NATIONAL OPEN UNIVERSITY OF NIGERIA

INTRODUCTION TO SCIENCE: TPM 106

FACULTY OF MANAGEMENT SCIENCES

COURSE GUIDE

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CONTENT

Introduction

The course, Introduction to Science (TPM 106) is a first semester core course, which carries two credit units for first year level students in the Faculty of Management Sciences at the National Open University, Nigeria. This coursework will be useful in your academic pursuit by introducing you to science and its philosophies.

This course guide is built partially on prerequisite knowledge in elementary physics and chemistry. However, its simplicity will make the student assimilate faster. The practice questions at the end of each unit will also prepare the student for the examination purposes. The courseware suggests some general guidelines for the amount of time required of users on each unit in order to achieve the course aim and objectives successfully. In addition, it provides users with some guidance on their tutor marked assignments (TMAs) as contained herein.

Course Content

The course is made up of fourteen units (four modules) spread across eight (10) lecture hours, six (6) hours of laboratory practical demonstrations/experiments covering areas such as;

i Mechanics

ii Space and Time

iii Units and Dimensions

iv Vectors Kinematics and Newton’s Laws of motion

v Galilean Invariance

vi Static Dynamic of Particles

vii Universal gravitation

viii Thermal Physic

ix Thermal Properties including Elementary Thermodynamics and Kinetic Theory. Atomic Structure and the Periodic Classification of Elements

x Introduction to Gas Kinetic

xi Introduction to Nuclear Chemistry

xii Solid and Lattice Structure

xiii General Principles of Extraction of metal.

Course Aims and Objectives

This courseware and lecture note have been prepared with the primary aim of introducing Basic science to 100 Level engineering students as basic science preparatory. The course attempts to explain the concepts of philosophy of science. The objective is to impart training and help the students develop engineering thinking skill. This course also aims at inculcating respect for principles of physical work, and laboratory investigations in addition to some amount of value addition by being exposed to interdisciplinary science and engineering domains. In addition, the course is prepared in a way in which the users would easily enhance their previous knowledge of elementary physics and chemistry. The course further aim to develop critical thinking skills in students by learning how to carry out basic introductory aspects of basic science. It further focuses on steering up the students' ability to visualize and recognize units and dimension of objects, parts, models or systems.

However, the overall objectives of the course will be achieved by:

i. Introducing and familiarizing students with basic science of mechanics and kinematics.

ii. Development of force concepts, gravitation and Laws of motion.

iii. Discussing atomics structures, lattice structure and kinetics.

iv. Introducing students to Thermal Physics, Nuclear Chemistry and extraction of metals.

Working through the Course

To successfully complete this course, students/users are required to read the study units, references and other materials on the course.

Each unit contains self-assessment exercises called Student Assessment Exercises (SAE). At some points in the course, you will be required to submit assignments for assessment purposes. At the end of the course there is a final examination. This course should take about **10 weeks** to complete and some components of the course are outlined under the course material sub-section.

Course Material

The major component of the course and what you have to do and how you should allocate your time to each unit in order to complete the course successfully on time are listed as follows:

1. Course guide
2. Study unit
3. Textbooks
4. Assignment file
5. Presentation schedule

Study Unit

There are 14 units covered in 4 modules in this course, which should be studied carefully and diligently.

Module 1: Equilibrium of bodies, Units and Dimensions, Vector kinematics and Galilean invariance

Unit 1: Mechanics of Space and Time
Unit 2: Units and Dimensions
Unit 3: Vectors Kinematics and Newton’s Laws of Motion
Unit 4: Galilean Invariance

Module 2: Statics/Dynamics properties, Thermal Physics and Gravitation of Bodies

Unit 1: Static/Dynamic Properties of Particles
Unit 2: Kinetic Energy, Heat, Work and Behaviour of Gases
Unit 3: Gravitation of Bodies

Module 3: Thermal Properties, Atomic Structure and Gas Kinetics

Unit 1: Thermal Properties of Elements
Unit 2: Elementary Thermodynamics
Unit 3: Atomic Structure and the Periodic Classification of Elements
Unit 4: Introduction to Gas Kinetic

Module 4: Nuclear Chemistry, Lattice Structure and Extraction of Metals

Unit 1: Introduction to Nuclear Chemistry
Unit 2: Solids and Lattice Structures
Unit 3: General Principles of Extraction of Metal

References and Other Resources

Every unit contains a list of references and further reading guide. Try to get as many as possible of those textbooks and materials listed. The textbooks and materials are meant to deepen your knowledge and understanding of the course.

Assignment File

There are assignments on this course and you are expected to do all of them by following the schedule prescribed for them in terms of when to attempt them and submit same for grading by your tutor. The marks you obtain for these assignments will count toward the final mark you obtain for this course. Further information on assignments will be found in the Assignment File itself and later in this Course Guide in the section on Assessment.

There are five assignments in this course. The five course assignments will cover: Assignment 1 - All TMAs’ question in Units 1 - 4 of Module 1
Assignment 2 - All TMAs' question in Units 1 - 3 of Module 2
Assignment 3 - All TMAs' question in Units 1 - 4 of Module 3
Assignment 4 - All TMAs' question in Units 1 - 3 of Module 4
Assignment 5 - All TMAs' question in Units 1 - 4 of Module 5

Presentation Schedule

The presentation schedule included in your course materials gives you the important dates for this year for the completion of tutor-marking assignments and attending tutorials. Remember, you are required to submit all your assignments by due date. You should guide against falling behind the schedule.

Assessment

There are two types of assessment of the course. First are the tutor-marked assignments; second, there is a written examination.

In attempting the assignments, you are expected to apply information, knowledge and techniques gathered during the course. The assignments must be submitted to your tutor for formal assessment in accordance with the deadlines stated in the Presentation Schedule and the Assignments File. The work you submit to your tutor for assessment will count for 30 % of your total course mark.

At the end of the course, you will need to sit for a final written examination of three hours duration. This examination will also count for 70% of your total course mark.

Tutor-Marked Assignments (TMAs)

There are five tutor-marked assignments in this course. You will submit all the assignments. You are enjoined to work all the questions thoroughly. The TMAs constitute 30% of the total score.

Assignment questions for the units in this course are contained in the Assignment File. You will be able to complete your assignments from the information and materials contained in your text books, reading and study units. However, it is desirable that you demonstrate that you have read and researched more widely than the required minimum. You should use other references to have a broad viewpoint of the subject and also to give you a deeper understanding of the subject.

When you have completed each assignment, send it, together with a TMA form, to your tutor. Make sure that each assignment reaches your tutor on or before the deadline given in the Presentation File. If for any reason, you cannot complete your work on time, contact your tutor before the assignment is due to discuss the possibility of an extension. Extensions will not be granted after the due date unless there are exceptional circumstances.

Final Examination and Grading

The final examination will be of three hours' duration and have a value of 70% of the total course grade. The examination will consist of questions which reflect the types of self-assessment practice exercises and tutor-marked problems you have previously encountered. All areas of the course will be assessed

Use the time between finishing the last unit and sitting for the examination to revise the entire course material. You might find it useful to review your self-assessment exercises, tutor-marked assignments and comments on them before the examination. The final examination covers information from all parts of the course.

Course Marking Scheme

The table presented below indicate the total marks (100%) allocation.

Assessment	**Marks**
Assignment (Best three assignment out of the five marked)	30%
Final Examination	70%
Total	**100%**

How to Get the Most from This Course

In distance learning the study units replace the university lecturer. This is one of the great advantages of distance learning; you can read and work through specially designed study materials at your own pace and at a time and place that suit you best.

Think of it as reading the lecture instead of listening to a lecturer. In the same way that a lecturer might set you some reading to do, the study units tell you when to read your books or other material, and when to embark on discussion with your colleagues. Just as a lecturer might give you an in-class exercise, your study units provides exercises for you to do at appropriate points.

Each of the study units follows a common format. The first item is an introduction to the subject matter of the unit and how a particular unit is integrated with the other units and the course as a whole. Next is a set of learning objectives. These objectives let you know what you should be able to do by the time you have completed the unit.

You should use these objectives to guide your study. When you have finished the unit you must go back and check whether you have achieved the objectives. If you make a habit of doing this you will significantly improve your chances of passing the course and getting the best grade.

The main body of the unit guides you through the required reading from other sources. This will usually be either from your text books or from a readings section. Some units require you to undertake practical overview of historical events. You will be directed when you need to embark on discussion and guided through the tasks you must do.

The purpose of the practical overview of some certain historical economic issues are in twofold. First, it will enhance your understanding of the material in the unit. Second, it will give you practical experience and skills to evaluate economic arguments, and understand the roles of history in guiding current economic policies and debates outside your studies. In any event, most of the critical thinking skills you will develop during studying are applicable in normal working practice, so it is important that you encounter them during your studies.

Self-assessments are interspersed throughout the units, and answers are given at the ends of the units. Working through these tests will help you to achieve the objectives of the unit and prepare you for the assignments and the examination. You should do each self-assessment exercises as you come to it in the study unit. Also, ensure to master some major historical dates and events during the course of studying the material.

The following is a practical strategy for working through the course. If you run into any trouble, consult your tutor. Remember that your tutor's job is to help you. When you need help, don't hesitate to call and ask your tutor to provide it.

Read this Course Guide thoroughly.

- Organize a study schedule. Refer to the `Course overview' for more details. Note the time you are expected to spend on each unit and how the assignments relate to the units. Important information, e.g. details of your tutorials, and the date of the first day of the semester is available from study centre. You need to gather together all this information in one place, such as your dairy or a wall calendar. Whatever method you choose to use, you should decide on and write in your own dates for working breach unit.
- Once you have created your own study schedule, do everything you can to stick to it. The major reason that students fail is that they get behind with their course work. If you get into difficulties with your schedule, please let your tutor know before it is too late for help.
- Turn to Unit 1 and read the introduction and the objectives for the unit.
- Assemble the study materials. Information about what you need for a unit is given in the `Overview' at the beginning of each unit. You will also need both the study unit you are working on and one of your text books on your desk at the same time.

- Work through the unit. The content of the unit itself has been arranged to provide a sequence for you to follow. As you work through the unit you will be instructed to read sections from your text books or other articles. Use the unit to guide your reading.
- Up-to-date course information will be continuously delivered to you at the study centre.
- Work before the relevant due date (about 4 weeks before due dates), get the Assignment File for the next required assignment. Keep in mind that you will learn a lot by doing the assignments carefully. They have been designed to help you meet the objectives of the course and, therefore, will help you pass the exam. Submit all assignments no later than the due date.
- Review the objectives for each study unit to confirm that you have achieved them. If you feel unsure about any of the objectives, review the study material or consult your tutor.
- When you are confident that you have achieved a unit's objectives, you can then start on the next unit. Proceed unit by unit through the course and try to pace your study so that you keep yourself on schedule.
- When you have submitted an assignment to your tutor for marking do not wait for it return `before starting on the next units. Keep to your schedule. When the assignment is returned, pay particular attention to your tutor's comments, both on the tutor-marked assignment form and also written on the assignment. Consult your tutor as soon as possible if you have any questions or problems.
- After completing the last unit, review the course and prepare yourself for the final examination. Check that you have achieved the unit objectives (listed at the beginning of each unit) and the course objectives (listed in this Course Guide).

Tutors and Tutorials

There are some hours of tutorials (1-hour sessions) provided in support of this course. You will be notified of the dates, times and location of these tutorials. Together with the name and phone number of your tutor, as soon as you are allocated a tutorial group.

Your tutor will mark and comment on your assignments, keep a close watch on your progress and on any difficulties, you might encounter, and provide assistance to you during the course. You must mail your tutor-marked assignments to your tutor well before the due date (at least two working days are required). They will be marked by your tutor and returned to you as soon as possible.

Do not hesitate to contact your tutor by telephone, e-mail, or discussion board if you need help. The following might be circumstances in which you would find help necessary.

Contact your tutor if.

i You do not understand any part of the study units or the assigned readings
ii. You have difficulty with the self-assessment exercises
iii. You have a question or problem with an assignment, with your tutor's comments on an assignment or with the grading of an assignment.

You should try your best to attend the tutorials. This is the only chance to have face to face contact with your tutor and to ask questions which are answered instantly. You can raise any problem encountered in the course of your study. To gain the maximum benefit from course tutorials, prepare a question list before attending them. You will learn a lot from participating in discussions actively.

Summary

This course, Introduction to Science (TPM 106), exposes the students to Mechanics, Space and Time, Units and Dimensions, Vectors Kinematics Newton's Law; Galilean Invariance; Static Dynamic of Particles; Universal gravitation, Thermal Physic, Thermal Properties including Elementary Thermodynamics and Kinetic Theory. Atomic Structure and the Periodic Classification of Elements; introduction to Gas Kinetic; Introduction to Nuclear Chemistry, Solid and Lattice Structure. General Principles of Extraction of metal.

On successful completion of this course, the students would have been introduced to basic rudiments of Elementary science. You would have known the principles of structural mechanics and kinematics. Also, you would have developed crucial thinking skills to visualize, draft and analysed the motion and trajectory of objects which form part of engineered devices in a complex systems designed to solve the problem of mankind using basic mathematical tools. This courseware is written in very simple language so that even an average student can easily grasp the subject matter. With detailed illustrations throughout and simple, clear language, this is a practical introduction to what can be a very complex subject.

However, to benefit maximally from the course, please try to apply anything you learn in the course to the arrangement and assembly of engineering parts in the physical projects around you and other engineering courses. We wish you success in the course and hope that you will find this material both interestingly instructive and intuitively functional.

MODULE ONE

EQUILIBRIUM OF BODIES, UNITS AND DIMENSIONS, VECTOR KINEMATICS AND GALILEAN INVARIANCE

UNIT 1: EQUILIBRIUM OF BODIES AND KINEMATICS OF PARTICLES

CONTENTS

1.0 Introduction

This unit is to discuss the mechanics of bodies including static equilibrium of bodies and kinematics of particles. It deals with forces acting on a particle which does not move, i.e. is in equilibrium. The important concept is the resolution of forces to obtain the equations determining equilibrium. Motion in mechanics is understood as change of position of a mechanical body in space over time, relative to other bodies.

2.0 Objectives

At the end of this unit student should be able to

- Understand the equilibrium of bodies
- understand the mechanics of bodies in stable equilibrium
- Explain the kinematics of bodies in motion

3.0 Main Content

3.1 Static Equilibrium of Particles

Mechanics studies motion and equilibrium of physical bodies. Motion in its simplest form is, change of position of a body relative to other bodies. Newton in his Principia (first published in 1687) was the first to formulate the system of principles of mechanics, and although he had many great predecessors, such as Galileo (1564-1642), Huygens (1629-1695) and others, Newton is regarded as a founder of modern physics (and, as a matter of fact, a co-founder of differential-integrable calculus, which provides a natural mathematical language to express Newton's laws and their consequences).

For 200 years after Newton, in spite of the industrial revolution of the XIX century, principles of Newtonian mechanics \worked" in all areas of human endeavour and needed not to be revised. The revisions came only in the beginning of the XX century and concerned atomic length scales and speeds comparable to the speed of light, which before the end of the XIX century had been simply out of reach.

3.1.1 Statics - equilibrium of a particle

This lecture deals with forces acting on a particle which does not move, i.e. is in equilibrium. The important concept is the resolution of forces to obtain the equations determining equilibrium. It is essential when solving such problems to start with a good diagram showing all the forces.

The idea of limiting friction is introduced: this occurs when something is just on the point of slipping. The key concepts here include;

i Reduction of a number of forces to one resultant force by vector addition.
ii Condition for equilibrium: the resultant force is zero.
iii Resolution of forces in orthogonal directions to determine an unknown force.
iv Frictional force; limiting friction and relation via the coefficient of friction to the normal reaction.

a) Forces

We consider here the situation of a stationary particle acted on by a number of forces. It is not very useful to attempt to define exactly what we mean by a force: examples of forces will suffice. But we can think of a force as something that tends to produce motion. A force is therefore obviously a vector quantity.

There are (as far as is known at present) four fundamental forces: gravity, electromagnetism, weak nuclear force and strong nuclear force. Each force is accompanied by a theory and a set of equations governing the behaviour of the force and objects affected by the force.

All other forces are derived from these forces. Examples in no particular order are friction, tension in strings, normal reaction forces, air resistance, viscosity, magnetism, gravity, van der Waals forces between molecules, etc.

For example, for a particle on a rough horizontal table (Figure 1.1) being pulled by a string (though not hard enough to make the particle move), the forces are as shown in the diagram. There are two external forces, namely the applied (pulling) force acting along the string, and the weight acting downwards. The table exerts two forces on the particle: one is the force of friction which tends to oppose motion; the other is the reaction of the table on the particle that stops the particle falling through the table. This latter force is normal to the surface of the table and is called the normal reaction.

