw

(i) An Ogive   (ii) An histogram, to represent the distribution

6.2   The Profit made by the three income centers of an hotel for a period of three consecutive years are as follows:

Income Centers	1998	1999	2000
A	8.0	9.0	11.5
B	5.0	5.5	6.5
C	3.0	4.5	6.0

Represent these data on:

(i)   A simple bar chart
(ii)   A component bar chart
(iii)   A multiple bar chart

7.0   Reference and Other Resources

Ajayi, J.K. (1997) Elements of Business Statistics. Unpublished Monograph Ondo State Polytechnic, Owo

Daniel W.W and Terrel J.C (1979) Business Statistics: Basic Concepts and Methodology 2"ᵃ ed. Houghton Mifflin Co. Boston

Levin R.T (1990) Statistics for Nonageomeric 4`" ed., Prentice-Hall of India Private L.t.d. New Delhi.

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UNIT 5

MEASURE OF CENTRAL TENDENCY I
THE ARITHMETIC MEAN

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1.0   Introduction

In unit 4, you learned how to construct graphs from frequency distribution and charts from raw data. The graphs and the charts gave us the trend and the patterns in the data. As beautiful and useful as these graphs and charts are, they do not give us any precise understanding of the reality of the information contained in the data.

We need single measures that can describe certain characteristics of the data and that give us a more precise understanding of the information the graphs and charts can give. This will help us make quicker and better decisions without the need to consult our original observation.

One of such measures is the arithmetic mean In this unit you will learn how to compute the mean from ungrouped and grouped data and how to compute weighted mean. You will also learn the characteristics of the arithmetic mean.

2.0   Objective

At the end of this unit, you should be able to compute the mean of the grouped data with or without a frequency distribution, and

• Compute the mean of grouped data

• List the properties, advantages and disadvantages of the arithmetic mean

• Compute weighted mean for some data

3.1 The Arithmetic Mean

3.1.1 The Arithmetic Mean of Ungrouped Data

We use average many times to mean ~he arithmetic mean. We compute arithmetic mean for both ungrouped and grouped data. We also compute arithmetic mean, which we henceforth in this unit call mean, for both the sample and the population from where the sample is drawn. Mean computed for the sample is called a statistic and it is donated by $\overline{x}$. The mean computed for the population is called a parameter and is denoted by μ You should note in this course that any measure computed for the sample is called statistic and any measure computed for the population is called parameter.

The mean of ungrouped data is the summation of the values in the set of data divided by the number of values in the set of data.

For a sample the number of values is denoted by n, that is the sample size, and for the population the population size is given as N.

Mean of ungrouped data (if a sample) is $\overline{x}$

Where x = $\dfrac{\sum\limits_{i=1}^{n} \overline{\times i}}{n}$

$\sum$ = Summation

$\times i$ = Different values of a variable

n = Sample size

Example 5.1

Compute the mean for the following:

Scores:7, 5, 8, 10, 11, 6, 3 ,

   4, 10, 9,13, 2.

Mean= $\bar{x}$ = $\dfrac{7+5+8+10+11+6+3+4+10+9+13+2}{12}$

= $\dfrac{88}{12}$ = 7.33

You will realize that there is no frequency distribution for this example. Suppose there is a frequency distribution for the values of a variable, then how do we calculate the mean?

This is simple. If we are computing the mean for a sample, that is x, the mean

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} \; or \; \frac{\sum f_i x_i}{n}$$

f_i = different frequencies of values in the set of data

x_i = different values in the set of data

$\sum f_i$ or n is equal to the number of values in the set of data.

Example 5.2

The table below gives the frequency distribution of the mark scored by 20 students in a test conducted in Statistics for Management

Scores	Frequency

11	2
12	3
13	5
14	7
15	2
16	1

You are required to calculate the mean of the scores

To solve the problem, you should

(i)   Multiply each score by its respective frequency to obtain $fixi$

(ii)   Add up the products of each score and its respective frequency to obtain $\sum fixi$

(iii)   Add up all the frequencies to obtain $\sum fi$ or n.

(iv)   Divide $\sum fixi$ by $\sum fi$ to obtain the mean.

Scores (xi)	Frequency (Fi)	Fixi
11	2	22
12	3	36
12	5	65
14	7	98
15	2	30
16	1	16
Total	20	276

$$\text{Mean} = x = \sum_{i=1}^{n} \frac{\sum fixi}{\sum fi} = \frac{276}{20} = 13.35$$

There can be another method of computing the mean apart from using

$$x = \frac{\sum fixi}{\sum fi}$$

we use the assumed mean, A, then the mean $x = A + \dfrac{\sum fidi}{\sum fi}$

where A = Assumed mean

di = xi-A for all the values of xi

Example 5.3: For the example 5.2, let us compute the mean again using an assumed mean of 13.

We will then have the table below for the solution of the problem.

Scores (Xi)	fi	di = Xi – A	fidi
11	2	11-13 =-2	-4
12	3	12-13 = -1	-3
13	5	13-13 = 0	0
14	7	14-13 = 1	7
15	2	15-13 = 2	4
16	1	16-13 = 3	3
Total	20		7

Where Assumed mean A = 13

Mean = x = $13 + \dfrac{7}{20}$ = 13 + 0.35 = 13.35

You will see that with the method of assumed mean, we should obtain the same value of mean we had before.

3.1.2   The Arithmetic Mean of Grouped Data

You have learned how to compute the arithmetic mean of ungrouped data.

When the values are many in a set of data, there is the need to group them into class intervals. You learned about this in unit 3. We need to take some time to compute the mean of grouped data.

Example 5.4

The earning per share (in kobo) of some firms is presented below with the frequency distribution.

Earning per share (in kobo)	Frequency
65-69	3
70-74	4
75-79	11
80-84	15
85-89	9
90-94	5
95-99	3

You are required to calculate the mean of the distribution.

To solve this question, we need to compute the class mark for the class intervals. The class mark becomes the Xi we will use in the computation. Immediately this is done, the whole distribution is reduced to the form of an ungrouped data with frequency distribution. You should recall that class mark is the mean of the upper and lower class boundaries (or Limits) of a class interval.

Class Mark I	Frequency	$\sum XIfI$
Xi	fi	Fixi
67	3	201
72	4	288
77	11	847
82	15	1230
87	9	783
92	5	460
97	_3	291
TOTAL	50	4100

Mean x = $\dfrac{4100}{50}$ = 82

The average earning per share of the firms is 82 kobo.
From the computation, you will still realize that the mean of a grouped data is given as

$$\frac{\sum fi \times i}{\sum fi}$$

$x =$ the x_i here are the various values of the class marks of the

class intervals.

The assumed mean method can also be used here.

Example 5.5: Using the assumed mean method, compute the mean of the distribution in example 5.4

We will still make use of the frequency distribution of the class mark

Class Mark xi	Frequency fi	A=77 di=xi-77	Fidi
67	3	-10	-30
72	4	-5	-20
77	11	0	0
82	15	5	75
87	9	10	90
92	5	15	75
97	3	20	60
TOTAL	50		250

We assume a mean of 77

$$\bar{x} = A + \frac{\sum fidi}{\sum fi}$$

$\bar{x} = 77 = \dfrac{250}{50} = 82$

We still obtain the same mean as before.

Example 5.1

For the distribution below, compute the mean, using the two methods we have used in this unit.

Class Intervals	Frequency
0-9	4
10-19	6
20-29	18
30-39	11
40-49	6
50-59	5

3.2   The Weighted Mean

Some times we are interested in showing the relative importance of some items. When we do, we attach weights, apart from the real values of the items. For instance in the Open University, some courses in a programme are core courses while the others are auxiliary courses. Core courses tend to have more credit unit than the others.

When we attach weighs to items we can only compute the weighted mean.

The weighted mean is given as: $\bar{\times}_w = \dfrac{\sum wi \times i}{\sum wi}$

Where xi = values of the items
Wi = relative weights of the item
Xw = Weighted mean

Example 5.6

The percentage scores of a student in some courses and the credit units of the courses are given below:

Courses	% Score	Credit Unit
HEM 101	52	3.0
HEM 102	64	3.0
HEM 103	71	3.0
HEM 104	45	3.0
GNS 101	82	2.0
GNS 126	55	1.0

Calculate the weighted percentage mean for the scores. xi = Percentage Scores

Xi = Percentage Scores
wi = Credit Units

Xi	wi	wixi
52	3.0	156
64	3.0	192
71	2.0	142
45	3.0	135
82	2.0	164
55	1.0	55
	14	884

The weighted percentage mean = $\dfrac{884}{14}$ = 60.29

Exercise 5.2

The scores of a student in a university in the courses he took in the first semester of the year and the units of the courses are presented as follows:

Courses	% Scores	Unit
CSC 101	75	3.0
CSC 102	70	3.0
CSC103	42	2.0

CSC104	45	3.0
GNS 101	52	2.0
GNS 128	63	1.0

The University is interested in computing weighted grade point average. The academic standard of the University shows the following scores, grades and grade points

Score	Grades	Grade Point
Above 75	A	4.0
70-74	AB	3.5
65-69	B	3.25
60-64	B C	3.0
55-59	C	2.75
50-54	CD	2.5
45-49	D E	2.25
4044	F	2.0
below 40		0.0

With this information, compute the weighted grade point average for the student to two decimal places.

3.3   Advantages And Disadvantages Of Arithmetic Mean

3.3.1 Advantages of Arithmetic Mean.

The arithmetic mean has the following advantages

(i)   Mean is the best known of all the averages
(ii)   Mean can be used for further mathematical process. Mean is used to perform statistical procedures such as estimation and hypothesis testing.
(iii)   Mean is unique, unlike mode (this will be discussed later], because a set of data has one and only one mean.

3.3.2 The disadvantages of mean

Arithmetic mean has the following disadvantages.

(i)   Since all the values in a set of data are used to compute the mean, the mean can be influenced by extreme values

For instance the mean of 3, 4, 5, 6 and 7 is $\frac{3+4+5+6+7}{5} = 5$

There is no extreme value here. Suppose we have 3, 4, 5, 6, 7, 19, we then have an extreme value, 19. The mean becomes

$$\frac{3 + 4 + 5 + 6 + 7 + 19}{5} = \frac{44}{5} = 7.33$$

The extreme value has greatly influenced the mean.

(ii)   A mean may result into an impossible value where the data is discrete. For instance, If the number of female students in five programmes at the National Open University are 35, 38, 42, 53, and 66, the mean value of the female students will be

$$\frac{35+38+42+53+66}{5} = \frac{234}{5} = 46.8 \text{ Students}$$

This is an impossible value.

(iii)   We are unable to compute mean for data in which there are open-ended classes either at the beginning of the distribution or at the end of the distribution. It will be difficult to know the class mark of the open-ended class.

4.0   Conclusion

This unit teaches the computation of the arithmetic mean from grouped and ungrouped data. It also teaches the computation of the weighted mean. Some advantages and disadvantages of the arithmetic means were given in the unit. Subsequent unit will teach other measures of central tendency.

5.0   Summary

You learned in this unit the computation of arithmetic mean and weighted mean. The advantages and disadvantages of mean were also learned. Future units will discuss other measures of central tendency.

6.0   Tutor Marked Assignment

From the frequency distribution table shown below

(i)　　Compute the mean using the two methods in this unit
(ii)　Compare your result and comment

Class	Frequency
8.0-8.9	5
9.0-9.9	7
10.0-10.9	10
11.0-11.9	13
12.0-12.9	15
13.0-13.9	5
14.0-14.9	3
15.0-15.9	2

7.0　References, Further Reading and other Resources

Daniel W.W and Terrel J.C (1979) Business Statistics: Basic Concepts and Methodology 2nd ed. Houghton Mifflin Co. Boston

Harper W.M (1982) Statistics. 4th ed. Macdonald and Evans

Levin R.1 (1990)Statistics for Management 4th ed. Prentice - Hall of India Private L.t.d

UNIT 6

MEASURES OF CENTRAL TENDENCY 2
GEOMETRIC MEAN AND HARMONIC MEAN.

Table Of Contents	

1.0   Introduction

In unit 5 you learned low to compute the Arithmetic mean, weighted mean and the advantages and disadvantages of arithmetic mean. You also learned that the arithmetic mean is the most commonly used of all the means. Many times we are dealing with some quantities that change over a period of time. Examples of such cases are growth rates in the population, the growth factors of interest values.

In all those situations, the computation of the arithmetic mean maybe inappropriate. The appropriate means in those situations are geometric and harmonic means. This unit will discuss these types of mean. Examples will be worked. Exercises will also be provided to show your understanding of the unit.

2.0   Objective

By the end of the unit, you should be able to:

   Compute harmonic mean for a set of values
   Compute harmonic mean for a set of values
   List the uses of geometric and harmonic means.

3.1   GEOMETRIC MEAN

3.1.1   Computation Of Geometric Mean

For values x_1, x_2, x_3, . . . x_n the geometric mean is the nth root of the product of the values. The geometric mean is denoted as GM therefore

$$GM = \sqrt[n]{x_1, X_2, x_3, \ldots x_n}$$

Where GM is geometric mean x_1, X_2, x_3, ... x_n are values of the variable of interest, while n represents the sample size.

(i)   Multiply the values all together to get the product
(ii)   Take the nth root of the product

Example 6.1

What is the geometric mean of 3, 5, 6, and 7?

$$GM \quad \sqrt[4]{3x5x6x7x}$$

$$= \sqrt[4]{630}$$

$$= 5.01$$

When the values are many this may be difficult to compute. We can also compute the geometric mean as follows;

The formula for geometric mean is:

$$GM = \sqrt[n]{x_1.x_2.x_3\ldots x_n}$$

We can take the logarithm of both sides and obtain log GM = long GM

$$= \sqrt[n]{x_1.x_2.x_3\ldots x_n}$$

Under this situation we have

$$Log\ GM = \frac{f_2 \log x_1 + f_2 \log x_2 \ldots f_n \log x_n}{n}$$

If the values have frequency distribution, the geometric means will be given as:

$$\text{Log GM} = \frac{f_2 \log x_1 + f_2 \log x_2 \ldots f_n \log x_n}{\sum f_1}$$

After getting the values of the logarithm and we have done the addition of all the logarithmic values we will take the anti-log of the value to obtain the geometric mean.

Example 6.2

Using logarithm, compute the geometric mean of 3, 5, 6 and 7

$$\text{GM} = \sqrt[4]{3 \times 5 \times 6 \times 7}$$

$$\text{Log GM} = \frac{\log 3 + \log 5 + \log 6 + \log 7}{4}$$

$\text{Log} 3 = 0.4771$

$\text{Log} 5 = 0.6990$

$\text{Log} 6 = 0.7782$

$\text{Log} 7 = 0.8451$

$\text{Total} = 2.7994$

$$\text{Log GM} = \frac{2.7994}{4} = 6998$$

Anti-log $0.6994 = 5.01$

Suppose there is a frequency distribution for the value. We can still compute the geometric mean.

Example 6.3

Compute the geometric mean for the distribution

X_i	fi
5	2

6	3
7	6
8	4
9	3
10	2

In computing the geometric mean we have GM =

$$^{20}\sqrt{5^2 \times 6^3 \times 7^3 \times 8^4 \times 9^3 \times 10^2}$$ we can use logarithm to solve the problem. Since we know that:

Log GM = $\dfrac{2 \log 5 + 3\log 6 + 6\log 7 + 4\log 8 + 3\log 9 + 2\log 10}{20}$

x_i	fi	Log x_i	$f_i \log x_i$
5	2	0.6990	1.398
6	3	0.7982	2.3346
7	6	0.8451	5.0706
8	4	0.9031	3.6124
9	3	0.9542	2.8626
10	2	1.0	2.000
			17.2782

Log GM = $\dfrac{17.2782}{20}$

= .86391

anti-log of 0.86391
= 7.31 The geometric mean is 7.31
Let us discuss some practical examples of geometric mean.

Example 6.4

Suppose the rates of growth in the sales of a product over a period of 5years are 8%, 10%, 12%, 18% and 24%. If the sales in year 0 is N100, 000,000 what is the sales at the end of the fifth year?

We may wish to find the mean rate of growth as

$$\frac{8 + 10 + 15 + 18 + 24}{5} = 14.4\%$$

Having got this we can get the sales at the end of the fifth year to be equal to 100,000,000 x 1.134 x 1.144 x 1.144 x 1.144 x 1.144 =N195,943,166.6. You will see that we use the factor of 1.144 when the rate of growth is 14.4%. This is correct because we will need to add 100% and 14.4% to get the new value for a year

$$100\% + 14.4\% = \frac{100}{100} + \frac{14.4}{100} = 1.144$$

The factor obtained here using arithmetic mean may not be appropriate. The appropriate factor should be obtained using geometric mean.

The appropriate growth rate is:

$$GM = \sqrt[5]{1.08 \times 1.1 \times 1.12 \times 1.18 \times 1.24} = 1.1425$$

The true growth rate is therefore 14.25% and not 14.4%. That we got initially. The sales at the end of the fifth year will then be equal to 100,000,000 x 1.1425 = N194,687,539.2

3.1.2   Uses of Geometric Mean

The geometric mean is used mainly in connection with index numbers. It is also used for arranging ratios.

Exercise 6.1

The table below shows the percentage increase in the net worth of a business over a period of 6 years.

Year	1994	1995	1996	1997	1998	1999
C/O increase	4%	9.5%	8.0%	5%	65%	75%

What is the average increase in the net worth over the period of 6 years?

3.2 HARMONIC MEAN

3.2.1 Computation Of Harmonic Mean.

For values $x_l, x_2, x_3 \ldots x_n$

The harmonic mean is given as

$$\frac{n}{\sum \frac{1}{x_1}} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \ldots \frac{1}{x_n}}$$

For this we do the following:

(1) We add up the reciprocals of the values
(2) Divide the sum into the number of items

Example 6.5

Compute the harmonic mean of 5, 6 and 7

$$\text{Harmonic mean} = \frac{3}{\frac{1}{5} + \frac{1}{6} + \frac{1}{7}}$$

$$= \frac{3}{0.2 + .0167 + 0.143} = \frac{3}{0.5099} = 5.88$$

The harmonic mean can also be used to obtain the average of different speeds

Example 6.6

A motorist moves from point p to Q, a distance of X kilometers with a speed of 100km/hour and returns to point P with a speed of 80km/hour. What is the average speed?

$$\text{The average speed} = \frac{2}{\frac{1}{100} + \frac{1}{80}} = \frac{3}{.00225} = 88.89 \text{ km/hour}$$

3.2.2   Uses Of Harmonic Mean

The harmonic mean is used to average ratios, speeds etc. It is used mostly in engineering.

Exercise 6.2

Compute the harmonic mean of the following values
5, 7, 9, 11, 13, and 15

4.0   Conclusion

In this unit, you have learned how to compute geometric and harmonic means. Geometric mean has been given for values $x_1, x_2, x_3, \ldots x_n$ as $^n x\sqrt{x_2.x_3...x_n}$ and as

Log GM = log XI + log x2 + log x3... + log xn n

The harmonic mean has been given for values xl, X2, x3, ... Xn as n n

$$\frac{n}{\sum \frac{1}{x_1}} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \ldots \frac{1}{x_n}}$$

Some of the uses of geometric and harmonic means have been given in this unit. Progress exercises are also given. The future units will still teach some other measure of central tendency.

5.0   Summary

This unit has shown the computation of both the geometric and harmonic means. It has also given some uses of these means

6.0   Tutor Marked Assignment

6.1   The rates of inflation in four consecutive years in a country were 6%, 8% 16% and 30%

What was the average rate of inflation per year?

6.2   Find the harmonic mean of 8, 9, 12, 15

7.0 REFERENCES AND OTHER RESOURCES

Hannagan T.J (1982) Mastering Statistics. The Macmillan Press Ltd.

Harper W.M (1982) Statistics 4'h Ed. Macdonald and Evans Handbook Series

Unit 7

MEASUREMENT OF CENTRAL TENDENCY 3 - MEDIAN AND MODE

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1.0   Introduction.

In the last two units, you learned about some measures of central tendency, their computation, their advantages and disadvantages. In this unit you will still learn about more measures of central tendency, their computation, advantages and disadvantages. We are often very interested in what is happening to the majority of the population and we show concern for this,.

In this unit two important measures, median and mode, concerned with what happens to the center of the set of values will be worked and exercises will be provided to test your understanding of the content of the unit

2.0   Objectives

At the end of this unit, you should be able to:

•   Compute the median for both the ungrouped and grouped data.
•   Estimate median from the cumulative frequency curve.
•   Compute the mode for both the ungrouped and group data.
•   Estimate the mode from a histogram.
•   List the advantages and the disadvantages of median and mode.

3.1   MEDIAN

3.1.1   Computation Of Median From Ungrouped And Grouped Data.

The median is a measure that shows the most central item in a set of data. It is a single value computed from the set of data that measures the central item in the data. If median is the most central item in the set of data, half of the values in the set must lie below the median, and the other half above the median

To calculate the median, there is a need to have a mathematical definition of the median

Median = the $\left(\dfrac{n+1}{2}\right)^{th}$ item in the set of data. Where n is the number of values in the set of data.

For an ungrouped data with odd number of values, the item in the middle constitutes the median.

Example 7.1

What is the median of 17, 12, 13, 15, 18? These values are not arranged in any order. To find the median, we need to order the arrangement of the values. as 12, 13, 15, 17, 18. The value in the middle is 15 and that is the median.

Using our formula for median

Median = the $\left(\dfrac{n+1}{2}\right)^{th}$ item

$\left(\dfrac{5+1}{2}\right)^{th}$ item

 = 3rd item
The third item is 15

Suppose we have an even number of values such as 8, 4, 3, 5,9,2,11,7., what will be the median?. We need to order the arrangement of the values as 2,3,4,5,7,8,9,11

There are 8 values. The median is therefore the $\left(\dfrac{n+1}{2}\right)^{th}$ item

$$\left(\dfrac{8+1}{2}\right)^{th} \text{ item}$$

$$= 4.5 \text{ item}$$

The 4.5^{th} item an only be between 5 and 7. Therefore the median will be equal to

$$\left(\dfrac{5+7}{2}\right)^{th} = 6$$

Suppose we have ungrouped data with frequency distribution, how do we compute the median?

We still compute the median the same way we did the two previous examples

Example 7.2

For the distribution below, compute the median

Scores	Frequency
2	3
3	4
4	6
5	7
6	5
7	3
8	2

The sum of the frequencies here is 30. There are people examined. The

median should now be the $\left(\dfrac{30+1}{2}\right)^{th}$ item

$= 15.5^{th}$

The median (the 15.5^{th} value) is $\dfrac{5+5}{2} = 5$

To obtain the median for grouped data, we still do the same thing we did previously.
However, we can decide to adopt the use of formula for the median.
Some authors use the formula shown below

Median $= L_1 + \dfrac{J}{fi} \times w$

Where $L_1 =$ Lower class boundary of the median class
$fi =$ frequency of class that contain the median
$w=$ class width of class containing the median
$J = \dfrac{n}{2}$ minus sum of all the frequencies up to, but not including the medium

class.

$n =$ sum of all the frequencies in the set of values $\sum fi$

Some other authors uses the formula below for median

Sample median $= \left(\dfrac{\dfrac{30+1}{2} - (f+1)}{fm}\right) w + Lm$

Where $m =$ total number of items in the distribution
$f =$ sum of all the class frequencies up to, but not including, the median class
$fm =$ frequency of median class
$w =$ class width
$Lm =$ lower limit of median class interval for the purpose of this course, we will continue to use

Median $= L_1 + \dfrac{J}{fl} \times w$

Example 7.3

For the distribution below, compute the median

Class interval	Frequency
0-499	229
500-999	306
1000-1499	398
1500-1999	298
2000-2499	127
2500-2999	66
3000-more	26

The sum of the frequency here is 1450 the median is the $\left(\dfrac{1450 + 1}{2}\right)^{th}$ item

The median class is then 1000 -1499
Since the 725.5[th] item lies in the class. From the first two classes we obtain (229 + 306) items = 535 items.
In the third class containing 398 items, we can get 190.5 items to make up for the 725.5 items we are looking for.
Since the median class is 1000 -1499, the lower class boundary = 1000,the fi = frequency of the median class = 398.
The class boundaries for the median class interval are 1000 - 1500. The class width is equal to 500

$$J = \frac{1450}{2} - (229 + 306) = 190$$
$$Median = 1000 + \frac{190}{398} \times 500 = 1238.69$$

3.1.2 Estimation Of Median Using Ogive

We can also estimate the median by plotting ogive for the set of data.

Example 7.4

Plot an ogive for the table below and estimate the median from the ogive

Class I	fi
1-1.99	2
2-2.99	5
3-3.99	6
4.499	7
5.599	5
6-6.99	3
7.799	2

We will need to obtain the cumulative frequency distribution to plot the ogive.

Upper Class	Cumulative frequency
1.00	0
2.00	2
3.00	7
4.00	13
5.00	20
6.00	25
7.00	28
8.00	30

To obtain the ogive, we plot the cumulative frequency against the upper class boundary as we discussed previously

FIGURE 7.1   Ogive for Example 7.4
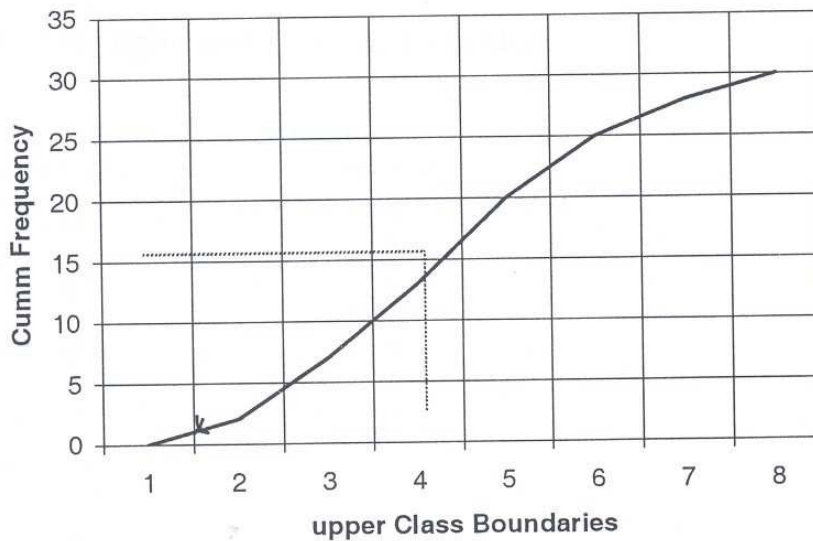
The median is taken as $\left(\dfrac{n}{2}\right)^{th}$ the median is taken as 15th value here.

Locate the 15.0 value along the vertical axis and draw it to meet the curve. At the point it meets the curve, draw a vertical line to meet the horizontal at the point the line meets the horizontal axis is the median. The estimate of the median here is 4.5 (from ogive)

3.13   Advantages and Disadvantages of median

The median cannot be influenced by extreme values as in the case of the means since not all the values are involved in the calculation of the median.

(ii)   The median can be calculated from incomplete records and for open ended classes if the median does not fall into the open ended class.

(iii)   It is not difficult to recognize the median

Median also has a number of disadvantages

(1)   Median cannot be subjected to further statistical processing as it is with mean.

(2)   To compute median we need to order the values. This can consume time.

Exercise 7.1

For the table below,

i.   Compute the median
ii.   Draw the ogive and estimate the median

Class Interval	Frequency
21-30	2
31-40	6
41-50	9
51-60	9
61-70	11
71-80	6
81-90	4
91-100	3

3.2   Mode

3.2.1   Computation of Mode from Ungrouped and Grouped Data

Mode is another measure of central tendency. It is simply defined as the value with the highest frequency in a set of values. Unlike mean and median, the mode can have more than one value in a set of data.

Example 7.5

From the distribution below, find the mode

X_1 (ages in years I	Frequency
5	5
6	7
7	8
8	11

10	3
11	2
12	1

The mode is 8 because it has the highest frequency of 11. Suppose we replace the frequency of those in age of 10 years by 11, the distribution will have two modes, that is 8 and 10 respectively.

We can also calculate the mode from grouped data. To do this, we will use the formula.

Mode = Ll + $\frac{d_i \times w}{d_i + d_2}$ where Li = Lower class boundary of the modal class

d_1 = frequency of the modal class minus frequency of the class just before the modal class

d_2 = frequency of the modal class minus frequency of the class just above the modal class

W = Class width

Example 7.6 For the distribution below computer the mode.

Class I	Frequency
22-26.9	5
27-31.9	8
32-36.9	14
37-41.9	23
42-46.9	19
47-51.9	7
52-56.9	4

The modal class in this question is 37-41.9.

L_1 = lower class boundary = 37

W = class width = 42 -37 = 5
fi = frequency of the modal class = 23
fo = frequency of class before the modal class = 14

f2 = frequency of class after the modal class = 19

d_l=fi-fo=23-14=9

d_2=f_1-f_2=23-19=4

mode = 37 + $\underline{9x5}$ = Li + $\underline{d1xW}$
                  9+4   dl +d2

= 40.46
= 40.5

3.2.2   Estimation Of Mode Using Histogram

By plotting the histogram, as was discussed in unit 4, we can also estimate the mode of the distribution.

Example 7.7

Using the table in example 7.6, plot the histogram and estimate the mode.

Class Boundaries	Frequency
22-27	5
27-32	8
32-37	14
37-42	23
42-47	19
47-52	7
52-57	4

For histogram, you will recall that you still plot the frequency against the class boundaries

FIGURE 7.2: Histogram for Example 7.7
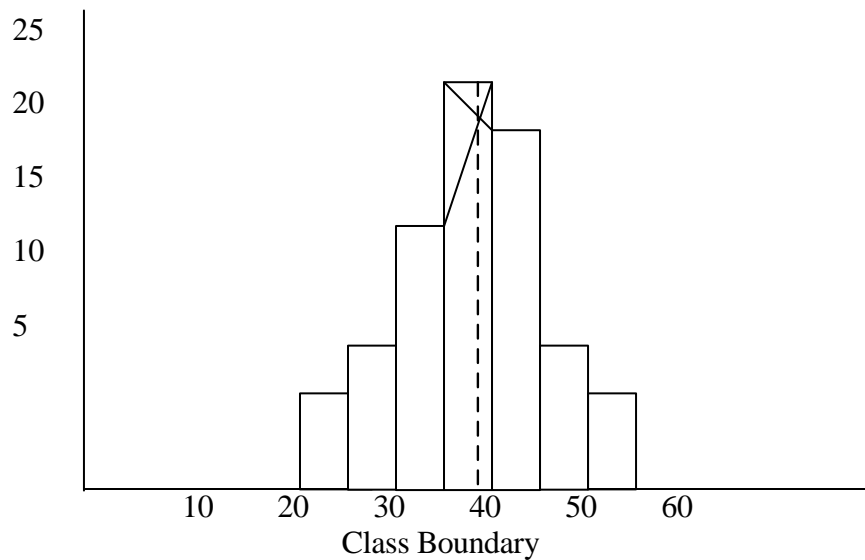
The mode obtained from the histogram is 40.0. This is a good estimate of the computed mode.

3. 2.3   Advantages And Disadvantages Of Mode

Modes has a number of advantages:

i.   It can be used as central location, just as in the case of median for both qualitative date

ii.   Since its computation does not involve all the values in a set of data, it can be computed from an incomplete record and it is not influenced by extreme values as it is the case with mean

Mode also has a number of disadvantages:

i   It is not as commonly used, as much as means are, to measure central tendency

ii   There may not be mode in a distribution if every value has the same frequency as others.

iii.   There can be many modes in a distribution resulting into a problem of interpretation.

Exercise 7.2

For the table in exercise 7.1,

i.   Compute the mode
ii.   Draw the histogram and estimate the mode

4.0   Conclusion

In this unit, you learned that median of an ungrouped data is the median

should now be the $\left(\dfrac{n+1}{2}\right)^{th}$ item in the distribution.

You also learned that the median of a grouped data is given by $Li + \dfrac{JxW}{ft}$

All the notations were properly defined in the unit. The median was also estimated from the ogive drawn from a frequency distribution.
Mode is defined as the value with the highest frequency for ungrouped data.
For grouped data, mode is defined as equal to $L_1 + \dfrac{dixw}{d_1 + d_2}$

All notifications are carefully defined in the unit.
The unit gives both the advantages and disadvantages of median and mode.

5.0   Summary

The unit teaches median and mode and their computation from ungrouped and grouped data. It also shows the estimation of measures graphically.
Subsequent units will teach measure of dispersion.

6.0   Tutor Marked Assignment

From the table below:

i.   Compute the median and mode
ii.   Plot a histogram and estimate the mode
iii.   Plot an ogive and estimate the median
iv.   Compare the values in (ii) and (iii) and common.

Class intervals	Frequency
10-19	5
20-29	8
30-39	15
40-49	11
50-59	7
60-69	4

7.0　References and other Resources

Daniel W. W. Terrel J. C (1979) Business Statistics. Basic Concepts and Methodology 2n^d ed., Houghton Co., Boston

Levin R.1 (1990) Statistics for Management 4th ed., Prentices Hall of India Private Ltd., New Delhi.

UNIT 8

FRACTILES, SKEWNESS AND KURTOSIS

1.0.   Introduction

In the previous unit, you learned about mean and median. These are location parameters. These measures can be located on the horizontal axis of a distribution when the distribution is graphed. There is also interest in some other location parameters such as quartiles, deciles and percentiles. These show the value of item which are given fractions of the way through a distribution.

The skewness further shows the relative positions of the mode, median and mean in the distribution. The Kurtosis also reveals the peakedness of a distribution.
In this unit, you will learn about the various characteristics of a distribution before proceeding to the measures of dispersion.

2.0   Objectives

At the end of this unit, you should be able to:

•   Compute different types of fractiles in a distribution
•   Describe different types of skewness.
•   Compute the degree of skewness in a distribution
•   Locate the relative positions of mode, median and mean in a distribution.
•   Explain measures of Kurtosis of a distribution.

3.1   Fractiles

3.1.1   Quartiles

A Fractile is defined as the value of the item which is a given fraction of the way through a distribution as given below. Suppose we have a distribution given below 163, 175, 184, 193, 206, 209, 219,233,224,228,231,235 the value of an item one quarter of the way through the distribution will be $\underline{12}$ item.
$$4$$

This is equal to the third.

Item in the distribution, and the value is 184. The value of an item three quarters of the way through the distribution will be $\underline{3}$ x 12TH item. This is the
$$4$$
9th item. The value from the distribution will be 224.

One of the important fractiles is the quartile in a distribution - the lower and the upper quartiles. The lower quartile tile is the value of item and it is designated as Q1. The upper quartile is the value of item three quartile the way through the distribution and it is designated as Q3.
To compute lower and upper quartiles for grouped data, we still adopt the formula used for calculation of the median.

$$\text{Quartile} = Li + \frac{J}{fi} \text{ x } W$$

Where L_1 = Lower class boundary of the class containing the quartile.
W = Class width of the class interval
fi = frequency of the class containing the quartile
J = Total frequency multiplied by the percentage value of the quartile minus the sum of all class frequencies up to, but not including, the quartile class

Example 8.1

Using the frequency distribution table below, compute the lower and the upper quartiles for the distribution

Class intervals	3-7	8-12	13-17	18-22	23-27	28-32
Frequency	6	8	16	10	7	3

In this distribution, the total frequency is 50.
25% of the item (for first quartile) is 12.5th . 75% of the items (for third quartile) that is upper quartile is $\frac{5}{100}$ x   50 = 37.5th item

For the first quartile, that is lower quartile, we are looking for the value of the 12.5`" item.

From the distribution, the 12.5th item can be found in class interval 8-12.
Therefore L1= 17.5, fi = 10,W = 22.5 - 17.5 = 5
J=37.5-30=7.5

Upper quartile = $17.5 + \frac{7.5}{10} \times 5$

=21.25

You have learned that the class width is the difference between the upper class and the lower class boundaries. For class interval 18 -22, the lower class boundary remains 17.5 and the upper class boundary 22.5 hence class width is 22.5 - 17.5 = 5.

3.1.2   Decile

This is another fractile showing the tenths of the way through a distribution

Example 8.2

Suppose we have a distribution given below, let us show and compute some decile.

Class intervals	Frequency	Cumulative frequency
20-40	11	11
40-60	27	3 8
60-80	32	70
80-100	26	96
100-120	14	110
120-140	10	120

The first decile is the value of the item $\frac{1}{10}$ of the way through the distribution

and our table, is the $\frac{1}{10}$ X 120th item = 12th item.

You should know that the total number of items (11 + 32 x 26 + 13 + 10) is 120.

The eight decile in the distribution is $\frac{8}{10}$ x 120th item = 96th item.

The first decile will be found in class interval 40 - 60 and the eight decile in class interval 80-100.
From the table. Let us compute the second and the seventh deciles.

Second Decile

The position of item = $\frac{2}{10}$ x 120 = 24th item.

This is located in class interval 40 - 60.

To compute the decile, we use the same formula we used for the median and quartile.

Decile = $L_1 + \frac{J}{fi}$ x W

Where
L_1 = lower class boundary of the class containing the decile.
W = class width of the class interval
fi = frequency of the class containing the decile

J = Value of item of the decile minus the cumulative frequency.

For second decile, the class interval containing the 24th value is 40 - 60 therefore L_1= 40, fi=27,W=20J=24-11=13.

You should know that the class boundaries are used to describe the class interval here since the class-intervals overlap.

Second decile $\frac{40}{1}$ + $\frac{13}{27}$ x $\frac{20}{1}$ = 49.63

The Seven decile

This is the value of $\frac{7}{10}$ x 120th item = 84th item.

The 84`h item falls into 80 -110
Therefore L1 80, fi =26, W = 20 J= 84 - 70 = 14
Seventh decile = $\frac{80 + 14 \times 20}{26}$ = 90.77

3.1.3 Percentile

These are fractiles that relate to the hundredths of the way through distribution. The 60th percentile is denoted as P_{60}. This is $\frac{85th}{100}$ of the way through the

distribution.

The formula for computing percentile is P^{th} percentile = L_1 + $\frac{J}{fi}$ x W

Where, L_1= lower class boundary of the class containing the percentile
        W= class width of the class interval
        fi = frequency of the class containing the percentile.

J = P^{th} percent of the total frequency minus cumulative frequency of the class between the class containing the percentile.

Example 8.3

Using the table in example 8.2, calculate the 60th percentile (P 60) and 85th percentile (P85)

Let us rewrite the table here

Class intervals I	Frequency	Cumulative frequency
20-40	11	11
40-60	27	38
60-80	32	70
80-100	26	96
100-120	14	110
120-140	10	120

60th percentile (P_{60})

$\frac{60}{100}$ x 120 [72th] item

the class containing P_{60} = 80 – 100
L_1=80, f_i=26, W=20, J=72-70=2

$P_{60} = \frac{80 \times 20}{26}$ = 81.54

85th percentile (P_{85})

$\frac{85 \times 120}{110}$ = 102th item

the class containing the 102th item is 100 – 120

Therefore L_1= 110, f_i =14, W = 20 J= 102 - 96 = 6

85th percentile = $P_{85} = \frac{110 + 6 \times 20}{14}$ = 108.57

Exercise 8.1

Salaries (N 000)	No of Workers
1-3	8
3-5	12
5-7	16
7-9	22
9-11	18
11-13	14
13-15	10

Using the table above, compute

(i)   Lower and upper quartiles
(ii)   Fourth and the sixth deciles
(iii) 45th and 78th percentile

3.2   Skewness

3.2.1   Symmetry and Skewness
In unit 4, you drew histogram. The histogram drawn from a frequency distribution can reveal certain features about the distribution. It can reveal whether the distribution is symmetry or skewed.

A symmetry distribution has a peak at the middle of the distribution with the slopes on the either side of the peak virtually equal to each other (see fig. 8.1) When the distribution is not symmetrical skewness is revealed. There can be positive and negative skews.

3.2.2   Direction Of The Skews

The direction of the skew depends on the relationship of the peak of the distribution to the centre (mean) of the histogram.
For positive skew [see figure 8.2 the peak lies to the left of the centre of the distribution. For negative skew [see figure 8.3] the peak lies to the right of the centre of the distribution.
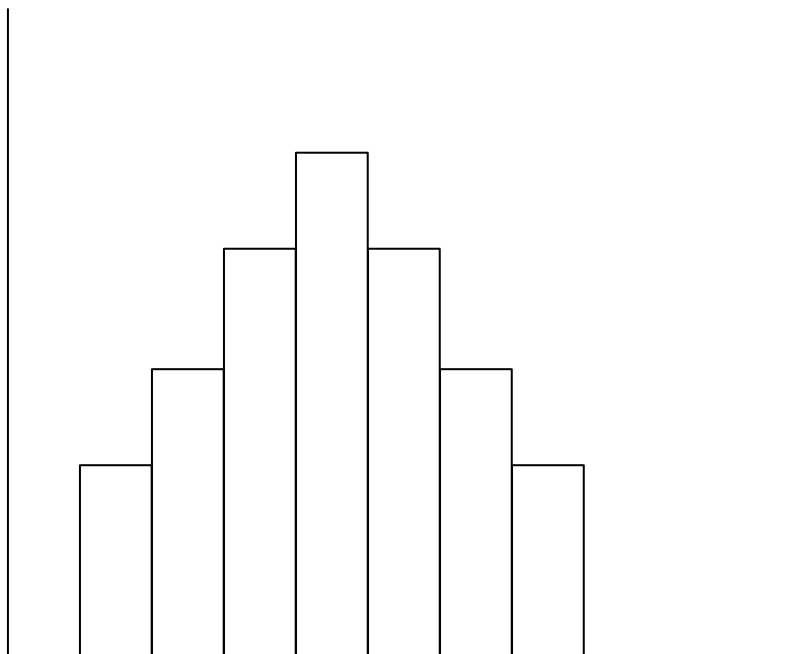
FIGURE 8.1 Symmetrical distribution
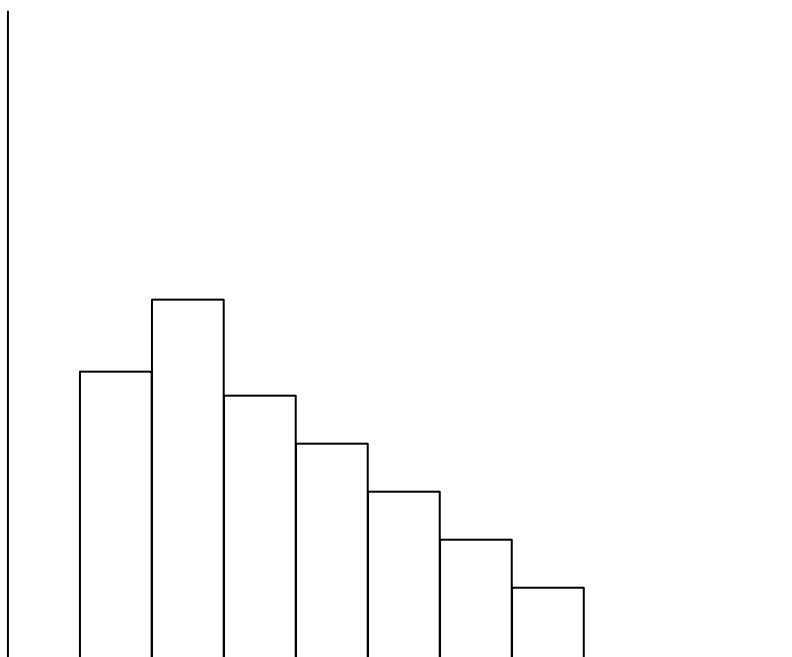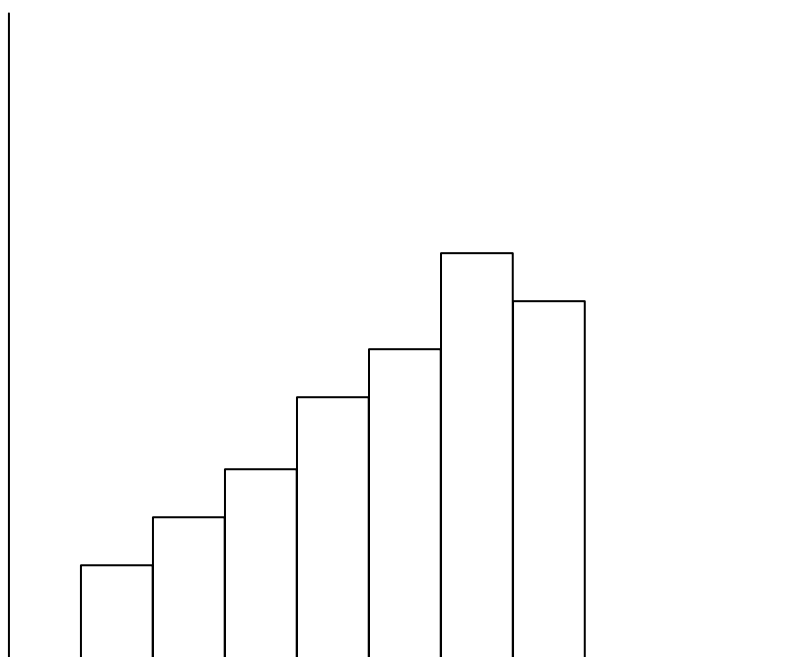
FIGURE 8.2 Positive Skew

FIGURE 8.3 Negative Skew (915)

3.2.3   Degree Skew

There is more than one way of obtaining the degree of skewness. In this unit, we will adopt the pearson coefficient of skewness denoted as SK

SK = 3 (mean – median) If the SK computed is positive, there is positive skew
        standard deviation

If it is negative the skew is negative. If the value is 0, there is a symmetrical distribution.

3.2.4   Averages in a Skewed Distribution

It gives better understanding of a frequency distribution to know the relative position of mode, median and mean in skewed distribution. Figure 8.5 shows the position of the measures in a symmetrical distribution. Figure 8.5 is a positively Skewed distribution and figure 8.6 is negatively skewed distribution.
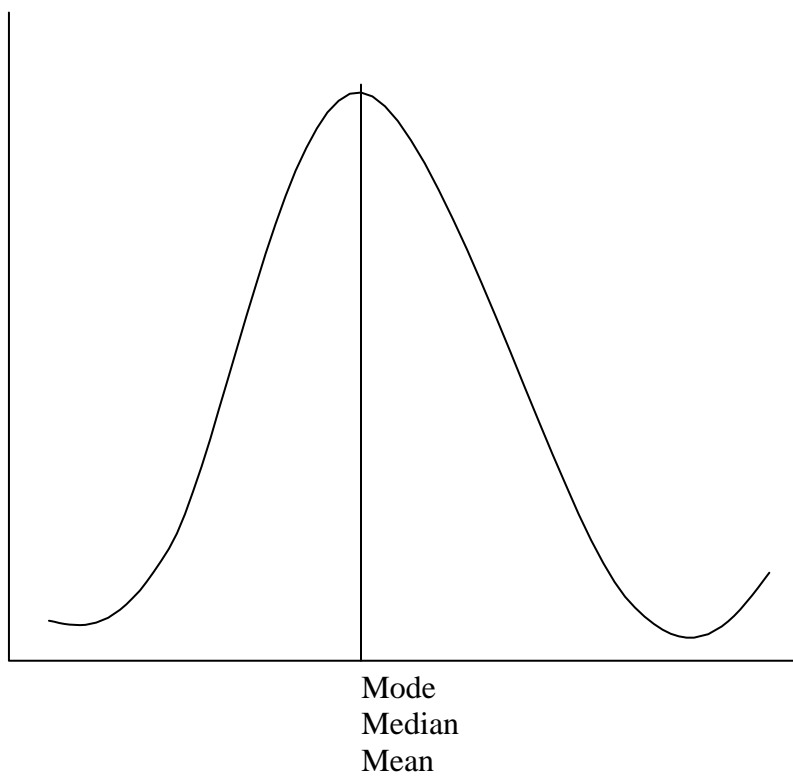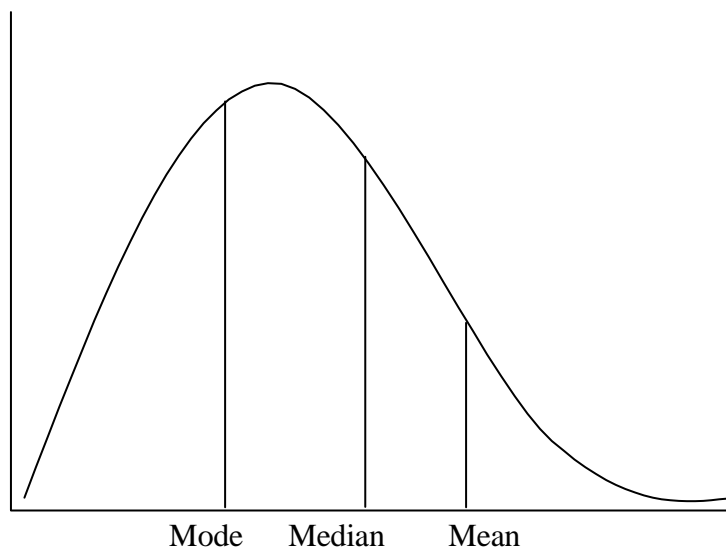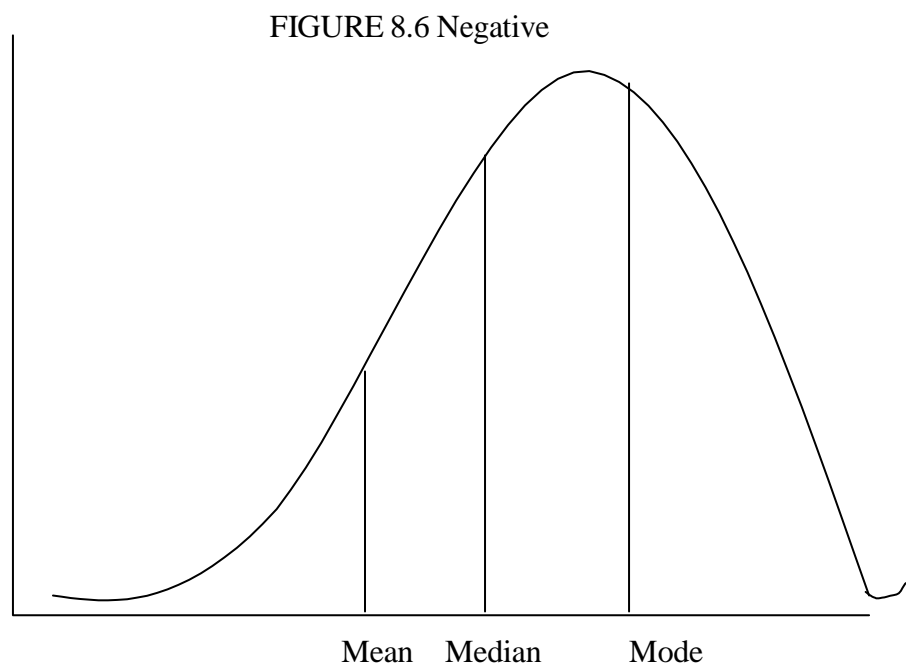
FIGURE 8.4 Symmetrical

Mode
Median
Mean

FIGURE 8.5 Positive Skew

Mode   Median   Mean

FIGURE 8.6 Negative

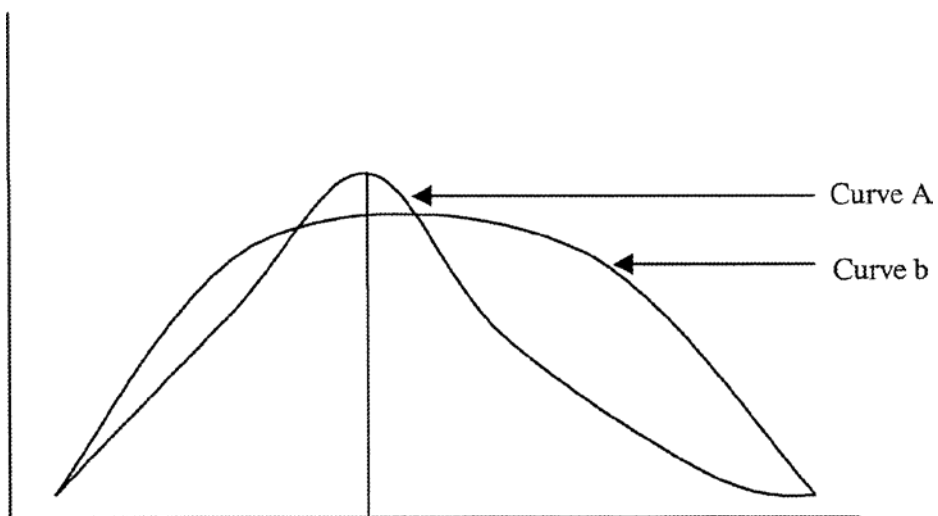Mean   Median   Mode

Exercise 8.2
How can you estimate the degree and direction of skewness of a distribution?

3.3 Kurtosis

3.3.3 Measures of Kurtosis

When we are interested in the peakness of a distribution, we are measuring the Kurtosis of the distribution

FIGURE 8.7: Difference Peaks of Distribution

The two curves here have the same central location but different Kurtosis.
Curve A is more peaked than Curve B.
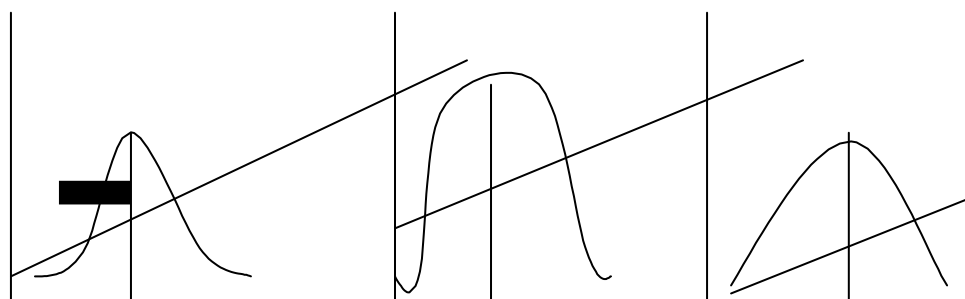On the basis of the peakedness of the distribution we can show three different types of distribution
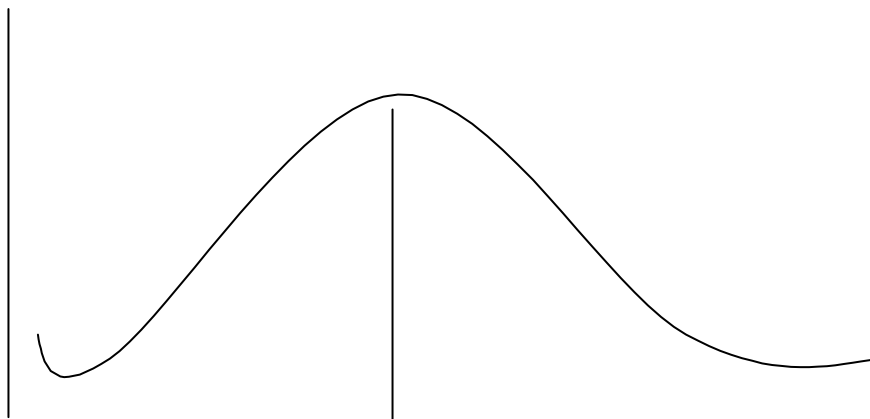
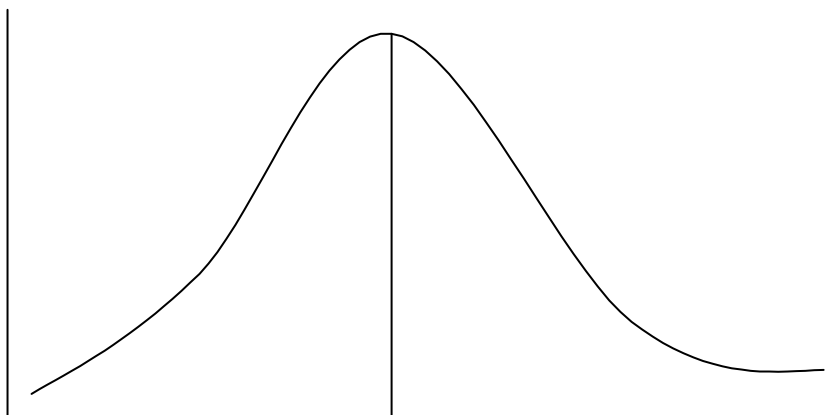FIGURE 8.8 (Mesokurtic (Intermediate)

FIGURE 8.9 Leptokurtic (Slender)

FIGURE 8.10 Platy Kurtic (Broader Flat)
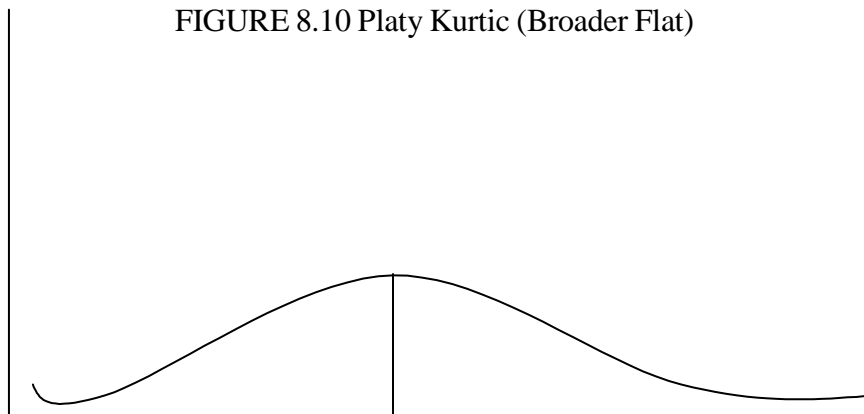
4.0 Conclusion

You learn how to compute the fractile that is quartiles, deciles and percentiles, in this unit you also learned skewness and Kurtosis in the unit. The exercises given, when undertaken will give you a better understanding of what is contained in the unit.

5.0 Summary

The unit teaches the computation of different types of fratiles -the quartiles, decile and percentile. The unit also shows the positions of mode, median and mean on different shapes of distribution. The subsequent unit will teach measures of dispersion to give more understanding about distribution.

6.0 Tutor Marked Assignment

Using the distribution shown below, compute
(i)   The lower and the upper quartile
(ii)   The 48th and 72th percentile
(iii)   The 9th and the 5th decile

With the idea of histogram drawn from the task, show the type of skewness of the distribution.

Class	Frequency
15-19	9
20-24	12
25-29	8
30-34	7
35-39	5
40-44	4
45-49	3
50-50	2

7.0   References and other Resources

Daniel W. W. Terrel J.C (1979) Business Statistics: Basic Concepts and Methodology2nd ed., Houghton Mifflin Co, Boston.

Harper W.M. (1982) Statistics. 4th ed., Macdonald and Evans Handbook Series.

Levin R. T. (1990)   Statistics for Management 4`h ed., Prentice Hall of India Private Ltd., New Delhi.

UNIT 9

MEASURES OF DISPERSION

1.0   Introduction

In unit 5, you learned how to compute the means and indeed you did compute the means. The means of the distribution computed do not show whether the values of the distribution are clustered closely together or they spread. It is even not unusual to see two sets of values with same mean but different degrees of spread.

You realize that the per capital income (mean income/person) is important to the economist as well as the distribution of income. The per capital income may be high and majority of the populace is still very poor. While per-capital income shows the mean of incomes, the distribution of the income shows the spread.

The emphasis in this unit is on computation of variation between the values in a distribution and the mean of the distribution. This leads us to the computation of the following measures of distribution; namely, range, semi-inter quartile range, mean deviation, standard deviation and co-efficient of variation.

2.0   Objectives

At the end of this unit, you will be able, after going through the worked, examples and the exercise,

to Compute the range for the distribution
Compute the semi -inter quartile range for a distribution Compute mean deviation for a distribution
Compute standard deviation for a distribution Compute the co-efficient of variation

3.1.1   Range

The range of a distribution is the difference between the highest and the lowest values in a distribution.

Example 9.1

What is the range of the following values?
2,3,5,6,8,9,11,22,34,
Range =34-2=32. The range shows how far apart the lowest and the highest value in the distribution.
It can be influenced by extreme values. It is however the simplest measure of dispersion.

3.1.2   Mean deviation

The interest here is on how far away are the values in a distribution from the mean of the distribution. The mean deviation is computed by taking the sum of the absolute value of the deviations of the values in the distribution from the mean of the distribution and dividing the sum by number of value in the distribution.

$$\text{Mean Deviation} = \frac{\sum |X_i - \bar{x}|}{n}$$

Example 9.2

Find the mean deviation of 4,6,8,9,11,12, and 13.

$$\text{The mean} = x = \frac{4+6+8+9+11+1}{9} = 9$$

X_i	$X_i - x$	$\sum [X_i - \bar{x}]$
4	-5	
6	-3	3
8	-1	1
9	0	0
11	2	2
12	3	3
13	4	4
		18

$|X_i - \bar{x}|$ is the absolute value of $X_i - \bar{x}$ and is equal to the positive value of the result obtained.

For example $|5-9| = 4$

Mean deviation $= \dfrac{18}{7} = 2.57$

If a group data constitutes the distribution then the

$$\text{Mean deviation} = \frac{\sum f_1 |X_i - \bar{x}|}{\sum f_i}$$

Where x_1 here is the class mark of individual class interval

Example 9.3

Compute the mean deviation of the distribution below

Class intervals	Frequency
10-14	3
15-19	5
20-24	7
25-29	3
30-34	2

The solution is as follows

x_i(class marks)	f_i	$f_i x_i$	$\|X_i - \bar{X}\|$	$(f_i \| X_i - \bar{X}\|)$
12	3	36	9	27
17	5	85	4	20
22	7	154	1	7
27	3	81	6	18
32	2	64	11	22
Total	20	420		94

Mean $= \dfrac{420}{20} = 21$

Mean deviation $= \dfrac{\sum f_i |X_i - \bar{x}|}{\sum f_i} = \dfrac{94}{21} = 4.48$

The mean deviation is a better measure of dispersion than the ranges because calculation use all the values in the distribution. It also shows how far, on the average, each of the values, lies away from the mean.

3.1.3 Quartile Deviation

In unit 8, we computed values of quartile from distribution. You may need to go back to this computation.

We have interquartile range and quartile deviation. Quartile deviation can also be called semi- interquartile range.

While interquartile range measures how far we need to go from the median on either sides before we can include one half the values of the set of data, quartile deviation or semi-interquartile range measures the average range of one-fourth of the data. Therefore,

Interquartile Range = Q3 - Q1
Quartile Deviation $= \dfrac{Q3 - Q1}{2}$
Where Q3 = upper quartile
       Q1 = lower quartile

Example 9.4

The annual electricity bills in a household are given below and arranged in ascending order. The unit is in Naira.
250,540, 470, 590, 620, 650, 680, 690, 730, 750, 770, 810.
Calculate the Interquartile range and quartile deviation of the distributions.
The problem can be solved as follows:

The lower quartile the value of the
$1/4$ x 12th item = 3rd item.
Lower = ₦570
The upper quartile is the value of the $3/4$ x 12th item = 9" item.
Interquartile range = 730 - 570 = 160
Quartile deviation = $\dfrac{730 - 570}{2}$ = 80

3.1.3 Standard Deviation

A measure of dispersion that enables us determine, with high degree of accuracy, where the values of a frequency distribution are located in relation to the mean, is standard deviation. We can compute it for both the sample and the population from where the sample is taken.

When it is from the samples, it is a statistic denoted by S. When it is from the population, it is a parameter denoted by ∂

The variance of a distribution is the square of the standard deviation of the distribution.

Variance (when it is sample) = 5^2

Variance (population) = ∂^2

The variance of the sample s2 is computed from this formula.
For a group of data:

$$S^2 = \frac{\sum f_1(X_i - x)^2}{n-1}$$

$$S^2 = \frac{^n\Sigma f_1 X_i^2 (X_i - x_i)^2}{n(n-1)}$$

Where
s^2= variance of the sample
n = total number of values or sample size f_i= respective frequency of xl

x_i= class mark of the class interval for the population, the standard deviation

$$\partial^2 = \sum f_1(X_i - \bar{x})^2$$

Where ∂^2 = population variance
N =   population size
x_i =   respective values of elements in the population
μ =   population mean.

Example 9.4

From the table below, compute the sample standard deviation.

Intervals	3-7	8-12	13-17	18-22	23-27	28-23
Frequency	6	8	16	10	7	3

Solution is as follows

Intervals	x_i= Class Mark	F_i	$F_i x_i$	$F_i x_i^2$
3-7	5	6	30	180
8-12	10	8	80	640
13-17	15	16	240	3840
18-22	20	10	200	4000
23-27	25	7	175	4375
28-23	30	3	90	2700
		50	815	15735

You should note that class mark (x,) is the mean of the upper and the lower class boundaries or limits.

For this example, we shall compute the variance first from

$$S^2 = \frac{n\sum f_i X_1^2 (\sum f \times_i)^2}{n(n-1)}$$

Where n = $\sum f_i$

$$S^2 = \frac{50 \times 15735 - 815^2}{50(50-1)}$$

$$= \quad \frac{78675 \text{ -}664225}{2450} \qquad = 50.01$$

The standard deviation S = $\sqrt{50.01}$ = 7.072

3.1.4   Coefficient of Variation.

There may be the need to compare two distributions in terms of their variability. The standard deviation of the distribution alone can not be used to achieve comparison without knowing the means of the distribution. Again the unit of values used in the distribution may not be the same. One unit may be in kilograms another unit may be in litre. This makes comparison of the variability not easy since it is not easy to compare two things with different units.

We need to solve this problem by obtaining the coefficient of variation

$$= \frac{\partial}{x} \text{ x } 100\%$$

Exercise 9.5

Suppose the distribution of samples A and B have $\overline{X}_B$ = 5.2, $\overline{X}_B$ = 6.5 respectively.

A has a standard deviation of 1.5 and B 2.5. Which of the distributions shows greater degree of variability?

Coefficient of variation of A $\frac{1.5}{5.2}$ x 100 = 28.85

Coefficient of variation for B = $\frac{2.5}{6.5}$ x 100 = 38.50%

Distribution B has greater variability than A.

Exercise 9.1

For the table below compute

(a)   (i) the mean (ii) mode (iii) median and (iv) standard deviation.

(b)   From the measure in a above compute

(i)   Coefficient of variation

(ii)   Pearson Coefficient of skewness

Marks	10-19	20-29	30-39	40-49	50-59
No of Student	10	16	30	14	10

4.0   Conclusion

You learn in this unit different measures of dispersion namely, range, mean deviation, quartile deviation, interquartile range and coefficient of variations. The distance of the two values of the average from the median was the focus of this unit. A measure, coefficient of variation that determines the variability of the distribution and that is use to compare two distribution in term of their variability was discussed.

5.0   Summary

The unit reveals the need to have a better understanding on variability exhibited by distribution by computing the values of coefficient of variation, semi-interquartile range in addition to the standard deviation. The unit shows the difference in the computation of population variance and sample variance.

6.1   Tutor Marked Assignment

6.1.   From the table below calculate (i) standard deviation and (ii) the coefficient of variation.

Salaries	1000-3000	3000-5000	5000-7000	7000-9000	9000-11000	11000-1300
No of workers	10	14	18	23	19	14

7.0 References And Other Resources.

Daniel W.W and Terrel J.C (1979) Business Statistics: Basic concepts and Methodology 2nd ed. Houghton Mifflin Co. Boston

Harper W.M (1982) Statistics .4th ed., Macdonald and Evans.

Levin R.1 (1990) Statistics For Management 4`h ed., Prentice-Hall of India Private Limited New Delhi.

UNIT 10

SET THEORY

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1.0   Introduction

In this unit, you will learn about some ideas and notations for set theory that will aid your understanding and calculation of probability that will constitute greater part of many of the subsequent units to study in this course. In this unit you will learn the basic concepts in the set theory and solve some problems involving the use of Venn diagrams.

2.0   Objectives

- •   By the end of the unit, you should be able to
- •   Understand set and the basic concept in set theory.
- •   Perform basic operations in set theory
- •   Use Venn Diagram to solve some problems involving set theory.

3.1   Definition and Basic Concepts in Set Theory

3.1.1   Definition

A Set is a collection of definite, distinct objects called
elements or members of the set.

For the set theory, the set is written in Capital letter, Set A. After each
element there must be a comma. The common bracket is not allowed.

A = { a, e, i, o, u} . This is a correct representation of Set A.
A = (a, e, i, o, u). This representation is not correct. The correct bracket is not
used.
A = {a, e, i, o, u) is a Set of English vowels.   Set B = {2, 3, 5, 7} is a Set of
prime numbers between 0 and 10

3.1.2   Basic Concepts in Set Theory

For a good understanding of Set theory, there is a need to
define some Concepts you will need in this unit.

(i)   Unit Set: is a Set with only one element. Example of this are
         B={a}

         C = {p}

(ii)   Null Set: is a Set that contains no element. It is an empty set.
         Example is B={ } or B = Ø

         When you use 0 for null set, the bracket should be avoided.