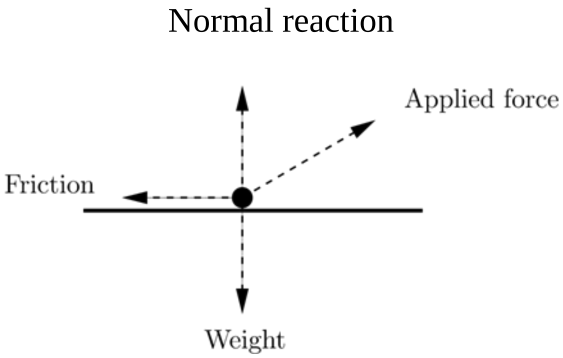

Figure 1.1: A body acted upon by forces

b) Equilibrium

A particle or body is said to be in equilibrium when all the forces acting on it balance and it is not in motion. Algebraically, this just means that the vector sum of the forces is zero:

$$\sum_i F_i = 0 \tag{1.1}$$

or, equivalently, the components of the vectors in three directions (which must be linearly independent, of course, but not necessarily orthogonal) sum to zero.

Geometrically, this means that the vectors representing the forces (in both direction and magnitude) can be joined to form a closed polygon in Figure 1.2.

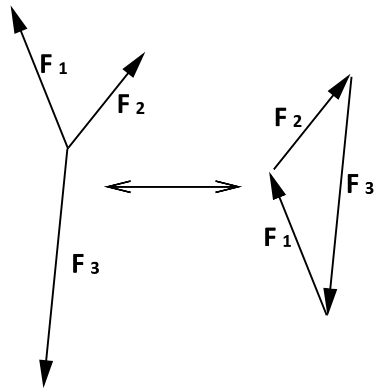

Figure 1.2: Force polygon

Example

A particle of weight, W lies on a fixed rough plane inclined at angle ® to the horizontal. It is held in position by a force of magnitude *T* acting up the line of greatest slope of the plane. Find the frictional force, *F*, in terms of *W*, *R* and *T*.

Solution: Before anything else, we must draw a good diagram showing all the forces. The importance of a diagram is seen immediately: as soon as we try to draw in the frictional force F we realise that we don't know which way it acts — up or down the plane.

As was stated earlier, the frictional force opposes the motion, so if, in the absence of friction, the force T is large enough to pull the particle up the plane, friction acts down the plane. If, in the absence of friction, the weight is enough to pull the particle down the plane, then friction acts up the plane. We assume first the former: friction acts down the plane Figure 1.3.

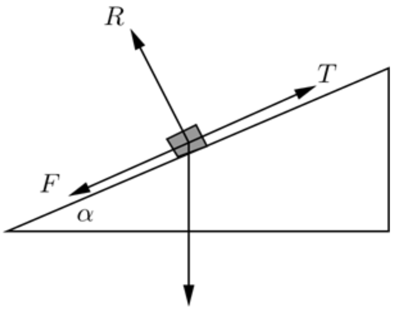

Figure 1.3: Forces acting on the body

The resolution is as follows;

$$T = F + W \sin\alpha \qquad (1.2)$$

$$R = W \cos\alpha \qquad (1.3)$$

$$F \leq \mu R \qquad (1.4)$$

$$T \leq W(\sin\alpha + \mu\cos\alpha) \qquad (1.5)$$

$$T \geq W(\sin\alpha - \mu\cos\alpha) \qquad (1.6)$$

For a given value of frictional coefficient,

$$W(\sin\alpha - \mu\cos\alpha) \leq T \leq W(\sin\alpha + \mu\cos\alpha) \qquad (1.7)$$

Self Assessment Exercise:

Explain the concept of static equilibrium of a body acted upon by forces

3.2 Equilibrium of a Solid Body

This sub-unit deals with forces acting on a body at rest. The difference between the particle of unit 1 and the body in this unit is that all the forces on the particle act through the same point, which is not the case for forces on an extended body. The important concept, again, is the resolution of forces to obtain the equations determining its equilibrium.

The simplest examples involve essentially one-dimensional bodies such as ladders. Again, it is essential start with a good diagram showing all the forces.

3.2.1 Resolving the Forces

The difference between forces acting on a particle and forces acting on an extended body is immediately obvious from the intuitive in equivalence of the two situations below: for an extended body, it matters through which points the forces act — i.e. on the position of the line of action of the force, Figure 1.4.

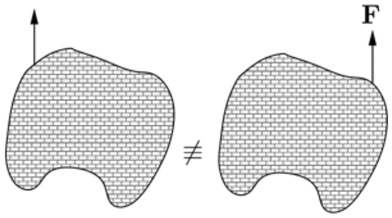

Figure 1.4: Equilibrium of solid

In general, each force acting on a body can be thought of as having two effects: a tendency to translate the body in the direction parallel to the line of action of the force; and a tendency to rotate the body. Clearly, for the body to be in equilibrium these effects must separately balance.

For the translational effects to balance, we need (as in the case of a particle) the vector sum of the forces to be zero:

$$\sum_i F_i = 0 \qquad (1.1)$$

i) Moment of a force

In two dimensions, or in three dimensions in the case of a planar body and forces acting in the same plane as the body, any force tends to rotate the body within the plane or, in other words, about an axis perpendicular to the plane. In this case, we define:

Moment of a force about a point P = magnitude of the force × the shortest distance between the line of action of the force and P, with account taken of the direction of the effect: either clockwise or anticlockwise Figure 1.5. i.e. moment of F about P is *F* x *d*

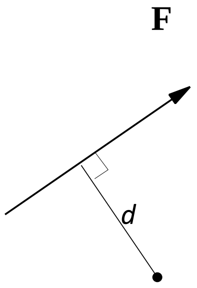

Figure 1.5: Moment of force about a point

where r is the position vector from P to any point on the line of action of F. Vector moment of the forces about any point is given by;

$$\equiv \sum_i r_i \times F_i = 0 \tag{1.8}$$

Self Assessment Exercise: Explain moment of a force about a point in a body

3.3 Kinematics of Particles

Kinematics studies motion of particles regardless of its cause. The venue for kinematics is space-time, i.e. an event, an instant of reality, is characterised by where and when it has occurred. The main object for kinematics is a particle - a point in space which has no size, and whose radius-vector r is a function of time t, or the point moves along its trajectory. Kinematics is concerned with the most basic question - how does one describe a trajectory.

Kinematics is the study of particle motion without reference to mass or force. In some ways, studying kinematics is rather artificial: in almost all realistic situations, the motion would have been produced by forces and the problem can only be solved by investigating the equations of motion appropriate to the forces acting. The study of motion produced by forces is called Dynamics. Note that we deal with particles, which, by definition, are point-like; they can have mass (though that is not needed in kinematics) but they have no internal structure, so they cannot, for example, spin. The example of projectile flight is important, historically and in terms of applications.

3.3.1 Projectiles

Projectiles are normally particles fired in the Earth's gravitational field. Properly, this should be treated as a problem in dynamics, since it involves forces but, since the gravitational field may be treated as uniform, the problem reduces to one of constant acceleration and the mass of the particle does not matter.

Example

A particle is projected from a point on a horizontal plane at speed u and at angle of projection α. Find the equation of the trajectory.

***First step*: Draw a diagram**

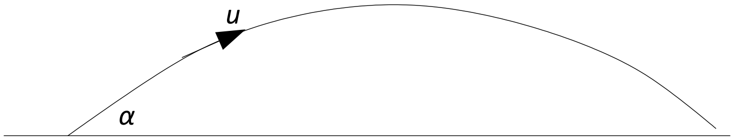

Figure 1.6: The projectile path

In most mechanics' problems, the first step is to draw a good diagram with all the information in it. This may just be a good way of keeping the data in front of you, but it may also give some important insight into the problem.

Equations of motion

The next thing to do is to write down the equations of motion, which are in this case just the mathematical expression of the fact that the acceleration the particle is given:

$$\frac{d^2 r}{dt^2} = a \qquad (1.9)$$

where a is the gravitational acceleration, which is constant in magnitude and direction.

Choice of axes

The next thing to do, as in most similar problems, is to choose suitable axes and coordinates. We choose Cartesian axes with origin at the point of projection. We are free to choose the orientation of the axes, so let the z axis be vertically upwards and let the x-axis be aligned with the initial velocity. With this choice, the initial velocity is (u cos α, 0, u sin α) and the acceleration a (due to gravity) is (0, 0, −g), where g is a positive constant.

Set out the problem in the chosen axes. We now write down the equations of motion in our chosen axes:

$$a \equiv (\ddot{x}, \ddot{y}, \ddot{z}) = (0,\ 0, -g) \qquad (1.10)$$

This gives us three differential equations; $\ddot{x} = 0, \ddot{y} = 0,\ \ddot{z} = -g$

Solve the equations with the given initial conditions. Integrating each one twice and using the initial conditions x = y = z = 0 and $\dot{x}$ = u cos α, $\dot{y}$ = 0, $\dot{z}$ = u cos α, at t = 0 gives

$$x = ut\cos\alpha,\ \ y = 0,\ \ z = -\frac{1}{2}gt^2 + ut\sin\alpha \qquad (1.11)$$

It can be seen by eliminating t, that, this is a parabola

$$t = \frac{x}{u\cos\alpha}, z = -\frac{g}{2u^2cos^2\alpha}x^2 + xtan\alpha \qquad (1.12)$$

$$x = \frac{1}{2}at^2 + ut + x_0 \qquad (1.13)$$

Equation (1.11) s the dynamical (or kinematic) equation of the trajectory, a parabola parametrised by time t. Equation (1.12) is the geometric equation: it describes a geometric object with no sense of motion. We could, of course, have just substituted the initial conditions u = (u cos α, 0, u sin α) and x_0 = 0 into the formula (1.13). It is probably better, though, to start from the equations of motion (1.13) rather than to quote elaborate formulae.

Self Assessment e-Exercise:

Explain the concept of kinematics in the study of particle motion.

4.0 Conclusion

We conclude that a body will remains at rest or in a state of stable equilibrium until a force pulls of pushes it out of equilibrium. Motion in its simplest form is, change of position of a body relative to other bodies as result of the action of internal or external forces. Differentiation of a vector is the same as differentiation of its Cartesian components. In all, we have studied the mechanics of particles and rigid bodies with constraints that require progressively more involved kinematics.

5.0 Summary

In this study unit, we attempt to introduce and familiarize students with the concept of forces, moment of forces as well as static equilibrium of bodies when subjected to forces. We also introduce the students to dynamics, as the study of motion produced by forces. Elementary analysis of forces by resolution using statics and differential calculus were also covered.

6.0 Tutor-Marked Assignment

a) Explain the concept of static equilibrium of a body acted upon by forces

b) Explain the moment of a force about a point in a body.

c) Explain the concept of kinematics in the study of particle motion.

7.0 References/Further Readings

Feynman, R.P., Leighton, R.B., and Matthew, S.M. (1963). The Feynman Lectures on Physics, Addison-Wesley, Reading, Massachusetts, p. 12-20.

Pratap, R., and Ruina, A. (2001). Statics and Dynamics, Oxford University Press.

Pratap, R., and Ruina, A. (2009). Introduction to Statics and Dynamics, Oxford University Press.

UNIT 2: UNITS AND DIMENSIONS

CONTENTS

1.0 Introduction

This unit will be looked at units and measurements of dimensions. The comparison of any physical quantity with its standard unit is called measurement. A definite amount of a physical quantity is taken as its standard unit. The standard unit as well as dimension of quantities to be measured, should be clearly marked, easily reproducible, unambiguous and internationally accepted.

2.0 Objective

At the end of this unit student should be able to

i Know the standard units of measurement

ii Understand the importance of proper units and scale of measurement.

iii Explain the standard fundamental units of dimensions in scientific measurements.

3.0 Main Content

3.1 Units of Measurement

3.1.1 Fundamental Units

Those physical quantities which are independent to each other are called fundamental quantities and their units are called fundamental units. The international system of units is the modern form of the metric system. It is the only system of measurement with an official status of international acceptance. It is sometimes referred to as the '*SI* units' of measurements. They are basically summarized in Table 1.1 below.

Table 1.1: SI units of measurement

S/No	Fundamental Quantity	Fundamental Unit	Symbol
1	Length	METRE	m
2	Mass	KILOGRAM	kg
3	Time	SECOND	S
4	Temperature	KELVIN	kg
5	Electric current	AMPERE	A
6	Luminous intensity	CANDELA	Cd
7	Amount of substance	MOLE	mol

3.1.2 Definition of fundamental SI units

The seven fundamental units of SI are defined as follows.

1. 1 ***kilogram*** – A cylindrical prototype mass made of platinum and iridium alloys of height

39 mm and diameter 39 mm. It is mass of 5.0188 x 10^{25} atoms of carbon-12.

2. 1 ***metre*** – 1 metre is the distance that contains 1650763.73 wavelength of orange-red light of Kr-86.

3. 1 ***second*** – 1 second is the time in which cesium atom vibrates 9192631770 times in an atomic clock.

4. 1 ***kelvin*** – 1 kelvin is the (1/273.16) part of the thermodynamics temperature of the triple point of water.

5. 1 ***candela*** –1 candela is (1/60) luminous intensity of an ideal source by an area of cm’ when source is at melting point of platinum (1760°C).

6. 1 ***ampere*** – 1 ampere is the electric current which it maintained in two straight parallel conductor of infinite length and of negligible cross-section area placed one metre apart in vacuum will produce between them a force 2 x 10^{-7} N per metre length.

7. 1 ***mole*** – 1 mole is the amount of substance of a system which contains a many elementary entities (may be atoms, molecules, ions, electrons or group of particles, as this and atoms in 0.012 kg of carbon isotope ${}_6C^{12}$.

3.1.3 Supplementary Fundamental Units

Radian and steradian are two supplementary fundamental units. It measures plane angle and solid angle respectively as shown in Table 1.2.

Table 1.2: Supplementary fundamental units

S/No	Suppl. Fundamental Quantity	Suppl. Fund. Unit	Symbol
1	Plane angle	RADIAN	rad
2	Solid angle	STERADIAN	Sr

3.1.4 Derived Units

Those physical quantities which are derived from fundamental quantities are called derived quantities and their units are called derived units. e.g., velocity, acceleration, force, work etc.

3.1.5 System of Units

A system of units is the complete set of units, both fundamental and derived, for all kinds of physical quantities. The common system of units which is used in mechanics are given below:

1. *CGS System* – In this system, the unit of length is centimetre, the unit of mass is gram and the unit of time is second.

2. *FPS System* – In this system, the unit of length is foot, the unit of mass is pound and the unit of time is second.

3. *MKS System* – In this system, the unit of length is metre, the unit of mass is kilogram and the unit of time is second.

4. *SI System* – This system contain seven fundamental units and two supplementary fundamental units.

Self Assessment Exercise: What do you understand by general workshop safety maintenance?

3.2 Dimensions of Measured Quantities

Dimensions of any physical quantity are those powers which are raised on fundamental units to express its unit. The expression which shows how and which of the base quantities represent the dimensions of a physical quantity, is called the dimensional formula.

3.2.1 Dimensional Formula of some physical quantities

The formula for the dimensions of some physical quantities are shown is Table 1.3 below

Table 1.3: Dimensional formula of some physical quantities

S/No	Physical Quantity	Dimensional formula	Unit
1	Area	$[L^2]$	m^2
2	Volume	$[L^3]$	m^3
3	Velocity	$[LT^{-1}]$	ms^{-1}
4	Acceleration	$[LT^{-2}]$	ms^{-2}
5	Force	$[MLT^{-2}]$	Newton (N)
6	Work or Energy	$[ML^2T^{-2}]$	joule (J)
7	Power	$[ML^2T^{-3}]$	J s^{-1} or watt
8	Pressure or Stress	$[ML^{-1}T^{-2}]$	Nm^{-2}
9	Linear momentum or Impulse	$[MLT^{-1}]$	kg ms^{-1}
10	Density	$[ML^{-3}]$	kg m^{-3}
11	Strain	Dimensionless	Unitless
12	Modulus of elasticity	$[ML^{-1}T^{-2}]$	Nm^{-2}
13	Surface tension	$[MT^{-2}]$	Nm^{-1}
14	Velocity gradient	T^{-1}	$second^{-1}$
15	Coefficient of velocity	$[ML^{-1}T^{-1}]$	kg $m^{-1}s^{-1}$
16	Gravitational constant	$[M^{-1}L^3T^{-2}]$	Nm^2/kg^2
17	Moment of inertia	$[ML^2]$	kg m^2
18	Angular velocity	$[T^{-1}]$	rad/s
19	Angular acceleration	$[T^{-2}]$	rad/S^2
20	Angular momentum	$[ML^2T^{-1}]$	kg m^2S^{-1}
21	Specific heat	$[L^2T^{-2}\theta^{-1}]$	kcal $kg^{-1}K^{-1}$
22	Latent heat	$[L^2T^{-2}]$	kcal/kg
23	Planck’s constant	$[ML^2T^{-1}]$	J^{-s}
24	Universal gas constant	$[ML^2T^{-2}\theta^{-1}]$	J/mol-K

3.2.2 Application of Dimension

1. To check the accuracy of physical equations.

2. To change a physical quantity from one system of units to another system of units.

3. To obtain a relation between different physical quantities.

Self Assessment Exercise: Discuss the application of dimension of quantities measured.

4.0 Conclusion

The unit established the importance of using clearly marked, easily reproducible, unambiguous and internationally accepted standard units and dimensions in all measured quantities. The SI units, which acceptance and application transcend geographical borders should be used in all measured quantities.

5.0 Summary

This unit explained the concept of a standard internationally accepted metric system of measurement, known as SI units. The seven fundamental units of SI metric system include; Length in metre, Mass in kilogram, time in seconds, temperature in kelvin, Electric current in ampere, Luminous intensity in candela and Amount of substance in mole. The unit also elucidated standard dimensions of any physical quantity as those powers which are raised on fundamental units to express its unit.

6.0 Tutor-Marked Assignment

a) Define the basic Fundamental SI units of measurement.
b) Discuss the application of dimension of quantities measured

7.0 References/Further Readings

Feynman, R.P., Leighton, R.B., and Matthew, S.M. (1963). The Feynman Lectures on Physics, Addison-Wesley, Reading, Massachusetts, p. 12-20.

Pratap, R., and Ruina, A. (2001). Statics and Dynamics, Oxford University Press.

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Press.

UNIT 3: VECTOR KINEMATICS AND NEWTON'S LAWS OF MOTION

CONTENTS

1.0 Introduction

Motion draws our attention. Motion itself can be beautiful, causing us to marvel at the forces needed to achieve spectacular motion, such as that of a dolphin jumping out of the water, or a pole vaulter, or the flight of a bird, or the orbit of a satellite. The study of motion is kinematics, but kinematics only describes the way objects move - their velocity and their acceleration. Dynamics considers the forces that affect the motion of moving objects and systems. Newton's laws of motion are the foundation of dynamics. These laws provide an example of the breadth and simplicity of principles under which nature functions. They are also universal laws in that they apply to similar situations on Earth as well as in space.

2.0 Objective

At the end of this unit student should be able to

- Understand dynamics and vector kinematics of motion
- Understand Newton's laws of motion

- Apply Newton's laws of motion to solve problems involving a variety of forces.

3.0 Main Content

3.1 Vector Kinematics

Kinematics is the mathematical description of motion. The term is derived from the Greek word kinema, meaning movement. In order to quantify motion, a mathematical coordinate system, called a reference frame, is used to describe space and time. Once a reference frame has been chosen, we can introduce the physical concepts of position, velocity and acceleration in a mathematically precise manner. Figure 1.7 shows a Cartesian coordinate system in one dimension with unit vector i pointing in the direction of increasing x -coordinate. The displacement vector of an object over a time interval is the vector difference between the two position vectors as shown in Figure 1.8.

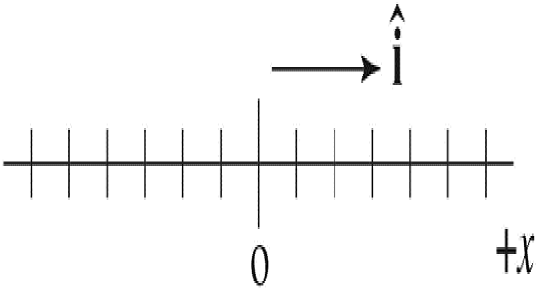

Figure 1.7: A one-dimensional Cartesian coordinate system

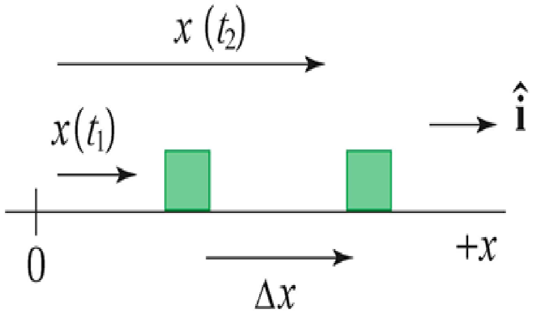

Figure 1.8: The displacement vector

We denote time by t, position by x, velocity by u or v and acceleration by a. Sometimes, displacement from the original position of the particle is denoted by s. We might write, for example, x(t) to emphasise that x is a function of time. Velocity, by definition, is rate of change of position, so

$$v = \frac{dx}{dt} \equiv \dot{x} \tag{1.14}$$

The overdot always denotes differentiation with respect to time. Acceleration, by definition, is rate of change of velocity, so

$$a = \frac{dv}{dt} \equiv \dot{v} = \frac{d^2y}{dx^2} \equiv \dot{x} \tag{1.15}$$

With respect to an origin and in standard Cartesian axes, we write

$$r = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{1.16}$$

or, to save space, just (x, y, z). Velocity, by definition, is rate of change of position, so

$$v = \frac{dr}{dt} \equiv \dot{r} \equiv \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} \tag{1.17}$$

This last equality (equivalence), obvious though it seems, actually needs proving. Is differentiating a vector the same as differentiating its components? The answer is yes, provided that the axes are fixed. Speed is the magnitude of the velocity vector |x|, which is non-negative. As in one dimension, acceleration is rate of change of velocity, so

$$a = \frac{dv}{dt} \equiv \dot{v} = \frac{d^2x}{dt^2} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} \tag{1.18}$$

We can obtain standard results for constant acceleration by (as is often the case) writing down the definitions and integrating the resulting differential equations. We have

$$\ddot{x} = a \tag{1.19}$$

where a is constant, so integrating with respect to time t once gives,

$$\dot{x} \equiv v = at + u \qquad (1.20)$$

where u is a constant of integration corresponding to the velocity at t = 0. Integrating again gives

$$x = \frac{1}{2}at^2 + ut + x_0 \qquad (1.21)$$

where x0 is a constant of integration corresponding to the position at t = 0. Sometimes, this is written as

$$\boxed{s = \frac{1}{2}at^2 + ut} \qquad (1.22)$$

where s is displacement from the initial position. We can find ***distance*** as a function of velocity by using (1.21) to eliminate time from (1.22):

$$t = \frac{(v-u)}{a} = s = \frac{1}{2}a\left(\frac{v-u}{a}\right)^2 + u\left(\frac{v-u}{a}\right) \qquad (1.23)$$

which simplifies to

$$\boxed{2as = v^2 - u^2} \qquad (1.24)$$

Self Assessment Exercise:

Explain the concept of kinematics to describe the position of object in motion.

3.2 Dynamics: Force and Newton's Laws of Motion

Dynamics is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. Our intuitive definition of force—that is, a push or a pull—is a good place to start. We know that a push or pull has both magnitude and direction (therefore, it is a vector quantity) and can vary considerably in each regard.

3.2.1 Newton's First Law of Motion: Inertia

Newton's First Law of Motion, often called the law of inertia, states that a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force. i.e. that an object at rest will remain at rest if left alone, and that an object in motion tends to slow down and stop unless some effort is made to keep it moving. Newton's first law of motion describes what happens when zero net force acts, which means that all forces acting must cancel out.

3.2.2 Newton's Second Law of Motion: Concept of a System

Newton's second law of motion states that, the acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass. The law is a cause and effect relationship among three quantities that is not simply based on their definitions. Mathematically, it is expressed as,

$$a = \frac{F_{net}}{m} \qquad (1.25)$$

This is often written in the more familiar form

$$F_{net} = ma \qquad (1.26)$$

3.2.3 Newton's Third Law of Motion

Newton's third law of motion states that, whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts, i.e. action and reaction are equal and opposite in direction.

Example 1 on Newton 2nd Law of Motion

Suppose that the net external force (push minus friction) exerted parallel to the ground on a lawn mower weighing 24 kg is 51 N. What is its acceleration?

Solution:

Using equation 1.25, $a = \frac{F_{net}}{m}$; a = $\frac{51}{24} = 2.1$ m/s^2; So, its acceleration is 2.1 m/s^{2}Ans.

Example 21 on Newton 2nd Law of Motion

Calculate the magnitude of force exerted by each rocket, called its thrust T, for the four-rocket propulsion system and it free body diagram shown in Figure 1.9. The sled's initial acceleration is

49 m/s^2, the mass of the system is 2100 kg, and the force of friction opposing the motion is known to be 650 N.

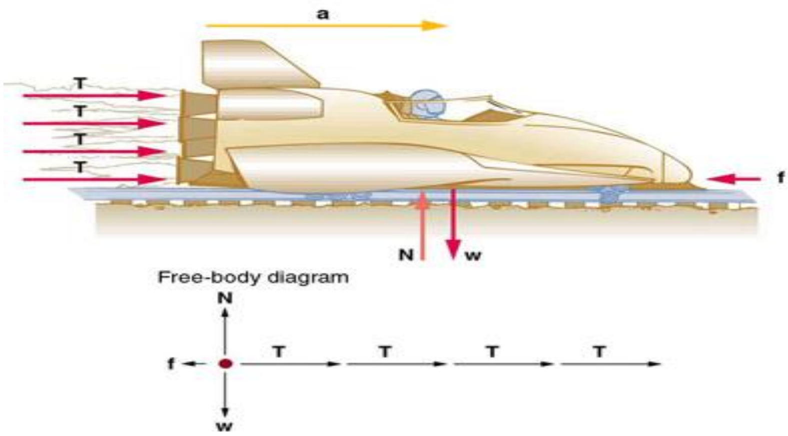

Figure1.9: Rocket thrust accelerating the sled

Solution:

$$F_{net} = ma$$

$$F_{net} = 4T - f$$

$$F_{net} = ma = 4T - f$$

$$4T = ma + f$$

$$4T = ma + f = (2100kg)(49ms^{-2}) + 650N$$

$$4T = 1.0 \times 10^5 N$$

$$T = \frac{1.0\times10^5 N}{4} = 2.5 \times 10^4 N \text{ Ans}$$

Examples of Newton’s third law are easy to find. As a professor paces in front of a whiteboard, she exerts a force backward on the floor. The floor exerts a reaction force forward on the professor that causes her to accelerate forward. Similarly, a car accelerates because the ground pushes forward on the drive wheels in reaction to the drive wheels pushing backward on the ground. You can see evidence of the wheels pushing backward when tires spin on a gravel road and throw rocks backward. In another example, rockets move forward by expelling gas backward at high velocity. This means the rocket exerts a large backward force on the gas in the rocket combustion chamber, and the gas therefore exerts a large reaction force forward on the rocket. This reaction force is called thrust. It is a common misconception that rockets propel themselves by pushing on the ground or on the air behind them. They actually work better in a vacuum, where they can readily expel the exhaust gases.

Helicopters similarly create lift by pushing air down, thereby experiencing an upward reaction force. Birds and airplanes also fly by exerting force on air in a direction opposite to that of whatever force they need. For example, the wings of a bird force air downward and backward in order to get lift and move forward.

Self Assessment Exercise:

State Newton's laws of motion and give examples of their applications.

4.0 Conclusion

We conclude that applying kinematics and Newton's laws of motion give a comprehensive description of the how objects under the action of forces (internal and or external) move. These laws provide an example of the breadth and simplicity of principles under which nature functions. They are also universal laws in that they apply to similar situations on Earth as well as in space.

5.0 Summary

This unit deals with kinematics, as the description of the way objects move - their velocity and their acceleration. Dynamics considers the forces that affect the motion of moving objects and systems. Newton's laws of motion were x-rayed as the foundation of dynamics. Newton's first, second and third laws were used to describe the velocity and acceleration of objects in motion. Real life applications of Newton's laws were also given.

6.0 Tutor-Marked Assignment

a) Explain the concept of kinematics to describe the position of object in motion

b) State Newton's laws of motion and give examples of their applications.

7.0 References/Further Readings

Feynman, R.P., Leighton, R.B., and Matthew, S.M. (1963). The Feynman Lectures on Physics, Addison-Wesley, Reading, Massachusetts, p. 12-20.

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UNIT 4: GALILEAN INVARIANCE AND RELATIVITY

CONTENTS

1.0 Introduction

Galilean invariance or Galilean relativity states that the laws of motion are the same in all inertia (or non-accelerating) frames. Galileo Galilei first described this principle in 1632 using the example of a ship travelling at constant velocity, without rocking on a smooth sea; any observer doing experiments below the deck would not be able to tell whether the ship was moving or stationary. The fact that the earth orbits around the sun at approximately 30 km/s offers a somewhat more dramatic example, though it is technically not an inertia reference frame. In other words, given two observers A and B moving at a constant velocity with respect to one another, it is not possible by any experiment whatsoever to determine which of the observers is ‘at rest’ or which is ‘in motion’.

2.0 Objective

At the end of this unit student should be able to

- Describe Galilean invariance or Galilean relativity.

i. Explain the use of reference frame in describing the motion of object with respect to space and time.

ii. Understand the translational properties of moving object and their relativity in space and time with respect to reference axes.

3.0 Main Content

3.1 Frames of Reference

Physical processes either directly or indirectly involve the dynamics of particles and/or fields moving or propagating through space and time. As a consequence, almost all of the fundamental laws of physics involve position and time in some way or other e.g. Newton's second law of motion earlier stated in eqn. 1.26

$$F_{net} = ma \qquad (1.26)$$

when applied to a particle responding to the action of a force will yield the position of the particle as a function of time. Likewise, Maxwell's equations will yield the wave equation

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 \qquad (1.27)$$

for the propagation of a light wave through space and time. Implicit in these statements of these fundamental physical laws is the notion that we have at hand some way of measuring or specifying or labelling each point in space and different instants in time. So, in order to describe in a quantitative fashion the multitude of physical processes that occur in the natural world, one of the important requirements is that we be able to specify where and when events take place in space and time.

Of course, for the purposes of formulating a mathematical statement of a physical law describing, say, the motion of a particle through space, or the properties of an electromagnetic or some other field propagating through space, a precise way of specifying the where and when of events is required, that is, the notion of a frame of reference, or reference frame, must be more carefully defined.

3.1.1 Construction of Arbitrary Reference Frame

A frame of reference can be constructed in essentially any way, provided it meets the requirements that it labels in a unique fashion the position and the time of the occurrence of any event that might occur. A convenient way of imagining how this might be done is to suppose that all of space is filled with a three dimensional (2-D projected) lattice or scaffolding – something like a fishing net, perhaps.

3.1.2 Event

The idea and use of the term ‘event’ here simply to connote a point in space and time. With this point specified by the spatial coordinates of the point in space, and the reading of a clock at that point. An event will have different coordinates in different reference frames. It is then important and useful to be able to relate the coordinates of events in one reference frame to the coordinates of the same event in some other reference frame. In Newtonian physics, this relation is provided by the Galilean transformation equations, and in special relativity by the Lorentz transformation.

3.1.3 Reference Frame or Frame of Reference

As we have just seen, a reference frame can be defined in a multitude of ways, but quite obviously it would be preferable to use the simplest possible, which brings to mind the familiar Cartesian set of coordinate axes. Thus, suppose we set up a lattice work of rods as illustrated in Figure 1.10 in which the rods extend indefinitely in all directions. Of course, there will be a third array of rods perpendicular to those in the figure is the Z direction.

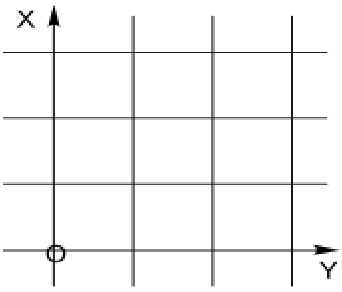

Figure 1.10: Cartesian coordinate system in 2-dimensions

Therefore, we can use frames of reference to specify the where and when of things other than the position of a particle at a certain time. For instance, the point in space-time at which an explosion occurs, or where and when two particles collide etc., can also be specified by the four numbers (x, y, z, t) relative to a particular frame of reference. In fact any event occurring in space and time can be specified by four such numbers whether it is an explosion, a collision or the passage of a particle through the position (x, y, z) at the time t. For this reason, the four numbers (x, y, z, t) together are often referred to as an event.

Self Assessment Exercise:

Briefly state Galilean invariance and the importance of reference frame.

3.2 Galilean Transformation and Newtonian Relativity

Galilean invariance state that Newton’s laws holds in all inertia frame. Newtonian mechanics assumes that there exist an absolute space and that time is universal. The arguments in the previous sub-unit do not tell us whether there is one or many inertial frames of reference, nor, if there is more than one, does it tell us how we are to relate the coordinates of an event as observed from the point-of-view of one inertial reference frame to the coordinates of the same event as observed in some others. These transformation laws are essential if we are to compare the mathematical statements of the laws of physics in different inertial reference frames. The transformation equations that are derived below are the mathematical basis on which it can be shown that Newton’s Laws are consistent with the principle of relativity. In establishing the latter, we can show that there is in fact an infinite number of inertial reference frames.