(iii)   Universal Set: is a Set that contains all the elements in a study or a
         given discussion. Examples are
         S = { male, female}. This is a universal set for sexes of human
         beings
         B   = [a, b,c,d................. x, y, z].This is   a universal set for the
         English Alphabets. Universal Set is designated as

(iv)   Subset: Set A is a subset of B if and only if every element in A is
         contained in B. All Sets are subsets of the universal set. A null set is
         a subset of all sets: Look at this example

         A= {a, e,i,o,u}
         B= {a, b, d, e, g,i,j,m,o,p,u,r}

A is a Subset of B because all the elements in A are contained in B

(v)   Equal Sets: Sets A and B are equal if and only if they contain the same elements.

(vi)   Compliment of a Set: The complement of set A, designated as A^1 is a set consisting all the elements in the universal set that are not in A. For example

   = {the English Alphabets} B= {a, e,i,o,u}
B compliment = B^1 = fall the consonants)

3.2   Operations In Set Theory

3.2.1   Union of Sets

The union of two sets A and B is another set consisting of all elements either in A or B or both. These are all the elements in A and B without repetition of any element. The union is designated by
Union of sets A and B is given as A   B.

A   B= nA+nB-nA   B.

Where A   B is A intersection B. This is the set of element(s) that are common to both set A and set B.

nA = number of elements in A
nB = number of elements in B
nA   B = number of elements that are common to both A and B.
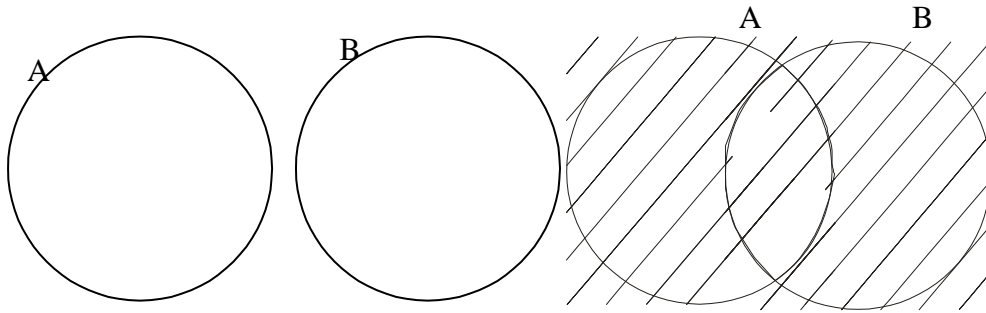
Example10.1

Suppose

   =   { Set of all integers}
A =   {0, 1, 2, 3,-----}
B =   {1,3,5,7,------}
C=   {---6,-4,-2,0)
Find   (i) A   B, (ii)   A   C (iii)   B C^1

(i)   A   B all elements in A and B without repetition of elements.

A   B ={0,1,2,3------- }=A

B is a subset of A, therefore A   B = A.

(ii)   A   C= f-----6,-4,-2, 0, 1, 2 3------}

(iii)   For B C^1, we will need to find C^1 first. $C^1$is the compliment of C^1
           C^1 is the set of all elements in the universal set that are not contained
           in C

           C^1= {------- 5,-3,-1}

        B   C={------- 5,-3,-1, 1,3,5,7-------}

Example 10.2

Suppose there are 300 employees in an hotel and 120 of them have worked for the hotel for more than 5 years. What are the two sets in this hotel in terms of working years in the hotel?

A ={ 120 employees who worked for more than 5 years }

A^1= {180 employees who worked for 5 or less than 5 years in the hotel)
The two sets cannot overlap. Such sets that do not overlap are called disjoint sets.

Example 10.3

There are three factories J, K, L supplying goods to warehouses A, B, C, and D, the amount of supplies from the factories to the warehouses are shown in the table below. Obtain   (i) J ∪ A,   (ii)   K ∪ D   (iii)   C ∪ L

Warehouses / Factory	A	B	C	D	Total
J	72	16	15	50	153
K	38	18	13	22	91
L	50	32	22	43	147
Total	160	66	50	115	391

From the table

n A =160, n B=66, n C = 50, n D = 115

n J =153, n K = 91, n L=147

Also n (J ∩ B) = 16, n(K ∩ D) = 22, n(l ∩ C) = 22, n(D ∩ L) = 43

For the intersection you look at the point where the two sets intersect. Therefore

(i)   J ∪ A = nJ+nA - n(J ∩ A) =153+160-72=241

(ii)   K ∪ D = nK+nD - n(K ∩ D) = 91+115-22=175

(iii)   C ∪ L = nC + nL = n{C ∩ L) = 50+147-22=175

3.2.2   Intersection

The intersection of Sets A and B is a set of all the elements that are common to both sets A and B.

= {Set of all integers}

A = {0, 1, 2, 3-----}

B= {1,3,5,7-----}

A   B = {1, 3 ,5, 7----}

The intersection of two sets in symbolized by "   "

Example 10.4

For the table in example 10.3, find. (i) A   L (ii) K   C (iii) C   J

A   L = 50
K   C = 13
C   J = 15

You need to know before going further that the intersection of sets A and B can be shown graphically as follows:
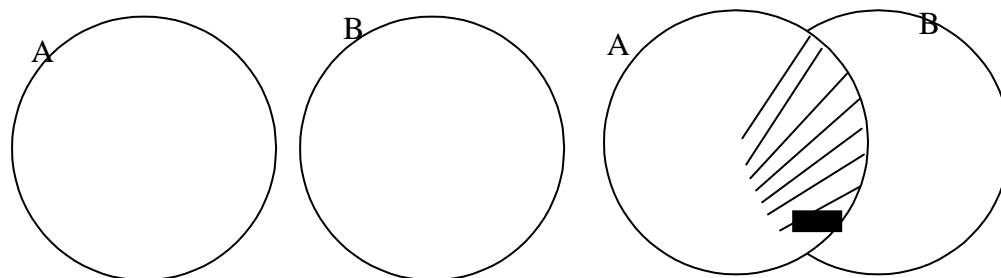
The shaded portion shows the intersection of sets A and B.

3.3   Venn Diagram

3.3.1   Use of Venn Diagram

Venn Diagram is used to solve problems involving set theory.
Let us go through this aspect of set theory with an example.

Example 10.5

Suppose there are 75 students in a class with the students offering at least one of the subjects Chemistry Physics and Biology: 34 students offer Physics, 36 students offer Chemistry, and 41 students offer Biology. 13 students offer both Physics and Chemistry14 offer Physics and Biology and 15 offer

Biology and Chemistry. Represent the data on a Venn diagram and find the number that offer:

(i)   All subjects
(ii)   Biology only
(iii)   A subject only
(iv)   Two subjects only
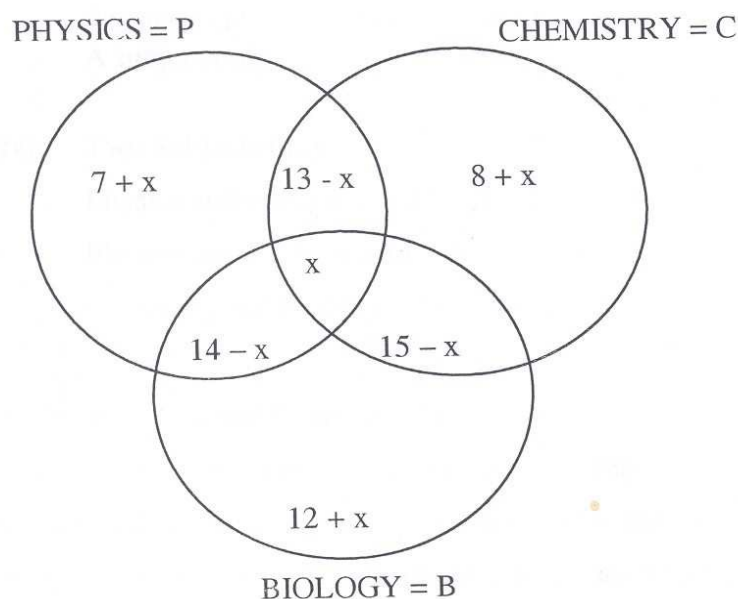
Let the set of P   C   B be represented by x.
The rest part of P   C = 13 – x
The rest part of P   B = 14 – x

The rest part of B   C=5-x

Physics only =34-[13-x+ 14-x+x] =34-13+x-14+x-x 7+x

Chemistry only = 36 - [13 - x + 15 - x + x] =8+x

Biology only = 41- [14 - x + 15 - x + x] = 12+x

Now that we have obtained the values of all the components, we will add up the values.

The sum of the values will be equal to 75.
7+x+8+x+12+x+13-x+14 x+15-x+x=75
69+x=75 x=6

(i)   The number that offer the 3 subjects is 6
(ii)   Biology only = 12 + x = 12 + 6 =18
(iii)   A Subject

    Biology only =12+x=12+6=18

    Chemistry only = 8 + x-- 8 + 6 = 14

    Physics only = 7 + x = 7 + 6 = 13

    A subject only = 45

(iv)   Two Subjects only

    Physics and chemistry = 13 - X = 3 - 6 = 7

    Physics and Biology = 14 - X = 14 - 6 = 8

    Chemistry and Biology = 15 - X = 15 - 6 = 9

    Total   = 24

Student Assessment Exercise 10.1

There are 85 participants at a national Conference on Child nutrition. 35 Yoruba, 38 women and 41 nutritionists attended the conference. 13 were both Yoruba and women, 16 were both women and nutritionists and 15 were both nutritionists and Yoruba. 13 of the Yoruba were neither women nor nutritionists, using Venn diagram, Calculate:

(i)   The Yoruba participants that were women and nutritionists.
(ii)   The number that were neither Yoruba nor nutritionists
(iii)   The number that were neither Yoruba nor women nor nutritionists

4.0   Conclusion

In this unit, you learned the definition of set and some basic concepts in set theory. You also learned the union and the intersection of sets. Venn diagram

is treated to solve problems in set theory. Exercises and examples given are to assist you further in the understanding of the unit.

5.0 Summary

Set has been defined as collection of definite, distinct objects called elements or members of the set. The union and intersection of sets are treated to give a good understanding of how numerical problems in set theory could be solved.

6.0 Tutor Marked Assignment
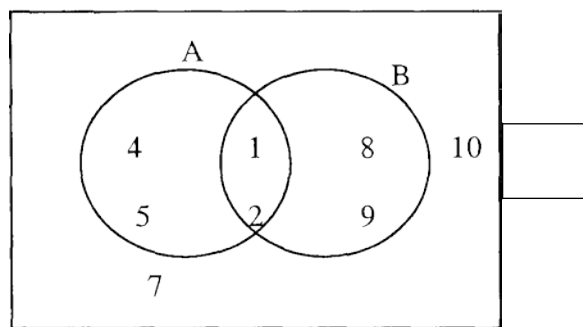
Define Sets A, B, U, A ∪ B, A ∩ B, B[1], A[1], (A ∩ B)[1]

Given

A = {1, 3, 5, 9, 11, 13, 15, 17, 19}

U = {1,2,3,---------------20}

C = {3,6,9,12,15,18}

D = {2, 3, 5, 7, 11, 13, 17, 19)

Find:

(i) A ∪ C (ii) A[1] (iii) (A ∪ C) n (C ∪ D)[1]

7.0 References, Further Reading and Other Resources

Ajayi J.K (1997) Elements of Business Statistics, Unpublished Monograph, Ondo State Polytechnic Owo

UNIT 11

PERMUTATIONS AND COMBINATIONS

Table of Content

1.0   Introduction

In unit 10 of this course you learned about set theory, which is an aspect of probability concepts. In this unit you will learn other aspects of probability concepts, such as permutation and combination. Many times, the focus is on computing the probability of some event or that of events. Some times, the total number of possible events is large. Under this condition, you need to have some method for counting the number of such events. The permutation and combination that this unit presents are useful techniques for counting the number of events comprising the numerator and/or the denominator of a probability.

2.0   Objectives

At the end of this unit, you should be able to:

Compute factorials of values
•
•   Compute permutations for objects that are different
•   Compute permutation for objects that are not all different
•   Compile value of combination for objects.

3.1   Factorials

3.1.1   Computation of Values of factorial

Let us define factorial. If we have a positive integer n, the product of all the whole numbers from n down to 1, is called n factorial and it is written as n! Therefore

n! = n! (n-1)(n-2)(n-3)(n-4).................. I

The values of n, since it is a positive integer are n =0, 1,2,3,4 ....∞
Infinity:.

Let us use numerical values for n

5!   =   5x4x3x2x1
5!   =   5x4!
5!   =   5x4x3!
5!   =   5x4x3x2!
4!   =   4x3x2x1
4!   =   4x3!
4!   =   4x3x2!

Mathematically

1!   = !
O!   = !!

With the use of factorials, you can solve questions on the number of ways, objects or persons can be arranged in a line.

Example 2.1

Suppose there are 3 waiters on a restaurant, standing in a line against a wall, waiting for quests. How many standing arrangements are possible?
If any of the waiters can stand in any position in the line

The answer is 3! = 3x2x1=6
We can also use a graphical means to solve the problem with the aid of a tree diagram. We shall designate the positions as first, second, and third and the waiters as A, B and C, we have a tree diagram presenting the possible arrangements

FIGURE 11.1 **Tree Diagram**

Ways to fill The first position	Ways to fill Second position	Way to Fill third position	Number of Arrangements
A	B	C	1
	C	B	2
			3
B	A	C	
	C	A	4
			5
C	A	B	
	B	A	6
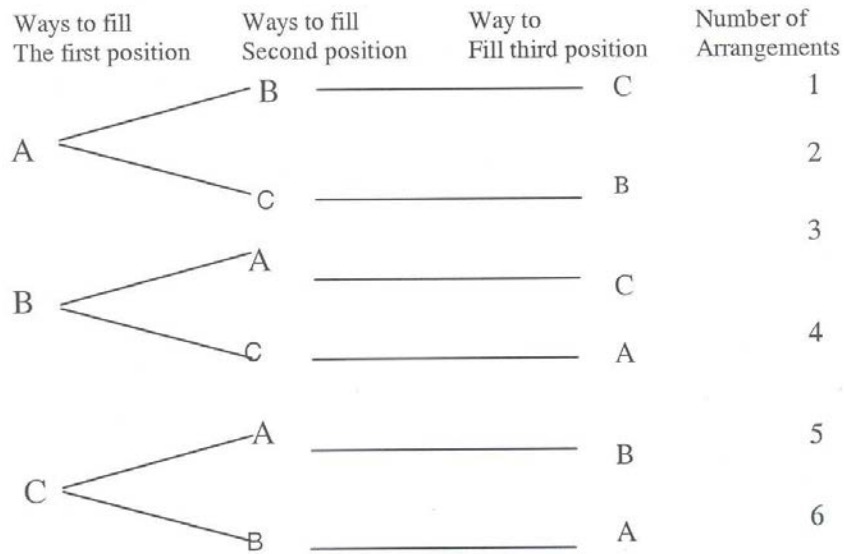
We now find out the number of possible arrangements is 6.
If instead of 3 waiters, we have 4 waiters, the number of possible arrangements will become

4! = 4x3x2x1 = 24 ways

3.2   PERMUTATIONS

3.3   Computation of permutation for objects that are different

Permutation can be defined as an ordered arrangement of objects. The tree diagram Fig. 11.1 that shows possible arrangements of three objects taken three at a time gives the possible permutation of 3 objects taken 3 at a time. In some situations, as in many cases in Nigeria, there more people applying

for the number of vacancies available. For instance, we may wish to fill three positions and 15 people are applying. How then do will go about it?

There are some ways we can solve the problem. There can be the use of logical reasoning and factorial method.

We have 15 people available and 3 positions to fill. The first position can be filled in one of 15 ways, the second in one of the 14 ways and the third in one of 13 ways. The resulting possible ways will be the product of 15,14 and 13 that is 15 x 14 x 13 = 2730 ways.

You should realize that once the first position is filled, there are 14 persons remaining looking for 2 positions. When the second position is filled, there are 13 persons remaining looking for 1 position and there is one of the 13 ways to fill the third position.

The example we are considering can be solved mathematically if we make distinction between the three positions to be filled. What we are saying here is that we are having permutations of objects that are all different.

To fill three positions, when 15 persons are available, we have 15 permutation 3 symbolically we write it as

$$15P_3 = \frac{15!}{(15-3)!}$$

Generally, n permutation r, that is taking r items at a time from n items available is given as $nPr = \dfrac{n!}{(n-r)!}$

For our example

$$15P_3 = \frac{15!}{(15-3)!} = \frac{15!}{12!} = \frac{15 \times 14 \times 13 \times 12!}{12!} = 2730$$

Example 2.2

Suppose there 10 different items to be stocked in the store and we have 6 bins for the storage, what are the possible arrangements?

We will now determine the permutation of 10 items taken 6 at a time

$$10P_6 = \frac{10!}{(10-6)!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4!} = 604,800 \text{ ways}$$

3.2.2. Calculation of Permutations from Objects That Are Not All Different.

You have learned the permutations of objects when all the objects are different. This is not always the case in every situation. In some instances one or more subsets of the items cannot be distinguishable. In permutation we are required to determine how many distinguishable arrangements are possible. This situation we are discussing could be represented as follows:

$$nP \, n_1, n_2 \, n_3 \quad n_k \quad = \quad \frac{n!}{n_1! \, n2! \cdots n_k!}$$

In this case there are n objects from which the permutations is made, we have items, nr n2, ... $n/_k$ that are distinguishable.

Example 2.3

There are 6 vegetables to be served in a restaurant, 1 of them is of type A, 2 of them of type B and 3 of them of type C. How many arrangements of the vegetables are possible? n=6

nr=1,n2=2,n3=3

$$nPn_1 \, n_2 \, n_3 = \frac{n!}{nl! \, n2! \, n3!}$$

$$6 \, p_{1,2,3} = \frac{6!}{1! \, 2! \, 3!}$$

$$= \frac{6 \times 5 \times 43!}{1 \times 2 \times l \times 3!} \quad = 60 \text{ ways}$$

Example 2.4

There are 6 positions available to be tilled by 6 persons from 3 tribes in Nigeria on equal basis. How many distinguishable arrangements are possible based on tribes?

$$6P_{2,2,2} = \frac{6!}{2!, \, 2! \, 2!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{2 \times 1 \times 2 \times l \times 2!} \quad = 90 \text{ ways}$$

Student Assessment Exercise 11.1

A sales person has 9 products to display in a trade fair but he can display only 4 at a time, how many displays can he make if the order in which he displays is important?

3.3 COMBINATIONS

3.3.1   Computation of values of combination
Combination is an arrangement of objects without regard to order. The number of combinations of n objects taken r at a time is written as n!

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

$$= \frac{n\ factorials}{}$$

r factorials ((n-r) factorials.)

For instance if we have 3 objects A, B and C and we take 2 objects at a time, the permutations will be

AB   BA
AC   CA
BC   CB

There are 6 ways of doing the arrangement. This is $3P_2 = \frac{3!}{(3-2)!} = 6$

However, in combination, where there is no regard to order

AB is the same as BA
AC is the same as CA and
BC is the same as CB hence

The number of combinations will just be 3.

$$\binom{3}{2} = \frac{3!}{2!(3-2)!} = \frac{3 \times 2!}{2!\,x!} = 3\ ways$$

Example 2.5

A food-processing firm has 8 brands of seasoning agents from which it wishes to prepare a gift package containing 5 seasoning agents. How many combinations of seasoning agents are available?

Solution is $= \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \dfrac{8!}{5!(8-5!)} = \dfrac{8 \times 7 \times 6 \times 5}{5! \times 3 \times 2 \times 1} = 56$

Student Assessment Exercise 11.2

Find the value of (i) $\begin{pmatrix} 10 \\ 2 \end{pmatrix}$ (ü) $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ (iii) $\begin{pmatrix} 18 \\ 5 \end{pmatrix}$

4.0   Conclusion

This unit teaches the definitions of factorials, permutations and combinations. This unit also teaches computation of values of some factorials, permutations and combinations. It also shows the difference in the computation of permutations when the objects are all different and when the objects are not all different. Some time is devoted to this unit because of the importance of the concepts in this unit to the study of the subsequent units in this course

5.0   Summary

The unit gives the permutations of r object, from n objects as $\dfrac{n!}{(n-r)}$ when all the objects are different. It also gives the permutation of objects r_1, r_2, r_3 .... r_k from n objects to be $\dfrac{n!}{r_1! r_2! r_{3}....... r_k!}$ when the objects are not all different.

The unit gives the combinations of r objects from n objects as $\dfrac{n!}{r!(n-r)!}$

Examples are worked in the unit and exercises are given to facilitate your understanding of the unit.

6.0   Tutor Marked Assignment

6.1   In how many different ways can MISSISSIPPI be arranged with distinction placed on letters

6.2   If we want to arrange 15 objects picking 5 at once, how many combinations are possible?

7.0   References. Further Reading and Other Resources

Ajay J.K (1997) Elements of Statistics Unpublished monograph, Ondo State Polytechnic, Owo

Daniel W.W. and Terrel J.C. (1979) Business Statistics; Basic Concepts and Methodology 2"a ed. Houghton Mifflin Co. Boston.

Unit 12

SOME ELEMENTARY PROBABILITY CONCEPTS

1.0   Introduction

The activities of man and the firms consist of occurrence of events. While some events are certain to occur some others can never occur. Between these extreme lie most of the events of human activities. There is always a lot of uncertainty in the occurrence of these events. The unit therefore introduces you to probability that deals with concepts and measurement of uncertainty. The unit therefore provides you the opportunity to understand some of the variability and complexity of the business world.

2.0   Objectives

At the end of this unit, you should be able to:

* Understand some views of probability
* Understand elementary properties of probability
* Compute probability of an event.

3.1   Different Views Of Probability

3.1.1   Definition Of Probability

Probability is defined as a value between 0 and 1 that shows the likelihood of occurrence of an event. If the value is 0, the event can never occur. For instance the probability that a Mango tree will bear an orange fruit is 0. If the probability is 1, it means the event is sure to occur. The probability that a man born of a woman will die one day is 1. Death for any man is imminent.

Many other events have the values of the probability in between 0 and 1. This raises the question of uncertainty. There are two types of probability:
a.   Objective and
b.   Subjective probabilities

The objective probability can be subdivided into

(i)   The classical or prior probability
(ii)   The relative frequency concept or posterior probability

Classical probability is defined as follows "If some event can occur in N mutually exclusive and equally likely ways and if m of these possess characteristic E, the probability of the occurrence of E is equal to $\frac{M}{N}$

(Daniel & Terrel, 1979)

That is $P(E) = \frac{M}{N}$

P(E) is probability of characteristic E.

Example 3.1

If a couple makes five trials at birth and records 3 male children. What is probability of having a success if the birth of a male child is regarded as a success.

P(success) $\frac{M}{N}$

M=3
N=5

P(success) $= \frac{3}{5} = 0.6$

The relative frequency concept is also used to define probability. For instance if we have a variable x; and we can draw a frequency distribution for the variable, then the relative frequency of a value of the variable is approximately equal to the probability of occurrence of the variable.

Example 12.2

Suppose a test is conducted for twenty students in the course Statistics for Management and the results of the test are as presented in the frequency distribution table below. Calculate the probabilities of the students scoring a particular score in the distribution.

Score	Frequency
2	1
3	2
4	5
5	6
6	3
7	2
8	1

To solve the problem we need to compute the relative frequency of each of the scores.

Total number of students is 20

Relative frequency of score $2 = \dfrac{1}{20}$

This is the frequency of 2 divided by the total frequency.   We   can   now prepare the relative frequency table for the distribution as follows

Scores	Frequency	Relative frequency
2	1	0.05
3	2	0.10
4	5	0.25
5	6	0.30
6	3	0.15
7	2	0.10
8	1	0.5

From the table the probability that a student scores 6 is approximately equal to 0.15 that is $\frac{3}{20}$

The relative frequency distribution table can also be used to find the values of many other probabilities. For instance what is the probability that a student scores more than 5. This will be the probability that a student scores 6 or 7 or 8. This will be equal to the sum of these: Probabilities = 0.15 + 0.10 + 0.05 = 0.30

3.1.2 Mutually Exclusive And Equally Likely Events

Two events are mutually exclusive when they cannot occur together. Examples of these are:

(i)　　Sex of a child in a birth
(ii)　　Values of throw when throwing a die
(iii)　　Head and tail in a coin
(iv)　　Win or loss when playing a match in which a winner must emerge.

When two events are mutually exclusive, the intersection of the events will be zero
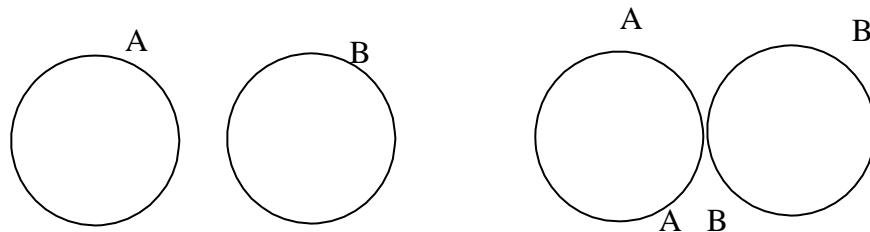
These sets 'A and B are mutually exclusive.

Equally likely events are those events in which there is no reason to expect one event rather than the others to occur. For instance, if a coin is not biased the head and tail are equally likely. If a die is not biased, 1, 2, 3, 4, 5, 6 are equally likely to occur.

Indeed, if there is no dominant effect of heredity on the sex of a child, male and female are equally likely events.

3.2   Properties Of Probability

There are three properties of probability.

a.   The first one is derived from the definition. Probability of an event is defined as a value between 0 and 1 that shows the likelihood of occurrence of the event. Therefore probability of an event cannot be negative that is less than 0. The probability can also not be more than 1. Therefore the probability of an event with characteristic E is $0 \leq P(E) \leq 1$. This is the first property of probability.

b.   If events are both mutually exclusive and collectively exhaustive then the sum of the probabilities of the events is equal to 1. The events that are mutually exclusive and collectively exhaustive are as follows.

   (i)   Male and female as sexes of a child in a birth.
   (ii)   Head and tail in a throw of a coin
   (iii)   1, 2, 3, 4, 5, and 6 in a throw of a die

   With these examples, no other event can occur in each of the cases. The events are therefore collectively exhaustive. Since the events cannot occur simultaneously, the events are said to be mutually exclusive.

   Therefore if events El, EZ, E3 ... E_n are both mutually and collectively exhaustive then

   $$\Sigma P(E_1) = P(E_1) + P(E_2) + P(E_3).-P(E_n) = 1$$

c.   The occurrence of either of two events A or B or both is given as
   $P(A \; B) = P(A) + P(B) - P(A \; C)$
   But this holds when the two events can occur simultaneously. However, if the two events are mutually exclusive then $P(A \; B) = P(A) + P(B)$

You should note that if two events are mutually exclusive, their intersection will be zero. i.e. (A   B)=0

3.3   CALCULATING THE PROBABILITY OF AN EVENT

3.3.1 Conditional And Unconditional Probabilities

It is necessary at this stage to introduce the use of the concepts introduced previously in this unit in solving of problems involving calculation of probabilities. In this section we will treat two types of probabilities - the unconditional and conditional probabilities. Let us explain these probabilities by solving a question that will bring the understanding of unconditional and conditional probabilities out properly.

Example 12.3

Suppose there are 70 Secretaries, cross-classified as by sex and marital status as follows: There are 20 male and 25 married Secretaries. 5 of the married Secretaries are male. With this information, we will prepare a table with which we can calculate the various probabilities required.

Marital Status	Sex		
	Male (S1)	Female (S2)	Total
Single (Ml)	15	30	45
Married (M2)	5	20	25
Total	20	50	70

Let us now look at the unconditional probabilities. Total number of secretaries is 70. The total married secretaries are 25, total single is 45, total male is 20 and total female is 50.

With these information, we can compute the following unconditional probabilities. Probability of getting a female secretary is given by $\frac{50}{70}$

$= 0.7143$ similiarly

P (male) $= \frac{20}{70} = 0.2857$

P (single) $= \frac{45}{70} = 0.6429$

70

P (married) $= \dfrac{25}{70} = 0.3571$

All these probabilities are unconditional probabilities The conditional probability can be treated as follows; From the table, some male are single and some are married hence male and single are not mutually exclusive; so also male and married. The probability of getting a male given that he is single is given as:

P (male/single) $= \dfrac{\text{P(male} \quad \text{single)}}{\text{P(single)}}$ or

P(male/single) $= \dfrac{\text{n(male} \quad \text{single)}}{\text{n(single)}}$

Where P(male single) is probability of male intersection single and n(male n single) is the number that is common to both male and single. This probability is a conditional probability

Example 3.4

Using the table in example 3.1, calculate the following probabilities.

(i)   Of getting a male given that is married
(ii)   Of getting a single given that is female
(iii)   Of getting married secretary given that he is a male
(iv)   Of getting a female secretary given that she is single.

1.   P(male/married) $= \dfrac{\text{n(male} \quad \text{married)}}{\text{n(married)}} = \dfrac{5}{25}$
$= 0.2$

Another method is P(male/married) $= \dfrac{\text{P(male} \quad \text{married)}}{\text{P(married)}}$

P(male married) is equal to the number that is common to both male and married divided by the total number of secretaries. In this example n(male married) = 5 total number of secretaries = 70 therefore P(male married) $= \dfrac{5}{70}$

P(married) $= \dfrac{25}{70}$

$\dfrac{5}{}$

Therefore $\underline{P(male \quad wedded)} = \quad \underline{70}$
$\quad\quad\quad\quad\quad P(married) \quad\quad 25\Big/$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad /70 \quad = 0.2$

You may wish to use any of the methods, though 1 personally show preference for

$P(A/B) = \underline{n(A \quad B)}$ for any events A and B
$\quad\quad\quad\quad (B)$

2 $P(single/female) = \underline{n(single \quad female)}\ \underline{30}$
$\quad\quad\quad\quad\quad\quad\quad\quad n(female) \quad\quad 50 = 0.6$

3 $P(married/male) = \underline{n(married \quad male)} = 5 = \underline{0.25}$
$\quad\quad\quad\quad\quad\quad\quad\quad n(male) \quad\quad\quad\quad\quad 20$

4. $P(female/single)\ \underline{n(female \quad single)}\ \underline{30} = 0.67$
$\quad\quad\quad\quad\quad\quad\quad\quad n(single) \quad\quad 45$

Student Assessment Exercise 12.1

Suppose there are four types of appliances in the kitchen denoted by A_1, A_2, A_3 and A_4 and the colours of the appliances are of 3 types C_1, C_2, C_3. If the table below shows the cross-classification of the appliances by both the types and colours, Calculate the probabilities listed after the table
Colour

Types	C 1	C2	C3
A_1	45	35	70
A_2	60	15	25
A_3	40	50	30
A_4	75	25	30

From the table calculate probability that an appliance picked at random is of.

a) (i)   Type A_1

    (ii)   Type A_2

    (iii)   Type A_3

    (iv)   Type A_4

(b)   (i)   Colour C_1 and Colour C_3

(c)   (i)   Type A_1 given colour C_3

     (ii)   Type A_3 given colour C_2

     (iii)   Colour C_2 given type A_4

4.0   Conclusion

The unit stated the properties of probability. The unit presented the calculation of the probability of an event and worked out problems involving conditional and unconditional probabilities.

5.0   Summary

You learned the three different points of view, classical, relative frequency and subjective, from which we can discuss probability. You also learned the properties of probability as:

(i) $0 \leq P(E) \leq 1$ for events $E1, E2, E3 ... E_n$

(ii)   For mutually exclusive and collectively exhaustive events $El, E2, E3 ... E_n$ that $\sum P(E;) = 1$

     That is $P(E_1) + P(E_2) + P(E_3)$ ------ $P(E_n) = 1$

(iii) $P(E_1$ or E_2 or both$) = P(E_1) + P(E_2) - P(E_1 \quad E_2)$ but for mutually exclusive events $P(E_1 \quad E_2) = P(E_1) + P(E_2)$.

The unit also taught the conditional probability of A given that is of type B to be
$$P(^A/_B) = \frac{n(A \quad B)}{n(B)}$$

The future units will build on these.

6.0   Tutor Marked Assignment

The table below shows the programmes in the National Open University and the types of Employers that will employ the graduates of this programmes. It also shows the number of graduates the employer's employ.

Programmes	Types of Employer				
	Teaching	Civil Service	Manufact-uring	Merchand-ising	Others
	El	E2	E3	E4	Es
Business Admin. D1	30	20	10	14	16
Hotel & Catering D2	20	15	18	13	14
E - Banking (D3)	15	17	12	4	2
E - Library D4	15	13	25	8	9
Co-op Study D5	20	15	8	20	17
Others D6	90	5	7	11	17

From the table, compute the following probabilities

a   (i) $P(D_4)$ (ii) $P(D_5)$ (iii) $P(D_3)$

b   (i) $P(D_5 \quad E_3)$ (ii) $P(E_2 \quad D_4)$ (iii) $\left[n_1 \middle/ E_5 \right]$

7.0   References. Further Reading and Other Resources

Daniel W.W. and Terrel J.C. (1979) Business Statistics: Basic Concepts and
        Methodology 2nd ed., Houghton Mifflin Co; Boston.

UNIT 13

PROBABILITY RULE, EVENTS AND BAYES' THEOREM

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1.0   Introduction

In the unit 12 we discussed that probability is used to measures uncertainty in the activities of man and the firms.

There is a need to know the rules that are used in probability so that we can effectively solve problems involving probability. This unit helps us to do this. The unit will be useful in identifying some events that are important probability concepts. The unit will also discuss Bayes' Theorem that will assist the solutions to some problems.

2.0   Objectives

At the end of his unit, you should be able to:

*   Understand the addition and multiplication rules in probability
*   Explain independent and complimentary events
*   Solve problems involving joint probability and
*   Apply Bayes' Theorem to solving of questions involving probability.

3.1   Probability Rules

3.1.1 Addition Rule

In unit 12, under properties of probability, you learned that the probability that event A or event B or both will occur is

$$P(A \cup B) = P(A) + P(B) = P(A \cap B).$$

This is true when the events are not mutually exclusive, then $P(A \cup B) = P(A) + P(B)$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ is known as the addition rule. When the two events are mutually exclusive $P(A \cap B) = 0$

Example 13.1

Let us go back to the table we had for 3.3 in unit 12 to illustrate numerically the addition rule.

SEX

Marital Status	Male (S 1)	Female (S2)	Total
Single (M1)	15	30	45
Married (M2)	5	20	25
Total	20	50	70

The probability of picking either S_1 or M_2 or both is $P(S_1 \cup M_2) = P(s1) + P(M_2) - P(S_1 \cap M_2)$ from the table,

$$P(S_1) \ \frac{20}{70}, \quad P(M_2) \ \frac{25}{70}$$

$$(S_1 \cap M_2) = 5$$

$$P(S_1 \cap M_2) = \frac{5}{70}$$

5 is the value at the intersection of S_1 and M_2 and 70 is the total number of people involved

$$P(S_1 \cup M2) = P(S_P + P(M_2) - P(S_1 \cap M_2) = \frac{20}{70} + \frac{25}{70} - \frac{5}{70} = \frac{40}{70} = \frac{4}{7}$$

3.1.2   The Multiplication Rule

This is another rule that is useful in the computation of the probability of an event.

In unit 12 under conditional probability, we discussed that the probability of event A given B is equal to $P(^A/_B) = P(A \cap B)$ for $P(B) \neq 0$

This relationship holds if $P(B)$ is not equal to 0. If $P(B)$ is equal to 0, the definition of a probability $0 \leq P(\Sigma) \leq 1$ will be violated. If we have $P(^A/_B) = P(A \cap B) P(B)$

Then $P(A \cap B) = P(B) P(^A/_B)$. This is obtained by rearranging the preceding equation. $P(A \text{ n } B)$ is the probability of the joint occurrence of event A and event B.

The probability of joint occurrence of event A and event B is equal to the conditional probability of a given B that is $P(^A/_B)$ times the marginal probability of B, that is $P(B)$.
Therefore $P(A \cap B) = P(B) P(^A/_B)$.

Example 13.2

What is the probability of joint occurrence of M_1 and S_2 in the table used for

example 4.1. $P(M_1 \cap S_2) = P(B) P\left[m_1 \Big/ S_2 \right]$

$$P(S_2) = \left[\frac{50}{70} \right]$$

$$P\left[\frac{m_1}{S_2} \right] = \frac{(m_1 \cap S_2)}{n(S_2)} = \frac{30}{50}$$

$$P(M_1 \cap S_2) = \frac{50}{70} \times \frac{30}{50} = \frac{30}{70} = -\ 0.4286$$

3.2   Probability Of Events

3.2.1 Independent Events

You learned in our discussion before that $P(A \cap B) = P(B) P(^A/_B)$.

Suppose the probability of event A is the same whether event B occurs or not then events A and B will be said to be independent event therefore $P(^A/_B)$ = P(A) $P(^A/_B) = P(B)$

$P(A \cap B) = P(A) P(B)$

You will recall that for all cases. But if the events are independent then P(A n B) = P(A) P(B).

Example 13.3

Suppose there are 50 secretaries in the National Open University and 20 of them are single and 20 of them are male. If 8 of the male are single, what is the probability that a secretary picked at random is married given her is female.

Let us prepare the required table for this required question.

	Single (S1)	Married (S_2)	Total
Male (M_1)	8	12	20
Female (M_2)	12	18	30
Total	20	30	50

What we are interested in is to show that when events A and B are independent, the $P(^A/_B) = P(A)$

$P(\text{married}) = \dfrac{30}{50} = 0.6$

$P(\text{Married/ Female}) = \dfrac{P(\text{married} \cap \text{Female})}{P(\text{Female})}$

$P(\text{married} \cap \text{Female}) = \dfrac{18}{50}$

$P(\text{Female}) = \dfrac{30}{50}$

Therefore $P(\text{married /Female}) = \dfrac{\frac{18}{50}}{\frac{30}{50}}$ = $\dfrac{18}{50} \times \dfrac{50}{30}$ = $\dfrac{18}{30} = 0.6$

$$\frac{30}{50} \quad 50 \quad 30 \quad 30$$

With this example, you will see that P(married) = P(married/ female).

It means that the fact that the married secretary is a female does not affect the probability that the secretary is married; therefore the two events are independent.

3.2.2 Complimentary Events

Two events A and B are complimentary if and only if P(B) = 1 - P(B). That is if (A) + P(B) = 1.

Examples of complimentary events are
(1)   Male and female
(ii)   Defective and not defective.
(iii)   Failure and success.

3.2.3 Joint Probability

Having treated multiplication rule there is a need to compute joint probability of events.

Example 13.4

It is known from experience that for a particular type of seedling, 3 out of 5 them transplanted will survive. Suppose two of the seedlings are picked at random, what is the probability that:

(i)   Both of them survived
(ii)   The first survives and the second does not survive.
(iii)   The first does not survive and the second survives
(iv)   None of the them survives

Let S represents survival and f represents non - survival.

$P(S) = {}^3/_5 = .6$
$P(f) = (1-0.6) = 0.4$

(i)   Both survive = P(S) P(S) = 0.6 x 0.6 = 0.36

(ü)   First survives and second does not survive = P(S) P(f) = 0.6 x 0.4 = 0.24

(iii) First fails and second survives = P(f) P(s) = 0.4 x 0.6 = 0.24

(iv) Both do not survive = P(f)(P(f) = 0.4 x 0.4 = 0.16

Exercise 13.4

1.   For the table below, compute
      (1) P(S$_1$ M$_1$)(ii) P(M$_2$ v S$_2$) (iii) P(M$_1$ n S$_2$)

	Si	Sz
M,	20	30
MZ	40	50

2.   If the probability that a hunter will hit a target is $^2/_3$ and the hunter aims at the target two times, what is the probability that:

      (i)   He hits it the two times?
      (ii)   He hits it once?
      (iii)   He hits first and misses second?
      (iv)   He misses it the two times?

3.3   Bayes Theorem

The Baye theorem is stated as follows:
Given B$_1$, B$_2$, B$_3$ - - - - B$_n$, Mutually exclusive events whose union is the universe, and let A be an arbitrary event in the Universe, such that P(A) ≠ 0 then

$$P(B_j/A) = \frac{P(A/B_j)\ P(B_j)}{\Sigma(PA/B_j)\ P(B_j)}$$

Where J = 1, 2, 3------ n (Daniel & Terrel, 1979)

$$P(B_j/A) = \frac{P(A\quad B_j)}{P(A\quad BI) + P(A\quad B_2) ---- + P(A/B_n)}$$

You should recall that (P(A   B) = P(A/B) P(B)

Example 13.5

Three tutors are assigned to mark the assignments of Hotel and Catering Management Students in the National Open University. The tutor B_1 marks 45 % of the assignment, the second tutor, B_2, marks 35% of the assignment and the third tutor B_3 marks 20% of the assignment.

The course coordinator vetted the marking and it is formed that the first tutor has an error of 0.03, the second 0.05 and the third 0.04. If a script selected is found to have an error, what is the probability that, the script was marked by the first second or third tutor respectively?

We are going to apply B ayes theorem in the problem.

$$P(B_1) = \frac{45}{100} = 0.45$$

$$P(B_2) = \frac{35}{100} = 0.35$$

$$P(B_3) = \frac{20}{100} = 0.20$$

$P(A/B_1) = 0.03$
$P(A/B2) = 0.05$
$P(A/B3) = 0.04$

Let us draw a table for easy computation.

Events	P(B)	$P(A/B_j)$	$P(A \cap B_j)$	$P(B_j/A)$
B_1	0.45	0.03	0.0135	0.3462
B_2	0.35	0.05	0.0175	0.4487
B_3	0.20	0.04	0-008	0.2051
Total			0.039	1.00

You should note that

$$P(A \cap B_j) = P(A/B_j)\,P(B_j)$$

$$P(B_j / A) = \frac{P(A \cap B_j)}{\sum P(A \cap B_j)}$$

For B$_1$, P(B$_j$/A) = $\underline{00135}$ = 0.3462
                          0.039

B$_2$, P(B$_j$/A)= $\underline{0.0175}$ = 0.4487
                      0.039

B$_3$, P(B$_j$/A) = $\underline{0.008}$ = 0.2051
                      0.039

Exercise 13.2

Two students S$_1$ and S$_2$ are asked to record scores in an examination. Student S 1 recorded 60% of the scores and student S$_2$ recorded 40%. It is found that the error rates of the students S$_1$ and S$_2$ are 0.03 and 0.02 respectively. What is the probability that an error detected in the recording is committed by S$_1$ or S$_2$ respectively.

4.0   Conclusion

This unit teaches the application of addition and multiplication rules in probability. It also teaches the meaning of independent and complimentary events. Problems on joint probability are also taught. There is also the application of Bayes theorem to solving of problems in probability.

5.0   Summary

The unit reveals that
P(A   B) = P(A) + P(B) - P(A   B) and that P(A/B) = $\underline{P(P(A \quad B)}$
                                                          P(B)

for P(B) $\neq 0$

For complimentary events A and B, (P(A) = 1 - P(B)

In the application of Bayes theorem the unit gives

P(B$_j$/A) = $\underline{P(A \quad B_j)}$ for j = 1, 2, ~ n)
                   ΣP(A/B$_j$)P(B$_j$)   = 1, 2, ~ n.

P(B$_j$ /A) $\underline{P (A \quad B_j)}$
                   ΣP(A   B$_j$)

6.0   Tutor Marked Assignment

6.1   Two hunters A and B aim at a target. The probability that A hits the target is and that B hits the target is 3/a. What is the probability that:

(i)   They both hit the target.
(ii)   They both miss the target
(iii)   One of them hits the target
(iv)   At least one of them hits the target?

6.2   In an area in Oshodi - Oke, 35% of the households use brand A, 25% of them brand B and 40% of them brand C of a seasoning agent.
The proportion of the people who learned about the product from radio advertisement is 0.03 for brand A, 0.08 for brand B and 0.05 for brand C. the household contacted reveals that the households learned about the product through radio advertisement.

What is the probability that the brand of the seasoning agent used in the household is (1) A?   (ii) B? (iii) C?

7.0   References. Further Reading and Other Resources

Daniel W.W. and Terrel J.C. (1979) Business Statistics: Basic Concepts and Methodology 2"a ed., Houghton Mifflin Co, Boston.

UNIT 14

PROBABILITY DISTRIBUTION OF A DISCRETE RANDOM VARIABLE

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1.0   Introduction

You learned in unit 12 of this course the basic concepts of probability theory. You also learned the method of computing the probability of an event. In unit 1 of module 1 of this course, you learned the meaning of a discrete random variable. You leaned that a discrete random variable is a variable that can only assume whole numbers.

This unit will build on the techniques of calculating probability of an event you learned before by showing how to calculate probability under a more complicated situation.

2.0   Objectives

By the end of this unit, you should be able to

•   Construct a probability distribution of a discrete random variable from raw data.
•   Compute the mean and variance of the probability distribution of a discrete random variable.

3.1   RELATIVE FREQUENCY DISTRIBUTION

3.1.1 Relative Frequency Distribution

In unit 12, you learned about the various views of probability. You learned that the relative frequency of a value of a variable is approximately equal to the probability of that value. This is the relative frequency definition of probability.

In that unit 12 you also learned that the probability of an event, say $X = X_{1o}$, is given as $P(X_1 = X)$ and that

$0 \leq P(x = xl) \leq 1$. In the unit 12 we have made use of E and we gave that $0 \leq P(E) \leq 1$. This is the first property of probability.

Also you learned that for events X_1 X_2, X_3 ---- X_n that are both mutually exclusive and collectively exhaustive, $\sum P(X = X_1) = 1$

For this unit the probability of a value of variable x, that is equal to x will be given as $P(x, = x)$. This should be noted throughout. If $X_1 = 5$
then we will give the probability as $P(X_1 = 5)$ Let us have a distribution of a throws of a die The events in a die are, 1,2,3 4,5,6

Example 5.1

Suppose a die is thrown 20 times and the frequency distribution of the throw is shown below.

X_1	F_1
1	2
2	2
3	4
4	6
5	4
6	2

The next thing to do is find the probability distribution of this discrete random variable. The probability of 1, given as

$$P(X_1 = 1) = \frac{\text{Frequency l}}{\text{Totalfrequency}} = \frac{2}{20} = 0.10$$

That of 2, $P(X_1 = 2) = \dfrac{2}{20} = 0.10$

That of 4, $P(X_1 = 4) = \dfrac{6}{20} = 0.30$

You have learned that the relative frequency of an event is approximately equal to the probability of the event.

The relative frequency distribution of the discrete random variable is given as follows:

X_1	F_1	$P(X - X_1)$
1	2	0.1
2	2	0.1
3	4	0.2
4	6	0.3
5	4	0.2
6	2	0.1
	20	1.00

With this probability distribution obtained, we can make a number of probability statements. We can even generate a number of questions from the distribution.

For instance, what is the probability that a throw will give value of 3?

The answer is $P(x = 3) = \dfrac{4}{20} = 0.2$.

Another one is what is the probability of getting in a throw 4 or 5. The answer is $P(X = 4$ or $5)$

This is equal to $P(x = 4) + P(x = 5) = 0.3 + 0.2$ (from table) $= 0.5$

Another question is what is the probability that the value of the throw will be more than 3.

The answer is $P(x \geq 4) = P(x = 4) + P(x = 5) + P(x = 6) = 0.3 + 0.2 + 0.1 = 0.6$

3.1.2. Cumulative Relation Frequency Distribution

In view of the fact that the cumulative relative frequency distribution is important in computing the probabilities of some events, we will need to construct the cumulative relative frequency of the discrete random variable.

For a discrete random variable X the cumulative relative frequency of x_1 is given as is $P(x \leq x_1)$ for $x_1 = 0,1,2,\text{-----} n$

hence $P(x \leq 4)$ is equal to $P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4)$ For our example 5.1, the cumulative frequency is given as follow.

X_1	F_1	$P(x = X_1)$	Cumulative Relative Frequency $P(x \leq x_1)$
1	2	0.1	0.1
2	2	0.1	0.2
3	4	0.2	0.4
4	6	0.3	0.7
5	4	0.2	0.9
6	2	0.1	1.00

We can also make a number of probability statements on the cumulative relation frequency distribution.

The probability of a value that is at most 4 is given as $P(x \leq 4) = 0.7$ The probability of getting a value between 2 and 5 inclusive is given by
$P(x \leq 5) - P(x \leq 1) = 0.9-0.1 = 0.8$
Or $P(x=2)+P(x=3)+P(x=4)+P(x=5)=0.1+0.2+0.3+0.2=0.8$

You will realize that we did not say that the probability is equal to
$P(x \leq 5) - P(x \leq 2)$.

Rather we say that it is

$P(x \leq 5) - P(x \leq 1)$ because the question state that the probability of 2 is inclusive.

3.2   Measures Of Probability Distribution

3.2.1   Mean of probability distribution of a discrete Random Variable.

The mean as a concept was introduced in Unit 5. You learned that mean could be computed for both the population and the sample taken from the population. This unit treats the concept in terms of probability distribution of a discrete random variable.

The mean of the probability distribution is the expected value of the random variable that has the specified distribution.

Here we will find the expected value of X which is

$E(X) = \sum xi \, P(x = xi)$

Example 14.2

Using the table showing the frequency distribution of the values in a die thrown 20 times.

X_1	f_1	$P(x = xi)$	$X_1 \, P(x \leq xi)$
1	2	0.1	0.1
2	2	0.1	0.2
3	4	0.2	0.4
4	6	0.3	0.7
5	4	0.2	0.9
6	2	0.1	1.00
		1.00	3.7

E(x)=1 x 0.1 +2 x 0.1 + 3 x 0.2 + 4 x 0.3 x 5 x 0.2 + 6 x 0.1
=0.1+0.2+0.6+1.2+1.2+1.0+0.6=3.7

You will also realize that

$X_i \, P(x = x_i) = X; \, x \, P(x = x_i)$

The E(x) obtained is the mean. If we compute it for population it is designated as μ Hence $E(x) = μ$

Let us use the formula we used previously to compute the mean, that is

$\mu = \dfrac{\sum f_i x_i}{\sum . f_i}$

Therefore $\mu = \dfrac{1\times2 + 2\times2 + 3\times4 + 4\times6 + 5\times4 + 6\times2}{20}$

This is equal to $\dfrac{74}{20} = 3.7 = E(x)$

3.2.2   Variance of Probability Distribution of A Discrete Random Variable

You also learned about variance when you studied measures of dispersion in unit 9 of this course. This variance in this unit will involve that of the probability distribution of a discrete random variable. The variance is given as the mathematical expectation of
$(x - \mu)^2$ which is equal to

$E(x_l - \mu)^2 = \sum(x_i - \mu)^2 P(x = x_i)$

Example 14.3

Using the table in example 5.2, compute the variance for the distribution.

X_i	f_i	$P(X = X_l)$	$(x_i - \mu)$	$(x_i - \mu)^2$	$(X_i - \mu)^2 P(X = X_i)$
1	2	0.1	-2.7	7.29	0.79
2	2	0.1	-1.7	2.89	0.29
3	4	0.2	-0.7	0.49	0.098
4	6	0.3	0.3	0.09	0.027
5	4	0.2	1.3	1.69	0.338
6	2	0.1	2.3	5.29	0.529
Total	20				2.01

The variance of the distribution is 2.01

The computation appears tedious. There is a short cut to it. It is known that E
$(x - \mu)^2 = E(X)2 - [E(X)]^2$
We have computed $E(x)$ before, hence we can easily obtain $[E(x_i)]^2$

What it remains is for us to compute $E(x)^2$. This can be done as follows

X_i	f_i	P(X - Xi)	X_i	$X_i{}^2$ P(X - X^i)
1	2	0.1	1	0.1
2	2	0.1	4	0.4
3	4	0.2	9	1.8
4	6	0.3	16	4.8
5	4	0.2	25	5.0
6	2	0.1	36	3.6
Total				15.7

The $E(x)^2$ $\quad = \sum x^2 P(x = xi) = 15.7$

Using $E(x - \mu)^2 = E(X)^2 - [E(X_i)]^2$

The variance, $E(x - \mu)^2 = 15.7 - 3.7^2 = 15.7 - 13.69 = 2.01$

The result is the same when we calculated the variance using

$E(x - \mu)^2 = \sum(x - \mu)^2 P(x = X_1)$

Therefore Variance $= E(x^2) - (E(x))^2 = E(x - \mu)^2$

Student Assignment Exercise 14.1

Given the table below,

X_i	f_i
0	2
1	2
2	4
3	5
4	6
5	9
6	7
7	5

8	4
9	2
10	1

(a) Prepare a probability distribution table for the data

(b) Prepare a cumulative frequency curve

(c) From the table prepared, estimate

(i) $P(3 \leq x \leq 8)$ (ii) $P(x > 5)$ (iii) $P(x \leq 4)$

(d) Compute (i) $E(x)$ (ii) $E(x_2)$ (iii) $E(x - \mu)^2$

4.0   Conclusion

This unit has built more on the concepts and techniques of probability by showing the application of these concepts to the relative frequency of distribution of a discrete random variable. The unit has also giver the means of computing mean and variance of the probability distribution of a discrete random variable.

Future units will show more of the types of probability distributions both of discrete and continuous random variables.

5.0   Summary

You learned in the unit that the relative frequency of an even x; is approximately equal to its probability, $P(x = x_1)$, for a discrete random variable.

The mean of the probability distribution of a discrete random variable, which is the expected value of x given as $E(x)$ is equal to $\sum x_i P(x = x_1)$. You also learned that the variance of the distribution, given as $E(x_1 - \mu)^2$ is equal to $\sum (x_1 - \mu)^2 P(x = X_1)$.

It is also discussed in the unit that $\sum(x - \mu)^2 = E(x^2) - (E(x))^2$ and this is the variance of the distribution.

6.0   Tutor Marked Assignment

The table below shows the number of industrial accidents suffered by 50 employees of a large manufacturing firm in a year.

Number of Accidents	Number of Employees
0	5
1	10
2	16
3	8
4	6
5	3
6	2

(a) (i) Construct the probability distribution for the table, that is the relative frequency distribution.

(ii) Construct the cumulative relative frequency for the distribution $(P x = x_1)$

(b) What is the probability that a randomly selected employee will be the one who had:

(i) 4 accidents
(ii) more than 2 accidents
(iii) At least an accident
(iv) Between 2 and 5 accidents
(v) 2 or 3 accidents

(c) (i) the mean of the distribution
(ii) The variance of the distribution

(Use probability method)

Answers to Student Assessment Exercise 14.1

X_i	f_i	$P(X = x_i)$	$Px \leq X_1$	$XP(X = X_1)$	X^2_1	$X^2_1(Px=x_1)$
0	2	0.04	0.04	0	0	0

1	4	0.08	0.12	0.08	1	0.08
2	5	0.10	0.22	0.20	4	0.40
3	6	0.12	0.34	0.36	9	1.08
4	9	0.18	0.52	0.72	16	2.88
5	7	0.14	0.66	0.70	25	3.50
6	5	0.10	0.76	0.60	36	3.60
7	4	0.08	0.84	0.56	49	3.92
8	4	0.08	0.92	0.64	64	5.12
9	2	0.04	0.96	0.36	81	3.24
10	1	0.02	1.00	0.20	100	2.00
				4.42		25.82

(i) $P(\leq x \leq 8) = P(x \leq 8) - P(x \leq 2) = 0.92 - 0.22 = 0.70$
(ii) $P(x > 5) = 1 - P(x <- 5) = 1 - 0.66 = 0.34$
(iii) $P(x \leq 4) = 0.52$

From the table

$E(x) = 4.42, \quad E(x_2) = 25.82$

$E(x - \mu)^2 = E(x^2) - (E(x))^2 = 25.82 - 4.42^2 = 6.2836$

7.0   References And Other Recourse

Daniel W.W and Terrel J.C (1979) Business statistics: Basic Concept and Methodology 2nd edition Houghton Mifflin Co;, Boston

Levin R.1 (1990) Statistics for Management 4th edition, Prentice -Hall of India Private Ltd. New Delhi

UNIT 15

BINOMIAL DISTRIBUTION

1.0   Introduction

This unit treats Binomial distribution, which is one of the most widely used probability distributions in statistics. Binomial distribution is a discrete random variable distribution in which one of the two possible mutually exclusive outcomes occurs in a trial or an experiment. Binomial distribution is derived from a process called Bernoulli trial. A Bernoulli trial results from experiment that has two possible mutually exclusive outcomes and one and only one of the outcomes can occur in a trial.

The unit will treat the properties of Binomial distribution, Binomial tables and measures of central tendency and dispersion in the distribution.

2.0   Objectives

At the end of the unit you should be able to:

• Know the definition and properties of Binomial distribution.
• Solve problems involving binomial distribution
• Read Binomial Tables
• Compute measures for a binomial distribution.

3.1   Definition and Properties of Binomial Distribution.

A Binomial distribution is derived from a process known as a Bernoulli trial. A Bernoulli trial is said to be trial of some process or experiment that can result into one of the two possible mutually exclusive events such as

Male and Female
Head and tail
Present and absent
Correct and incorrect
Defective and not defective
Acceptable or not acceptable

When there is a sequence of the trials we have a process. For instance a couple trying to have children, may decide to make five trials at births. These trials will now constitute a process. To achieve the process; the following conditions must be met (Daniel and Terrel, 1979).

(i)   Each trial results in one of the two possible mutually exclusive outcomes. We denote one of the two outcomes as a success and the other as a failure.
(ii)   The probability of a success is arbitrarily denoted as P the probability of failure is 1 - P. which is denoted as q.
         q=1-porP+q=1
(iii)   The trials are independent, that is the outcome of a trial does not affect the outcome of any other trial.

In binomial distribution, there are trials, say n, where n = 1,2,3,4 --------- N
We also have success denoted as X where X = 0,1,2,3 -------------- N

For instance if you want to have four children and you regard the birth of a male as a success then the value of success will be 0,1,2,3,4. If there is no male it means there is 0 success. If there are four males, there are 4 successes. On the basis of this, for n trials we have 0,1,2,3,4 successes. If it is possible to know the probability of a success, p, we can calculate the probability of Binomial distribution if we know the number of trials.

You shall know that if we have n trials where n = 1,2,3,4 --------- n, we can have X successes where x=0,1,2,3 - - - - n. Suppose probability of a success is P, that of failure will be 1-P. Therefore the probability of X success will be

$$P(X_1 = X_2) = \binom{n}{X} P_x \, q^{n-x}$$

This is the general formula for binomial distribution. Previously in this course, you learned that

$$\binom{n}{r} \quad \frac{n\,!}{r!\,(n-r)!}$$

$$\binom{n}{r} \quad \frac{5!}{3!2!} = \frac{5 \times 4 \times 3!}{3! \times 2 \times 1} = 10$$

The formula is easy to learn. If you know the number of trials n and number of successor

X, you see $\binom{n}{x}$ and multiplying it by probability of success raised is

number of successor and multiply the product by probability of failure raised to the number of failure.
If there are n trials and x successors number of failure will be (n -x) failure.

Example 15.1

Suppose it is known that the probability that a person in a profession will be at home in particular time of the day is 0.6 or 60%. If a person calls on 5 persons in the profession at that particular time of the day, what is the probability of getting 2 of them at home?

Here number of trials is 5 = n
Number of successes is 2 = x
Probability of successor is 0.6 =
P. Probability of a failure is 1- 0.6 = 0.4 = q
Number of failure is (5 - 2) = 3

Using the formula for calculating probability of binomial distribution

$$p(x = 2) = 20.6^2 0.4^3$$

$$\left(\begin{matrix} 2 \\ 5 \end{matrix} \right. = \frac{5!}{2!(5-2)!} = \frac{5!}{2!3!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times l \times 3 \times 2 \times 1} = 10$$

$$0.6^2 = 0.36$$
$$0.4^3 = 0.064$$

$$p(x = 2) \binom{5}{2} \ 0.6^2 0.4 = 10 \times 0.36 \times 0.064 = 0.2304$$

Let us now solve the problem graphically. There are 5 trials and 2 successes. Let S represent success and frequency a failure. We can have the possible arrangements for the 2 successes. This is equal to

$$\binom{5}{2} \qquad \frac{5!}{2!(5-2)} = \frac{5!}{2!3!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 3 \times 2 \times 1} = 10$$

We will then have the following possible arrangements. The first arrangement is SSFFF.

The joint probability of this, if probability of success P and that of failure is q, is $P \times p \times g \times q = p^2 g^3$.
The whole arrangement will be

Numbers of Arrangement	Possible Arrangement	Probability
1	SSFFF	$P \times p \times q \times q \times q = p^2 q^3$
2	SFSFF	$P \times q \times p \times q \times q = p^2 q^3$
3	SFFSF	$P \times q \times q \times p \times q = p^2 q^3$
4	SFFFS	$P \times q \times q \times q \times p = p^2 q^3$
5	FFFSS	$q \times q \times q \times p \times p = p^2 q^3$
6	FFSSF	$q \times q \times p \times p \times q = p^2 q^3$
7	FSSFF	$q \times p \times p \times g \times q = p^2 q^3$
8	FFSFS	$q \times g \times p \times q \times q = p^2 q^3$
9	FSFFS	$q \times p \times q \times q \times p = p^2 q^3$
10	FSFSF	$q \times p \times q \times p \times q = p^2 q^3$
	Total	$= 10\, p^2 q^3$

The total is $10p^2g^3 = 10 \times 0.6^2 \times 0.4^6 = 0.2304$

You will realize that we also obtain the same value as we obtained before.
For n trials, X success and P as probability of success, we can have probability distribution for the binomial distribution.

For example if n=5,p=0.6, we can obtain probability values for 0,1,2,3,4, and 5 successes. The sequence of this value constitutes the distribution of interest.

$$p(x=0) = \binom{5}{0} 0.6^0 x0.4^{5-0} = 1x1x0.4^5 = 0.01024$$

$$p(x=1) = \binom{5}{1} 0.6^1 x0.4^{5-1} = 5x0.6^1 x0.4^4 = 0.0768$$

$$p(x=2) = \binom{5}{2} 0.6^2 0.4^3 = 10x0.6^3 0.4^2 = 0.2304$$

$$p(x=3) = \binom{5}{3} 0.6^3 0.4^2 = 10x0.6^3 0.4^2 = 0.3456$$

$$p(x=4) = \binom{5}{4} 0.6^4 0.4^1 = 5x0.6^4 0.4^1 = 0.2592$$

$$p(x=5) = \binom{5}{5} 0.6^5 0.4^0 = 1x0.6^5 x1 = 0.07776$$

For n=5 and p=0.6,

The probability distribution of the binomial distribution is given as

X = no of Success	Probability
0	0.01024
1	0.0768
2	0.2304
3	0.3456
4	0.2592
5	0.07776
Total	1.00

With this distribution, we can make a lot of probability statements. For instance, what is the probability of meeting more than two of the persons at home at that particular time of the day?

P(x>2) =P(x=3)+P(x=4)+P(x=5)
         =0.3456 x 0.2592 x 0.07776

=0.68256

Let us solve another,

What is the probability of getting at least a person at home?
P (at least a person) = P(x=1) + p(x=2) + P(x=3) + P(x=4) + P(x=5)
=1- P(x = 0) =1-0.01024
=0.98976

You should know that when we obtain probability distribution for all possible success, the sum of the probabilities is equal to 1.00

Student Assessment Exercise 15.1

Suppose the probability that a family will have a male child is given as 0.4. If a couple in the family decides to have five births, what is the probability of having?

(1)   3 male births
(ii)   Less than 3 male births
(iii)   More than three male births
(iv)   At least a male birth
(v)   At most 4 male births [prepare the probability distribution table first]

3.2   Binomial Tables

Binomial tables are used to solve problems involving probability of success. It is very useful when the number of trials is large, say 20. Calculation of probabilities using mere calculator may be time consuming hence the use of binomial tables.

In the textbooks, there are three types of the table namely

(i)   Tables of individuals probabilities
(ii)   Cummulative binomial table (px $\leq$ xi)
(ii)   Cummulative binomial; probability

For the purpose of this course, we will restrict ourselves to the most commonly used distraction, - that is the cumulative binomial probability distribution table [p(x $\geq$ xl)]

3.2.1   Cummulative Binomial Probability Distribution Table [p(x >_ xl)]

For this table we need to know the values of n and p. having known this we will locate the table of interest for the question being solved.

In the table, for any x, the probability of $x \geq x_l$ is given. For example, if we have five trials that is n=5 and the p=0.4, we will locate the table of n=5, p=0.4. We are expected to have 0,1,2,3,4,5 successes when we have 5 trials.

In the table we will have probabilities against x=1,2,3,4 and 5, that of 0 is not given. The value against 0 is 1.00 for all values of n and p.

For x=1, the value of probability against 1 is that of $p(x > 1)$ which is $p(x = 1) + p(x = 2) + p(x = 3) + p(x = 4) + p(x = 5)$ The value against 4 is $p(x \geq 4)$ Which is $p(x = 4) + p(x = 5)$ the value against 3 is $p(x \geq 3)$ which is $p(x = 3) + p(x = 4) + p(x = 5)$ You need to understand all these before you can apply the table.

You should note that the value against 0 is not given but it is 1.00. this is $p(x \geq 0) = 1 = p(x = 0) + p(x = 1) + p(x = 3) + p(x = 4) + p(x = 5)$

Example 15.2

Solve the problems in exercise 1 with the use of table if p=0.5 n=5, p=0.5
(i) $p(x = 3) = p(x \geq 3) - p(x \geq 4)$
        $= 0.5000 - 0.1875 = 0.3125$

Note that $p(x \geq 3) = p(x = 3) + p(x = 4) + p(x = 5)$
$p(x \geq 4) = p(x = 4) + p(x = 5)$

.; $p(x=3) = p(x>3) - p(x \geq 4) = p(x=3) + p(x=4) + p(x=5) - p(x=4) - p(x=5)$
$= p(x = 3)$

(ii)   Less than three births $= p(x < 3)$
        $p(x<3) = p(x = 0) + p(x = 1) + p(x = 2) = 1 - p(x > - 3)$ since
        $p(x=0) + p(x=1) + p(x=2) + p(x=3) + p(x=4) + (x=5) = 1$
        $p(x < 3) = 1 - p(x > - 3) = 1 - 0.5000 = 5000$

(iii)   More than 3 male birth
        $p(x > 3) = p(x = 4) + p(x=5) = p(x \geq 4)$
        $p(x > 4) = .1875$

(iv)   At least a male birth
        $p(x \geq 1) = p(x = 1) + p(x = 2) + p(x = 3) + p(x = 4) + p(x=5)$
        $p(x \geq 1) = .9688$

(v)　　At most 4 male births
　　　　$p(x \leq 4) = p(x = 0) + p(x = 1) + p(x = 2) + p(x=3) + p(x = 4)$
　　　　$=1-p(x>5)$
　　　　$1-0.0312$
　　　　$= 0.9688$

Student Assessment Exercise 15.2

Suppose a binomial distribution is defined by n=20 and P=0.3 What is the probability of getting?

(1)　　Exactly 15 successes
(ii)　　More than 15 successes
(iii)　　Between 5 and 11 successes
(iv)　　Less than 6 successes
(v)　　At least a success.

3.3　　MEASURES OF BINOMIAL DISTRIBUTION

3.1.1　Mean

The mean of a binomial distribution is given as n x p, that is the number of trials multiplied by the probability of success

For n = 5 and p = 0.4
The mean = 5 x 0.4 = 2.0
Variance = 5x0.4 x 0.6=1.2

3.1.1　Variance

The variance of a binomial distribution is given as n p q where n is number of trials, p is probability of a success and q=1-p is probability of a failure.
If n=5, p=0.4, q will be 1-0.4=0.6

4.0　　Conclusion

This unit treats a binomial distribution as a distribution in which one of the two possible mutually exclusive outcomes must occur in a trial that is derived from Bernoulli, process. The units teaches the computation of probability of successes given x=number of, successes, given n= number of trials and p= probability of success. It also introduces you to the use of binomial table and computation of measures for binomial distribution.

5.0　　Summary

The binomial distribution is said to be derived from Bernoulli trial in which one of the two possible mutually exclusive outcomes such as present and absent, correct and incorrect will occur in a trial. The unit gives the properties of a binomial distribution. It also gives that for n trials and p as probability of a success, (1-p) is the probability of a failure, and the probability of x success is given as

$$p(x = x_1) = \binom{n}{x} p^x q^{n-x}$$

Where n= 1,2,3………..n

x=0,1,2,3………n

Binomial table is also explained to solve problems involving binomial distribution.

The mean and the variance of binomial distribution are given as np and npq respectively.

6.0   Tutor Marked Assignment

6.1   If a binomial distribution is defined by n=25 and p=0.2 What is the probability of having?

(i)   Exactly 5 successes
(ii)   More than 6 successes
(iii)   Between 5 and 10 successes inclusive
(iv)   Less than 4 successes
(v)   At least a success

7.0   Reference and other resources

Daniel W.W and Terrel J.C (1979) Business Statistics: Basic Concepts and Methodology 2nd ed. Houghton Mifflin Co., Boston.

Harper W.M. (1982) Statistics. 4`h ed., Macdonald and Evans Handbook series.

Levin R.I (1990) Statistics for Management. 4th ed., Prentice-Hall of India Private Ltd., New Delhi

Stevenson W.J. (1978) Business Statistic: Concepts and Applications. Harper $ Row Publishers, New York.

UNIT 16

POISSON DISTRIBUTION

1.0   Introduction

In unit 15 you studied the binomial distribution - a probability distribution of a discrete random variable. Another very important distribution of a discrete random variable that is widely used is Poison distribution. This unit will teach the definition of Poisson distribution, the Poisson process, the use of Poisson table and use of Poisson distribution as an approximation to a binomial distribution.