3.2.1. Galilean Transformation

To derive these transformation equations, consider an inertial frame of reference *S* and a second reference frame S' moving with a velocity v_x relative to S as shown in Figure 1.11.

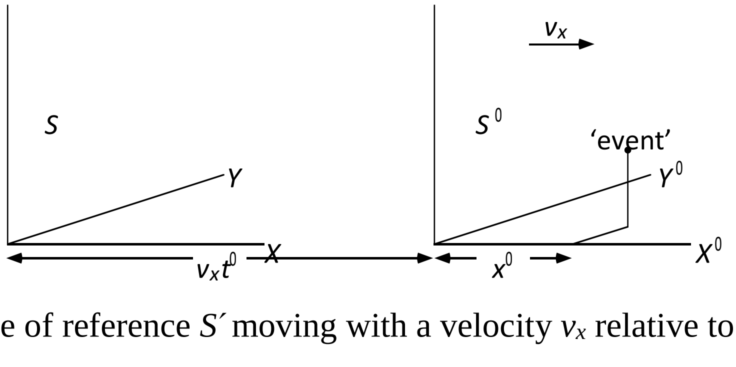

Figure 1.11: A frame of reference S' moving with a velocity v_x relative to the inertial frame *S*

Let us suppose that the clocks in S and S′ are set such that when the origins of the two reference frames O and O′ coincide, all the clocks in both frames of reference read zero i.e. $t = t' = 0$. According to ‘common sense’, if the clocks in S and S' 0 are synchronized at $t = t' = 0$, then they will always read the same, i.e. t = t′ always. This, once again, is the absolute time concept introduced by Einstein. Suppose now that an event of some kind, e.g. an explosion, occurs at a point (x′, y′, z′, t′) according to S′. Then, by examining Figure 1.11, according to S, it occurs at the point

$$x = x' + v_x t' ,\ y = y' ,\ z = z' \quad \textit{and at time } t = t' \qquad (1.28)$$

These equations together are known as the Galilean Transformation, and they tell us how the coordinates of an event in one inertial frame S are related to the coordinates of the same event as measured in another frame S′, which is moving with a constant velocity relative to frame S.

3.2.2 Newtonian Relativity

By means of the Galilean Transformation, we can obtain an important result of Newtonian mechanics which carries over in a much more general form to special relativity. We shall illustrate the idea by means of an example involving two particles connected by a spring. If the X coordinates of the two particles are x_1 and x_2 relative to some reference frame S then from Newton's Second Law the equation of motion of the particle at x_1 is

$$m_1 \frac{d^2x_1}{dt^2} = -k(x_1 - x_2 - l) \qquad (1.29)$$

where k is the spring constant, l the natural length of the spring, and m1 the mass of the particle. If we now consider the same pair of masses from the point of view of another frame of reference S′ moving with a velocity v_x relative to S, then

$$x_1 = x'_1 + v_x t' \quad \text{and } x_2 = x'_2 + v_x t' \qquad (1.30)$$

so that

$$\frac{d^2x_1}{dt^2} = \frac{d^2x'_1}{dt'^2} \qquad (1.31)$$

and

$$x_2 - x_1 = x'_2 - x'_1 \qquad (1.32)$$

Thus, substituting the last two results (1.31 and 1.32) into Eq. (1.29) gives

$$m_1 \frac{d^2x'_1}{dt'^2} = -k(x'_1 - x'_2 - l) \qquad (1.33)$$

Now according to Newtonian mechanics, the mass of the particle is the same in both frames i.e.

$$m_1 = m'_1 \qquad (1.34)$$

where m′ is the mass of the particle as measured in S′. Hence

$$m'_1 \frac{d^2x'_1}{dt'^2} = -k(x'_1 - x'_2 - l) \qquad (1.35)$$

which is exactly the same equation obtained in S, Eq. (1.29) except that the variables x_1 and x_2 are replaced by x'_1 and x'_2. In other words, the form of the equation of motion derived from Newton's Second Law is the same in both frames of reference. This result can be proved in a more general way than just masses on springs, and we are lead to conclude that the mathematical form of the equations of motion obtained from Newton's Second Law are the same in all inertial frames of reference.

Self Assessment Exercise:

State Galilean transformation equations and explain them

3.3 Einsteinian Relativity

The difficulties with the Newtonian relativity was overcome by Albert Einstein who made two postulates that lead to a complete restructuring of our ideas of space time, and the dynamical properties of matter.

3.3.1 Einstein's Postulates

The difficulty that had to be resolved amounted to choosing amongst three alternatives:

1. The Galilean transformation was correct and something was wrong with Maxwell's equations.

2. The Galilean transformation applied to Newtonian mechanics only.

3. The Galilean transformation, and the Newtonian principle of relativity based on this transformation were wrong and that there existed a new relativity principle valid for both mechanics and electromagnetism that was not based on the Galilean transformation.

3.3.2 Lorentz Transformation

In deriving this transformation, we will eventually make use of the constancy of the speed of light, but first we will derive the general form that the transformation law must take purely from kinematic/symmetry considerations. Doing so is based on two further assumptions which seem to be entirely reasonable:

Homogeneity: The intrinsic properties of empty space are the same everywhere and for all time.

In other words, the properties of the rulers and clocks do not depend on their positions in (empty) space, nor do they vary over time.

Spatial Isotropy: The intrinsic properties of space is the same in all directions. In other words, the properties of the rulers and clocks do not depend on their orientations in empty space.

There is a third, much more subtle condition:

No Memory: The extrinsic properties of the rulers and clocks may be functions of their current states of motion, but not of their states of motion at any other time.

Einstein concluded that the transformation equations represent a property that space and time must have in order to guarantee that light will always be observed to have the same speed c in all inertial frames of reference. But given that these transformation equations represent an intrinsic property of space and time, it can only be expected that the behaviour of other material objects, which may have nothing whatsoever to do with light, will also be influenced by this fundamental property of space and time. This is the insight that Einstein had, that the Lorentz transformation was saying something about the properties of space and time, and the consequent behaviour that matter and forces must have in order to be consistent with these properties.

Self Assessment Exercise:

State the Einstein postulates and discuss Einsteinian relativity

4.0 Conclusion

The principle of relativity can be understood as saying that the equations describing a law of nature take the same mathematical form in all frames of reference moving at a constant velocity with respect to each other. However, the velocity of the reference frame does not appear anywhere in these equations. But in order to guarantee that the principle of relativity holds true for all physical processes, including the postulate concerning the constancy of the speed of light, Einstein later proposed, along with a new perspective on the properties of space and time, modified versions of the familiar Newtonian concepts of force, momentum and energy, leading, amongst other things, to the famous equation $E = mc^2$.

5.0 Summary

The principle of relativity was accepted (in somewhat simpler form i.e. with respect to the mechanical behaviour of bodies) by Newton and his successors, even though Newton postulated that underlying it all was ‘absolute space’ which defined the state of absolute rest. This amounts to saying that any physical process taking place in space and time should proceed in a fashion that takes no account of the reference frame used to describe it. In other words, it ought to be possible to write down the laws of physics in terms of quantities that make no mention whatsoever of any particular reference frame. In accomplishing this task, Einstein was able to show that the force of gravity could be understood as a reflection of underlying geometrical properties of space and time that space and time can be considered as a single geometric entity that can exhibit curvature.

6.0 Tutor-Marked Assignment

a) State what you understand by Galilean invariance or Galilean relativity.

b) Explain the use of reference frame in describing the motion of object with respect to space and time.

c) State Galilean transformation equation and explain them

d) State the Einstein postulates and discuss Einsteinian relativity

7.0 References/Further Readings

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MODULE TWO

STATICS, DYNAMICS AND THERMAL PROPERTIES AND GRAVITATION OF BODIES

Unit 1: Statics and Dynamic Properties of Materials

Unit 2: Kinetic Energy, Heat, Work and Behaviour of Gases

Unit 3: Gravitation of Bodies

UNIT 1: STATIC AND DYNAMIC PROPERTIES OF MATERIALS

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1.0 Introduction

2.0 Objectives

3.0 Main Content

  3.1 Static Properties of Materials/Bodies

  3.2 Dynamic Properties of Materials/Bodies

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7.0 References/Further Readings

1.0 Introduction

Statics is the mechanics of things that don’t move. But everything does move, at least a little. So statics doesn’t exactly apply to anything. The statics equations are, however, a very good approximation of the more general dynamics equations for many practical problems. The statics equations are also easier to manage than the dynamics equations. When there is a net force there is acceleration of mass, and when there is acceleration of mass there is a net force and the body is in dynamic state.

2.0 Objective

At the end of this unit student should be able to

- Identify static properties of materials and objects
- Understand the dynamic properties materials.
- Relate the materials' inertia to their mobility function

3.0 Main Content

3.1 Static Properties of Materials/Bodies

In statics we set the right hand sides of (Eq. 1.36) to zero. The neglected terms involve mass times acceleration and are called the inertial terms. For statics we set the inertial terms $\vec{L}$ and $\vec{H}_C$ to zero. Thus, we replace the linear and angular momentum balance equations with their simplified forms

$$\sum_{\substack{All\ external\\ forces}} \vec{F} = \vec{0} \quad and \quad \sum_{\substack{All\ external\\ forces}} \overrightarrow{M_C} = \vec{0} \qquad (1.36)$$

which are called the force balance and moment balance equations and together are called the equilibrium equations. The forces to be summed are those that show on a free body diagram of the system. The torques that are summed are those due to the same forces (by means of $\overrightarrow{r_{1/C}} \times \vec{F_1}$) plus those due to any force systems that have been replaced with equivalent couples. If the forces on a system satisfy Eqn. 1.36, the system is said to be in static equilibrium or just in equilibrium.

A system is in static equilibrium if the applied forces and moments add to zero. Which can also be stated as – The forces on a system in static equilibrium, considered as a system, are equivalent to a zero force and a zero couple. The approximating assumption that an object is in static equilibrium is that the forces mediated by an object are much larger than the forces needed to accelerate it. The statics equations are generally reasonably accurate for;

a) Things that a normal person would call "still" such as a building or bridge on a calm day, and a sleeping person; for

b) Things that move slowly or with little acceleration, such as a tractor ploughing a field or the arm of a person holding up a book while seated in a smooth-flying airplane; and for

c) Parts that mediate the forces needed to accelerate more massive parts, such as gears in a transmission, the rear wheel of an accelerating bicycle, the strut in the landing gear of an airplane, and the individual structural members of a building swaying in an earthquake.

3.1.2 Procedure for solving static problems

The practice of statics involves:

1. Drawing free body diagrams of the system of interest and of appropriate subsystems;
2. Write out equations in (Eqn. 1.29) for each free body diagram; and
3. Use vector manipulation skills to solve for unknown features of the applied loads or geometry.

NOTE:

For one-force body, i.e., if only one force is acting on a body in equilibrium that force is zero. If a body in static equilibrium is acted on by two forces, (i.e. two-force body or two-force member) then those forces are equal, opposite, and have a common line of action. For a three-force body to be in equilibrium, the forces (a) must be coplanar, and (b) must either have lines of action which intersect at a single point, or the three forces are parallel.

3.1.3 Lack of Equilibrium as a Sign of Dynamics

Surprisingly, statics calculations often give useful information about dynamics. If, in a given problem, you find that forces cannot be balanced this is a sign that the related physical system will accelerate in the direction of imbalance. If you find that moments cannot be balanced, this is a sign of rotational acceleration in the physical system.

Self Assessment Exercise:

State the principles of static equilibrium of a body and the procedure for solving static problems.

3.2 Dynamic Properties of Materials/Bodies

In statics we neglected the inertial terms (the terms involving acceleration times mass) in the linear and angular momentum balance equations. In dynamics these terms are of central interest. In statics all the forces and moments cancel each other. In dynamics the forces and moments add to cause the acceleration of mass. As the names imply, statics is generally concerned with things that don’t move, or at least don’t move much, whereas dynamics with things that move a lot. How to quantify what is ‘still’ (statics) vs ‘moving’ (dynamics) is itself a dynamics question. A big part of learning dynamics is learning to keep track of motion of objects or bodies, i.e. kinematics.

3.2.1 Force and Motion in 1-D

We now limit our attention to the special case where one particle moves on a given straight line. For problems with motion in only one direction, the kinematics is particularly simple.

3.2.2 Position, Velocity, and Acceleration in 1-Dimension

If, say, we call the direction of motion the $\hat{\imath}$ direction, then we can call x the position of the particle we study Figure 2.1. Even though we are neglecting the spatial extent of the particle, to be precise we can define x to be the x coordinate of the particle’s centre.

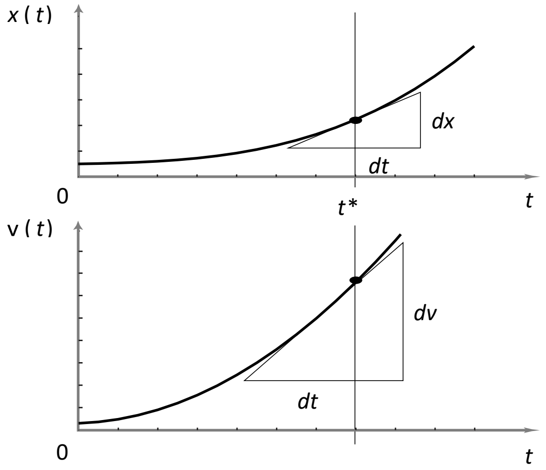

Figure 2.1: Graph of *x(t)* vs *v(t)*

We can write the position $\vec{r}$, velocity $\vec{v}$ and acceleration $\vec{a}$ as

$$\vec{r} = x\hat{\imath}\ \mathit{and}\ \vec{v} = v\hat{\imath} = \frac{dx}{dt}\hat{\imath} = \dot{x}\hat{\imath}\ \mathit{and}\ \vec{a} = a\hat{\imath} = \frac{dv}{dt}\hat{\imath} = \frac{d^2x}{dt^2}\hat{\imath} = \ddot{x}\hat{\imath} \qquad (1.37)$$

3.2.4 Differential Equations in Dynamics

A differential equation is an equation that involves derivatives. Thus the equation relating position to velocity is

$$\frac{dx}{dt} = v \text{ or more explicitly } \frac{dx(t)}{dt} = v(t) \qquad (1.38)$$

is a differential equation. An Ordinary Differential Equation (ODE) is one that contains ordinary derivatives (as opposed to partial differential equations which we will not use in this unit).

3.2.5 Equation of Dynamics

Linear momentum balance

For a particle moving in the x direction the velocity and acceleration are $\vec{v} = v\hat{\imath}$, and $\vec{a} = a\hat{\imath}$. Thus, the linear momentum and its rate of change are

$$\vec{L} \equiv \sum m_i \vec{v}_i = m\vec{v} = mv\hat{\imath} \quad \text{and } \vec{L} \equiv \sum m_i \vec{a}_i = m\vec{a} = ma\hat{\imath} \qquad (1.39)$$

Thus, the equation of linear momentum balance, or right hand part of Eqn. 1.39, or Newton's second equation of motion (Eqn. 1.26) can be thus be written as

$$F = ma, \;\; F\hat{\imath} = ma\hat{\imath} \qquad (1.40)$$

When applying the laws of mechanics, we must be sure that when we differentiate vectors we do so with respect to a Newtonian frame. In typically static situations, therefore, the applied force is not lost or dissipated or absorbed. Instead, it is opposed by the equal and opposite reaction force that results from the tendency of the material that has been deformed to return to its resting shape. No material is totally rigid; even blocks of the stiffest materials, such as metals and diamonds, deform when they are loaded. The reason that this deformation was such a hard discovery to make is that most structures are so rigid that their deflection is tiny; it is only when we use compliant structures such as springs or bend long thin beams that the deflection common to all structures is obvious. So, the principles of statics and dynamics applies to bodies or part in real life situations.

Self Assessment Exercise:

State the relationship of material's mass in the momentum and its acceleration when propelled by force.

4.0 Conclusion

In statics we neglected the inertial terms (the terms involving acceleration times mass) in the linear and angular momentum balance equations. In statics all the forces and moments cancel each other. As the names imply, statics is generally concerned with things that don't move, or at least don't move much, whereas dynamics with things that move a lot. When applying the laws of mechanics, we must be sure that when we differentiate vectors we do so with respect to a Newtonian frame.