2.0   Objectives

At the end of this unit, you will be able to:

- Explain Poisson distribution
- Compute probabilities involving Poisson process
- Use Poisson tables
- Use Poisson Distribution as an approximation of a binomial distribution

3.1   DEFINITION AND CHARACTERISTICS OF POISSON Distribution

3.1.1   Definition

A Poisson distribution is defined by x and λ, where x = 0, 1, 2, 3 and $\lambda = \mu$ = mean.

For x_i and λ given

The Poisson function f(x) = P(X=x$_1$) is given as

$$F(x) = \frac{\ell^{-\lambda} \lambda^x}{x_1}, \quad x = 0, 1, 2, \ldots\ldots$$

$$\sum f(x) = P(x = O) + P(x = 1) + P(x = 2)\ldots\ldots\ldots\ldots$$

Poisson distribution is used in many areas of the business especially when we are interested in reducing the waiting time on the queue at the petrol station, cafeteria etc. Some of the areas in which Poisson distribution has found application are:

(1)   Arrival pattern of students at a location of water tap.
(ii)   Arrival pattern of cars at a petrol station or of customers at a restaurant
(iii) Typographical errors in a book
(v)   Demand for a product.

3.2   Poisson Process

In Poisson distribution, there is said to be random occurrences of some event, and that this is brought about by the Poisson process.

Daniel and Terrel (1979) gave the characteristics of this process as follows:

1.   The occurrences of the event are independent, that is occurrence of an event in an interval of space or time has no effect on the probability of a second occurrence of the event in the same or any other interval.

2.   Theoretically an infinite number of occurrences of the event must be possible in the interval.

3.   The probability of a single occurrence of the event in a given interval is proportional to the length of the interval.

4.   In any infinitesimally small portion of the interval, the probability of more than one occurrence of the event is negligible.

       Another very interesting characteristic of Poisson distribution is that the mean and variance are both equal to   .

3.2   Poisson Table

Before you can solve any question on Poisson distribution, you need to understand the use of a Poisson Distribution Table. The table you will be using is that of cumulative Poisson Distribution. It is defined as $P(X \geq X; / \lambda)$ This is to say that for probability value against any x; we have $P(x \geq x_i)$ for a given .

For example, for value of x = 5 we will have a probability value showing
$P(x \geq 5) = P(x = 5) + P(x=6) + P(x=7)$......................

For x = 1, the probability value against it is:

$P(x >\_) = P(x = 1) + P(x = 2), P(x = 3)$. ……………..

Before you can use a cumulative Poisson Distribution table, you need to recognise the value of λ and then locate where the value is on the table. You also need to note that all probability values are given for x in such a way that the value shows $P(x \geq x_i)$, for the table. Let us now solve problems involving Poisson Distribution by using the table of cumulative Poisson Distribution.

Example 7.1

Suppose the number of guests entering into Klass Restaurant is Poisson distributed and the average number of the guests entering into the restaurant in an hour is 10, what is the probability that in a given hour:

(i)   2 guests enter the restaurant
(ii)   Exactly 7 guests enter the restaurant
(iii)   More than 8 guests enter the restaurant
(iv)   Less than 9 guests enter the restaurant
(v)   Between 6 and 12 guests inclusive enter the restaurant?

For this example   = 10
We locate the table showing   = 10

(i)   Probability that 2 guests enter the restaurant is given as f(x = 2)
         $f(x = 7) = p(x \geq 2) - p(x \geq 3)$
         =1.0 - 0.997
         = 0.003

(ii)   Exactly 7 guests enter the restaurant
         $f(x = 7)$ $p(x \geq 7) -= P(x \geq 8)$
         = 0.870 - 0.780
         = 0.090

(iii)   More than 8 guests enter the restaurant
         f(x=8) = P(x≥8)- P(x≥9)
         = 0.667

(iv)   Less than 9 guests enter the restaurant
         f(x < 9) =   1 - P(x ≥ 9)
         = 1 - 0.667   =   0.333

(v)   Between 6 and 12 guests enter the restaurant
         (That is 6,7,8,9,10,11, or 12 guests enter the restaurant)
         f (6≤ x≤ 12) = P(X ≥ 6) - P(X >-13)
         = 0.933 - 0.208 = 0.725

Student Assessment Exercise 16.1

Suppose the number of persons per car arriving at the entrance of a Tourist Centre is Poisson Distributed with   = 5.8, what is the probability that a car arriving at the entrance contains:

(i)   No persons?
(ii)   At least a person?
(iii)   More than 10 people?
(iv)   Between 5 persons and 10 persons inclusive?
(v)   Less than 3 persons?

3.3   Poisson Distribution as An Approximation to Binomial Distribution

Poisson distribution can be used to approximate the binomial distribution when n number of trials is large and p = probability of success is small.

When these conditions are met, we can express $\binom{n}{x} p^x q^{n-x}$ as equal to $\dfrac{e^{-\lambda}\lambda^x}{x!}$ = $\dfrac{e^{-\lambda}(np)^x}{x!}$

Here   = np = μ

np was given in unit 15 as equal to the mean of binomial distribution. How do we know when n is large and p is small? Generally it is accepted that for n to be large here n must be greater than or equal to 20, that is n ≥ 20. For p to "be small, it must be less than or equal to 0.05, that is p ≤ 0.05.

To show this approximation, we shall give a question involving binomial distribution with n very large and p very small. We will answer it using

concepts of binomial distribution and Poisson distribution and compare the results.

Example 16.2

If n = 25 and p = 0.02, for a binomial distribution, find:
(i)   P(x = 1)   (ii)   p(x ≥ 2)   and   (iii)   p(x < 3)

Let us use binomial concept first. When n = 25, P = 0.02
(i)   P(x=1) = p(x≥1) - p(x ≥2) = 0.3965-0.886 = 0.3079
(ii)   P(x≥2) = 0.0886
(iii)   P(x ≥3)   =   1 - p(x ≥ 3)
                           =   1-0.0132
                           =   0.9868

Let us now use Poisson Distribution. Since n = 25 is considered to be large (n > 20) and P = 0.02 is small (P < 0.05) then we can approximate the binomial distribution with Poisson distribution.

   = np = 25 x 0.02 =
        0.5
We locate the table for λ = 0.5
(i)   f(x = 1)   =   0.393 - 0.090 = 0.303
(ii)   f(x ≥ 2) = 0.090
(iii)   f(x<3) =   1 - f (x ≥ 3)

                           = 1 -014
                           = 0.986

If you compare the corresponding values, you will realize that the Poisson distribution can actually approximate binomial distribution when n is large n ≥ 20 and p is small p ≤ 0.05.

Student Assessment Exercise 16.2

Suppose for a binomial distribution n = 100 and p = 0.02, using Poisson distribution as an approximate of binomial distribution, calculate:

(i)   P(x = 4)
(ii)   P(x ≤ 6)
(iii)   P(≥ 4)
(iv)   P(3≤ x ≤ 7)

4.0   Conclusion

In this unit, you learned that the Poisson distribution is a distribution of a discrete random variable, which is widely used in business. You also learned the characteristics of a Poisson process. The unit gave a Poisson function f(x) as

$$f(x) = P(x = x_1) = \frac{\ell^{-\lambda} \lambda^x}{x!}$$

The unit taught the use of a Poisson distribution table $p(x \geq x; / \lambda)$ and also showed Poisson distribution as an approximation to the binomial distribution when $n \geq 20$ and $p \leq 0.05$.

5.0   Summary

The unit gave the probability function x = f(x) as:

$$f(x) = \frac{\ell^{-\lambda} \lambda^x}{x!}$$

Where x = 0, 1, 2. ....

The unit also gave the characteristics of Poisson distribution and taught the use of cumulative Poisson distribution table $p(x \geq x; /\ \ )$.

In the use of Poisson distribution as approximation to binomial distribution, when $n \geq 20$, that is large and when $p \leq 0.05$. that is small;   is given as np, the mean of binomial distribution.

6.0   Tutor Marked Assignment

1.   For a binomial distribution where n = 25 and p = 0.04 using both cumulative binomial distribution table and cumulative Poisson table, compute:
(i)   P(x = 5)   (ii) P(x $\leq$ 5)   (iii) P(x>2)

2.   The number of impurities per litre of a vegetable oil has been found to be Poisson distributed with   = 0.55.

If a litre of the oil is inspected what is the probability that the number of impurities observed will be:

(i) Less than 2 (ii) at least 1   (iii) exactly 3?

7.0   References and Other Resources

Daniel W.W. and Terrel J.C. (1979) Business Statistics: Basic

Concepts and Methodology. 2°d Edition Houghton Mifflin. Boston

Harper W.M. (1982) Statistics. 4`" Edition. Macdonald and Evans
Handbook Series.

Levin R.T. (1990) Statistics for Management 4`h Edition, Prentice-
Hall of India Private Ltd, New Delhi

Stevenson W.J. (1978) Business Statistics: Concepts and Application, Harper
and Row Publishers, New York.

UNIT 17

THE HYPERGEOMETRIC DISTRIBUTION

1.0   Introduction

In unit 15, Binominal distribution was discussed. The assumption made in binomial distribution is that the probability of a success, P must be constant for all trials.

This assumption can hold where there is infinite population that is sampled or when a finite population is sampled with replacement. In practice, replacement of elements is not done. This will then create problem with analysis of binomial distribution of finite population when replacement is not done. The hyper geometric distribution then solves the problem of unequal value of probability of success, p, that may occur when sampling finite population without replacement. This unit therefore treats the nature of hypergeometric function, solving of problems involving hypergeometric distribution and computation of measures for hypergeometric distribution.

2.0   Objectives

At the end of this unit, you should be able to
- Explain the concept of hypergeometric distribution
- Solve problems involving hypergeometric distribution
- Compute measures for hypergeometric distribution.

3.1   Nature of Hypergeometrc Distribution

As previously said in this unit, the binominal distribution assumes a constant probability of success; p, therefore for a distribution this assumption does not

have problem if the population sampled is an infinite one. However, if the population sampled is finite, P will not remain constant for every successive drawings of elements made, when the sampling is without replacement.

For instance if we have in a basket two types of balls, yellow and blue balls. They are 20 altogether. Suppose 12 of the balls are yellow and 8 of them are blue. If two balls are to be drawn successfully, the probability that the first ball will be yellow is $\underline{12}$. If the first ball is taken away and it is yellow, there
                    20
will remain 11 yellow balls and 8 blue balls. If the second ball is also yellow the probability of getting a second ball that is yellow is **11**
                                                         19
If the first ball is blue, and the second ball is yellow, the probability of getting yellow as the second ball will be $\underline{12}$. Both $\underline{11}$ and $\underline{12}$ are never equal to $\underline{12}$,
                              19   19   19   20
the probability that the first ball is yellow.

With this in mind the assumption made for binomial distribution may not be valid for finite population except sampling is done with replacement. Sampling with replacement is not practical with many real life situations, especially when investigation requires the destruction of elements.

To solve the problem identified above with binomial distribution when it comes to sampling finite population without replacement, we make use of hypergeometric distribution model. hypergeometric model is used in situations where we wish to determine the probability of a given number of success or failures.

3.2   Solution Of Problems Involving Hypergeometrie Distribution

3.2.1   Use Of Tree Diagrams

This is one of the ways to solve problems involving hyper geometric distribution. Let us illustrate the use of the tree diagram to a problem.

Example 8.1
There are twelve bottles of brand of beer in a carton. The carton contains 2 defective bottles and 10 not defective ones. If we select three bottles of the beer at random from the cartons, what is the probability that the sample contains exactly (1) 0 defective bottles (2) 1 defective bottle (3) 2 defective bottles? Let us use a tree diagram for this. We start by denoting defective by D and not defective by N. We will all put the successive probability of selecting the defective or not defective one in bracket. We will start with

2D and 1ON. If the first bottle is defective we have $P(D) = \dfrac{2}{12}$, if the second

after this is D, the probability becomes $\dfrac{1}{11}$ and so on

Fig 8.1 Tree diagram showing example 8.1
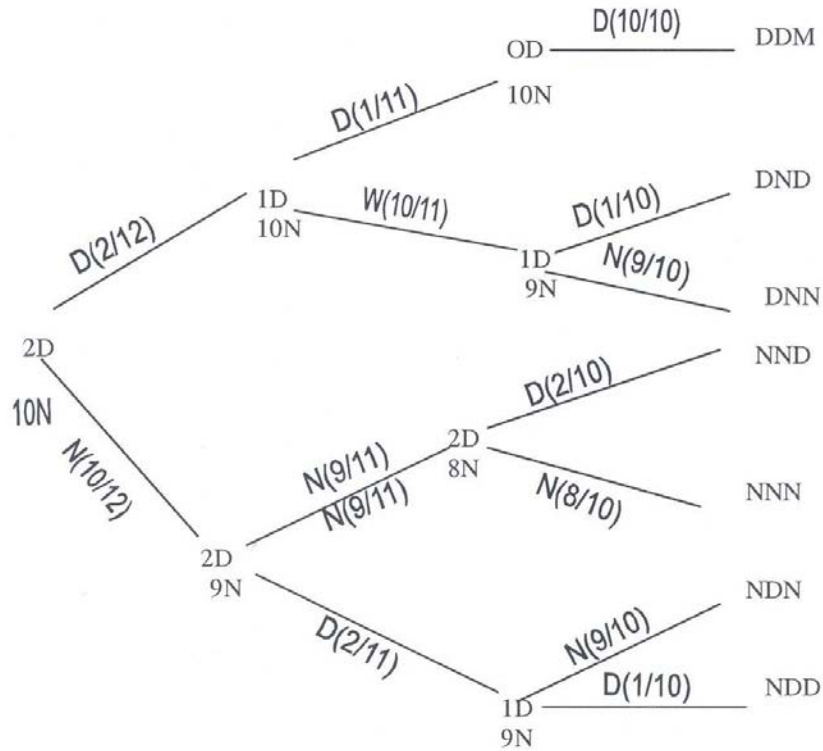
From the above tree diagram, we can identify the following arrangements with their probabilities.

$$DNN = \dfrac{2}{12} \text{ X } \dfrac{10}{11} \text{ X } \dfrac{9}{10} = 0.1365$$

$$NDD = \dfrac{10}{12} \text{ X } \dfrac{9}{11} \text{ X } \dfrac{2}{10} = 0.1365$$

$$NNN = \dfrac{10}{12} \text{ X } \dfrac{9}{11} \text{ X } \dfrac{8}{10} = 0.5455$$

$$NDN = \frac{10}{12} \times \frac{2}{11} \times \frac{9}{10} = 0.1364$$

$$NDD = \frac{10}{12} \times \frac{2}{11} \times \frac{1}{10} = 0.0152$$

We have now obtained the distribution let us answer the questions now

(1)   For O detective, we have NNN, that all of them are not defective. The required probability is 0.5455.

(2)   For 1 defective, we have
        DNN = 0.1365
        NND = 0.1365
        NDN = 0.1365
        TOTAL = 0.4092

        For 2 defectives, We have   DDN = 0.0152
                                         NNN = 0.0152
                                         TOTAL = 0.0456

3.2.2   Use Of Formula

Another way of solving problems involving hypergeometric distribution is by using formula

Daniel and Terrel (1999) stated that if from a population of size N consisting of N_1 elements of one type (successes) and N_2 element of another type (failure), select at random without replacement a sample of size n, determine the probability $P(X)$ that the sample will contain x successes.

The formula for $P(X)$ under these conditions is given as:

$$P(X) = \frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N}{n}} \quad for X = 6,1,2,3,........n$$

For our example, $N_1 = 2$, $N_2 = 10$ $N = N_1 + N_2 = 12$, $n = 3$, $x = 0, 1, 2, - - - -$

(i)   For 0 defective

$$P(0) = \frac{\binom{2}{0}\binom{10}{3}}{\binom{12}{3}}$$

$$\binom{2}{0} = 1$$

$$\binom{10}{3} = \frac{10!}{3!3!} = \frac{10x9x8x7!}{3x2x7!} = 120$$

$$\binom{12}{3} = \frac{12!}{3!9!} = \frac{12x11x10x9!}{3x2x9!} = 220$$

$$P(0) = \frac{1x120}{220} = 0.5455$$

(ii)   For $x = 1$

$$P(1) = \frac{\binom{2}{1}\binom{10}{2}}{\binom{12}{3}}$$

$$\binom{2}{1} = 2$$

$$\binom{10}{2} = \frac{10!}{2!8!} = \frac{10x9x8!}{2x8!} = 45$$

$$\binom{12}{3} = 220 \quad \text{(Already computed above)}$$

$$P(1) = \frac{2x45}{220} = 0.409$$

(iii)   For $x = 2$

(iii)   For x = 2

$$P(2) = \frac{\binom{2}{2}\binom{10}{1}}{\binom{12}{3}}$$

$$\binom{2}{2} = 1, \quad \binom{10}{1} = 10,$$

$$\binom{12}{3} = 220$$

$$P(2) = \frac{1 \times 10}{220}$$

$$= 0.0455$$

A comparison of the values obtained when we used the tree diagram with the values obtained when we used formula will show that the corresponding values are equal

3.3   Measure Of Hypergeometric Distribution

3.31   **Mean** $\mu = E(X) = n\left(\frac{N_1}{N}\right)$

The mean of a hypergeometric function is given as

For example 8.1

the mean is 3 x $\frac{2}{12}$ = 0.5

3.3.2   Variance

the variance of a geometric distribution is given as

$$\sigma^2 = n\left(\frac{N_1}{N}\right)\left(\frac{1 - N_1}{N}\right)\left(\frac{N - n}{N_1}\right)$$

$$\sigma 2 = \left(\frac{3x2}{12}\right)\left(\frac{1-2}{12}\right)\left(\frac{12-3}{12-1}\right)$$

$$= \left(\frac{1}{2}\right)\left(\frac{10}{12}\right)\left(\frac{9}{11}\right)$$

$$= 0.3409$$

Student Assessment Exercise 17.1

It has been found that out of 100 cocoa beans, 4 are defective. If four of the cocoa beans are selected, what is the probability that there are?

(i)   No defective bean?
(ii)   1 defective bean?
(iii)   2 defective beans?

4.0   Conclusion

In this unit, you have been thought how to overcome the problems encountered with the use of binomial distribution when sampling a finite population without replacement. The hypergeometric distribution in which the probabilities of taking successive elements from a population changes helps to solve the problem of constant value of probability of a success assumed in binomial distribution.

The unit has also given how problems involving hypergeometric distribution can be solved using tree diagram and formula for computing the mean and variance of hypergeometric distribution.

5.0   Summary

The unit teaches how the weakness of constant value of probability of a success in binominal distribution could be solved when sampling a finite population without replacement. The unit also gives the probability function of a hypergeometric distribution as

$$P(x) = \frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\dfrac{N}{n}} \quad \text{For } x = 0,1,2, - - - - -$$

Future unit will discus probability distribution of a continuous random variable.

6.0 Tutor Marked Assignment

In a crate of eggs containing 30eggs, 4 eggs are found to be defective. If 3 of such eggs are picked at random what is the probability of getting.

(i)   0   defective
(ii)   1   defective
(iii) 2 defective?

7.0 References And Other Resources

Daniel WW and Terrel J.C. (1979) Business Statistics: Basic Concepts and Methodology 2th ed., Houghton Mifflin Co; Boston.

UNIT 18

NORMAL DISTRIBUTION

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1.0   Introduction

In the last three units before this unit, you learned the probability distributions of discrete random variable - the Binomial, the Poisson and the Hyper-geometric Distributions. Before you learned those units, you have learned that discrete variables are characterized by `interruptions' between one value and the other. These `interruptions' are absent in distributions of continuous random variables. One of the distributions of the continuous random variables is the normal distribution, a distribution that is of most important in statistics. In this unit, you will learn this important probability distribution of continuous random variable and some of its applications.

2.0   Objectives

At the end of this unit, you should be able to:
•   List the characteristics of the Normal Distribution
•   Explain the use of the standard normal distribution
•   Read the areas under the curve of a normally distributed population
         using the normal curve areas
•   Transform X values to Z values
•   Use normal distribution model to calculate probabilities for
         continuous random variable.

3.1   Characteristics of the Normal Distribution

This is the most important distribution in statistics. The distribution is defined by the normal density function given by:

$$f(x) = \frac{1}{\sqrt{2\pi\delta}} \; \ell^{-(x-u)^2}/2\delta^2$$

For $\infty < x < \infty$

Where π is 3.14159 and e = 2.71828; δ is the population standard deviation. μ = Population mean.

A normal distribution is determined by the values of δ and μ, the standard deviation and mean of the population respectively. Any change in either δ or μ changes the distribution. The characteristics of a normal distribution are as follows:

(i)   The graph of the normal distribution is a bell-shaped curve.
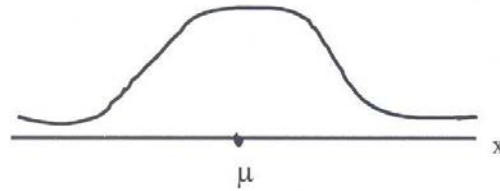
(ii)   The distribution is symmetrical above its mean μ. The curve on the either side of the mean is a mirror image of the other side.
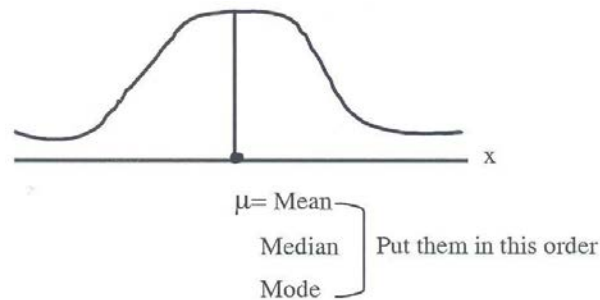
(iii)   The mean, the median and the mode are all equal.

(iv)   The total area under the curve above x-axis is equal to l. This means that 50% of the area is to the left of the mean and the other 50% to the right of the mean.
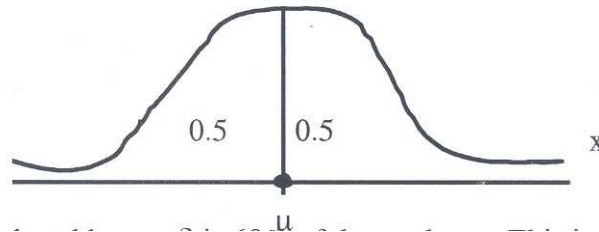
(v)   The area enclosed by $\mu \pm \delta$ is 68% of the total area. This is to say that 68% of the area is between 1 standard deviation from the mean in both directions.
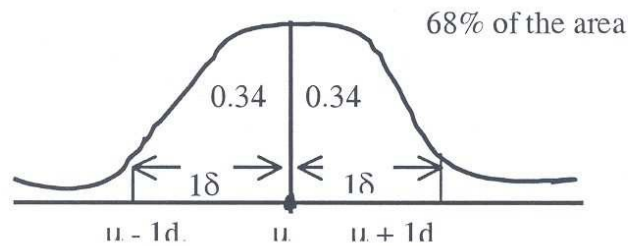
95% of the area under the curve is enclosed between 2 standard deviations from the mean in both directions.
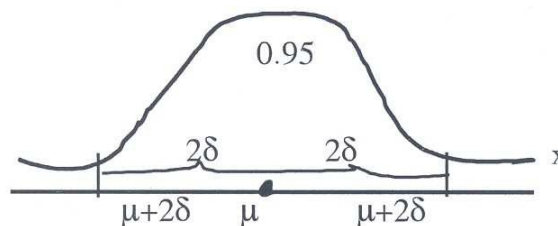
Also 99.7% of the area lies between 3 standard deviations from the mean in both directions.
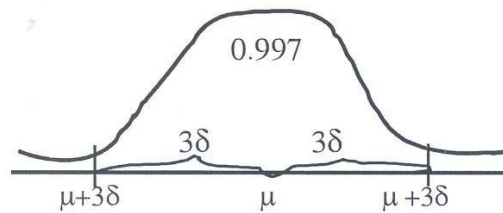
These are the characteristics of the normal distributions that will still be of serious use in the other courses that you may encounter in this programme.

3.2   The Standard Normal Distribution

The Normal Distribution, as already said is determined by the values of its δ and μ. Any change in the values of either δ or μ will give another normal distribution curve.

In view of this, there is a need to have a standard normal distribution that is determined by a mean of O and a standard deviation of 1.
With this, the X-axis in the normal distribution changes to Z- axis in the standard normal distribution.
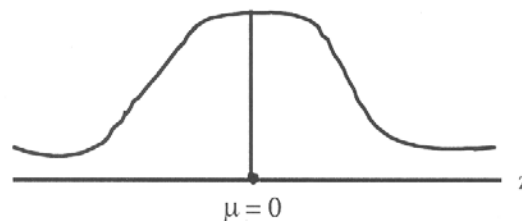
We then have:

$$f(z) = \frac{1}{\sqrt{2\pi\sigma}} \; \ell^{-2Z/2},$$

For $\infty < Z < \infty$

3.3   Normal Curve Areas

Now that the values of X in normal distribution has been transformed to values of Z in standard normal distribution, we still need to find the area under the curve for a given value of Z for this, we use the Normal Curve Areas.

In finding the area under the curve for a value of Z, we get the area between O and that given value of $Z = P (0 \leq Z \leq Z_I)$. For instance if we are looking for area under the curve when $Z = 2.5$. The area will be $P (0 \leq Z \leq 2.5)$.

We will read the area from the table of normal curve areas as follows; we look at the column for Z and get the value of 2.5 under .00. This is 0.4938. If the value of $Z = 0.55$, we will check $Z = 0.5$ under 0.05. This is equal to 0.2088.

For $Z = 1.43$, we look at the column 1.4 under 0.03. This is 0.4236.
For $Z = 1.435$, this cannot be easily read from the table. We will observe that 1.435 is equidistant from 1.43 and 1.44. Indeed it is the mean of the two. To find this probability value for $Z = 1.435$, look at:

Z = 1.43	=	0.4236
Z = 1.44	=	0.4251
We add up	=	0.8487
We divide by 2	=	0.4244

The area for $Z = 1.435$ is equal to 0.4244. Let us now apply the use of table to solving some problems.

Example 18.1
For a distribution, that is normal distribution, what is the probability the a Z picked as random from the population of Z's will have a value

(i) $Z = 2.3$
(ii) $Z < 2.3$
(iii) $Z = -1.75$
(iv) $1.75 \leq Z \leq 1.85$

You need to know that the value of $Z = 1.75$ is equal to value of $Z = -1.75$ The sign does not affect the value of the area covered by the value of Z.

Solution (i)
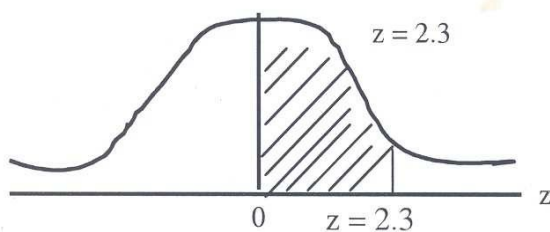
The area of interest is P (O < Z < 2.5) from the table, for Z = 2.3, the area is 0.4893.

(ii)   Z = 2.3
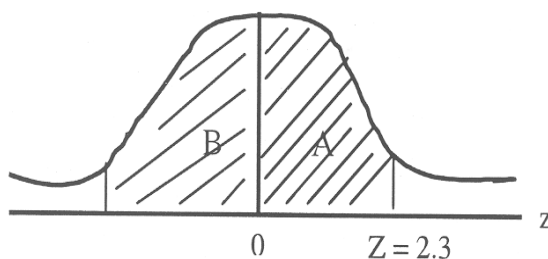
There are two areas involved here, the one to the right of the mean and the one to the left of the mean this is half of the area under the curve and is 0.5. The area to the left is P $0 \leq z \leq 2.3$which is equal to 0.4893 we obtained before.

The required area is 0.5 + 0.4893 = 0.9893.

(ii)   z = -1.75
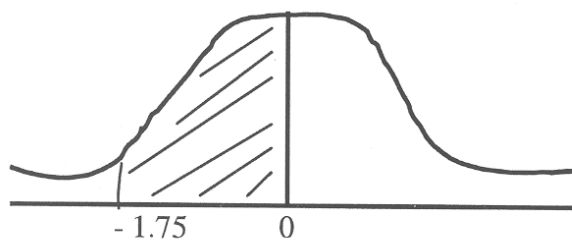
Area = P(-1.75 $\leq$ z $\leq$ 0) = 0.4599

(iv)   -1.75 $\leq$ Z $\leq$ 1.85
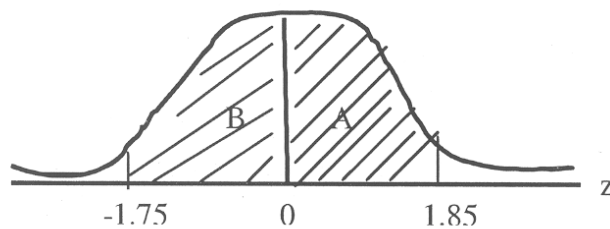
Area A = P(-1.75 $\leq$ Z $\leq$ 0) = 0.4599
Area B = P(0 $\leq$ Z $\leq$ 1.75) = 0.4678
Total Area   =   0.9277

(v)   1.75 $\leq$ Z $\leq$ 1.85
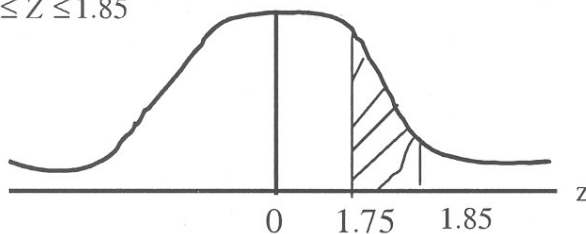
From 0 to Z = 1.85, the area is:
$$= P(0 \leq Z \leq 1.85) \quad = \quad 0.4678$$
Similarly P(0 $\leq$ Z $\leq$ 1.75) = $\underline{0.4599}$
Required Area   = $\underline{0.0079}$

In question (iv) the two values of Z are on different sides of the mean, hence we add up their areas to obtain the required area.

In question (v) the two values of Z are on the same side of the mean hence we subtract the value of the area of the smaller Z from that of the larger Z to get the required area.

3.4   The Applications of the Normal Distribution

The normal distribution is a very important distribution in Statistics especially when we will treat statistical inference in other statistics courses. The distribution is an ideal one since it is not possible to have a population of measurements that is normally distributed. However, if the population of measurements is approximately normally distributed with a known value of σ, standard deviation and a known value of µ , mean, then we can transform the measurements in X to the standard value in Z, since we have standard normal distribution as follows:

$$Z \quad = \quad \frac{x - \mu}{\delta}$$

Where x = is the x measurement
µ = Population mean
  = Population standard deviation.

Having known the standard value of Z, we can obtain the area covered by that value of x that gives the standard value of Z on the table of normal curve areas. Let us illustrate this with an example

Example of 18.2

Suppose the population of the weights of cubes of a seasoning agent is approximately normally distributed with a mean of 20 gm and a standard deviation of 1.50m. What is the probability that a cube selected at random from the population has a weight of:

(i)   20.5 gms
(ii)   Less than 20.5 gms
(iii)   More than 21 gms
(iv)   Between 19.5 and 20 gms

(i)   X = 20.5 gm

20gms   20.5gms
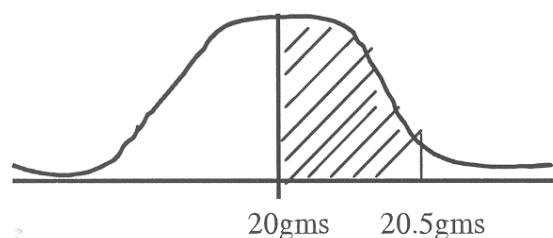
$$Z = \frac{x - \mu}{\delta} = \frac{20.5 - 20}{1.5}$$

$$= 0.33$$

Required probability = P(0≤Z≤0.33) = 0.1293

(ii)   X less than 20.5

20gm   20.5gm
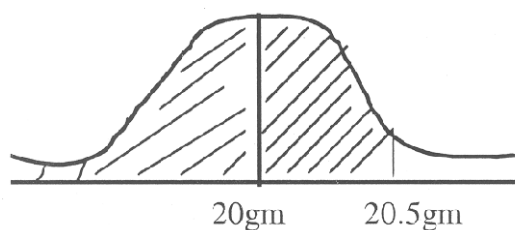
Z - $\frac{20.5 - 20}{1.5}$ = 0.33

$P(0 \leq Z \leq 0.33) = 0.1293$
The required probability   =   0.5 + 0.1293
                             =   0.6293

(iii)   More than 21
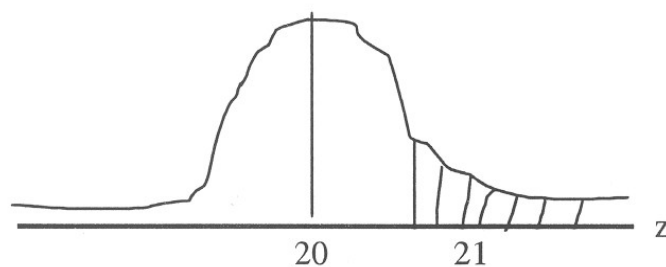
We will first calculate the area between 20gms and 21gms as follows:

Z   = $\dfrac{21-20}{1.5}$ = 0.67

P(0 ≤ Z ≤ 0.67) = 0.2486

The required probability   =   0.5-0.2486
                             =   0.2514

(iv).   Between 19.5gm and 20.75gms
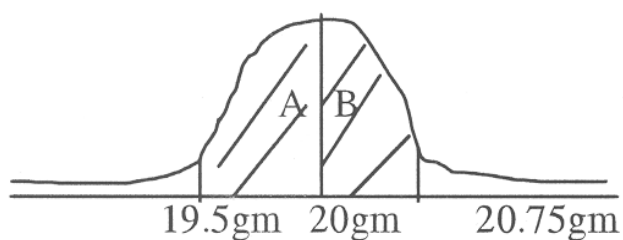
Z_1 = $\dfrac{19.5-20}{1.5}$ = 0.33

Z_2 = $\dfrac{20.75-20}{1.5}$ = 0.50

P(0 ≤ Z < 0.33)   =   0.1293

P(0 ≤ Z ≤ 0.5)   =   0.1915
Required probability   =   0.3208

Student Assessment Exercise 18.1

Suppose the ages of the pupils in a Primary School are normally distributed with a mean of 7.5 years and a standard deviation of 1.5years, what is the probability that a pupil selected at random has the age:

(i)   Of 9 years
(ii)   Between 6.75 years and 8.25 years
(iii)   Less than 6.75years
(iv)   More than 8.25 years.