5.0 Summary

The unit has introduced the principle of statics and dynamics of objects' motion. It can be summarized as, if I give a push to a ball that is initially at rest, it will accelerate in that direction at a rate proportional to the force and inversely proportional to its mass. The great step forward in Newton's scheme was that, together with the inverse square law of gravity, it showed that the force that keeps us down on earth is one and the same with the force that directs the motion of the planets.

6.0 Tutor-Marked Assignment

a. State the principles of static equilibrium of a body and the procedure for solving static problems.
b. Discuss some Non-destructive test methods for metallic and non-metallic materials
c. State the relationship of material's mass in the momentum and its acceleration when propelled by force.

7.0 References/Further Readings

Pratap, R., and Ruina, A. (2001). Statics and Dynamics, Oxford University Press.

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UNIT 2: KINETIC THEORY, HEAT, WORK AND BEHAVIOUR OF GASES

CONTENTS

1.0 Introduction

Almost all materials expand on heating – the most famous exception being water, which contracts as it is warmed from 0 degrees Celsius to 4 degrees. Heat can be transferred in various forms/ways such as conduction, convection and radiation. Bodies in “thermal contact” eventually come into “thermal equilibrium”, meaning they finally attain the same temperature, after which no further heat flow takes place. (i.e. the Zeroth Law of Thermodynamics)

2.0 Objective

At the end of this unit student should be able to

- Understand thermal properties of materials/medium.
- Explain thermal conductivity and thermodynamics of medium/materials
- Understand how thermal conductivity affects kinetic energy of materials

3.0 Main Content

3.1 Kinetic Theory of Gases

Daniel Bernoulli, in 1738, was the first to understand air pressure from a molecular point of view. He drew a picture of a vertical cylinder, closed at the bottom, with a piston at the top, the piston having a weight on it, both piston and weight being supported by the air pressure inside the cylinder. He described what went on inside the cylinder as follows Figure 2.2: “Let the cavity contain very minute corpuscles, which are driven hither and thither with a rapid motion; so that these corpuscles, when they strike against the piston and sustain it by their repeated impacts, form an elastic fluid which will expand of itself if the weight is removed or diminished…”

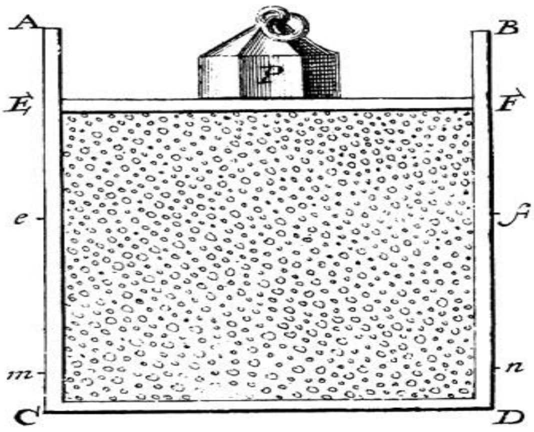

Figure 2.2: Bernoulli apparatus

Most scientists believed that the molecules in a gas stayed more or less in place, repelling each other from a distance, held somehow in the ether. Newton had shown that PV = constant followed if the repulsion were inverse-square, i.e. $P_1V_1=P_2V_2$.

3.1.1 Molecular Energy and Pressure

It is not difficult to extend Bernoulli’s picture to a quantitative description, relating the gas pressure to the molecular velocities. As a warm up exercise, let us consider a single perfectly elastic particle, of mass m, bouncing rapidly back and forth at speed v inside a narrow cylinder of length L with a piston at one end, so all motion is along the same line. For example what is the force on the piston shown in Figure 2.3?

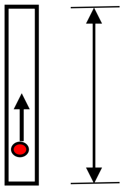

Figure 2.3: Relating gas theory to molecular velocity

Obviously, the piston doesn't feel a smooth continuous force, but a series of equally spaced impacts. However, if the piston is much heavier than the particle, this will have the same effect as a smooth force over times long compared with the interval between impacts. So what is the value of the equivalent smooth force? Using *Newton's law in the form force = rate of change of momentum, we see that the particle's momentum changes by 2mv* each time it hits the piston. The time between hits is $2L/v$, so the frequency of hits is $v/2L$ per second. This means that if there were no balancing force, by conservation of momentum the particle would cause the momentum of the piston to change by $2mv \times v/2L$ units in each second. This is the rate of change of momentum, and so must be equal to the balancing force, which is therefore $F = mv^2/L$.

We now generalize to the case of many particles bouncing around inside a rectangular box, of length L in the x-direction (which is along an edge of the box). The total force on the side of area A perpendicular to the x-direction is just a sum of single particle terms, the relevant velocity being the component of the velocity in the x-direction. The pressure is just the force per unit area, $P = F/A$. Of course, we don't know what the velocities of the particles are in an actual gas, but it turns out that we don't need the details. If we sum N contributions, one from each particle in the box, each contribution proportional to vx^2 for that particle, the sum just gives us N times the average value of vx^2. That is to say,

$$P = \frac{F}{A} = \frac{Nm\overline{v^2}}{LA} = \frac{Nm\overline{v^2}}{V} \qquad (2.1)$$

where there are N particles in a box of volume V. Next we note that the particles are equally likely to be moving in any direction, so the average value of v_x^2 must be the same as that of v_y^2 or v_z^2, and since $v^2 = v_x^2 + v_y^2 + v_z^2$, it follows that,

$$P = \frac{Nm\overline{v^2}}{3V} \qquad (2.2)$$

The macroscopic pressure of a gas relates directly to the average kinetic energy per molecule. Furthermore, it is well established experimentally that most gases satisfy the Gas Law over a wide temperature range:

$$PV = nRT \qquad (2.3)$$

for n moles of gas, that is, $n = N/N_A$, with N_A as Avogadro's number and R, the gas constant.

Introducing Boltzmann's constant k = R/N_A, it is easy to check from our result for the pressure and the ideal gas law that the average molecular kinetic energy is proportional to the absolute temperature,

$$\overline{E_k} = \overline{\frac{1}{2}mv^2} = \frac{3}{2}kT \qquad (2.4)$$

k is Boltzmann's constant, k = $1.38.10^{-23}$ joules/K.

3.2 Heat Work and Ideal Gas Behaviour

The Kinetic Theory (covered in the previous sub-unit), often called the ideal gas model, ignores interactions between molecules, and the finite size of molecules. In fact, though, these only become important when the gas is very close to the temperature at which it become liquid, or under extremely high pressure. In this lecture, we will be analyzing the behaviour of gases in the pressure and temperature range corresponding to heat engines, and in this range the Ideal Gas Model is an excellent approximation. Essentially, our program here is to learn how gases absorb heat and turn it into work, and vice versa. This heat-work interplay is called thermodynamics.

The simplest way to see what's going on is to imagine the gas in a cylinder, held in by a piston, carrying a fixed weight, able to move up and down the cylinder smoothly with negligible friction Figure 2.4 The pressure on the gas is just the total weight pressing down divided by the area of the piston, and this total weight, of course, will not change as the piston moves slowly up or down: the gas is at constant pressure.

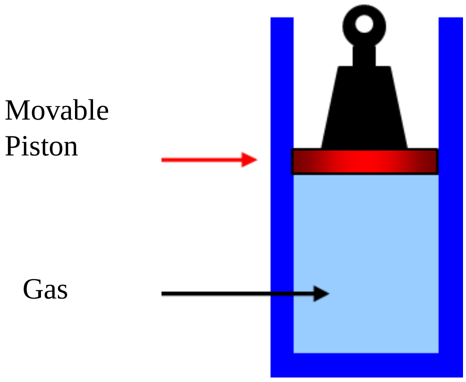

Figure 2.4: Gas at constant pressure in a chamber

3.2.1 The Gas Specific Heats C_V and C_P

Consider now the two specific heats of this same sample of gas, let's say one mole: Specific heat at constant volume, C_V (piston glued in place), Specific heat at constant pressure, C_P (piston free to rise, no friction). We already worked out C_V in the Kinetic Theory lecture above: at temperature T, recall the average kinetic energy per molecule is kT , so one mole of gas – Avogadro's number of molecules – w ill have total kinetic energy, which we'll label internal energy,

$$E_{int} = \frac{3}{2}kT.NA = \frac{3}{2}RT \qquad (2.5)$$

Therefore, $$C_v = \frac{3}{2}R \qquad (2.6)$$

If the area of piston is A, then the gas at pressure P exerts force P_A. If on heating through one degree the piston rises a distance Δh the gas does work

$$PA.\Delta h = P\Delta v \qquad (2.7)$$

And, for one mole of gas, $PV= RT$, so at constant P

$$P\Delta v = R\Delta T \qquad (2.8)$$

Therefore, the work done by the gas in raising the weight is just $RT\Delta$, the specific heat at constant pressure, the total heat energy needed to raise the temperature of one mole by one degree, and

$$C_P = C_v + R \qquad (2.9)$$

It's worth having a standard symbol for the ratio of the specific heats is:

$$\frac{C_P}{C_v} = \gamma \qquad (2.10)$$

3.2.2 Tracking a Gas in the (P, V) Plane: Isotherms and Adiabats

An ideal gas in a box has three thermodynamic variables: P, V, T. But if there is a fixed mass of gas, fixing two of these variables fixes the third from (for n moles). In a heat engine, heat can enter the gas, then leave at a different stage. The gas can expand doing work, or contract as work is done on it. To track what's going on as a gas engine transfers heat to work, say, we must follow the varying state of the gas. We do that by tracing a curve in the (P, V) plane.

3.2.3 Isothermal behaviour of Gas

The gas is kept at constant temperature by allowing heat flow back and forth with a very large object (a "heat reservoir") at temperature T. From $PV=nRT$, it is evident that for a fixed mass of gas, held at constant T but subject to (slowly) varying pressure, the variables P, V will trace a hyperbolic path in the (P, V) plane.

This path, $PV=nRT$, is called the isotherm at temperature T_1. Here are two examples of isotherms in Figure 2.5:

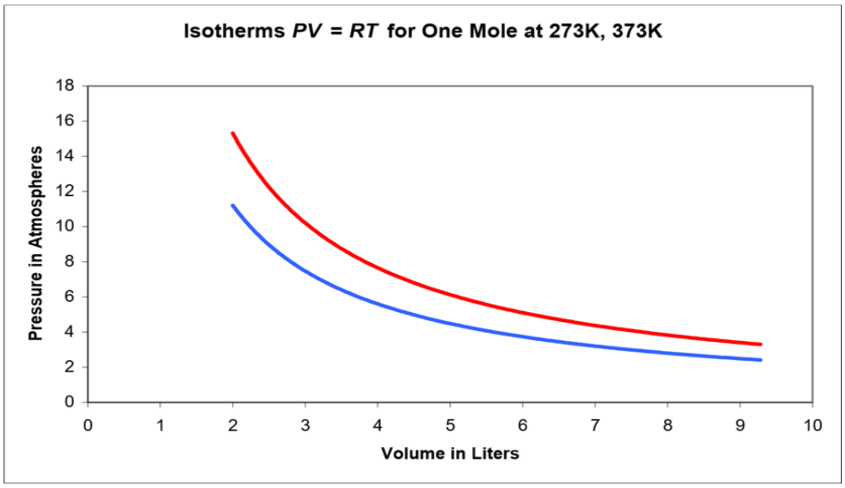

Figure 2.5: Isotherm behaviour in gas

3.2.4 Adiabatic behaviour of Gas

"Adiabatic" means "nothing gets through", in this case no heat gets in or out of the gas through the walls. So all the work done in compressing the gas has to go into the internal energy E_{int}.

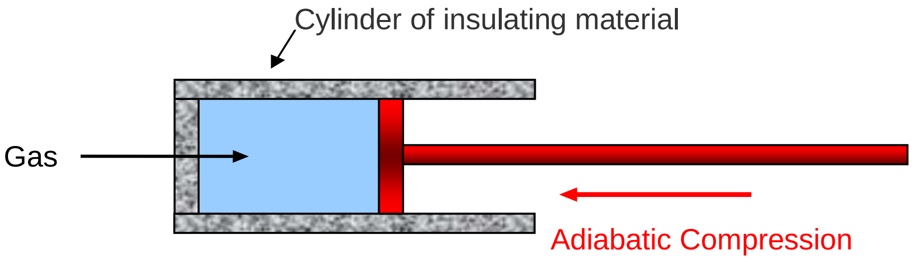

Figure 2.6: Adiabatic behaviour of gas confined in impregnable walls

As the gas is compressed, it follows a curve in the (P, V) plane called an adiabat. To see how an *adiabat* differs from an *isotherm*, imagine beginning at some point on the blue 273K isotherm on the above graph, and applying pressure so the gas moves to higher pressure and lower volume. Since the gas's internal energy is increasing, but the number of molecules remains the same, its temperature is necessarily rising, it will move towards the red curve, then above it. This means the *adiabats* are always steeper than the isotherms. Adiabats and isotherms are compared in Figure 2.7 below.

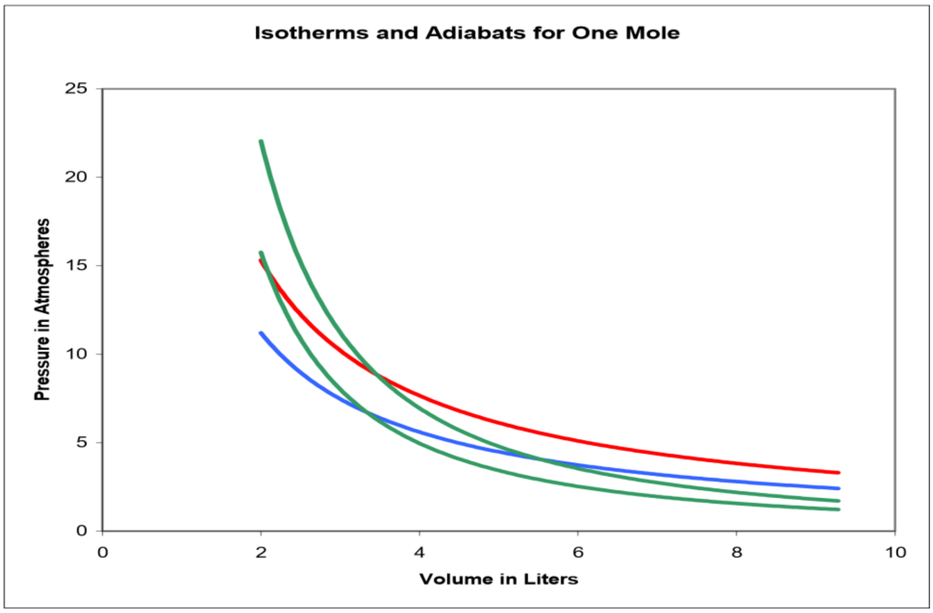

Figure 2.7: Isotherms and adiabats for one mole

Self Assessment exercise:

Differentiate between '*isotherm*' and '*adiabat*' in gas behaviour.

4.0 Conclusion

In this unit, we have learn and analyzed the behaviour of gases in the pressure and temperature range corresponding to heat engines, and in this range the Ideal Gas Model provided an excellent approximation and a case study. Essentially, our program made us learn how gases absorb heat and turn it into work, and vice versa. This heat-work interplay is called thermodynamics. The unit also introduced students to elementary thermodynamics.

5.0 Summary

The average kinetic energy of ideal gas is $3/2Kt$, and one mole of gas, of Avogadro number of molecules will have total kinetic energy $E_{\text{int}} = 3/2RT$. Heat can be transferred in various forms/ways such as conduction, convection and radiation. Bodies in “thermal contact” eventually come into “thermal equilibrium”, meaning they finally attain the same temperature, after which no further heat flow takes place (i.e. the Zeroth Law of Thermodynamics).

6.0 Tutor-Marked Assignment

a) Discuss the kinetic theory of gases
b) Relate gas pressure to molecular velocity
c) Differentiate between ‘isotherm’ and ‘adiabat’ in gas behaviour

7.0 References/Further Readings

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UNIT 3: GRAVITATION OF BODIES

CONTENTS

1.0 Introduction

The unit aims at studying the effects of gravity on the acceleration of the earth and the associated potential gravitational energy U_{grav}. The study further consider gravitation in a more general manner through the Law of Gravitation enunciated by Newton in 1687.

2.0 Objective

Under this unit student should be able to

- Understand Newton's law of gravitation.
- Understand gravitational interaction between spherically symmetrical bodies.
- Explain the relationship between weight and gravitational potential energy.

3.0 Main Content

3.1 Newton's Law of Gravitation

Newton pondered why things tended to fall toward the Earth, as opposed to other directions and what kept the moon orbiting around the Earth and the Earth around the sun. He theorized that the force of something he called “gravity” is inversely proportional to the square of the distance between the centres of the masses. Because of his Third Law of Motion, Newton also realized that forces exist in pairs, meaning if the Earth pulls on objects, those objects pull on Earth as well. He then theorized that the force of gravity is directly proportional to the product of the masses of the objects. Simply put, Therefore, Newton Universal law of gravitation, which applies to all objects states that;

Every particle of matter in the universe attracts every other particle with a force that is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them.