3.5   The Normal Approximation to Binomial

For binomial distribution, the table of cumulative Binomial probability distribution does not show values for n that is very large. With this, it is difficult to estimate the probability of the values when n is large using binomial distribution.

This approximation of normal distribution to binomial distribution is only possible when n is large and p, probability of a success is not too close to 0 or 1.

The mean, μ will be equal to np. The standard deviation a will be equal to

$$\sqrt{npq} \quad \text{or} \quad \sqrt{np(1-p)}$$

Let us illustrate this with an example.

Example 18.3
Suppose a binomial distribution is defined by n = 25 and p = 0.4, what is the probability of getting:

(i)   Exactly 7 successes
(ii)   Between 7 and 12 successes

        Use binomial distribution first and normal distribution again for the solution. Compare the sets or values obtained in the two exercises.
        Let us use binomial first. Use the table of binomial distribution.

        For n=25, P=0.4

        $P(x=7) = P(x \geq 7) - P(x \geq 8)$
                 $= 0.9264 - 08464$

$$= 0.0800$$

(ii)   For $7 \leq x \leq 12$

$$P(7 \leq x \leq 12) = P(x \geq 7) - P(x \geq 13)$$
$$= 0.9264 - 0.1538$$
$$= 0.7726$$

Let us now use the normal distribution. The binomial distribution is a distribution of a discrete variable while the normal distribution is a distribution of a continuous variable. Before we can convert the value of x to z we need to introduce the continuity correction by subtracting 0.5 from the lower value and adding 0.5 to the higher value when we are considering an interval; say $7 \leq x \leq 12$. We will now have $6.5 \leq x \leq 12.5$.

If we consider a point, say x = 7, we shall introduce the continuity correction by saying 7 is between 6.5 and 7.5 that is $7 \pm 0.5 = 6.5, 7.5$. It is these values of X that we will convert to values of z.

For our example,

For P(x=7), we have $P(6.5 \leq x \leq 12.5)$
$$\mu = np = 25 \times 0.4 = 10$$

$$= \sqrt{25 \times 0.4(1 - 0.4)} \qquad = 2.45$$

For $P(6.5 < x < 12.5)$

For $P(6.5 \leq x \leq 12.5)$
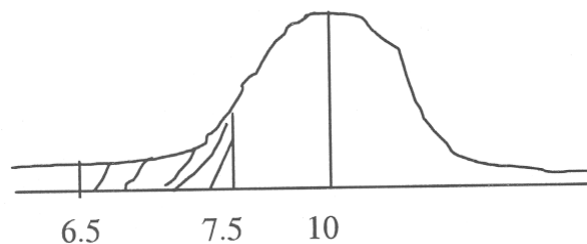
$$Z_1 = \frac{6.5 - 10}{2.45} = 1.43$$

$Z_2 = \dfrac{7.5-10}{2.45} = 1.02$

$P(-1.43 \leq Z \leq 0)$ = 0.4236
$P(4.02 \leq Z \leq 0)$ = 0.3465
Required area = 0.0771

For between 7 and 12
We will introduce the continuity correction, and obtain $6.5 \leq x \leq 12.5$
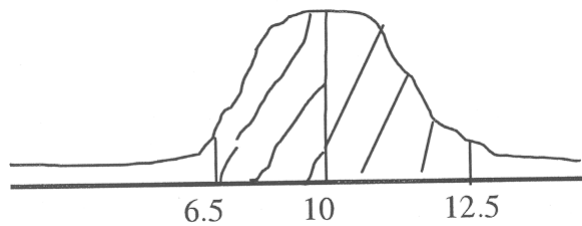
$Z_1 = \dfrac{6.5 - 10}{12.5} = -1.43$

$Z^2 = \dfrac{12.5-10}{2.45} = -1.02$

$P(-1.43 \leq Z \leq 0) = 0.4236$
$P(-1.02 \leq Z \leq 0) = 0.3465$
Required Area = 0.7701

If you compare the values in binomial with the corresponding values in normal distribution; you will realize that they are very close. For very large values of n when p is not too close to 0 or 1, we always omit the continuity correction, when finding probabilities associated with intervals such as $P(x^1 \leq x \leq x^2)$.

Student Assessment Exercise 18.2

A binomial distribution is defined by n = 50 and p = 0.3, Using normal distribution find the probability of getting.

(i) More than 8 successes
(ii) Between 10 and 20 successes.

4.0   Conclusion

This unit treated the probability distribution of a continuous random variable - normal distribution. It also treated the transformation of normal distribution to a standard normal distribution with a mean of 0 and standard deviation of 1. The x values are transformed to 2 values as follows:

$$Z = \frac{x - \mu}{}$$

You learned the application of this to some problems. You also learned normal approximation to binomial distribution when n is large and p is not too close to 0 or 1.

5.0   Summary

You learned in this unit, the last unit of this course about the distribution of continuous random variable given as normal distribution.

You also learned the transformation of normal distribution to standard normal distribution. Some problems involving normal distributions were solved and some were given as student Assessment Exercise. This distribution you have just learnt will find more useful application in Business Statistics course to follow

6.0   Tutor Marked Assignment

The weights of packages of a particular drug item are approximately normally distributed with a mean of 30gms and a standard deviation of 2.5gms. What is the probability that a package of the drug has a weight of:

(i)   28.5 gins
(ii)   Between 28.5gms and 32.5gms
(iii)   More than 32.5gms
(iv)   More than 27.5gms

7.0   References and other Resources

Daniel W.W. and Terrel J.C. (1979) Business Statistics; Basic Concepts and

Methodology 2nd Ed. Houghton Mifflin Co; Boston.

Harper W.M (1982) Statistics. 4`b Edition, Macdonald and Evans Handbook
     Series.

Levin R. 1. (1990) Statistics for Management 4`" Edition, Prentice Hall of
     India Private Ltd., New Delhi.

Stevenson W.J. (1978) Business Statistics: Concepts and Application. Harper
     and Row, Publishers, New York.

APPENDIX.

TABLE C Cumulative Binomial Probability Distribution

P (x 315,0.40) = 0.3174
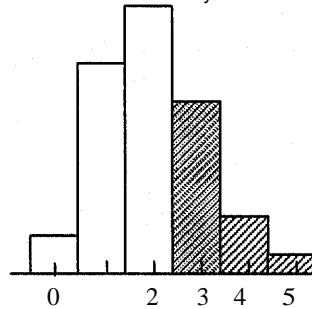
$$p_{(xx} \quad |_{n,p)} = X\{ \quad p^x q^{n-x}$$

	n=5									
	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
1	.0490	.0961	.1413	.1846	.2262	.2261	.3043	.3409	.3760	.4095
2	.0010	.0038	.0085	.0148	.0226	.0319	.0425	.0544	.0674	.0815
3	.0000	.0001	.0003	.0006	.0012	.0020	.0031	.0045	.0063	00.86
4	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0003	.0005
	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	.4416	.4723	.5016	.5296	.5563	.5818	.6061	.6293	.6513	.6723
2	.0965	.1125	.1292	.1467•	.1648	.1835	.2027	.2224	.2424	.2627
3	.0112	.0143	.0179	0220	.0266	.0318	.0375	.0437	.0505	.0579
4	.0007	.0009	.0013	.0017	.0022	.0029	.0036	.0045	.0055	.0067
5	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0002	.0002	.0003
	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30 .
1	.6923	.7113	.7293	.7464	.7627	.7781	.7927	.8065	.8196	.8319
2	.2833	.3041	.3251	.3461	.3672	.3883	.4093	.4303	.4511	.4718
3	.0659	.0744	.0836	.0933	.1035	.1143	.1257	.1376	.1501	.1631
4	.0081	.0097	.0114	.0134	.0156	.0181	.0208	.0238	.0272	.0308
5	.0004	.0005	.0006	.0008	.0010	.0012	.0014	.0017	.Q021	.0024
	.31	.32	.33	.34	.35	.36	.37	.38	.39	40
1	.8436	.8546	.8650	.8748	.8840	.8926	.9008	.9084	.9155	.9222
2	.4923	.5125	.5325	.5522	.5716	.5906	.6093	.6276	.6455	.6630
3	.1766	.1905	.2050	.2199	.2352	.2509	.2670	.2835	.3003	.3174
4	.0347	.o39b	.0436	.0486	.0540	.0598	.0660	.0726	.0796	.0870
5	.0029	.0034	.0039	.0045	.0053	.0068	.0069	.0079	.0090	.0102
	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50
1	.9285	.9344	.9398	.9449	.9497	.9541	.9582 .	.9620	.9655	.9688
2	.6801	.6967	.7129	.7286	.7438	.7585	.7728	.7865	.7998	.8125
3	.3349	.3525	.3705	.3886	.4069	.4253	.4439	.4625	.4813	.5000
4	.0949	.1033	.1121	.1214	.1312	.1415	.1522	.1635	.1752	.1875
5	.0116	.0131	.0147	.0165	.0185	.0206	.0229	.0255	.0282	.0312

n=6

X	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
1	.0585	.1142	.1670.	.2172	.2649	.3101	.3530	.3936	.4321	.4686
2	.0015	.0057	0125	.0216	.0328	.0459	.0608	.773	.0952	.1143
3	.0000	.0002	.0005	.0012	.0022	.0038	.0058	.0085	.0118	.0158
4	.0000	.0000	.0000	.0000	.0001	.0002	.0003	.0005	.0008	.0013
5	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

X	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	.5030	.5356	.5664	.5954	6.229	.6487	.6731	.6760	.7176	.7379
2	.1345	.1556	.1776	.2003	.2235	.2472	.2713	.2956	.3201	.3446
3	.0206	.0261	.0324	.0395	.0476	.0560	.0655	.0759	.0870	.0989
4	.0018	.0025	.0034	.0045	.0059	.0075	.0094	.0116	.0141	.0170
5	.0001	.0001	.0002	.0003	.0004	.0005	.0007	.0010	.0013	.0016
6	**.0000**	**.0000**	**.0000**	**.0000**	**.0000**	**.0000**	**'.0000**	**.0000**	**.0000**	**.OOOi**

X	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30
1	.7569	.7748	.7916	.8073	.8220	.8358	.8487	.86007	.8719	.8824
2	.3692	.3937	.4180	.4422	.4661	.4896	.5128	.5356	.5580	.5798
3	.1115	.1250	.1391	.1539	.1694	.1856	.2023	.2196	.2374	.2557
4	.0202	.0239	.0280	.0326	.0376	.0431	.0492	.0557	.0628	.0705
5	.0020	.0025	.0031	.0038	.0046	.0056	.0062	.0079	.0093	.0109
6	.000i	.0001	.0001	.0002	.0002	.0003	.0004	.0005	.0006	.0007

X	.31	.32	.33	.34	.35	.36	.37	.38	.39	40
1	.8921	.9011	.9095	.9173	.9246	.9313	.9375	.9432	.9485	.9533
2	.6012	.6220	.6422	.6619	.6809	.6994	.7172	.7343	.7508	.7667
3	.2744	.2936	.3130	.3328	.3529	.3732	;3937	.4143	.4350	.4557
4	.0787	.0875	.0969	.2019	.1174	.1286	.1404	.1527	.1657	.1792
5	.0127	.0148	.0170	.0195	.0223	.0254	.0228	.0325	.0365	.0410
6	.0009	.0011	.0013	.0015	.0018	.0022	.0026	.0030	.0035	.0041

X	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50
1	.9578	.9619	.9657	.9692	.9723	.9752	.9778	.9802	.9824	.9844
2	.7819	.7965	.8105	.8238	.8364	.8485	.8599	.8707	.8810	.8906
3	.4764	.4971	.5177	.5382	.5585	.5786	.5985	.6180	.6373	.6562
4	.1933	.2080	.2232	.2390	.2553	.2721	.2893	.3070	.3252	.3438
5	.0458	.0510	.0566	.0627	.0692	.0762	.0837	.0917	.1003	.1094
6	.0048	.0055	.0063	.0073	.0083	.0095	.0108	.0122	.0138	.0156

n=7

X	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
1	.0679	.1319	.1920	.2486	.3017	.3515	.3983	.4422	.4832	.5217
2	.0020	.0019	.0171	.0294	.0444	.0618	.0813	.1026	.1255	.1497
3	.0000	.0003	.0009	.0020	.0038	.0063	.0097	.0140	.1093	.0257
4	.0000	.0000	.0000	.0001	.0002	.0004	.0007	.0012	.00018	.0027
5	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	..0001	.0002

X	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	.5577	.5913	.6227	.6521	;6794	.7049	.7286	.7507	.7712	.7903
2	.1750	.2012	.2281	.2556	.2834	.3115	.3396	.3677	.3956	.4233
3	.0331	.0416	.0513	.0620	.0738	.0866	.1005	.1154	.1313	.1480
4	.0039	.0054	.0072	.0094	.0121	.0153	.0189	.0231	.0279	.3333
5	.0003	.0004	.0006	.0009	.0012	.0017	.0022	.0029	.0037	.0047

6	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002	.0003	.0004

	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30
1	.8080	.8243	.8395	.8535	.8665	.8785	.8895	.8997	.9090	.9176
2	.4506	.4775	.5040	.5298	.5551	.5796	.6035	.6266	.6490	.6706
3	.1657	.1841	.2033	.2231	.2436	.2646	.2861	.3081	.3304	.3529
4	.0394	.0461	.0536	.0617	.0706	.0802	.0905	.1016	.1134	.1260
5	.0058	.0072	.0088	.0107	.0129	.0153	.0181	.0213	.0248	.0288
6	.0005	.0006	.0008	.0011	.0013	.0017	.0021	.0026	.0031	.0038
7	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0002	.0002

	.31	.32	.33	.34	.35	.36	.37	.38	.39	40
1	.9255	.9328	.9394	.9454	.9510	.9560	.9606	.9648	.9686	.9720
2	.6914	.7113	.7304	.7487	.7662	.7828	.7987	.8137	.8279	.8414
3	.3757	.3987	.4217	.4447	.4677	.4906	.5134	.5359	.5581	.5801
4	.1394	.1534	.1682	.1837	.1998	.2167	.2341	.2521	.2707	.2898
5	.0332	.0380	.0434	.0492	.0556	.0625	.0701	.0782	.0869	.0963
6	.0046	.0055	.0065	.0077	.0090	.0105	.0123	.0142	.0164	.0188
7	.0003	.0003	.0004	.0005	.0006	.0008	.0009	.0011	.0014	.0016

n=8

	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
1	.0773	.1492	.2163	.2786	.3366	.3904	.4404	.4868	.5297	.5695
2	.0027	.0103	.0223	.0381	.0572	.0792	.1036	.1298	.1577	.1869
3	.0001	.0004	.0013	.0031	.0058	.0096	.0147	.0211	.0289	.0381
4	.0000	.0000	.0001	.0002	.0004	.0007	.0013	.0022	.0034	.0050
5	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0003	.0004

	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	.6063	.6404	.6(18	.7008	.7275	.7521	.7748	.7956	.8147	.8322
2	.2171	.2480	.2794	.3111	.3428	.3744	.4057	.4366	.4670	.4967
3	.0487	.0608	.0743	.0891	.1052	.1226	.1412	.1608	.1815	.2031
4	.0071	.0097	.0129	.0168	.0214	.0267	.0328	.0397	.0476	.0563
5	.0007	.0010	.0015	.0021	.0029	.0038	.0050	.0065	.0083	.0104
6	.0000	.0001	.0001	.0002	.0002	.0004	.0005	.0007	.0009	.0012
7	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001

	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30
1	.8483	.8630	.8764	.8887	.8999	.9101	.9194	.9278	.9354	.9424
2	.5257	.5538	.5811	.6075	.6329	.6573	.6807	.7031	.7244	.7447
3	.2255	.2486	.2724	.2967.	.3215	.3465	.3718	.3973	.4228	.4482
4	.0659	.0765	.0880	1004	.1138	.1281	.1433	.1594	.1763	.1941
5	.0129	.0158	.0191	.0230	.0273	.0322	.0377	.0438	.0505	.0580
6	.0016	.0021	.0027	.0034	.0042	.0052	.0064	.0078	.0094	.0113
7	.0001	.0002·	.0002	.0003	.0004	.0005	.0006	.0008	.0010	.0013
8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001

	.31	.32	.33	.34	.35	.36	.37	.38	.39	40

1	.9486	.9543	.9594	.9640	.9681	.9719	.9752	.9782	.9808	.9832
2	.7640	.7822	.7994	.8156	.8309	.8452	.8586	.8711	.8828	.8936
3	.4736	.4987	.5236	.5481	.5722	.5958	.6189	.6415	.6634	.6846
4	.2126	.2319	.2519	.2724	.2936	.3153	.3374	.3599	.3828	.4059
5	.0661	.0750	.0846	.0949	.2011	.1180	.1307	.1443	.1586	.1737
6	.0134	.0159	.0187	.0218	.0253	.0293	.0336	.0385	.0439	.0498
7	.0016	.0020	.0024	.0030	.0036	.0043	.0051	.0061	.0072	.0085
8	.0001	.0001	.0001	.0002	.0002	.0003	.0004	.0004	.0005	.0007

	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50
1	.9853	.9872	.9889	.9903	.9916	.9928	.9938	.9947	.9954	.9961
2	.9037	.9130	.9216	.9295	.9435	.9435	.9496	.9552	.9602	.9648
3	.7052	.7250	.7440	.7624	.7966	.7966	.8125	.8276	.8419	.8555
4	.4292	.4527	.4762	.4996	.5463	.5463	.5694	.5922	.6146	.6367
5	.1895	.2062	.2235	.2416	.2798	.2798	.2999	.3205	.3416	.3633
6	.0563	.0634	.0711	.0794	.0982	.0982	.1086	.1198	.1318	.1445
7	.0100	.0117	.0136	.0157	.0208	.0208	.0239	.0272	.0310	.0352
8	.0008	.0010	.0012	.0014	.0020	.0020	.0024	.0028	.0033	.0039

n – 9

	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
1	.0865	.1663	.2398	.3075	.3698	.4270	.4796	.5278	.5721	.6126
2	.0034	.0131	.0282	.0478	.0712	.0978	.1271	.1583	.1912	.2252
3	.0001	.0006	.0020	.0045	.0084	.0138	.0209	.0298	.0405	.0530
4	.0000	.0000	.0001	.0003	.0006	.0013	.0023	.0037	.0057	.0083
5	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0003	.0005	.0009
6	.0000	.0000	.0!>00	.0000	.0000	.0000	.0000	.0000	.0000	.0001

	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	.6496	.6835	.7145	.7427	.7684	.7918	.8131	.8324	.8499	.8658
2	.2599	.2951	.3304	.3657	.4005	.4348	.4685	.5012	.5330	.5638
3	.0672	.0833	.1009	.1202	.1409	.1629	.1861	.2105	.2357	.2618
4	.0117	.0158	.0209	.0269	.0339	.0420	.0512	.0615	.0730	.0856
5	.0014	.0021	.0030	.0041	.0056	.0075	.0098	.0125	.0158	.0196
6	.0001	.0002	.0003	.0004	.0006	.0009	.0013	.0017	.0023	.0031
7	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0002	.0003

	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30
1	.8801	.8931	.9048	.9154	.9249	.9335	.9411	.9480	.9542	.9596
2	.5934	.6218	.6491	.6750	.6997	.7230	.7452	.7660	.7856	.8040
3	.2885	.3158	.3434	.3713	.3993	.4273	.4829	.5102	.5372	
4	.0994	.1143	.1304	.1475	.1657	.1849	.2050	.2260	.2478	.2703
5	.0240	.0291	.0350	.0416	.0489	.0571	.0662	.0762	.0870	.0988
6	.0040	.0051	.0065	.0081	.0100	.0122	.0149	.0179	.0213	.0253
7	.0004	.0006	.0008	.0010	.0013	.0017	.0022	.0028	.0035	.0043
8	.0000	.0000	.0001	.0001	.0001	.0001	.0002	.0003	.0003	.0004

	.31	.32	.33	.34	.35	.36	.37	.38	.39	40
1	.9645	.9689	.9728	.9762	.9793	.9820	.9844	.9865	.9883	.9899
2	.8212	.8372	.8522	.8661	.8789	.8908	.9017	.9118	.9210	.9295
3	.5636	.5894	.6146	.6390	.6627	.6856	.7076	.7287	.7489	.7682
4	.2935	.3173	.3415	.3662	.3911	.4163	.4416	.4669	.4922	.5174
5	.1115	.1252	.1398	.1553	.1717	.1890	.2072	.2262	.2460	.2666

6	.0298	.0348	.0404	.0467	.0536	.0612	.0696	.0787	.0886	.0994
7	.0053	.0064	.0078	.0095	.0112	.0133	.0157	.0184	.0215	.0250
8	.0006	.0007	.0009	.0011	.0014	.0017	.0021	.0026	.0031	.0038
9	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0002	.0002	.0003
	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50
1	.9913	.9926	.9936	.9946	.9954	.9961	.9967	.9972	.9977	.9980
2	.9372	.9442	.9505	.9563	.9615	.9662	.9704	.9741	.9775	.9805
3	.7866	.8039	.8204	.8359	.8505	.8642	.8769	.8889	.8999	.9102
4	.5424	.5670	.5913	.6152	.6386	.6614	.6836	.7052	.7260	.7461
5	.2878	.3097	.3322	.3551	.3786	.4024	.4265	.4509	.4754	.5000
6	.1109	.1233	.1366	.1508	.1658	.1817	.1985	.2161	.2346	.2539
7	.0290	.3334	.0383	.0437	.0498	.0564	.0637	.0717	.0804	.0898
8	.0046	.0055	.0065	.0077	.0091	.0107	.0125	.0145	.0169	.0195
9	.0003	.0004	.0005	.0006	.0008	.0009	.0011	.0014	.0016	.0020

n = 10

	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
1	.0956	.1829	.2626	.3352	.4013	.4614	.5160	.5656	.6201	.6513
2	.0043	.0162	.0345	.0582	.0861	.1176	.1517	.1879	.2254	.2639
3	.0001	.0009	.0028	.0062	.0115	.0188	.0283	.0401	.0540	.0702
4	.0000	.0000	.0001	.0004	.0010	.0020	.0036	.0058	.0088	.0128
5	.0000	.0000	.0000	.0000	.0001	.0002	.0035	.0006	.0010	.0016
6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001
	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	.6882	.7215	.7516	.7787	.8031	.8251	.8448	.8626	.8784	.8926
2	.3028	.3417	.3804	.4184	.4557	.4920	.5270	.5608	.5932	.6242
3	.0884	.1087	.1308	.1545	.1798	.2064	.2341	.2628	.2922	.3222
4	.0178	.0239	.0313	.0400	.0500	.0614	.0741	.0883	.1039	.1209
5	.0025	.0037	.0053	.0073	.0099	.0130	.0168	.0213	.0266	.0328
6	.0003	.0004	.0006	.0010	.0014	.0020	.0027	.0037	.0049	.0064
7	.0000	.0000	.0001	.0001	.0001	.0002	.0003	.0004	.0006	.0009
8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001
	.21	.22	.23	.24	.25	.26	.27	.28	.29	30
1	.9053	.9166	.9267	.9357	.9437	.9508	.9570	.9626	.9674	.9718
2	.6536	.6815	.7079	.7327	.7560	**.7778**	**.7981**	**.8170**	**.8347**	**.8507**
3	.3526	.3831	.4137	.4442	.4744	.5042	.5335	.5622	.5901	.6172
4	.1391	.1587	.1794	.2012	.2241	.2479	.2726	.2979	.3239	.3504
5	.0399	.0479	.0569	.0670	.0781	.0904	.1037	.1181	.1337	.1503
6	.0082	.0104	.0130	.0160	.0197	.0239	.0287	.0342	.0404	.0473
7	.0012	.0016	.0021	.0027	.0035	.0045	.0056	.0070	.0087	.0201
8	.0001	.0002	.0002	.0003	.0004	.0006	.0007	.0010	.0012	.0016
9	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001
	.31	.32'	.33	.34	.35	.36	.37	.38	.39	40
1	.9755	.9789	.9818	.9843	.9865	.9885	.9902	.9916	.9929	.9940
2	.8656	.8794	.8920	.9035	.9140	.9236	.9323	.9402	.9473	.9536
3	.6434	.6687	.6930	.7162	.7384	.7595	.7794	.7983	.8160	.8327
4	.3772	.4044	.4316	.4589	.4862	.5132	.5400	.5664	.5923	.6177
5	.1679	.1867	.2064	.2270	.2485	.2708	.2939	.3177	.3420	.3669
6	.0551	.0637	.0732	.0836	.0949	.1072	.1205	.1348	.1500	.1662
7	.0129	.0155	.0185	.0220	.0260	.0305	.0356	.0413	.0477	.0548
8	.0020	.0025	.0032	.0039	.0048	.0059	.0071	.0086	.0103	.0123
9	.0002	.0003	.0003	.0004	.0005	.0007	.0009	.0011	.0014	.0017

10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001
	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50
1	.9949	.9957	.9964	.9970	.9975	.9979	.9983	.9986	.9988	.9990
2	.9594	.9645	.9691	.9731	.9767	.9799	.9827	.9852	.9874	.9893
3	.8483	.8628	.8764	.8889	.9004	.9111	.9209	.9298	.9379	.9453
4	.6425	.6665	.6898	.7123	.7340	.7547	.7745	.7933	.8112	.8281
5	.3922	.4178	.4436	.4696	.4956	.5216	.5474	.5730	.5982	.6230
6	.1834	.2016	.2207	.2407	.2616	.2832	.3057	.3288	.3526	.3770
7	.0626	.0712	.0806	.0908	.1020	.1141	.1271	.1410	.1560	.1719
8	.0146	.0172	.0202	.0236	.0274	.0317	.0366	.0420	.0480	.0547
9	.0021	.0025	.0031	.0037	.0045	.0054	.0065	.0077	.0091	.0107
10	.0001	.0002	.0002	.0003	.0003	.0004	.0005	.0006	.0008	.0010

n = 11

X	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
1	.1047	.1993	.2847	.3618	.4312	.4937	.5499	.6004	6456	.6862
2	.0052	.0195	.0413	.0692	.1019	.1382	.1772	.2181	.2601	.3026
u	.VVV&.	.VVi&.?	.0037	.0083	.Vi6&.?	.0248	.0370	.0519	.0695'	.0896
4	.0000	.0000	.0002	.0007	.0016	.0030	.0053	.0085	.0129	.0185
5	.0000	.0000	.0000	.0000	.0001	.0003	.0005	.0010	.0017	.0028
6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0003
	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	.7225	.7549	.7839	.8097	.8327	.8531	.8712	.8873	.9015	.9141
2	.3452	.3873	.4286	.4689	.5078	.5453	.5811	.6151	.6474	.6779
3	.1120	.1366	.1632	.1915	.2212	.2521	.2839	.3164	.3494	.3826
4	.0256	.0341	.0442	.0560	.0694	.0846	.1013	.1197	.1397	.1611
5	.0042	.0061	.0087	.0119	.0159	.0207	.0266	.0334	.0413	.0504
6	.0005	.0008	.0012	.0018	.0027	.0037	.0051	.0068	.0090	.0117
7	.0000	.0001	.0001	.0002	.0003	.0005	.0007	.0010	.0014	.0020
8	.0000	.0000	.0b00	.0000	.0000	.0000	.0001	.0001	.0002	.0002
	.21	.22	.23	.24	.25	.26	.27	.28	.29	30
1	.9252	.9350	.9436	.9511	.9578	.9636	.9686	.9730	.9769	.9802
2	.7065	.7333	.7582	.7814	.8029	.8227	.8410	.8577	.8730	.8870
3	.4158	.4488	.4814	.5134	.5448	.5753	.6049	.6335	.6610	.6873
4	.1840	.2081	.2333	.2596	.2867	.3146	.3430	.3719	.4011	.4304
5	.0607	.0723	.0851	.0992	.1146	.1313	.1493	.1685	.1888	.2103
6	.0148	.0186	.0231	.0283	.0343	.0412	.0490	.0577	.0674	.0782
7	.0027	.0035	.0046	.0059	.0076	.0095	.0119	.0146	.0179	.0216
8	.0003	.0005	.0007	.0009	.0012	.0016	.0021	.0027	.0034	.0043
9	.0000	.0000	.0001	.0001	.0001	.0002	.0002	.0003	.0004	.0006
	.31	.32	.33	.34	.35	.36	.37	.38	.39	40
1	.9831	.9856	.9878	.9896	.9912	.9926	.9938	.9948	.9956	.9964
2	.8997	.911	.9216	.9310	.9394	.9470	.9537	.9597	.9650	.9698
3	.7123	.7361	.7587	.7799	.7999	.8186	.8360	.8522	.8672	.8811
4	.4598	.4890	.5179	.5464	.5744	.6019	.6286	.6545	.6796	.7037
5	.2328	.2563	.2807	.3059	.3317	.3581	.3850	.4122	.4397	.4672
6	.0901	.1031	.1171	.1324	.1487	.1661	.1847	.2043	.2249	.2465
7	.0260	.0309	.0366	.0430	.0501	.0581	.0670	.0768	.0876	.0994
8	.0054	.0067	.0082	.0101	.0122	.0148	.0177	.0210	.0249	.0293
9	.0008	.0010	.0013	.0016	.0020	.0026	.0032	.0039	.0048	.0059
10	.0001	.0001	.0001	.0002	.0002	.0003	.0004	.0005	.0006	.0007

	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50
1	.9970	.9975	.9979	.9983	.9986	.9989	.9991	.9992	.9994	.9995
2	.9739	.9776	.9808	.9836	.9861	.9882	.9900	.9916	.9930	.9941
3	.8938	.9055	.9162	.9260	.9348	.9428	.9499	.9564	.9622	.9673
4	.7269	.7690	.7700	.7900	.8089	.8266	.8433	.8588	.8733	.8867
5	.4948	.5223	.5495	.5764	.6029	.6288	.6541	.6787	.7026	.7256
6	.2690	.2924	.3166	.3414	.3669	.3929	.4193	.4460	.4729	.5000
7	.1121	.1260	.1408	.1568	.1738	.1919	.2110	.2312	.2523	.2744
8	.0343	.0399	.0461	.0532	.0610	.0696	.0791	.0895	.1009	.1133
9	.0072	.0087	.0104	.0125	.0148	.0175	.0206	.0241	.0282	.0327
10	.0009	.0012	.0014	.0018	.0022	.0027	.0033	.0040	.0049	.0059
11	.0001	.0001	.0001	.0001	.0002	.0002	.0002	.0003	.0004	.0005

n = 12

	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
1	.1136	.2153	.3062	.3873	.4596	.5241	.5814	.6323	.6775	.7176
2	.0062	.0231	.0486	.0809	.1184	.1595	.2033	.2487	.2948	.3410
3	.0002	.0015	.0048	.0107	.0196	.0316	.0468	.0652	.0866	.1109
4	.0000	.0001	.0003	.0010	.0022	.0043	.0075	.0120	.0180	.0256
5	.0000	.0000	.0000	.0001	.0002	.0004	.0009	.0016	.0027	.0043
6	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0003	.0005
7	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	.7530	.7843	.8120	.8383	.8578	.8766	.8931	.9076	.9202	.9313
2	.3867	.4314	.4648	.5166	.5565	.5945	.6304	.6641	.6957	.7251
3	.1377	.1667	.1977	.2303	.2642	.2990	.3344	.3702	.4060	.4417
4	.0351	.0464	.0597	.0750	.0922	.1114	.1324	.1552	.1795	.2054
5	.0065	.0095	.0133	.0181	.0239	.0310	.0393	.0489	.0600	.0726
6	.0009	.0014	.0022	.0033	.0046	.0065	.0088	.0116	.0151	.0194
7	.0001	.0002	.0003	.0004	.0007	.0010	.0015	.0021	.0029	.0039
8	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0003	.0004	.0006
9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

	.31	.32	.33	.34	.35	.36	.37	.38	.39	40
1	.9884	.9902	.9918	.9932	.9943	.9953	.9961	.9968	.9973	.9978
2	.9256	.9350	.9435	.9509	.9676	.9634	.9685	.9730	.9770	.9894
3	.7704	.7922	.8124	.8313	.8487	.8648	.9795	.8931	.9054	.9166
4	.5381	.5681	.5973	.6258	.6533	.6799	.7053	.7296	.7528	.7747
5	.3032	.3308	.3590	.3876	.4167	.4459	.4751	.5043	.5332	.5618
6	.1343	.1521	.1711	.1913	.2127	.2352	.2588	.2833	.3087	.3348
7	.0458	.0540	.0632	.0734	.0846	.0970	.1201	.1253	.1411	.1582
8	.0118	.0144	.0176	.0213	.0255	.0304	.0359	.0422	.0493	.0573
9	.0022	.0028	.0036	.0045	.0056	.0070	.0086	.0104	.0127	.0153
10	.0003	*.0001*	.0005	.0007	.0008	.0011	.0014	.0018	.0022	.0028
11	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0002	.0002	.0003

	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50
1	.9982	.9986	.9988	.9990	.9992	.9994	.9995	.9996	.9997	.9998
2	.9834	.9860	.9882	.9901	.9917	.9931	.9943	.9953	.9961	.9968
3	.9267	.9358	.9440	.9513	.9579	.9637	.9688	.9733	.9773	.9807
4	.7953	.8147	.8329	.8498	.8655	.8801	.8934	.9057	.9168	.9270
5	.5899	.6175	.6443	.6704	.6956	.7198	.7430	.7652	.7862	.8062
6	.3616	.3889	.4167	.4448	.4731	.5014	.5297	.5577	.5855	.6128

7	.1765	.1959	.2164	.2380	.2607	.2843	.3089	.3343	.3604	.3872
8	.0662	.0760	.0869	.0988	.1117	.1258	.1411	.1575	.1751	.1938
9	.0183	.0218	.0258	.0304	.0356	.0415	.0481	.0555	.0638	.0730
10	.0035	.0043	.0053	.0065	.0079	.0095	.0114	.0137	.0163	.0193
11	.0004	.0005	.0007	.0009	.0011	.0014	.0017	.0021	.0026	.0032
12	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0002	.0002

n = 13

	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
1	.1225	.2310	.3270	.4118	.4867	.5526	.6107	.6617	.7065	.7458
2	.0072	.0270	.0564	.0932	.1354	.1814	.2298	.2794	.3293	.3787
3	.0003	.0020	.0062	.0135	.0245	.0392	.0578	.0799	.1054	.1339
4	.0000	.0001	.0005	.0014	.0031	.0060	.0103	.0163	.0242	.0342
5	.0000	.0000	.0000	.0001	.0003	.0007	.0013	.0024	.0041	.0065
6	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0003	.0005	.0009
7	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001

	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	.7802	.8102	.8364	.8592	.8791	.8963	.9113	.9242	.9354	.9450
2	.4270	.4738	.5186	.5614	.6017	.6396	.6751	.7080	.7384	.7664
3	.1651	.1985	.2337	.2704	.3080	.3463	.3848	.4231	.4611	.4983
4	.0464	.0609	.0776	.0967	.1180	.1414	.1667	.1939	.2226	.2527
5	.0097	.0139	.0193	.0260	.0342	.0438	.0551	.0681	.0827	.0991
6	.0015	.0024	.0036	.0053	.0075	.0104	.0139	.0183	.0237	.0300
7	.0002	.0003	.0005	.0008	.0013	.0019	.0027	.0038	.0052	.0070
8	.0000	.0000	.0001	.0001	.0002	.0003	.0004	.0006	.0009	.0012
9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002

	.21	.22	.23	.24	.25	.26	.27	.28	.29	30
1	.9533	.9604	.9666	.9718	.9762	.9800	.9833	.9860	.9883	.9903
2	.7920	.8154	.8 67	.8559	.8733	.8889	.9029	.9154	.9265	.9363
3	.5347	.5699	.6039	.6364	.6674	.6968	.7245	.7505	.7749	.7975
4	.28 9	.3161	.3489	.3822	.4157	.4493	.4826	.5155	.5478	.5794
5	.1173	.1371	.1585	.1816	.2060	.2319	.2589	.2870	.3160	.3457
6	.0375	.0462	.0562	.0675	.0802	.0944	.1099	.1270	.1455	.1654
7	.0093	.0120	.0154	.0195	.0243	.0299	.0365	.0440	.0527	.0624
8	.0017	.0024	.0032	.0043	.0056	.0073	.0093	.0118	.0147	.0182
9	.0002	.0004	.0005	.0007	.0010	.0013	.0018	.0024	.0031	.0040
10	.0000	.0000	.0001	.0001	.0001	.0002	.0003	.0004	.0005	.0007
11	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001

	.31	.32	.33	.34	.35	.36	.37	.38	.39	40
1	.9920	.9934	.9945	.9955	.9963	.9970	.9975	.9980	.9984	.9987
2	.9450	.9527	.9594	.9653	.9704	.9749	.9787	.9821	.9849	.9874
3	.8185	.8379	.8557	.8720	.8868	.9003	.9125	.9235	.9333	.9421
4	.6101	.6398	.6683	.6957	.7217	.7464	.7698	.7917	.8123	.8314
5	.3760	.4067	.4376	.4686	.4995	.5301	.5603	.5899	.6188	.6470
6	.1867	.2093	.2331	.2581	.2841	.3111	.3388	.3673	.3962	.4256
7	.0733	.0854	.0988	.1135	.1295	.1468	.1654	.1853	.2065	.2288
8	.0223	.0271	.0326	.0390	.0462	.0544	.0635	.0738	.0851	.0977
9	.0052	.0065	.0082	.0102	.0126	.0154	.0187	.0225	.0270	.0321
10	.0009	.0012	.0015	.0020	.0025	.0032	.0040	.0051	.0063	.0078
11	.0001	.0001	.0002	.0003	.0003	.0005	.0006	.0008	.0010	.0013
12	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001

	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50
1	.9990	.9992	.9993	.9995	.9996	.9997	.9997	.9998	.9998	.9999
2	.9895	.9912	.9928	.9940	.9951	;9960	.9967	.9974	.9979	.9983
3	.9499	.9569	.9630	.9684	.9731	.9772	.9808	.9838	.9865	.9888
4	.8492	.8656	.8807	.8945	.9071	.9185	.9288	.9381	.9464	.9539
5	.6742	.7003	.7554	.7493	.7721	.7935	.8137	.8326	.8502	.8666
6	.4552	.4849	.5146	.5441	.5732	.6019	.6299	.6573	.6838	.7095
7	.2524	.2770	.3025	.3290	.3563	.3842	.4127	.4415	.4707	.5000
8	.1114	.1264	.1426	.1600	.1788	.1988	.2200	.2424	.2659	.2905
9	.0379	.0446	.0520	.0605	.0698	.0803	.0918	.1045	.1183	.1334
10	.0096	.0117	.0141	.0170	.0203	.0242	.0287	.0338	.0396	.0461
11	.0017	.0021	.0027	.0033	.0041	.0051	.0063	.0077	.0093	.0112
12	.0002	.0002	.0003	.0004	.0005	.0007	.0009	.0011	.0014	.0017
13	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001

n = 14

\CI	.01	.02	--	.04	.-	.06	.07	.08	.09	.10
1	.1313	.2464	.4372	.4353	.5123	.5795	.6380	.6888	.7330	.7712
2	.0003	.0025	.0077	.0167	.0301	.0478	.0698	.0958	.1255	.1584
3	.0000	.0001	.0006	.0019	.0042	.0080	.0136	.0214	.0315	.0441
4	.0000	.0000	.0000	.0002	.0004	.0010	.0020	.0035	.0059	.0092
5	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0004	.0008	.0015
6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002

	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	.8044	.8330	.8577	.8789	.8972	.9129	.9264	.9379	.9477	.9560
2	.4658	.5141	.5599	.6031	.6433	.6807	.1752	.7469	.7758	.8021
3	.1938	.2315	.2708	.3111	.3521	.3932	.4341	.4744	.5138	.5519
4	.0594	.0074	.0979	.1210	.1465	.1742	.2038	.2351	.2679	.3018
5	.0137	.0196	.0, 69	.0359	.0467	.0584	.0741	.0907	.1093	.1298
6	.0024	.0038	.0057	.0082	.0115	.0157	.0209	.0273	.0349	.0439
7	.0003	.0006	.0009	.0015	.0022	.0032	.0046	.0064	.0087	.0116
8	.0000	.0001	.0001	.0002	.0003	.0005	.0008	.0012	.0017	.0024
9	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0003	.0004

	.21	.22	.23	.24	.25	.26	.27	.28	.29	30
1	.9631	.9691	.9742	.9786	.9822	.9852	.9878	.9899	.9917	.9932
2	.8259	.8473	.8665	.8837	.8990	.9126	.9246	.9352	.9444	.9525
3	.5887	.6239	.6574	.6891	.7189	.7467	.7727	.7967	.8188	.8392
4	.3366	.3719	.4076	.4432	.4787	.5136	.5479	.5813	.6137	.6448
5	.1523	.1765	.2023	.2297	.2585	.2884	.3193	.3509	.3832	.4158
6	.0543	.0662	.0797	.0949	.1117	.1301	.1502	.1718	.1949	.2195
7	.0152	.0196	.0248	.0310	.0383	.0467	.0563	.0673	.0796	.0933
8	.0033	.0045	.0060	.0079	.0103	.0132	.0167	.0208	.0257	.0315
9	.0006	.0008	.0011	.0016	.0022	.0029	.0038	.0050	.0065	.0083
10	.0001	.0001	.0002	.0002	.0003	.0005	.0007	.0009	.0012	.0017
11	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0002

	.31	.32	.33	.34	.35	.36	.37	.38	.39	40
1	.9945	.9955	.9963	.9970	.9976	.9981	.9984	.9988	.9990	.9992
2	.9596	.9657	.9710	.9756	.9795	.9828	.9857	.9881	.9902	.9919
3	.8577	.8746	.8899	.9037	.9161	.9271	.9370	.9457	.9534	.9602
4	.6747	.7032	.7301	.7556	.7795	.8081	.8226	.8418	.8595	.8757
5	.4486	.4813	.5138	.5458	.5773	.6080	.6378	.6666	.6943	.7207

6	.2454	.2724	.3006	.3297	.3595	.3899	.4208	.4519	.4831	.5141
7	.1084	.1250	.1431	.1626	.1836	.2059	.2296	.2545	.2805	.3075
8	.0381	.0458	.0545	.0643	.0753	.0876	.1012	.1162	.1325	.1501
9	.0105	.0131	.0163	.0200	.0243	.0294	.0353	.0420	.0497	.0583
10	.0022	.0029	.0037	.0048	.0060	.0076	.0095	.0117	.0144	.0175
11	.0003	.0005	.0006	.0008	.0011	.0014	.0019	.0024	.0031	.0039
12	.0000	.0001	.0001	.0001	.0001	.0002	.0003	.0003	.0005	.0006
13	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50
1	.9994	.9995	.9996	.9997	.9998	.9998	.9999	.9999	.9999	.9999
2	.9934	.9946	.9956	.9964	.9971	.9977	.9981	.9985	.9988	.9991
3	.9661	.9713	.9758	.9797	.9830	.9858	.9883	.9903	.9921	.9935
4	.8905	.9039	.9161	.9270	.9368	.9455	.9532	.9601	.9661	.9713
5	.7459	.7697	.7922	.8132	.8328	.8510	.8678	.8833	.8974	.9102
6	.5450	.5754	.6052	.6344	.6627	.6900	.7163	.7415	.7654	.7880
7	.3355	.3643	.3937	.4236	.4539	.4843	.5148	.5451	.5751	.6047
8	.1692	.1896	.2113	.2344	.2586	.2840	.3105	.3380	.3663	.3953
9	.0680	.0789	.0910	.1043	.1189	.1348	.1520	.1707	.1906	.2120
10	.0212	.0255	.0304	.0361	.0426	.0500	.0583	.0677	.0782	.0898
11	.0049	.0061	.0076	.0093	.0114	.0139	.0168	.0202	.0241	.0287
12	.0008	.0010	.0013	.0017	.0022	.0027	.0034	.0042	.0053	.0065
13	.0001	.0001	.0001	.0002	.0003	.0003	.0004	.0006	.0007	.0009
14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

n = 15

	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
1	.1399	.2614	.3667	.4579	.5367	.6047	.6633	.7137	.7570	.7941
2	.0096	.0353	.0730	.1191	.1710	.2262	.2832	.3403	.3965	.4510
3	.0004	.0030	.Q.094	.0203	.0362	.0571	.0829	.1130	.1469	.1841
4	.0000	.0002	.0008	.0024	.0055	.0104	.0175	.0273	.0399	.0556
5	.0000	.0000	.0001	.0002	.0006	.0014	.0028	.0050	.0082	.0127
6	.0000	.0000	.0000	.0000	.0001	.0001	.0003	.0007	.0013	.0022
7	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0003

	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	.8259	.8530	.8762	.8959	.9126	.9269	.9389	.9490	.9576	.9648
2	.5031	.5524	.5987	.6417	.6814	.7179	.7511	.7813	.8085	.8329
3	.2238	.2654	.3084	.3520	.3958	.4392	.4619	.5234	.5635	.6020
4	.0742	.0959	.1204	.1476	.1773	.2092	.2429	.2782	.3146	.3518
5	.0187	.0265	.0361	.0478	.0617	.0778	.0961	.1167	.1394	.1642
6	.0037	.0057	.0084	.0121	.0168	.0227	.0300	.0387	.0490	.0611
7	.0006	.0010	.0015	.0024	.0036	.0052	.0074	.0102	.0137	.0181
8	.0001	.0001	.0002	.0004	.0006	.0010	.0014	.0021	.0030	.0042
9	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0031	.0005	.0008
10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001

	.21	.22	.23	.24	.25	.26	.27	.28	.29	30
1	.9709	.9759	.9802	.9837	.9866	.9891	.9911	.9928	.9941	.9953
2	.8547	.8741	.8913	.9065	.9198	.9315	.9417	.9505	.9581	.9647
3	.6385	.6731	.7055	.7358	.7639	.7899	.8137	.8355	.8553	.8732
4	.3895	.42?4	.4650	.5022	.5387	.5742	.6086	.6416	.6732	.7031
5	.1910	.2195	.2495	.2810	.3135	.3469	.3810	.4154	.4500	.4845
6	.0748	.0905	.1079	.1272	.1484	.1713	.1958	.2220	.2495	.2784
7	.0234	.0298	.0374	.0463	.0566	.0684	.0817	.0965	.1130	.1311

8	.0058	.0078	.0104	.0135	.0173	.0219	.0274	.0338	.0413	.0500
9	.0011	.0016	.0023	.0031	.0042	.0056	.0073	.0094	.0121	.0152
10	.0002	.0003	.0004	.0006	.0008	.0011	.0015	.0021	.0028	.0037
11	.0000	.0000	.0001	.0001	.0001	.0002	.0002	.0003	.0005	.0007
12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001

	.31	.32	.33	.34	.35	.36	.37	.38	.39	40
1	.9962	.9969	.9975	.9980	.9984	.9988	.9990	.9992	.9994	.9995
2	.9704	.9752	.9794	.9829	.9858	.9883	.9904	.9922	.9936	.9948
3	.8893	.9038	.9167	.9281	.9383	.9472	.9550	.9618	.9678	.9729
4	.7314	.7580	.7829	.8060	.8273	.8469	.8649	.8813	.8961	.9095
5	.5187	.5523	.5852	.6171	.6481	.6778	.7062	.7332	.7587	.7827
6	.3084	.3393	.3709	.4032	.4357	.4684	.5011	.5335	.5654	.5968
7	.1509	.1722	.1951	.2194	.2452	.2722	.3003	.3295	.3595	.3902
8	.0599	.0711	.0837	.0977	.1132	.1302	.1487	.1687	.1902	.2131
9	.0190	.0236	.0289	.0351	.0422	.0504	.0597	.0702	.0820	.0950
10	.0048	.0062	.0079	.0099	.0124	.0154	.0190	.0232	.0281	.0338
11	.0009	.0012	.0016	.0022	.0028	.0037	.0047	.0059	.0075·	.0093
12	.0001	.0002	.0003	.0004	.0005	.0006	.0009	.0001	.0015	.0019
13	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002	.0002	.0003

	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50
1	.9996	.9997	.9998	.9998	.9999	.9999	.9999	.9999	1.0000	1.0000
2	.9958	.9966	.9973	.9979	.9983	.9987	.9990	.9992	.9994	.9995
3	.9773	.9811	.9843	.9870	.9893	.9913	.9929	.9943	.9954	.9963
4	.9125	.9322	.9417	.9502	.9576	.9641	.9697	.9746	.9788	.9824
5	.8052	.8261	.8454	.8533	.8796	.8945	.9080	.9201	.9310	.9408
6	.6274	.6570	.6854	.7131	.7392	.7641	.7875	.8095	.8301	.8491
7	.4214	.4530	.4847	.5164	.5478	.5789	.6095	.6394	.6684	.6964
8	.2374	.2630	.2898	.3176	.3465	.3762	.4065	.4374	.4686	.5000
9	.1095	.1254	.1'427	.1615	.1818	.2034	.2265	.2510	.2767	.3036
10	.0404	.0479	.0565	.0661	.0769	.0890	.1024	.1171	.1333	.1509
11	.0116	.0143	.0174	.0211	.0255	.0305	.0363	.0430	.0506	.0592
12	.0025	.0032	.0040	.0051	.0063	.0079	.0097	.0119	.0145	.0176
13	.0004	.0005	.0007	.0009	.0011	.0015	.0018	.0023	.0029	.0037
14	.0000	.0000	.0001	.0001	.0001	.0002	.0002	.0003	.0004	.0005

n =16										
	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
1	.1485	.2762	.3857	.4796	.5599	.6284	.6869	.7366	.7789	.8147
2	.0109	.0399	.0818	.1327	.1892	.2489	.3098	.3701	.4289	.4853
3	.0005	.0037	.0113	.0242	.0429	.0673	.0969	.1311	.1694	.2108
4	.0000	.0002	.0011	.0032	.0070	.0132	.0221	.0342	.0496	.0684
5	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0111	.0170
6	.0000	.0000	.0000	.0000	.0001	.0002	.0005	.0010	.0019	.0033
7	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0003	.0005
8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	.8450	.8707	.8923	.9105	.9257	.9386	.9493	.9582	.9657	.9719
2	.5386	.5885	.6347	.6773	.7161	.7513	.7830	.8115	.8368	.8593
3	.2545	.2999	.3461	.3926	.4386	.4838	.5277	.5698	.6101	.6482
4	.0907	.1162	.1448	.1763	.2101	.2460	.2836	.3223	.3619	.4019
5	.0248	.0348	.0471	.0618	.0791	.0988	.1211	.1458	.1727	.2018

6	.0053	.0082	.0120	.0171	.0235	.0315	.0412	.0527	.0662	.0817
7	.0009	.0015	.0024	.0038	.0056	.0080	.0112	.0153	.0204	.0267
8	.0001	.0002	.0004	.0007	.0011	.0016	.0024	.0036	.0051	.0070
9	.0000	.0000	.0001	.0001	.0002	.0003	.0004	.0007	.0010	.0015
10	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0002
	.21	.22	.23	.24	.25	.26	.27	.28	.29	30
1	.9770	.9812	.9847	.9876	.9900	.9919	.9935	.9948	.9958	.9967
2	.8791	.8965	.9117	.9250	.9365	.9465	.9950	.9623	.9686	.9739
u	.attln	.774ï3	.ïXA9u	.7ï78	.ovï3m	.8t3ñ-	.8·ïnñ1.	.8877	.8851	.nnvV';p
4	.4418	.4814	.5203	.5583	.5950	.6303	.6640	.6959	.7260	.7541'
5	.2327	.2652	.2991	.3341	.3698	.4060	.4425	.4788	.5147	.5501
6	.0992	.1188	.1405	.1641	.1897	.2169	.2458	.2761	.3077	.3402
7	.0342	.0432	.0536	.0658	.0796	.0951	.1125	.1317	.1526	.1753
8	.0095	.0127	.0166	.0214	.0271	.0340	.0420	.0514	.0621	.0744
9	.0021	.0030	.0041	.0056	.0075	.0098	.0127	.0163	.0206	.0257
10	.0004	.0006	.0008	.0012	.0016	.0023	.0031	.0041	.0055	.0071
11	.0001	.0001	.0001	.0002	.0003	.0004	.0006	.0008	.0011	.0016
12	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002'	.0003
	.31	.32	.33	.34	.35	.36	.37	.38	.39	40
1	.9974	.9979	.9984	.9987	.9990	.9992	.9994	.9995	.9996	.9997
2	.9784	.9822	.9854	.9880	.9902	.9921	.9936	.9948	.9959	.9967
3	.9144	.9266	.9374	.9467	.9549	9620	.9681	.9734	.9778	.9817
4	.7804	.8047	.8270	.8475	.8661	.8830	.8982	.9119	.9241	.9349
5	.5846	.6181	.6504	.6813	.7108	.7387	.7649	.7895	.8123	.8334
6	.3736	.4074	.4416	.4759	.5100	.5438	.5770	.6094	.6408	.6712
7	.1996	.2257	.2531	.2819	.3119	.3428	.3746	.4070	.4398	.4728
8	.0881	.1035	.1205	.1391	.1594	.1813	.2048	.2298	.2562	.2839
9	.0317	.0388	.0470	.0564	.0671	.0791	.0926	.1076	.1242	.1423
10	.0092	.0117	.0,148	.0185	.0229	.0280	.0341	.0411	.0491	.0583
11	.0021	.0028	.0037	.0048	.0062	.0079	.0100	.0125	.0155	.0191
12	.0004	.0005	.0007	.0010	.0013	.0017	.0023	.0030	.0038	.0049
13	.0001	.0001	.0001	.0001	.0002	.0003	.0004	.0005	.0007	.0009
14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001
	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50
1	.9998	.9998	.9999	.9999	.9999	.9999	1.0000	1.0000	1.0000	1.0000
2	.9974	.9979	.9984	.9987	.9990	.9992	.9994	.9995	.9997	.9997
3	.9894	.9876	.9899	.9918	.9934	.9947	.9958	.9966	.9973	.9979
4	.9444	.9527	.9600	.9664	.9719	.9766	.9806	.9840	.9869	.9894
5	.8529	.8707	.8869	.9015	.9147	.9265	.9370	.9463	.9544	.9616
6	.7003	.7280	.7543	.7792	.8024	.8241	.8441	.8626	.8795	.8949
7	.5058	.5387	.5711	.6029	.6340	.6641	.6932	.7210	.7476	.7728
8	.3128	.3428	.3736	.4051	.4371	.4694	.5019	.5343	.5665	.5982
9	.1619	.1832	.2060	.2302	.2559	.2829	.3111	.3405	.3707	.4018
10	.0687	.0805	.0936	.1081	.1241	.1416	.1607	.1814	.2036	.2272
11	.0234	.0284	.0342	.0409	.0486	.0574	.0674	.0786	.0911	.1051
12	.0062	.0078	.0098	.0121	.0149	.0183	.0222	.0268	.0322	.0384
13	.0012	.0016	.0021	.0027	.0035	.0044	.0055	.0069	.0086	.0201
14	.0002	.0002	.0003	.0004	.0006	.0007	.0010	.0013	.0016	.0021
15	.0000	.0000	.0000	.0000	.0001	.0001	.0000	.0001	.0002	.0003

	n =17									
	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
1	.1571	.2907	.4042	.5004	.5819	.6507	.7088	.7577	.7988	.8332
2	.0123	.0446	.0909	.1465	.2078	.2717	.3362	.3995	.4604	.5182
3	.0006	.0044	.0134	.0286	.0503	.0782	.1118	.1503	.1927	.2382
4	.0000	.0003	.0014	.0040	.0088	.0164	.0273	.0419	.0603	10826
5	.0000	.0000	.0001	.0004	.0012	.0026	.0051	.0090	.0145	.0221
6	.0000	.0000	.0000	.0000	.0001	.0003	.0007	.0015	.0027	.0047
7	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0004	.000S
8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	.8621	.8862	.9063	.9230	.9369	.9484	.9579	.9657	.9722	.9775
2	.5723	.6223	.6682	.7099	.7475	.7813	.8113	.8379	.8613	.8818
3	.2858	.3345	.3836	.4324	.4802	.5266	.5711	.6133	.6532	.6904
4	.087	.1383	.1710	.2065	.2444	.2841	.3251	.3669	.4091	.4511
5	.0321	.0446	.0598	.0778	.0987	.1224	.1487	.1775	.2087	.2418
6	.0075	.0114	.0166	.0234	.0319	.0423	.0548	.0695	.0864	.1057
7	.0014	.0023	.0037	.0056	.0083	.0118	.0163	.0220	.0291	.0377
8	.0002	.0004	.0007	.0011	.0017	.0027	.0039	.0057	.0080	.0109
9	.0000	.0001	.0001	.0002	.0003	.0005	.0008	.0012	.0018	.0026
10	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0003	.0005
11	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	.21	.22	.23	.24	.25	.26	.27	.28	.29	30
1	.818	.9854	.9882	.9906	.9925	.9940	.9953	.9962	.9970	.9977
2	.96	.9152	.9285	.9400	.9499	.9583	.9654	.9714	.9765	.9807
3	.49	.7567	.7859	.8123	.8363	.8578	.8771	.8942	.9093	.9226
4	.27	.5333	.5728	.6107	.6470	.6814	.7137	.7440	.7721	.7981
5	.66	.3128	.3,500	.3879	.4261	.4643	.5023	.5396	.5760	.6113
6	.1273	.1510	.1770	.2049	.2347	.2661	.2989	.3329	.3677	.4032
7	.0479	.0598	.0736	.0894	.1071	.1268	.1485	.1721	.1976	.2248
8	.0147	.0194	.0251	.0320	.0402	.0499	.0611	.0739	.0884	.1046
9	.0037	.0051	.0070	.0094	.0124	.0161	.0206	.0261	.0326	.0403
10	.0007	.0011	.0016	.0022	.0031	.0042	.0057	.0075	.0098	.0127
11	.0001	.0002	.0003	.0004	.0006	.0009	.0013	.0018	.0024	.0032
12	.0000	.0000	.0000	.0001	.0001	.0002	.0002	.0003	.0005	.0007
13	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001
	.31	.32	.33	.34	.35	.36	.37	.38	.39	40
1	.9982	.9986	.9989	.9991	.9993	.9995	.9996	.9997	.9998	.9998
2	.9843	.9872	.9896	.9917	.9933	.9946	.9957	.9966	.9973	.9979
3	.9343	.9444	.9532	.9608	.9673	.9728	.9775	.9815	.9849	.9877
4	.8219	.8437	.8634	.8812	.8972	.9115	.9241	.9353	.9450	.9536
5	.6453	.6778	.7087	.7378	.7652	.7906	.8142	.8360	.8559	.8740
6	.4390	.4749	.5105	.5458	.5803	.6139	.6465	.6778	.7077	.7361
7	.2536	.2838	.3153	.3479	.3812	.4152	.4495	.4839	.5182	.5522
8	.1227	.1426	.1642	.1877	.2128	.2395	.2676	.2971	.3278	.3595
9	.0492	.0595	.0712	.0845	.0094	.1159	.1341	.1541	.1757	.1989
10	.0162	.0204	.0254	.0314	.0383	.0464	.0557	.0064	.0784	.0919
11	.0043	.0057	.0074	.0095	.0120	.0151	.0189	.0234	.0286	.0348
12	.0009	.0013	.0017	.0023	.0030	.0040	.0051	.0066	.0084	.0201
13	.0002	.0002	.0003	.0004	.0006	.0008	.0011	.0015	.0019	.0025
14	.0000	.0000	.0000	.0001	.0001	.0001	.0002	.0002	.0003	.0005

15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50
1	.9999	.9999	.9999	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	.9984	.9987	.9990	.9992	.9994	.9996	.9997	.9998	.9998	.9999
3	.9900	.9920	.9935	.9948	.9959	.9968	.9975	.9980	.9985	.9988
4	.9610	.9674	.9729	.9776	.9816	.9849	.9877	.9901	.9920	.9936
5	.8904	.9051	.9183	.9301	.9404	.9495	.9575	.9644	.9704	,9755
6	.7628	.7879	.8113	.8330	.8529	.8712	.8878	.9028	.9162	.9283
7	.5856	.6182	6499	.6805	.7098	.7377	.7641	.7890	.8122	.8338
8	.3920	.4250	.4585	.4921	.5257	.5590	.5918	.6239	.6552	.6855
9	.2238	.2502	.2780	.3072	.3374	.3687	.4008	.4335	.4667	.5000
10	.1070	.1236	.1419	.1618	.1834	.2066	.2314	.2577	.2855	.3145
11	.0420	.0503	.0597	.0705	.0826	.0962	.1112	.1279	.1462	.1662
12	.0133	.0165	.0203	.0248	.0301	.0363	.0434	.0517	.0611	.0717
13	.0033	.0042	.0054	.0069	.0086	.0108	.0134	.0165	.0202	.0245
14	.0006	.0008	.0011	.0014	.0019	.0024	.0031	.0040	.0050	.0064
15	.0001	.0001	.0002	.0002	.0003	.0004	.0005	.0007	.0009	.0012
16	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001

n =18

	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
1	.1655	.3049	.4220	.5204	.6028	.6717	7292	.7771	.8169	.8499
2	.0138	.0495	.1003	.1607	.2265	.2945	.3622	.4281	.4909	.5497
3	.0007	.0052	.0157	.0333	.0581	.0898	.1275	.1702	.2168	.2662
4	.0000	.0004	.0018	.0050	.0109	.0201	.0333	.0506	.0723	.0982
5	.0000	.0000	.0002	.0006	.0015	.0034	.0067	.0116	.0187	.0282
6	.0000	.0000	.0000	.0001	.0002	.0005	.0010	.0021	.0038	.0064
7	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0006	.0012
8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002
	.11	.12	.1.3	.14	.15	.16	.17	.18	.19	.20
1	.8773	.8998	.9185	.9338	.9464	.9566	.9651	.9719	.9775	.9820
2	.6042	.6540	.6992	.7398	.7759	.8080	.8362	.8609	.8824	.9009
3	.3173	.3690	.4206	.4713	.5203	.5673	.6119	.6538	.6927	.7287
4	.1282	.1618	.1986	.2382	.2798	.3229	.3669	.4112	.4554	.4990
5	.0405	.0558	.0743	.0959	.1206	.1482	.1787	.2116	.2467	.2836
6	.0102	.0154	.0222	.0310	.0419	.0551	.0708	.0889	.1097	.1329
7	.0021	.0034	.0054	.0081	.0118	.0167	.0229	.0306	.0400	.0513
8	.0003	.0006	.0011	.0017	.0027	.0041	.0060	.0086	.0120	.0163
9	.0000	.0001	.0002	.0003	.0005	.0008	.0013	.0020	.0030	.0043
10	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0004	.0006	.0009
11	.0000	.000<f	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002
	.21	.22	.23	.24	.25	.26	.27	.28	.29	30
1	.9856	.9886	.9909	.9928	.9944	.9956	.9965	.9973	.9979	.9984
2	.9169	.9306	.9423	.9522	.9605	.9676	.9735	.9784	.9824	.9858
3	.7616	.7916	.8187	.8430	.8647	.8839	.9009	.9158	.9288	.9400
4	.5414	.5825	.6218	.6591	.6943	.7272	.7578	.7860	.8119	.8354
5	.3220	.3613	.4012	.4414	.4813	.5208	.5594	.5968	.6329	.6673
6	.1586	.1866	.2168	.2488	.2825	.3176	.3538	.3907	.4281	.4656
7	.0645	.0799	.0974	.1171	.1390	.1630	.1891	.2171	.2469	.2783

8	.0217	.0283	.0363	.0458	.0569	.0699	.0847	.1014	.1200	.1407
9	.0060	.0083	.0112	.0148	.0193	.0249	.0316	.0395	.0488	.0596
10	.0014	.0020	.0028	.0039	.0054	.0073	.0097	.0127	.0164	.0210
11	.0003	.0004	.0006	.0009	.0012	.0018	.0025	.0034	.0046	.0061
12	.0000	.0001	.0001	.0002	.0002	.0003	.0005	.0007	.0010	.0014
13	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002	.0003

	.31	.32	.33	.34	.35	.36	.37	.38	.39	40
1	.9987	.9990	.9993	.9994	.9996	.9997	.9998	.9998	.9999	.9999
2	.9886	.9908	.9927	.9942	.9954	.9964	.9972	.9978	.9983	.9987
3	.9498	.9581	.9652	.9713	.9764	.9807	.9843	.9873	.9897	.9918
4	.8568	.8759	.8931	.9083	.9217	.9335	.9439	.9528	.9606	.9672
5	.7001	.7309	.7598	.7866	.8114	.8341	.8549	.8737	.8907	.9058
6	.5029	.5398	.5759	.6111	.6450	.6776	.7086	.7379	.7655	.7912
7	.3111	.3450	.3797	.4151	.4509	.4867	.5224	.5576	.5921	.6257
8	.1633	.1878	.2141	.2421	.2717	.3027	.3349	.3681	.4021	.4366
9	.0720	.0861	.1019	.1196	.1391	.1604	.1935	.2084	.2350	.2632
10	.0264	.0329	.0405	.0494	.0597	.0714	.0847	.0997	.1163	.1347
11	.0080	.0104	.0133	.0169	.0212	.0264	.0325	.0397	.0480	.0576
12	.0020	.0027	.0036	.0047	.0062	.0080	.0102	.0130	.0163	.0203
13	.0004	.0005	.0008	.0011	.0014	.0019	.0026	.0034	.0044	.0058
14	.0001	.0001	.0001	.0002	.0003	.0004	.0005	.0007	.0010	.0013
15	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002	.0002

	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50
1	.9999	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	.9990	.9992	.9994	.9996	.9997	.9998	.9998	.9999	.9999	.9999
3	.9934	.9948	.9959	.9968	.9975	.9981	.9985	.9989	.9991	.9993
4	.9729	.9777	.9818	.9852	.9880	.9904	.9923	.9939	.9952	.9962
5	.9193	.9313	.9418	.9510	.9589	.9658	.9717	.9767	.9810	.9846
6	.8151	.8372	.8573	.8757	.8923	.9072	.9205	.9324	.9428	.9519
7	.6582	.6895	.7193	.7476	.7742	.7991	.8222	.8436	.8632	.8811
8	.4713	.5062	.5408	.5750	.6085	.6412	.6728	.7032	.7322	.7597
9	.2928	.3236	.3556	.3885	.4222	.4562	.4906	.5249	.5591	.5927
10	.1549	.1786	.2004	.2258	.2527	.2812	.3110	.3421	.3742	.4073
11	.0686	.0811	.0951	.1107	.1280	.1470	.1677	.1902	.2144	.2403
12	.0250	.0307	.0372	.0449	.0537	.0638	.0753	.0883	.1028	.1189
13	.0074	.0094	.0118	.0147	.0183	.0225	.0275	.0334	.0402	.0481
14	.0017	.0022	.0029	.0038	.0049	.0063	.0079	.01000	.0125	.0154
15	.0003	.0004	.0006	.0007	.0010	.0013	.0017	.0023	.0029	.0038
16	.0000	.0001	.0001	.0001	.0001	.0002	.0003	.0004	.0005	.0007
17	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001

n = 19

	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
1	.1738	.3188	.4394	.5396	.6229	.6914	.7481	.7949	.8334	.8649
2	.0153	.0546	.1100	.1751	.2453	.3171	.3879	.4560	.5202	.5797
3	.0009	.0061	.0183	.0384	.0665	.1021	.1439	.1908	.2415	.2946
4	.0000	.0005	.0022	.0061	.0132	.0243	.0398	.0602	.0853	.1150
5	.0000	.0000	.0002	.0007	.0020	.0044	.0085	.0147	.0235	.0352
6	.0000	.0000	.0000	.0001	.0002	.0006	.0014	.0029	.0051	.0086
7	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0004	.0009	.0017
8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0003

	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	.8908	.9119	.9291	.9431	.9544	.9636	.9710	.9770	.9818	.9856
2	.6342	.6835	.7277	.7669	.8015	.8318	.8581	.8809	.9004	.9171
3	.3488	.4032	.4568	.5089	.5587	.6059	.6500	.6910	.7287	.7631
4	.1490	.1867	.2275	.2708	.3159	.3620	.4085	.4549	.5005	.5449
5	.0502	.0685	.0904	.1158	.1444	.1762	.2107	.2476	.2864	.3267
6	.0135	.0202	.0290	.0401	.0537	.0700	.0891	.1110	.1357	.1631
7	.0030	.0048	.0076	.0113	.0163	.0228	.0310	.0411	.0532	.0676
8	.0005	.0009	.0016	.0026	.0042	.0061	.0089	.0126	.0173	.0233
9	.0001	.0002	.0003	.0005	.0008	.0014	.0021	.0032	.0047	.0067
10	.0000	.0000	.0000	.0001	.0001	.0002	.0004	.0007	.0010	.0016
11	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0003
12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