When considering the force of attraction between any two objects: The force is greater when the mass of either of the two objects is greater. Earth, with its huge mass has a relatively large attractive force with all of the objects near its surface. The moon has less mass than Earth, so the moon has less attraction for objects on its surface than Earth does. (Objects on the surface of the moon weigh less than on Earth because the gravitational force of the moon is less than the gravitational force of Earth.) The reason we do not notice the attraction between ordinary sized objects when we are on earth is that the force that the earth exerts on objects is so great; the force of attraction between other objects is very small. The closer the two objects are, the greater the force. When an object such as a space vehicle moves away from Earth, the gravitational attraction between Earth and the vehicle becomes less and less.

Mathematically, this law, and the magnitude of the force due to the gravitational interaction between two particles, is expressed as;

$$F_{grav} = \frac{G_{m1\,m2}}{r^2} \qquad (2.11)$$

where G = 6.67 ×10^{-11} $N{\cdot}m^2/kg^2$ is the universal gravitational constant, m_1 and m_2 the particles masses, and r the distance between them. Eqn. 2.11 is however not complete since the force due to gravitation is vectorial in nature. If we define the vectors r_1 and r_2 for the positions of m_1 and m_2, respectively, and the unit vector that points from r_1 to r_2 is $e_r = (r_2 - r_1)/\,|r_2 - r_1|$, then the force that m_2 exerts on m_1 is

$$F_{21} = F_{grav}\, e_r = \frac{Gm_1m_2}{r^2}\, e_r \qquad (2.12)$$

Conversely, accordingly to Newton's Third Law, the force that m_1 exerts on m_2 is

$$F_{12} = -F_{21} = -\frac{Gm_1m_2}{r^2}e_r \qquad (2.13)$$

Equations (2.12) and (2.13) taken together show that gravitation is an attractive force, i.e., the two masses are drawn to one another.

Self Assessment Exercise:

State and explain Newton's universal gravitational law

3.2 Gravitational Interactions with Spherically Symmetric Bodies

Although the related calculations are beyond the scope of our studies, it can be shown that gravitational interactions involving spherically symmetric bodies (e.g., uniform spheres, cavities, and shells) can be treated as if all the mass was concentrated at the centres of mass of the bodies (Figure 2.8). This property is especially useful when dealing with the earth, other planets and satellites, as well as the sun and stars, which can all be approximated as being spherically symmetric.

There are other interesting results that can be derived for spherically symmetric bodies. For example, the gravitational force felt by a mass m located at a radius r inside a spherical shell of radius R and mass M (i.e., with $r \leq R$) is zero. That is,

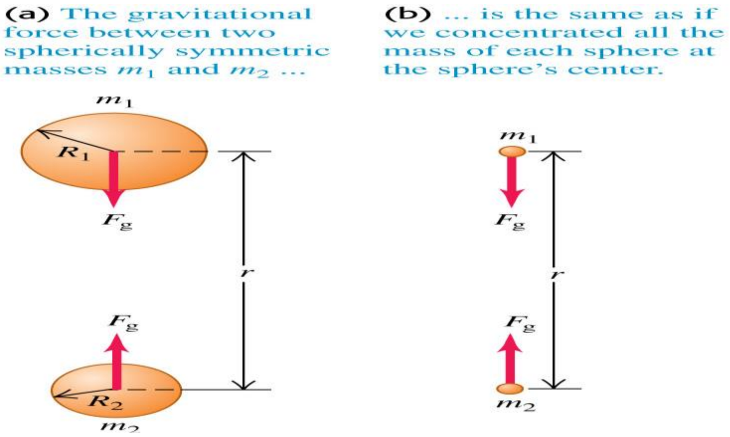

Figure 2.8: Gravitational attraction of two spherical symmetrical bodies

$$F_{grav} = \begin{cases} 0. & r < R \\ -\frac{G_m M}{r^2}e_r, & r \geq R \end{cases} \qquad (2.14)$$

with e_r the unit vector directed from the centre of the shell to the mass. It therefore follows from this that the gravitational force felt by a mass located inside/outside a uniform spherical shell can be shown to be equal to the portion of the mass of the sphere contained within the radius *r*. Mathematically this can expressed with

$$F_{grav} = \begin{cases} -\frac{G_m M r}{R^3} e_r , & r < R \\ -\frac{G_m M}{r^2} e_r , & r \geq R \end{cases} \tag{2.15}$$

The result when the mass is located inside the sphere, i.e., when $r < R$, can be explained by considering the relation between the mass *M*, the mass density ρ (units of kg/m^3), and the volume *V* (units of m^3) of the sphere. More precisely, we have that

$$M = \rho V \tag{2.16}$$

with

$$V = \frac{4}{3}\pi R^3 \tag{2.17}$$

for a sphere. If the sphere is uniform, then the density ρ is constant throughout. It follows that the mass M_r contained within a volume V_r (with a radius $r \leq R$) is

$$\begin{aligned} M_r &= \rho V_r & \qquad (2.16) \\ &= \left(\frac{4}{3}\pi r^3\right) & \\ &= \left(\frac{4}{3}\pi R^3\right)\left(\frac{r}{R}\right)^3 & \qquad (2.17) \\ &= \rho V \left(\frac{r}{R}\right)^3 & \end{aligned}$$

from Eqn. 2.16

$$M_r = M\left(\frac{r}{R}\right)^3 \tag{2.18}$$

We can then use equation (2.13) to write down the force felt by the mass when it is located inside the sphere (when $r \leq R$)

$$F_{grav} = -\frac{GmM_r}{r^2} e_r = -\frac{GmM}{r^2}\left(\frac{r}{R}\right)^3 e_r \tag{2.19}$$

and

$$F_{grav} = -\frac{GmM_r}{R^3} e_r \tag{2.20}$$

We thus recover the first of equations (2.15).

Self Assessment Exercise:

Obtain the gravitational force felt by a mass m located at a radius r inside a spherical shell of radius R and mass M (i.e., with $r \leq R$) is zero.

3.3 Weight and Gravitational Potential Energy

Recall the modelled force exerted by the earth on a particle of mass *m* by its weight as

$$w = mg \qquad (2.21)$$

with g the gravitational acceleration due to the earth. Referring to Problem 2 above, we can now easily evaluate this quantity by equating the weight with the gravitational force exerted by the earth at its surface. That is,

$$mg = \frac{GM_E m}{{R_E}^2} \quad (2.22), \quad or\ g = \frac{GM_E}{{R_E}^2} \quad (2.23)$$

$$g = \frac{GM_E}{{R_E}^2}$$

$$g = \frac{6.67\times 10^{-11} N.\frac{m^2}{kg^2}.5.97\times 10^{24}\ kg}{(6.37\times 10 m)^2} \qquad (2.24)$$

$$= 9.81\ m/s^2$$

as was expected. It should however be clear, and it is important to note, that *the gravitational acceleration and force exerted by the earth vary with the position of the particle above the surface of the earth; they both decrease as the particle is located further away from the surface (or the centre) of the earth*.

The gravitational force is only a function of the initial and final positions of the path traced by the particle. This in turn means that the gravitational force is (minus) the gradient of the gravitational potential energy *Ugrav*.

$$F_{grav} = -\nabla U_{grav} \qquad (2.25)$$

For a particle located at a position r ≥ RE the gravitational force exerted by the earth is

$$F_{grav} = -\frac{G m_E m}{r^2} e_r \qquad (2.26)$$

where m is once again the mass of the particle and e_r is the unit vector pointing from the centre of the earth to the position occupied by the particle. Combining equations (2.25) and (2.26), we write

$$\frac{dU_{grav}}{d_r} = \frac{Gm_E m}{r^2} \qquad (2.27)$$

It can readily be verified that

$$U_{grav} = -\frac{Gm_E m}{r} \qquad (2.28)$$

is a solution consistent with equations (2.25) and (2.27). Equation (5.28) is however different to the relation we have used so far for the gravitational potential energy

$$U_{grav} = mg\Delta_r \qquad (2.29)$$

where the quantity Δr can be conveniently chosen to be the position of the particle relative to the surface of the earth. We will now establish that this apparent disparity is solely due to our previously implied assumption that the particle is located very near the surface of the earth, where the gravitational acceleration can, to a good approximation, considered constant at *g*.

Let us then assume that

$$r = R_E + \Delta_r \qquad (2.30)$$

with $\Delta_r \ll R_E$. Inserting equation (2.30) in equation (2.28) yields

$$U_{grav} = -\frac{Gm_E m}{R_E + \Delta_r}$$

$$U_{grav} = -\frac{Gm_E m}{R_E(1 + \Delta_r/R_E)} \qquad (2.31)$$

$$U_{grav} = -\frac{Gm_E m}{R_E}\left(1 - \frac{\Delta_r}{R_E}\right)$$

since $\Delta_r \ll R_E$

$$= \frac{1}{1 + \Delta_r/R_E} \cong 1 - \frac{\Delta_r}{R_E} \qquad (2.32)$$

We now modify equation (2.31) to

$$U_{grav} = -\frac{Gm_E m}{R_E} + m\left(\frac{GM_E}{{R_E}^2}\right)\Delta_r \qquad (2.33)$$

We finally make two observations: i) the quantity in parentheses in the second term on the right-hand side of equation (2.33) is simply the gravitational acceleration at the surface of the earth (see equation (2.24)) and ii) as was stated above, the potential gravitational energy can only been defined up to constant, and thus the first term on the right-hand side of equation (2.33), which is constant, can be ignored (if one wishes) and the potential gravitational energy near the surface of the earth redefined to

$$U_{grav} = mg\,\Delta_r \qquad (2.29)$$

This justifies the relation we have used so far. When the particle moves from a point r_1 to another point at r_2 in the earth's gravitational field we have

$$W = \int_1^2 F_{grav} \,.\, d_r$$

$$= -Gm_E m \int_1^2 \frac{e_r}{r^2} \,.\, d_r \qquad (2.30)$$

$$= -Gm_E m \int_1^2 \frac{d_r}{r^2}$$

$$= \frac{Gm_E m}{r_2} - \frac{Gm_E m}{r_1}$$

since $\int \frac{d_r}{r^2} = -1/\,r$. Using equation (2.28) we then write

$$W = U_{grav}(r_1) - U_{grav}(r_2) \qquad (2.31)$$

This once again verifies that the gravitational force is indeed conservative, since the change in potential energy is not a function of the path taken between the initial and final points. Although these results were derived using the gravitational interaction with the earth as an example, they apply to any body in general.

Self Assessment Exercise:

Show that the change in potential energy of gravitational bodies is not a function of the path taken between the initial and final points between them.

4.0 Conclusion

The unit conclude that, every particle of matter in the universe attracts every other particle with a force that is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them. The closer the two objects are, the greater the gravitational force. Gravitational force is (minus) the gradient of the gravitational potential energy.

5.0 Summary

When considering the force of attraction between any two objects. The force is greater when the mass of either of the two objects is greater. The force of gravity is directly proportional to the product of the masses of the objects. All particles in the universe gravitationally attract all other particles, and that gravitational attraction is inversely proportional to the square of the distance between the objects and directly proportional to the product of the masses.

6.0 Tutor-Marked Assignment

a) State and explain Newton's universal gravitational law

b) Obtain the gravitational force felt by a mass m located at a radius r inside a spherical shell of radius R and mass M (i.e., with $r \leq R$) is zero.

c) Show that the change in potential energy of gravitational bodies is not a function of the path taken between the initial and final points between them.

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MODULE THREE

THERMAL PROPERTIES, ATOMIC STRUCTURE AND GAS KINETICS

Unit 1: Thermal Properties of Elements

Unit 2: Elementary Thermodynamics

Unit 3: Atomic Structure and Periodic Classification of Elements

Unit 4: Introduction to Gas Kinetic

UNIT 1: THERMAL PROPERTIES OF ELEMENTS

CONTENTS

1.0 Introduction

Thermal properties of elements include *Heat capacity*, *Thermal expansion*, *thermal conductivity*, and *Thermal stresses*. Thermal stresses can be generated due to restrained thermal expansion/contraction or temperature gradients that lead to differential dimensional changes in different part of the solid body. Stress and pressure induced by heat can result in plastic deformation or eventual fracture of the material.

2.0 Objective

At the end of this unit student should be able to

- Know the effect of heat on properties of elements.
- Explain Pressure and stress variation in Elements under heat.
- Understand the characteristics and behaviour of Elements when subjected to heat.

3.0 Main Content

3.1 Heat Capacity and Thermal Expansion of Elements

Thermal properties of elements include *Heat capacity*, *Thermal expansion*, *thermal conductivity*, and *Thermal stresses*. The heat capacity and thermal expansion sub-themes include;

Heat capacity

- atomic vibrations, phonons
- temperature dependence
- contribution of electrons

Thermal expansion

- ✦ connection to anharmonicity of interatomic potential
- ✦ linear and volume coefficients of thermal expansion

3.1.1 Heat capacity

The heat capacity, *C*, of a system is the ratio of the heat added to the system, or withdrawn from the system, to the resultant change in the temperature:

$$C = \frac{\Delta Q}{\Delta T} = \frac{dQ}{dT} \quad [J/deg], \qquad (3.1)$$

Heat capacity can be measured under conditions of constant temperature or constant volume. Thus, two distinct heat capacities can be defined:

$$C_v = \left(\frac{\delta q}{dT}\right)_v - heat\ capacity\ at\ constant\ volume$$

$$C_P = \left(\frac{\delta q}{dT}\right)_P - heat\ capacity\ at\ constant\ pressure$$

C_P is always greater than C_V - Why?

Hint: The difference between C_P and C_V is very small for solids and liquids, but large for gases. Heat capacity is a measure of the ability of the material to absorb thermal energy. Thermal energy = kinetic energy of atomic motions + potential energy of distortion of interatomic bonds.

The higher is T, the higher the mean atomic velocity and the amplitude of atomic vibrations, and the higher the thermal energy. The coupling of atomic vibrations of adjacent atoms results in waves of atomic displacements. Each wave is characterized by its wavelength and frequency. For a wave of a given frequency ν, there is the smallest "quantum" of vibrational energy, h_ν, called ***phonon.***

Thus, the thermal energy is the energy of all phonons (or all vibrational waves) present in the crystal at a given temperature. Scattering of electrons on phonons is one of the mechanisms responsible for electrical resistivity

3.1.2 Thermal expansion

Materials expand when heated and contract when cooled

$$\frac{l_f - l_0}{l_0} = \frac{\Delta l}{l_0} = \alpha_l(T_f - T_0) = \alpha_l \Delta T \qquad (3.2)$$

where l_o is the initial length at T_0, l_f is the final length at T_f. α_l is the linear coefficient of thermal expansion. Similarly, the volume change with T can be described as

$$\frac{V_f - V_0}{V_0} = \frac{\Delta l}{V_0} = \alpha_v(T_f - T_0) = \alpha_v \Delta T \qquad (3.3)$$

where α_V is the volume coefficient of thermal expansion.

Thermal expansion is related to the asymmetric (anharmonic) shape of interatomic potential. If the interatomic potential is symmetric (harmonic), the average value of interatomic separation does not change, i.e. no thermal expansion.

3.2 Thermal conductivity and Thermal stresses

Thermal properties of elements include Heat capacity, Thermal expansion, thermal conductivity, and Thermal stresses. Thermal stresses can be generated due to restrained thermal expansion/contraction or temperature gradients that lead to differential dimensional changes in different part of the solid body. Stress and pressure induced by heat can result in plastic deformation or eventual fracture of the material.

Thermal conductivity

- heat transport by phonons and electrons

Thermal stresses

✦ stress induced by heat exchange

3.2.1 Thermal conductivity

Thermal conductivity: heat is transferred from high to low temperature regions of the material.

$$q = -\frac{dT}{dx} \quad (3.4) \qquad - Fourier's\ law$$

where q is the heat flux (amount of thermal energy flowing through a unit area per unit time) and dT/dx is the temperature gradient, and k is the coefficient of thermal conductivity, often called simply thermal conductivity.

3.2.1 Thermal stress

Stresses due to change in temperature or due to temperature gradient are termed as thermal stresses($\sigma_{thermal}$).

$$\sigma_{thermal} = \alpha E \Delta T \qquad (3.5)$$

Thermal sty will be of compressive nature if it is heated, and vice versa. Engineering materials can be tailored using multi-phase constituents so that the overall material can show a zero thermal expansion coefficient. For example Zerodur – a glass-ceramic material that consists of 70-80% crystalline quartz, and the remaining as glassy phase. Sodium-zirconium-phosphate (NZP) have a near-zero thermal expansion coefficient.

Self Assessment exercise:

Discuss thermal energy and state C_P is always greater than C_V.

3.3 Mechanisms of heat conduction

Heat is transferred by phonons (lattice vibration waves) and electrons. The thermal conductivity of a material is defined by combined contribution of these two mechanisms:

$$k = k_l + k_e \qquad (3.6)$$

where k_l and k_e are the lattice and electronic thermal conductivities.

Lattice conductivity: Transfer of thermal energy phonons

Electron conductivity: Free (conduction band) electrons equilibrate with lattice vibrations in hot regions, migrate to colder regions and transfer a part of their thermal energy back to the lattice by scattering on phonons.

The electron contribution is dominant in metals and absent in insulators.

Since free electrons are responsible for both electrical and thermal conduction in metals, the two conductivities are related to each other by the Wiedemann-Franz law:

$$L = \frac{k}{\sigma T} \qquad (3.7)$$

where σ is the electrical conductivity and L is a constant

Self Assessment Exercise:

Discuss the mechanism of heat conduction.

4.0 Conclusion

Physical property of a solid body related to application of heat energy is defined as a thermal property. Thermal properties of elements include *Heat capacity*, *Thermal expansion*, *thermal conductivity*, and *Thermal stresses*.

5.0 Summary

Thermal conductivity is ability of a material to transport heat energy through it from high temperature region to low temperature region. Heat is transported in two ways – electronic contribution, vibrational (phonon) contribution. In metals, electronic contribution is very high. Thus, metals have higher thermal conductivities.

6.0. Tutor-Marked Assignment

a) Discuss thermal energy and state C_P is always greater than C_V.
b) Discuss thermal expansion in asymmetric objects.
c) Discuss the mechanism of heat conduction.

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UNIT 2: ELEMENTARY THERMODYNAMICS

CONTENTS

1.0 Introduction

Thermodynamics is the branch of physics that describes and correlates the physical properties of macroscopic systems of matter and energy. In other words, it is concerned with heat and related thermal phenomena. The various laws of thermodynamics enable us to accurately describe processes involved in heat energy.

2.0 Objective

At the end of this unit student should be able to

- Understand the basic concepts of thermodynamic systems.
- Explain the laws of thermodynamics.
- Understand the characteristics and behaviour of thermodynamic system due to heat exchange.

3.0 Main Content

3.1 Basic Concept of Thermodynamic Systems

A thermodynamic system is defined as a quantity of matter of fixed mass and identity on which attention is focused for study. Everything external to the system is the surroundings, and the system is separated from the surroundings by the system boundaries. These boundaries may be either movable or fixed.

3.1.1 Thermodynamic system

In Figure. 3.1, the gas in the cylinder is considered to be the system. When the cylinder is heated from below, the temperature of the gas will increase and the piston will rise. As the piston rises, the boundary of the system moves. Heat and work cross the boundary of the system during this thermodynamic process, but the matter that comprises the system can always be identified. Mass, as well as heat and work (and momentum), can flow across the control surface; Example is air compressor (Controlled volume).

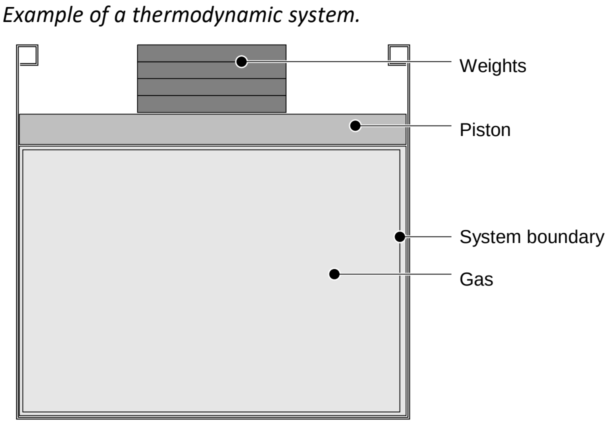

Figure 3.1: Example of thermodynamic system

3.1.2 Process and cycles

Whenever one or more of the properties of a system change, a Change in State has occurred. For example, when one of the weights on the piston in Figure 3.1 is removed, the piston rises and a change in state occurs, for the pressure decreases and the specific volume increases. The path of the succession of states through which the system passes is called the process.

3.1.3 Energy

One of the very important concepts in a study of thermodynamics is the concept of energy. Energy is a fundamental concept, such as mass or force and, as is often the case with such concepts, is very difficult to define. Energy is defined as the capability to produce an effect. It is important to note that energy can be stored within a system and can be transferred (as heat, for example) from one system to another.

3.1.4 Specific volume

The specific volume of a substance is defined as the volume per unit mass, and is given the symbol *v*.

$$v = \frac{\delta V}{\delta m}\left[\frac{m^3}{kg}\right] \qquad (3.8)$$

The density of a substance (r) is defined as the mass per unit volume, and is therefore the reciprocal of the specific volume.

$$\rho = \frac{1}{v}\left[\frac{kg}{m^3}\right] \qquad (3.9)$$

3.1.5 Pressure

Pressure is defined as the normal component of force per unit area.

$$P = \frac{\delta F_n}{A}\left[\frac{N}{m^2}\ or\ P\right] \qquad (3.10)$$

Most thermodynamics investigations are concerned with absolute pressure. Most pressure and vacuum gauges, however, read the difference between the absolute pressure and the atmospheric pressure existing at the gauge. This is referred to as gauge pressure.

3.1.5 Temperature

Although temperature is a familiar property, an exact definition of it is difficult. Thus, we define equality of temperatures. Two bodies have equality of temperatures if, when they are in thermal communication, no change in observable property occurs.

3.2 Basic Laws of Thermodynamics

Like all sciences, the basis of thermodynamics is experimental observations. In thermodynamics these findings from the experimental observations have been formalized into certain basic laws, which are known as the first, second, and third law of thermodynamics. In addition to these laws, the zeroth law of thermodynamics, which precedes the first law.

Self Assessment Exercise:

Discuss the basic concepts of thermodynamic systems.

3.2.1 The Zeroth Law of Thermodynamics

The zeroth law of thermodynamics states that when two bodies have equality of temperatures with a third body, they in turn have equality of temperatures with each other. Simply put, it states that, if two systems A and B are in thermal equilibrium with a third system C, then they are in equilibrium with each other i.e. A is in equilibrium with C or B is in equilibrium C. We can then determine the direction of heat flow when two systems (A and B) are put in contact. One system A is said to be hotter than another B if heat flows from the former (A) to the latter (B) when they are in thermal contact.

This then allows us to introduce a parameter, called an empirical temperature, which is the same for all bodies that are in thermal equilibrium with each other. This is done by constructing a system, called a thermometer which allows us to ascribe a number to the temperate.

3.2.1 The First Law of Thermodynamics

Also called the law of conservation of energy, i.e. energy is neither lost nor created. The first law of thermodynamics states that during any systemic process, the integral of the heat is equal to the integral of the work. The Law tells us that heat is a form of energy which is conserved. The 1st Law applies to closed systems, i.e. for a given amount of matter, and is expressed in the form of an energy balance equation:

$$du = dq + dw \qquad (3.11)$$

The above equation states that a small change (du) in the internal energy (u) of a closed system is the sum of a small amount (dq) of heat supplied to the system and the work performed (dw) on the system. Note that, dq may be positive or negative and that dw can assume many different forms depending on the type of action to which the system is subjected.

Here we will assume work done to be;

$$dw = -P_{ext} dV \qquad (3.12$$

where P_{ext} = external pressure applied in order to perform work, which causes a change in volume dV. The negative sign implies compression ($dV<0$) when dw should be positive.

Now Eqn. 3.11 becomes

$$du = dq - P_{ext}dV \qquad (3.13)$$

depending on whether the internal pressure, Pint of the system is higher or lower than P_{ext} respectively. In particular, if

$$P_{ext} = P_{int} = P \qquad (3.14)$$

The quantities or variables *U*, *P*, and *V* are called functions of state or state variables. The absolute temperature *T* (in Kelvin) is also a function of state.

The equation of state of a system can therefore be written as:

$$\frac{V}{n} = \overline{V} = \frac{RT}{P} \quad \textit{for ideal gas} \qquad (3.14)$$

$$\left(P + \frac{a}{\overline{v}^2}\right)(\overline{v} - b) = RT \quad \textit{for real gas e.g. van der Waals} \qquad (3.15)$$

where b is the effect of finite size of the molecule, $\frac{a}{\overline{v}^2}$ is the effect of the attractive (dispersion) forces. It follows that from the ideal gas model,

$$\overline{U} - \overline{U}(T) \qquad (3.16)$$

We can now write on equation of state in a *PV* – diagram where, for each *T*, the *P*-*V* relationship describes a curve, called an isotherm Figure 3.2. The *PV* – diagram can also be used to represent a reversible change of a system. Each point (*V*, *P*) corresponds to definite values of both *T* and *U* Figure 3.3.

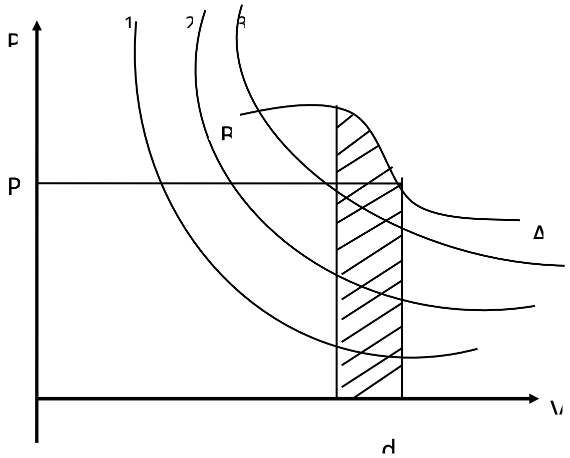

Figure 3.2: PV – diagram showing Isotherm

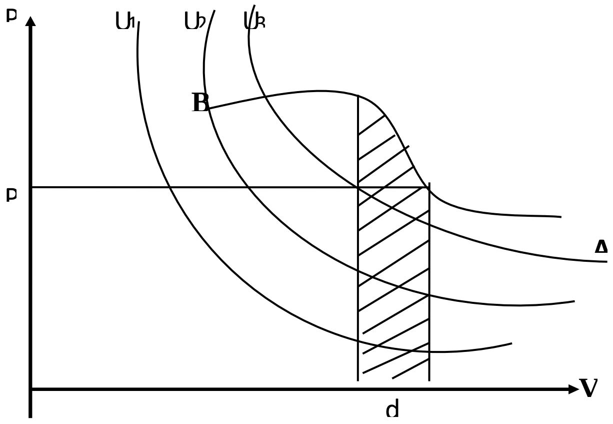

Figure 3.3: PV – diagram, reversible change

The specific heat can be defined at constant pressure or at constant volume. At constant volume, it is known as *Isochoric heat energy*, c_v, ($dV=0$) given by

$$c_v = \frac{dq_v}{dT} = \left(\frac{\partial U}{\partial T}\right)_v \qquad (3.17)$$

Or at constant pressure, it is known as *Isobaric heat energy*, c_p, given by

$$c_p = \frac{dq_p}{dT} = \left(\frac{dU+PdV}{dT}\right) = \left(\frac{\partial H}{\partial T}\right)_p \qquad (3.18)$$

3.2.2 The Second and Third Laws of Thermodynamics

There exist many statements of the 2nd Law of thermodynamics; all of them essentially, are equivalent. For example, Lord Kelvin postulated that: “No process is possible whose sole result is the conversion of heat to an equivalent amount of useful work” Work can of course always be converted to heat, for example in the form of frictional losses, but Kelvins statement tells us that it is impossible to retrieve this heat and convert it back entirely to work.

The Second Law acknowledges that processes proceed in a certain direction but not in the opposite direction. A hot cup of coffee cools by virtue of heat transfer to the surroundings, but heat will not flow from the cooler surroundings to the hotter cup of coffee. Or it may be stated that, during a spontaneous process, the total entropy of an isolated system increases. It remains constant for reversible changes of state.

Now, according to the 1st Law, the amount of *PV*- work *W*, performed by the gas on the surrounding must be equal to:

$$q_1 + q_2 = W \qquad (3.19)$$

Recall that along the isotherm the internal energy of a perfect gas is constant so that any heat absorbed (evolved) should exactly match the PV work done (absorbed). Therefore,

$$q_1 = \int_B^c PdV = \mathrm{nRT_1} \int_B^c \frac{dv}{V} = \mathrm{nRT_1}\, ln \frac{v_c}{v_B} \qquad (3.20)$$

And similarly,

$$q_1 = \int_B^c PdV = \mathrm{nRT_1} \int_B^c \frac{dv}{V} = \mathrm{nRT_1}\, ln \frac{v_D}{v_A} \qquad (3.21)$$

Let us now consider an infinitesimal adiabatic compression or expansion of the gas: Since

$$dq = 0 \qquad (3.22)$$

$$dq = -Pdv \qquad (3.23)$$

With small simultaneous change in P, V, and T, substituting to have

$$VdP = -\left[1 + \frac{nR}{c_v}\right] PdV = \left[-\frac{c_v + nR}{c_v}\right] PdV \qquad (3.24)$$

$$Vdp = -\frac{c_p}{c_v} PdV = -\gamma Pdv \qquad (3.25)$$

$$d[lnP] = d[lnV]^{-\gamma} \qquad (3.26)$$ or

$$d[\ln (PV^{\gamma}] = 0 \qquad (3.27)$$

Thus, along a reversible adiabatic:

$$PV^{\gamma} = constant \qquad (3.28)$$

Since γ is obviously > 1, adiabatics are always “steeper” then isotherms, for which PV = constant.

The third law of thermodynamics

The entropy of all pure substances can be assigned the value of zero at the absolute zero of temperature. When there is entropy difference between two points, it’s a bit like potential energy. Actually, though, there is a natural base point: a system at absolute zero temperature has zero entropy. This is sometimes called the Third Law of Thermodynamics, or Nernst’s Postulate, and can only be really understood with quantum mechanics, which is not covered in this course.

Self Assessment Exercise:

State the laws of thermodynamics and explain them.

3.3 The Steam Cycle

A *heat Engine* may be defined as a device that operates in a thermodynamics cycle and does a certain amount of net positive work through the transfer of heat from a high-temperature and to a low-temperature body. A simple steam power plant is an example of a heat engine Figure 3.4. The ideal cycle for a simple steam power plant is the Rankine cycle, shown in Figure 3.5.

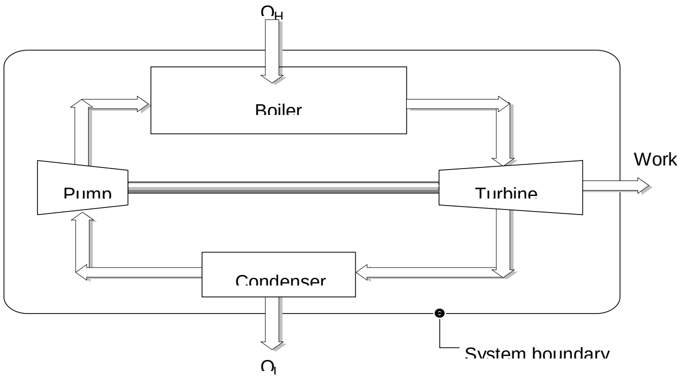

Figure 3.4: A heat engine in steady flow process

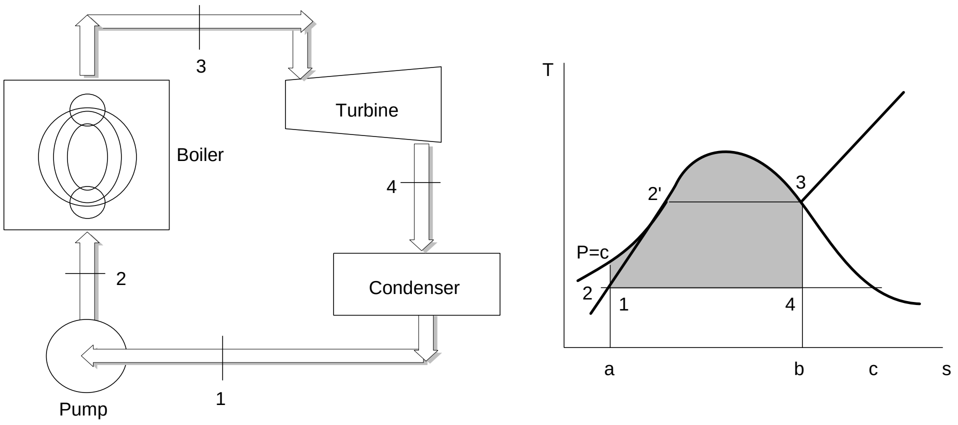

Figure 3.5: Steam cycle in a steam power plant

The processes that comprise the cycle are.

1-2: Reversible adiabatic pumping process in the pump.