	.21	.22	.23	.24	.25	.26	.27	.28	.29	30
1	.9887	.9911	.9930	.9946	.9958	.9967	.9975	.9981	.9985	.9989
2	.9313	.9434	.9535	.9619	.9690	.9749	.9797	.9837	.9869	.9896
3	.7942	.8222	.8471	.8692	.8887	.9057	.9205	.9333	.9443	.9538
4	.5877	.6285	.6671	.7032	.7369	.7680	.7965	.8224	.8458	.8668
5	.3681	.4100	.4520	.4936	.5346	.5744	.6129	.6498	.6848	.7178
6	.1929	.2251	.2592	.2950	.3322	.3705	.4093	.4484	.4875	.5261
7	.0843	.1034	.1249	.1487	.1749	.2032	.2336	.2657	.2995	.3345
8	.0307	.0396	.0503	.0629	.0775	.0941	.1129	.1338	.1568	.1820
9	.0093	.0127	.0169	.0222	.0287	.0366	.0459	.0568	.0694	.0839
10	.0023	.0034	.0047	.0066	.0089	.0119	.0156	.0202	.0258	.0326
11	.0005	.0007	.0011	.0016	.0023	.0032	.0044	.0060	.0080	.0105
12	.0001	.0001	.0002	.0003	.0005	.0007	.0010	.0015	.0021	.0028
13	.0000	.0000	.0000	.0001	.0001	.0001	.0002	.0003	.0004	.0006
14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

	.31	.32	.S3	.34	.35	.36	.37	.38	.39	40
1	.9991	.9993	.9995	.9996	.9997	.9998	.9998	.9999	.9999	.9999
2	.9917	.9935	.9949	.9960	.9969	.9976	.9981	.9986	.9989	.9992
3	.9618	.9686	.9743	.9791	.9830	.9863	.9890	.9913	.9931	.9945
4	.8856	.9022	.9169	.9297	.9409	.9505	.9588	.9659	.9719	.9770
5	.7486	.7773	.8037	.8280	.8500	.8699	.8878	.9038	.9179	.9304
6	.5641	.6010	.6366	.6707	.7032	.7339	.7627	.7895	.8143	.8371
7	.3705	.4073	.4445	.4818	.5188	.5554	.5913	.6261	.6597	.6919
8	.2091	.2381	.2688	.3010	.3344	.3690	.4043	.4401	.4762	.5122
9	.1003	.1186	.1389	.1612	.1855	.2116	.2395	.2691	.3002	.3325
10	.0405	.0499	.0608	.0733	.0875	.1035	.1213	.1410	.1626	.1861
11	.0137	.0176	.0223	.0280	.0347	.0426	.0518	.0625	.0747	.0885
12	.0038	.0051	.0068	.0089	.0114	.0146	.0185	.0231	.0287	.0352
13	.0009	.0012	.0017	.0023	.0031	.0041	.0054	.0070	.0091	.0116
14	.0002	.00m ,·	.0003	.0005	.0007	.0009	.0013	.0017	.0023	.0031
15	.0000	.0000	.0001	.0001	.0001	.0002	.0002	.0003	.0005	.0006
16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001

	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	.9994	.9995	.9996	.9997	.9998	.9999	.9999	.9999	.9999	1.0000
3	.9957	.9967	.9974	.9980	.9985	.9988	.9991	.9993	.9995	.9996
4	.9813	.9849	.9878	.9903	.9923	.9939	.9952	.9963	.9971	.9978
5	.9413	.9518	.9590	.9660	.9720	.9771	.9814	.9850	.9879	.9904

6	.8579	.8767	.8937	.9088	.9223	.9342	.9446	.9537	.9615	.9682
7	.7226	.7515	.7787	.8039	.8273	.8488	.8684	.8862	.9022	.9165
8	.5480	.5832	.6176	.6509	.6831	.7138	.7430	.7706	.7964	.8204
9	.3660	[1].4003	.4353	.4706	.5060	.5413	.5762	.6105	.6439	.6762
10	.2114	.2385	.2672	.2974	.3290	.3617	.3954	.4299	.4648	.5000
11	.1040	.1213	.1404	.1613	.1841	.2087	.2351	.2631	.2928	.3238
12	.0429	.0518	.0621	.0738	.0871	.1021	.1187	.1372	.1575	.1796
13	.0146	.0183	.0227	.0280	.0342	.0415	.0500	.0597	.0709	.0835
14	.0040	.0052	.0068	.0086	.0109	.0137	.0171	.0212	.0261	.0318
15	.0009	.0012	.0016	.0021	.0028	.0036	.0046	.0060	.0076	.0096
16	.0001	.0002	.0003	.0004	.0005	.0007	.0010	.0013	.0017	.0022
17	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002	.0003	.0004

n = 20

	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
1	.1821	.3324	.4562	.5580	.6415	.7099	.7658	.8113	.8484	.8784
2	.0169	.0599	.1198	.1897	.2642	.3395	.4131	.4831	.5484	.6083
3	.0010	.0071	.0210	.0439	.0755	.1150	.1610	.2121	.2666	.3231
4	.0000	.0006	.0027	.0074	.0159	.0290	.0471	.0706	.0993	.1330
5	.0000	.0000	.0003	.0010	.0026	.0056	.0107	.0183	.0290	.0432
6	.0000	.0000	.0000	.0001	.0003	.0009	.0019	.0038	.0068	.0113
7	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0006	.0013	.0024
8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0004
9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	.9028	.9224	.9383	.9510	.9612	.9659	.9759	.9811	.9852	.9885
2	.6624	.7109	.7539	.7916	.8244	.8529	.8773	.8982	.9159	.9308
3	.3802	.4369	.4920	.5450	.5951	.6420	.6854	.7252	.7614	.7939
4	.1710	.2127	.2973	.3041	.3523	.4010	.4496	.4974	.5439	.5886
5	.0610	.0827	.1083	.1375	.1702	.2059	.2443	.2849	.3271	.3704
6	.0175	.0260	.0370	.0507	.0673	.0870	.1098	.1356	.1643	.1958
7	.0041	.0067	.0103	.0153	.0219	.0304	.0409	.0537	.0689	.0867
8	.0008	.0014	.0024	.0038	.0059	.0088	.0127	.0177	.0241	.0321
9	.0001	.0002	.0005	.0008	.0013	.0021	.0033	.0049	.0071	.0100
10	.0000	.0000	.0001	.0001	.0002	.0004	.0007	.0011	.0017	.0026
11	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0004	.0006
12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001

	.21	.22	.23	.24	.25	.26	.27	.28	.29	30
1	.9910	.9931	.9946	.9959	.9968	.9976	.9982	.9986	.9989	.9992
2	.9434	.9539	.9626	.9698	.9757	.9805	.9845	.9877	.9903	.9924
3	.8230	.8488	.8716	.8915	;9087	.9237	.9365	.9474	.9567	.9645
4	.6310	.6711	.7085	.7431	.7748	.8038	.8300	.8534	.8744	.8929
5	.4142	.4580,·	.5014	.5439	.5852	.6248	.6625	.6981	.7315	.7625
6	.2297	.2657	.3035	.3427	.3828	.4235	.4643	.5048	.5447	.5836
7	.1071	.1301	.1558	.1838	.2142	.2467	.2810	.3169	.3540	.3920
8	.0419	.0536	.0675	.0835	.1018	.1225	.1455	.1707	.1982	.2277
9	.0138	.0186	.0246	.0320	.0409	.0515	.0640	.0784	.0948	.1133
10	.0038	.0054	.0075	.0103	.0139	.0183	.0238	.0305	.0385	.0480
11	.0009	.0013	.0019	.0028	.0039	.0055	.0074	.0100	.0132	.0171
12	.0003	.0003	.0004	.0006	.0009	.0014	.0019	.0027	.0038	.0051
13	.0000	.0000	.0001	.0001	.0002	.0003	.0004	.0006	.0009	.0013
14	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0003

	.31	.32	.33	.34	.35	.36	.37	.38	.39	40
1	.9994	.9996	.9997	.9998	.9998	.9999	.9999	.9999	.9999	1.0000
2	.9940	.9953	.9964	.9972	.9979	.9984	.9988	.9991	.9993	.9995
3	.9711	.9765	.9811	.9848	.9879	.9904	.9924	.9940	.9953	.9964
4	.9092	.9235	.9358	.9465	.9556	.9634	.9700	.9755	.9802	.9840
5	.7911	.8173	.8411	.8626	.8818	.8989	.9141	.9274	.9390	.9490
6	.6213	.6574	.6918	.7242	.7546	.7829	.8090	.8329	.8547	.8744
7	ꞁuꞁⱴ	ꞁuꞁⱴ	.5079	.5460	.5834	.6197	.6547	.6882	.7200	.7599
8	.2591	.2922	.3268	.3624	.3990	.4361	.4735	.5108	.5478	5841
9	.1340	.1568	.1818	.2087	.2376	.2683	.3005	.3341	.3688	.4044
10	.0591	.0719	.0866	.1032	.1218	.1424	.1650	.1897	.2163	.2447
11	.0220	.0279	.0350	.0434	.0532	.00645	.0775	.0923	.1090	.1275
12	.0069	.0091	.0119	.0154	.0196	.0247	.0308	.0381	.0466	.0565
13	.0018	.0025	.0034	.0045	.0060	.0079	.0102	.0132	.0167	.0210
14	.0004	.0006	.0008	.0011	.0015	.0021	.0028	.0037	.0049	.0065
15	.0001	.0001	.0001	.0002	.0003	.0004	.0006	.0009	.0012	.0016
16	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002	.0002	.0003

|	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	.9996	.9997	.9998	.9998	.9999	.9999	.9999	1.0000	1.0000	1.0000
3	.9972	.9979	.9984	.9988	.9991	.9993	.9995	.9996	.9997	.9998
4	.9872	.9898	.9920	.9937	.9951	.9962	.9971	.9977	.9983	.9987
5	.9577	.9651	.9714	.9767	.9811	.9848	.9879	.9904	.9924	.9941
6	.8921	.9078	.9217	.9340	.9447	.9539	.9619	.9687	.9745	.9793
7	.7780	.8041	.8281	.8501	.8701	.8881	.9042	.9186	.9312	.9423
8	.6196	.6539	.6868	.7183	.7480	.7759	.8020	.8261	.8482	.8684
9	.4406	.4771	.5136	.5499	.5847	.6207	.6546	.6873	.7186	.7483
10	.2748	.3064	.3394	.3736	.4086	.4443	.4804	.5166	.5525	.5881
11	.1480	.1705	.1949	.2212	.2493	.2791	.3104	.3432	.3771	.4119
12	.067	.0810	.0958	.1123	.1308	.1511	.1734	.1977	.2238	.2517
13	.0262	.0324	.0397	.0482	.0580	.0694	.0823	.0969	.1133	.1316
14	.0084	.0107	.0136	.0172	.0214	.0265	.0326	.0397	.0480	.0577
15	.0022	.0029	.0038	.0050	.0064	.0083	.0105	.0133	.0166	.0207
16	.0004	.0006	.0008	.0011	.0015	.0020	.0027	.0035	.0046	.0059
17	.0001	.0001	.0001	.0002	.0003	.0004	.0005	.0007	.0010	.0013
18	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0002

n =25

	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
1	.2222	.3965	.5330	.6396	.7226	.7871	.8370	.8756	.9054	.9282
2	.0258	.0886	.1720	.2642	.3576	.4473	.5304	.6053	.6714	.7288
3	.0020	.0132.	.0380	.0765	.1271	.1871	.2534	.3232	.3937	.4629
4	.0001	.0014'	.0062	.0165	.0341	.0598	.0936	.1315	.1831	.2364
5	.0000	.0001	.0008	.0028	.0072	.0150	.0274	.0451	.0686	.0980
6	.0000	.0000	.0001	.0004	.0012	.0031	.0065	.0123	.0210	.0334
7	.0000	.0000	.0000	.0000	.0002	.0005	.0013	.0028	.0054	.0095
a	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0005	.0011	.0023
9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0005
10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20
1	.9457	.9591	.9692	.9770	.9828	.9872	.9905	.9930	.9948	.9962
2	.7779	.8195	.8543	.8832	.9069	.9263	.9420	.9546	.9646	.9726

3	.5291	.5912	.6483	.7000	.7463	.7870	.8226	.8533	.8796	.9018
4	.2934	.3525	.4123	.4714	.5289	.5837	.6352	.6829	.7266	.7660
5	.1331	.1734	.2183	.2664	.3179	.3707	.4241	.4772	.5292	.5793
6	.0499	.0709	.0965	.1268	.1615	.2002	.2425	.2875	.3347	.3833
7	.0156	.0243	.0359	.0509	.0695	.0920	.1185	.1488	.1827	.2200
8	.0041	.0070	.0113	.0173	.0255	.0361	.0495	.0661	.0859	.1091
9	.0009	.0017	.0030	.0050	.0080	.0121	.0178	.0252	.0348	.0468
10	.0002	.0004	.0007	.0013	.0021	.0035	.0055	.0083	.0122	.0173
11	.0000	.0001	.0001	.0003	.0005	.0009	.0015	.0024	.0037	.0056
12	.0000	.0000	.0000	.0000	.0001	.0002	.0003	.0006	.0010	.0015
13	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0004
14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

	.21	.22	.23	.24	.25	.26	.27	.28	.29	30
1	.9972	.9980	.9985	.9990	.9992	.9995	.9996	.9997	.9998	.9999
2	.9789	.9838	.9877	.9907	.9930	.9947	.9961	.9971	.9979	.9984
3	.9204	.9360	.9488	.9593	.9679	.9748	.9804	.9848	.9883	.9910
4	.8013	.8324	.8597	.8834	.9038	.9211	.9358	.9481	.9583	.9668
5	.6270	.6718	.7134	.7516	.7863	.8174	.8452	.8696	.8910	.9095
6	.4325	.4816	.5299	.5767	.6217	.6644	.7044	.7415	.7755	.8065
7	.2601	.3027	.3471	.3927	.4389	.4851	.5308	.5753	.6183	.6593
8	.1358	.1658	.1939	.2349	.2735	.3142	.3565	.3999	.4440	.4882
9	.0614	.0788	.0993	.1228	.1494	.1790	.2115	.2465	.2838	.3231
10	.0240	.0325	.0431	.0560	.0713	.0893	.1101	.1338	.1602	.1894
11	.0082	.0117	.0163	.0222	.0297	.0389	.0502	.0636	.0795	.0978
12	.0024	.0036	.0053	.0076	.0107	.0148	.0199	.0264	.0345	.0442
13	.0006	.0010	.0015	.0023	.0034	.0049	.0069	.0096	.0130	.0175
14	.0001	.0002	.0004	.0006	.0009	.0014	.0021	.0030	.0043	.0060
15	.0000	.0000	.0001	.0001	.0002	.0003	.0005	.0008	.0012	.0018
16	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0003	.0005
17	:0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001

	.31	.32	.33	.34	.35	.36	.37	.38	.39	40
1	.9999	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	.99989	.9992	.9994	.9996	.9997	.9998	.9998	.9999	.9999	.9999
3	.9932	.9949	.9961	.9971	.9979	.9984	.9989	.9992	.9994	.9996
4	.9737	.9793	.9838	.9874	.9903	.9926	.9944	.9958	.9968	.9976
5	.9254	.9390	.9504	.9600	.9680	.9745	.9799	.9842	.9877	.9905
6	.8344	.8593'''	.8813	.9006	.9174	.9318	.9441	.9546	.9633	.9706
7	.6981	.7343	.7679	.7987	.8266	.8517	.8742	.8940	.9114	.9264
8	.5319	.5747	.6163	.6561	.6939	.7295	.7626	.7932	.8211	.8464
9	.3639	.4057	.4482	.4908	.5332	.5748	.6152	.6542	.6914	.7265
10	.2213	.2555	.2919	.3300	.3697	.4104	.4517	.4933	.5347	.5754
11	.1188	.1424	.1689	.1975	.2288	.2624	.2981	.3355	.3743	.4142
12	.0560	.0698	.0859	.1044	.1254	.1490	.1751	.2036	.2346	.2677
13	.0230	.0299	.0383	.0485	.0604	.0745	.0907	.1093	.1303	.1538
14	.0083	.0112	.0149	.0196	.0255	.0326	.0412	.0515	.0637	.0778
15	.0026	.0036	.0050	.0069	.0093	.0124	.0163	.0212	.0271	.0344

16	.0007	.0010	.0015	.0021	.0029	.0041	.0056	.0075	.0100	.0132
17	.0002	.0002	.0004	.0005	.0008	.0011	.0016	.0023	.0032	.0043
18	.0000	.0000	.0001	.0001	.0002	.0003	.0004	.0006	.0008	.0012
19	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.00001	.0002	.0003
20	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	.9997	.9998	.9998	.9999	.9999	1.0000	1.0000	1.0000	1.0000	1.0000
4	.9983	.9987	.9991	.9993	.9995	.9997	.9998	.9998	.9999	.9999
5	.9927	.9945	.9958	.9969	.9977	.9983	.9988	.9991	.9994	.9995
6	.9767	.9816	.9856	.9888	.9914	.9934	.9950	.9963	.9972	.9980
7	.9394	.9505	.9599	.9677	.9742	.9796	.9840	.9876	.9904	.9927
8	.8692	.8894	.9071	.9227	.9361	.9477	.9575	.9658	.9727	.9784
9	.7593	.7897	.8177	.8431	.8660	.8865	.9046	.9205	.9343	.9461
10	.6151	.6535	.6902	.7250	.7576	.7880	.8160	.8415	.8648	.8852
11	.4548	.4956	.5363	.5765	.6157	.6538	.6902	.7249	.7574	.7878
12	.3029	.3397	.3780	.4174	.4574	.4978	.5382	.5780	.6171	.6550
13	.1797	.2080	.2387	.2715	.3063	.3429	.3808	.4199	.4598	.5000
14	.0941	.1127	.1336	.1569	.1827	.2109	.2413	.2740	.3086	.3450
15	.0431	.0535	.0656	.0797	.0960	.1145	.1353	.1585	.1841	.2122
16	.0171	.0220	.0280	.0353	.0440	.0543	.0663	.0803	.0964	.1148
17	.0058	.0078	.0103	.0134	.0174	.0222	.0281	.0352	.0438	.0539
18	.0017	.0023	.0032	.0044	.0058	.0077	.0102	.0132	.0170	.0216
19	.0004	.0006	.0008	.0012	.0016	.0023	.0031	.0041	.0055	.0073
20	.0001	.0001	.0002	.0003	.0004	.0005	.0008	.0011	.0015	.0020
21	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0002	.0003	.0005
22	.0000	.0000	.0()00	.0000	.0000	.0000	.0000	.0000	.0001	.0001

SOURCE: Daniel & W.W & T Terrel J.C (1979) Business Statistics Basic Concepts And Methodology. 2nd Ed. Houghton Miftljn Co. Boston Pg 600 - 630

TABLED Cumulative Poisson Distribution.

$p(X \geq x \mid A.)$

1000 times the Probability of x or more Occurrences of Event That Has Average Number of Occurrences Equal to A.

$P(x \geq 2 \mid A = 1.00) = 0.264$
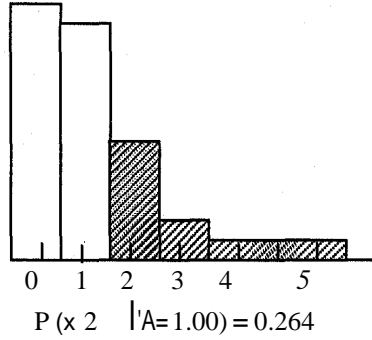
	.02	.04	.06	.08	.10	.15	.20	.25
0	1000	1000	1000	1000	1000	1000	1000	1000
1	020	039	058	077	095	139	181	221
2		001	002	003	005	010	018	026
3						001	001	002

	.30	.35	.40	.45	.50	.55	.60	.65
0	1000	1000	1000	1000	1000	1000	1000	1000
1	259	295	330	362	393	423	451	478
2	037	049	062	075	090	201	122	139
3	004	006	008	011	014	018	023	028
4			001	001	002	002	003	004
5								001

	.70	.75	.80	.85	.90	.95	1.0	1.1
0	1000	1000	1000	1000	1000	1000	1000	1000
1	503	528	551	573	593	613	632	667
2	156	173	191	209	228	246	264	301
3	034	041	047	055	063	071	080	100
4	006	007	009	011	013	016	019	026
5	001	001	001	002	002	003	004	005
6							001	001

	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
0	1000	1000	1000	1000	1000	1000	1000	1000
1	699	727	753	777	798	817	835	850
2	337	373	408	442	475	507	537	566
3	121	143	167	191	217	243	269	296
4	034	043	054	066	079	093	109	125
5	.008	011	014	019	024	030	036	044
6	002	002	003	004	006	008	010	013
7			001	001	001	002	003	003

8							001	001
	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4
0	1000	1000	1000	1000	1000	1000	1000	1000
1	865	889	909	926	939	950	959	967
2	594	645	692	733	769	801	829	858
3	323	377	430	482	531	577	620	660
4	143	181	221	264	308	353	397	442
5	053	072	096	123	152	185	219	256
6	017	025	036	049	065	084	105	129
7	005	007	012	017	024	034	045	058
8	001	002	003	005	008	013	015	

	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0
0	1000	1000	1000	1000	1000	1000	1000	1000
1	973	978	982	985	988	990	992	993
2	874	893	908	922	934	944	952	960
3	*69i*	*i31*	762	790	815	837	857	875
4	485	527	567	605	641	674	706	735
5	294	332	371	410	449	487	524	560
6	156	184	215	247	280	314	349	384
7	073	091	111	133	156	182	209	238
8	031	040	051	064	079	095	113	133
9	012	016	021	028	036	045	056	068
0	004	006	008	011	015	020	025	032
1	001	002	003	004	006	008	010	014
2	001	001	001	001	002	003	004	005
3					001	001	001	002
4								001

	5.2	5.4	5.6	5.8	6.0	6.2	6.4	6.6
0	1000	1000	1000	1000	1000	1000	1000	1000
1	999	995	996	997	998	998	998	999
2	966	971	976	979	983	985	988	990
3	891	905	918	928	938	946	954	960
4	762	787	809	830	849	866	881	895
5	594	627	658	687	715	741	765	787
6	419	454	488	522	554	586	616	649
7	268	298	330	362	394	426	458	489
8	155	178	203	229	256	284	313	342
9	082	097	114	133	153	174	197	220
0	040	049	059	071	084	098	114	131
1	018	023	028	035	043	051	061	073
2	007	010	012	016	020	025	031	037
3	003	,004	005	007	009	011	014	018
4	001	001	002	003	004	005	006	008
5			001	001	001	002	003	003
6					001	001	001	001
7								001

	6.8	7.0	7.2	7.4	7.6	7.8	8.0	8.5
0	1000	1000	1000	1000	1000			
1	999	999	999	999	999	1000	1000	1000
2	991	993	994	995	996	996	997	998
3	966	970	975	978	981	984	986	991

4	907	918	928	937	945	952	958	970
5	808	827	844	860	875	888	900	926
6	673	699	742	747	769	790	809	850
7	520	550	580	608	635	662	687	744
8	372	401	431	461	490	519	547	614
9	245	271	291	324	352	380	407	477
0	150	170	190	212	235	259	283	347
1	085	099	113	129	146	165	184	237
2	045	053	063	074	085	098	112	151
3	022	027	033	039	046	055	064	091
4	010	013	016	020	024	029	034	051
5	004	006	007	009	011	014	017	027

	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5
1	1000	1000						
2	999	999	1000	1000	1000	1000	1000	
3	994	996	997	998	999	999	999	1000
4	979	985	990	993	995	997	998	998
5	945	960	971	979	985	989	992	995
6	884	911	933	950	962	972	980	985
7	793	835	870	898	921	940	954	965
8	676	731	780	821	857	886	910	930
9	544	608	667	721	768	809	845	875
10	413	478	542	603	659	711	758	799
11	294	355	417	479	540	598	653	703
12	197	248	303	361	421	480	538	594
13	124	164	208	258	311	367	424	481
14	074	102	136	175	219	267	318	372
15	041	060	083	112	146	185	228	275
16	022	033	049	068	093	122	156	194
17	011	018	027	040	056	076	101	131
18	005	009	014	022	032	046	063	084
19	002	004	007	012	018	026	037	052
20	001	002	003	006	009	014	021	031
21		001	002	003	005	008	012	017
22			001	001	002	004	006	009
23				001	001	002	003	005
24						001	001	002
25							001	001
								001

	13.0	13.5	14.0	14.5	15	16	17	18
3	1000	1000						
4	999	999	1000	1000	1000			
5	996	,997	998	999	999	1000	1000	
6	989	992	994	996	997	999	999	1000
7	974	981	986	990	992	996	998	999
8	946	959	968	976	982	990	995	997
9	900	921	938	952	963	978	987	993
10	834	865	891	912	930	957	974	985
11	748	789	824	855	882	923	951	970
12	647	696	740	780	815	873	915	945
13	537	591	642	689	732	807	865	908
14	427	482	536	587	637	725	799	857
15	325	377	430	482	534	632	719	792

16	236	282	331	381	432	533	629	713
17	165	202	244	289	336	434	532	625
18	110	139	173	210	251	341	436	531
19	070	092	117	147	181	258	345	438
20	043	058	077	099	125	188	264	349
21	025	035	048	064	083	132	195	269
22	014	020	029	040	053	089	139	201
23	008	011	017	024	033	058	095	145
24	004	005	009	014	019	037	063	101
25	002	003	005	008	011	022	041	068
26	001	002	003	004	006	013	025	045
27		001	001	002	003	007	015	028
28			001	001	002	004	009	017
29				001	001	002	005	010

|	19	20	21	22	23	24	25
6	**1000**						
7	999	1000					
8	998	999	1000	1000			
9	996	998	999	999	1000		
10	991	995	997	998	999	1000	1000
11	982	989	994	996	998	999	999
12	965	979	987	992	996	997	999
13	939	961	975	985	991	995	997
14	902	934	957	972	983	989	994
15	850	895	928	923	969	980	988
16	785	843	889	883	948	966	978
17	708	779	837	831	918	944	962
18	622	703	773	768	877	913	940
19	531	619'	698	694	825	872	908
20	439	530	616	613	762	820	866
21	353	441	529	528	690	757	815
22	275	356	440	444	611	686	753
23	207	279	360	363	528	608	682
24	151	213	284	288	445	527	606
25	107	157	218	223	365	446	527
26	073	112	162	168	292	368	447
27	049	078	117	123	228	296	371
28	031	052	083	087	173	232	300
29	020	034	056	060	127	177	237
30	012	022	037	041	092	132	182
31	007	013	024	027	064	096	137
32	004	008	'015	017	044	068	100
33	002	005	009	011	029	047	071
34	001	\' 003	006	006	019	031	050
35	001	001	003	004	012	021	034
36		001	002	002	007	013	022
37			001	001	004	008	015
38			001	001	003	005	009
39					001	003	006
40					001	002	003
41						001	002
42						001	001
43							001

TABLE:E   Normal Curve Areas
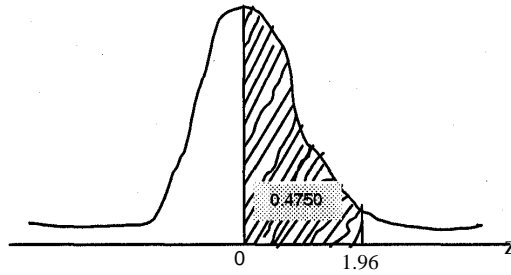
Entries in the body of the table give area under the standard normal curve from 0 to z.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.D160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.2014	.1480	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1844	.1517
0.4	.1554	**.1591**	**.1626**	**.1664**	.1700	.1736	.1772	.ana	.1100	**.1679**
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2517	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2823	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.3201	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3365	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3599	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3810	.3621"
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3997	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.4162	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4306'	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4429	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4535	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4625	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4699	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4761	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4812	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.49&6	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4988	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Appendix 1

:I"ABLE H   Percentiles of the Chi- Square Distribution.
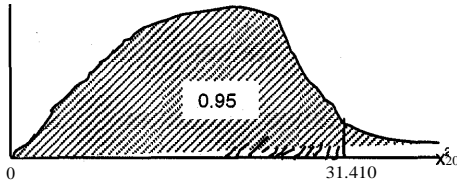
f	>to.oos	;?o.o2s	;?o.os	;?o.9o	;?o.9s	;?o.975	>to.99	;?o.995
1	0.0000393	0.0000982	0.00393	2.706	3.841	5.024	6.635	7.879
2	0.0100	0.0506	0.103	4.605	5.991	7.378	9.210	10.597
3	0.0717	0.216	0.352	6.251	7.815	9.348	11.345	12.838
4	0.207	0.484	0.711	7.779	9.488	11.143	13.277	14.860
5	0.412	0.831	1.145	9.236	11.070	12.832	15.086	16.750
6	0.676	1.237	1.635	10.645	12.592	14.449	16.812	18.548
7	0.989	1.690	2.167	12.017	14.067	16.013	18.475	20.278
8	1.344	2.180	2.733	13.362	15.507	17.535	20.090	21.955
9	1.735	2.700	3.325	14.684	16.919	19.023	21.666	23.589
10	2.158	3.247	3.940	15.987	18.307	20.483	23.209	25.188
11	2.603	3.816	4.575	17.275	19.675	21.920	24.725	26.757
12	3.074	4.404	5.226	18.549	21.026	23.336	26.217	28.300
13	**3.565**	**5.009**	**5.892**	꭯ouꞁ'	ꞮꞮꞮꞮꞮꞮꞮꞮꞮꞮ1	ꞌ..ꞏ44.ꞯ"1	**27.688**	nꞮꞮꞮꞮ.æꞯ11ꞯ ::::ꞮꞏꞯꞏꞮꞮꞯ
14	4.075	5.629	6.571	21.064	23.685	26.119	29.141	31.319
15	4.601	6.262	7.261	22.307	24.996	27.488	30.578	32.801
16	5.142	6.908	7.962	23.542	26.296	28.845	32.000	34.267
17	5.697	7.564	8.672	24.769	27.587	30.191	33.409	35.718
18	6.265	8.231	9.390	25.989	28.869	31.526	34.805	37.156
19	6.844	8.907	10.117	27.204	30.144	32.852	36.191	38.582
20	7.434	9.591	10.851	28.412	31.410	34.170	37.566	39.997
21	8.034	10.283	11.591	29.615	32.671	35.479	38.932	41.401
22	8.643	10.982	12.338	30.813	33.924	36.781	40.289	42.796
23	9.260	11.688	13.091	32.007	35.172	38.076	41.638	44.181
24	9.886	12.401	13.848	33.196	36.415	39.364	42.980	45.558
25	10.520	13.120	14.611	34.382	37.652	40.646	44.314	46.928
26	11.160	13.844	15.379	35.563	38.885	41.923	45.642	48.290
27	11.808	14.573	16.151	36.741	40.113	43.194	46.963	49.645
28	12.461	15.308	16.928	3.7916	41.337	44.461	48.278	50.993
29	13.121	16.047	17.708	39.087	42.557	45.722	49.588	52.336
30	13.787	1,6.791	18.493	40.256	43.773	46.979	50.892	53.672
35	17.192	20.569	22.465	46.059	49.802	53.203	57.342	60.275
40	20.707	24.433	26.509	51.805	55.758	59.342	63.691	66.766
45	24.311	28.366	30.612	57.505	61.656	65.410	69.957	73.166
50	27.991	32.357	34.764	63.167	67.505	71.420	76.154	79.490
60	35.535	40.482	43.188	74.397	79.082	83.298	88.379	91;952
70	43.275	48.758	51.739	85.527	90.531	95.023	100.425	104.215
80	51.172	57.153	60.391	96.578	101.879	201.629	112.329	116.321
90	59.196	65.647	69.126	107.565	113.145	118.136	124.116	1:2.8.299
100	67.328	74.222	77.929	118.498	124.342	129.561	135.807	140.169

TABLB F   Random Digits

```
85967   73152   14511   85285   36009   95892   36962   67835   63314   50162
07483   51453   11649   86348   76431   81594   95848   36738   25014   15460
96283   01898   61414   83525   04231   13604   75339   11730   85423   60698
49174   12074   98551   37895   93547   24769   09404   76548   05393   96770
97366   39941   21225   93629   19574   71565   33413   56087   40875   13351

90474   41469   16812   S1542   81652   45554   27931   93994   22375   00953
28599   64109   09497   76235   41383   31555   12639   00619   22909   29563
25254   16210   89717   65997   82667   74624   36348   44018   64732   93589
28785   02760   24359   99410   77319   73408   58993   61098   04393   48245
84725   86576   86944   93296   10081   82454   76810   52975   10324   15457

41059   66456   47679   66810   15941   84602   14493   65515   19251   41642
67434   41045   82830   47617   36932   46728   71183   36345   41404   81110
72766   68816   37643   19959   57550   49620   98480   25640   67257   18671
92079   46784   66125   94932   64451   29275   57669   66658   30818   58353
29187   40350   62533   73603   34075   16451   42885   03448   37390   96328

74220   17612   65522   80607   19184   64164   66962   82310   18163   63495
03786   02407   06098   92917   40434   60602   82175   04470   78754   90775
75085   55558   15520   27038   25471   76107   90832   10819   56797   33751
09161   33015   19155   11715   00551   24909   31894   37774   37953   78837
75707   48992   64998   87080   39333   00767   45637   12538   67439   94914

21333   48660   31288   00086   79889   75532   28704   62844   92337   99695
65626   50061   42539   14812   48895   11196   34335   60492   70650   51108
84380   07389   87891   76255   89604   41372   10837   66992   93183   56920
46479   32072   80083   63868   70930   89654   05359   47196   12452   38234
59847   97197   55147   76639   76971   55928   36441   95141   42333   67483

31416   11231   27904   57383   31852   69137   96667   14315   01007   31929
82066   83436   67914   21465   99605   83114   97885   74440   99622   87912
01850   42782   39202   18582   46214   99228   79541   78298   75404   63648
32315   89276   89582   87138   16165   15984   21466   63830   30475   74729
59388   42703   55198   80380   67067   97155   34160   85019   03527   78140

58089   27632   50987   91373   07736   20436   96130   73483   85332   24384
61705   57285   30392   23660   75841   21931   04295   00875   09114   32101
18914   98982   60199   99275   41967   35208   30357   76772   92658   62318
11965   94089   34803   48941   69709   16784   44642   89761   66864   62803
85251   48111   80936   81781   93248   67877   16498   31924   51315   79921

66121   96986   84844   93873   46352   92183   51152   85878   30490   15974
53972   96642   24199   58080   35450   03482   66953   49521   63719   57615
14509   16594   78883   43222   23093   58645   60257   89250   63266   90858
37700   07688   65533   72126   23611   93993   01848   03910   38552   17472
85466   59392   72722   15473   73295   49759   56157   60477   83284   56367

52969   55863   42312   76842   05673   91878   82738   36563   79540   61935
42744   68315   17514   02878   97291   74851   42725   57894   81434   62041
26140   13336   67726   61876   29971   99294   96664   52817   90039   53211
95589   56319   14563   24071   06916   59555   18195   32280   79357   04224
39113   13217   59999   49952   83021   47709   53105   19295   88318   41626

41392   17622   18994   98283   07249   52289   24209   91139   30715   06604
54684   53645   79246   70183   87731   19185   08541   33519   07223   97413
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