2-3: Constant-pressure transfer of heat in the boiler.

3-4: Reversible adiabatic expansion in the turbine

4-1: Constant-pressure transfer of heat in the condenser.

If changes of kinetic and potential energy are neglected, heat transfer and work may be represented by various areas on the T-S diagram. The heat transferred to the working fluid is represented by area a-1-2-2′-3-b-a, and the heat transferred from the working fluid by area a-1-4-b-a. From the first law we conclude that the area representing the work is the difference between these two areas, area 1-2-2′-3-4-1. The thermal efficiency is defined by the relation

$$n_{th} = \frac{w_{net}}{q_H} = \frac{area\ 1 - 2 - 2' - 3 - 4 - 1}{area\ a - 2 - 2' - 3 - b - a} \quad (3.29)$$

Self Assessment Exercise:

Discuss the concept of steam cycle.

4.0 Conclusion

Heat, like work, is a form of energy transfer to or from a system. Total energy, including heat energy, is always conserved. It is impossible to devise an engine which, working in a cycle, shall produce no effect other than the extraction of heat from a reservoir and the performance of an equal amount of mechanical work.

5.0 Summary

Thermodynamics is the science that deals with heat and work and these properties of substances that bear a relation to heat and work. Like all sciences, the basis of thermodynamics is experimental observation. In thermodynamics these findings have been formalized into certain basic laws, which are known as the first, second, and third law of thermodynamics. In addition to these laws, the zeroth law of thermodynamics, which in the logical development of thermodynamics precedes the first law, has been set forth.

6.0. Tutor-Marked Assignment

d) Discuss the basic concepts of thermodynamic systems.
e) State and explain the laws of thermodynamics.
f) Discuss the concept of steam cycle.

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UNIT 3: ATOMIC STRUCTURE AND THE PERIODIC CLASSIFICATION OF ELEMENTS

CONTENTS

1.0 Introduction

The classic model of an atom (the Bohr model) has protons and neutrons in the middle, surrounded by circling electrons. An atom has protons and neutrons in the middle, surrounded by circling electrons. In the Bohr model, electrons are stuck in their orbits unless something pushes them out. These orbits are called “shells,” or “energy levels”. Each shell can only have certain a certain number of electrons. The outermost shell contains special electrons called valence electrons. These electrons are very important because they determine how reactive the atom is. Elements are organized in the Periodic Table. Printed on a Periodic table are several important pieces of information, which described the number of protons and electron of the particular element.

2.0 Objective

At the end of this unit student should be able to

- Understand the basic structure of the atoms making up the element
- Understand the interaction of the atomic make-ups of elements

- Understand and explain the basis of grouping and classification of elements.

3.0 Main Content

3.1 Atomic Structure of Elements

According to Rutherford, Atoms must consist of a small, but very massive, positively charged nucleus in order to explain the observed scattering of α-particles on gold atoms. However, the first model that could account for the spectra of atomic hydrogen was that of Bohr in 1913. The classic model of an atom (the Bohr model) has protons and neutrons in the middle, surrounded by circling electrons. In the Bohr model, electrons are stuck in their orbits unless something pushes them out. These orbits are called “shells,” or “energy levels”. Each shell can only have certain a certain number of electrons. The outermost shell contains special electrons called valence electrons. These electrons are very important because they determine how reactive the atom is. A full shell means the atom cannot react.

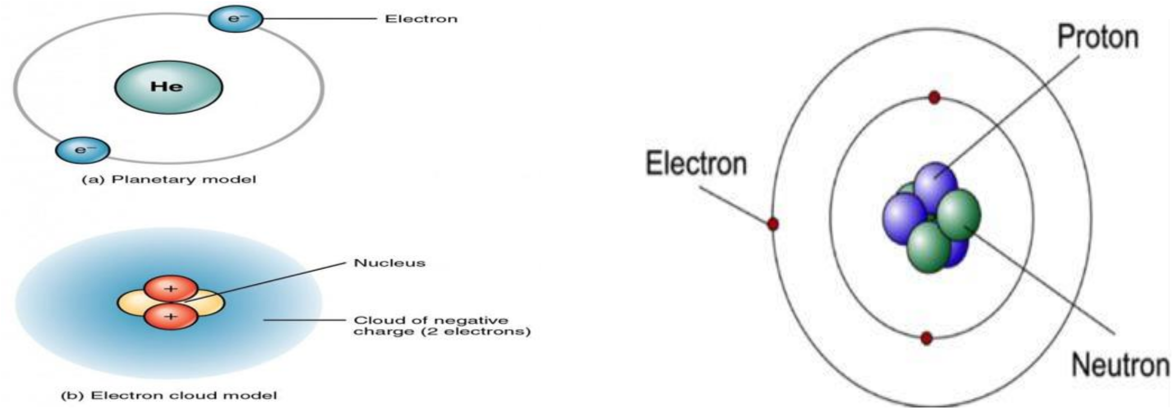


Figure 3.6: The Structure of an atom

Atomic spectra of hydrogen is not continuous but consists of discrete lines. Bohr suggested that the electron can adopt only certain distances r (orbits) where k is a constant (Bohr radius = 52.9 pm, also a_0), and *n* is any integer = QUANTUM NUMBER of the orbit

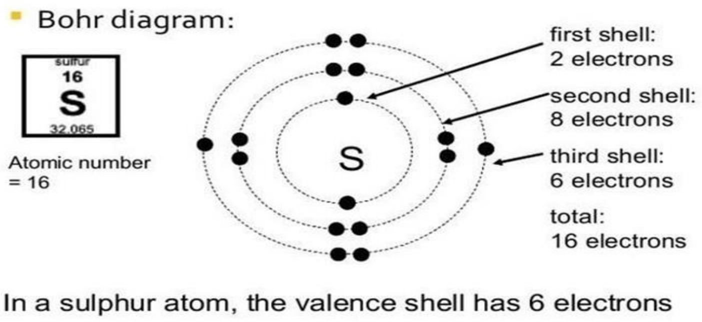


Figure 3.7: The outer shells of an atom (Bohr)

Each allowed orbit corresponds to a different energy level:

$$E_n = -\frac{me^4Z^2}{8{\varepsilon_0}^2h^2n^2} = -\frac{k'}{n^2} \qquad (3.30)$$

from centrifugal force = coulombic attraction; $4\pi\varepsilon_0 rmv^2 = {Z_e}^2$ (3.31)

quantizing of the angular momentum $= mvr = \frac{nh}{2\pi}$ (3.31)

Energy = kinetic energy + potential energy, $E = \frac{1}{2}mv^2 - \frac{1}{4\pi\varepsilon_0}.\frac{{Z_e}^2}{r}$ (3.32)

With these three equations, the radius, the energy and the velocity of the electron of the H atom with quantum number n (and nuclear charge Z) can be calculated.

3.1.1 Emission and absorption spectra

Atoms are excited either via electrical discharge (A, above) or with a white light source (B, below) in Figure 3.8. After passing through a prism the absorbed energy appears as discrete lines. The excited sample and emission spectra is upper mode. While the unexcited prism and absorption spectra mode is below.

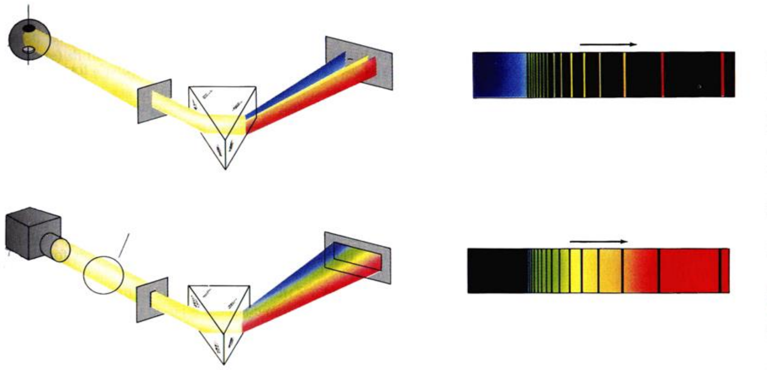


Figure 3.8: Emission and absorption spectra

Ionization energy is proportional to Z^2 (nuclear charge). Radius of hydrogen atom in ground state (n =1) is 52.9 pm (Bohr radius). The radius is inversely proportional to Z. The excited state radius is proportional to n^2. In the ground state the electron has a velocity of v = 2.187x10^8 cms^{-1}. Bohr’s model allows accurate prediction of the hydrogen atom spectrum BUT fails to describe atoms or ions with more than ONE electron.

3.1.2 Electron: Particle or Wave?

Depending on the experimental conditions, electrons appear either as particles, or show properties usually associated with waves. Electrons are diffracted by crystalline materials, just as observed with X-rays according to De Broglie relationship: h: Planck's constant, $m_e v$: momentum of electron. In addition, the photoelectric effect revealed a linear relationship between the kinetic energy of the photon and frequency:

$$E = \frac{1}{2} m_e v^2 = h(v - v_0), \qquad \Delta E = hv \ (3.33)$$

3.1.3 Periodic trends in atomic properties

The trends in the atomic properties of elements include:

a) Effective nuclear charge
b) Ionization energies
c) Electron affinities
d) Covalent and ionic radii
e) Bond strength
f) Electronegativity
g) Orbital energies
h) Promotion energies
i) Common oxidation states
j) Relativistic effects

3.1.4 Ionization energy

1) Energy required to remove an electron from an atom or ion

1st ionization energy or ionization potential (IE or IP)

2nd ionization energy or ionization potential (IE or IP)

• Depends on the effective nuclear charge experienced by the electron and the average distance from the nucleus.

• –> with increasing Z_{eff} increases *IP*

- –> with increasing distance decreases *IP*

- Note: Distance increases as principle quantum number increases (*n*)

3.1.5 Electron affinity

- Energy change associated with addition of an electron to an atom or ion *1st electron affinity (EA)*
- Favourable process for most elements
- Influenced by *Zeff* and atom size (principle quantum number)
- Note: Positive sign per definition:

$F + e^- \rightarrow F^-$ EA = + 328 $kJmol^{-1}$ $\Delta H = -328\ kJmol^{-1}$

- Trends in *EAs* parallel those of *IPs*, but for one lower *Z* value
- Exceptions:
  - EA of F is lower than of Cl
  - EA of N is lower than of P
  - EA of O is lower than of S;

  Smaller size of F (and N or O) cause greater electron-electron repulsion

3.2 Periodic Classification of Elements

Different kinds of atoms make different elements. Elements are organized in the Periodic Table. Printed on a Periodic Table are several important pieces of information, which described the number of protons and electron of the particular element.

Periodic table is arranged according to the valence electron at the outer shell of the element according to Figure 3.9. Elements are classified into ***Groups*** on the vertical (columns) and ***Periods*** horizontally (rows). There are Three Broad classifications, namely; ^{1}Metals, ^{2}Metalloids and ^{3}Non-metals. There are 8 Groups, I – VII, and O.

Pe
Ta

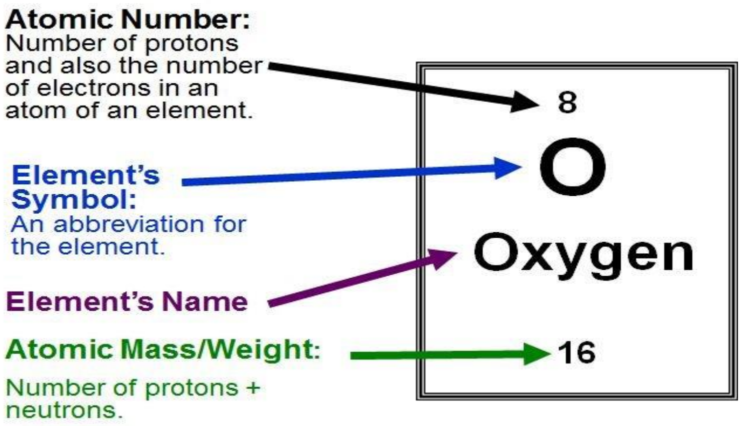
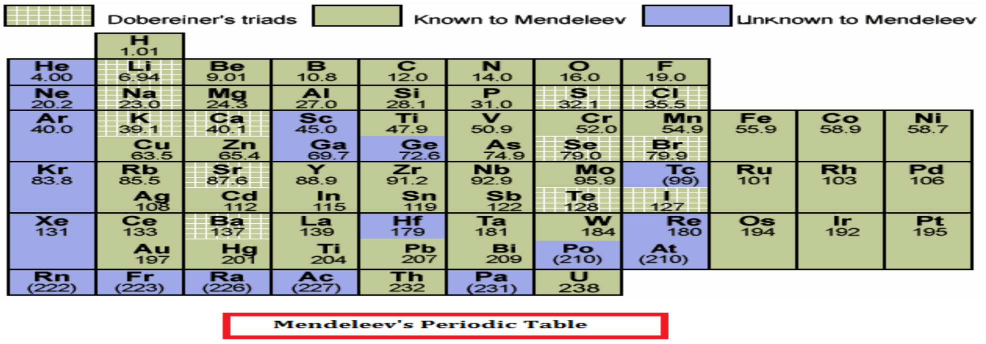


Figure 3.9: Atomic weight of element

3.2.1 *Mendeleev periodic table*

Mendeleev believed that the atomic mass of an element was the most fundamental property in classifying elements. Mendeleev arranged elements in the increasing order of their atomic masses and observed that the elements showed repetition after certain intervals in their physical and chemical properties. He arranged the known elements in the increasing order of their atomic masses in horizontal rows, till he encountered an element which had properties similar to the first element. Mendeleev placed the element below the first element and started the second row of elements. Proceeding in this way, he created the first periodic table containing 63 elements, arranged according to their properties. Mendeleev's Periodic Law: The physical and chemical properties of elements are a periodic function of their atomic masses as shown in Table 3.1.

Table 3.1: Mendeleev's Periodic table of element



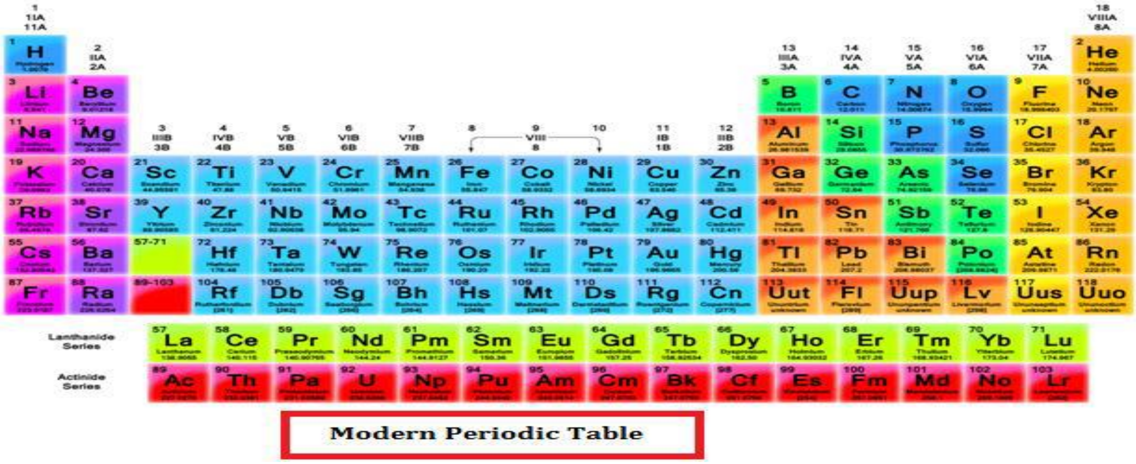
	H 1.01									
He 4.00	Li 6.94	Be 9.01	B 10.8	C 12.0	N 14.0	O 16.0	F 19.0			
Ne 20.2	Na 23.0	Mg 24.3	Al 27.0	Si 28.1	P 31.0	S 32.1	Cl 35.5			
Ar 40.0	K 39.1 Cu 63.5	Ca 40.1 Zn 65.4	Sc 45.0 Ga 69.7	Ti 47.9 Ge 72.6	V 50.9 As 74.9	Cr 52.0 Se 79.0	Mn 54.9 Br 79.9	Fe 55.9	Co 58.9	Ni 58.7
Kr 83.8	Rb 85.5 Ag 108	Sr 87.6 Cd 112	Y 88.9 In 115	Zr 91.2 Sn 119	Nb 92.9 Sb 122	Mo 95.9 Te 128	Tc (99) I 127	Ru 101	Rh 103	Pd 106
Xe 131	Ce 133 Au 197	Ba 137 Hg 201	La 139 Ti 204	Hf 179 Pb 207	Ta 181 Bi 209	W 184 Po (210)	Re 180 At (210)	Os 194	Ir 192	Pt 195
Rn (222)	Fr (223)	Ra (226)	Ac (227)	Th 232	Pa (231)	U 238				

Mendeleev's Periodic Table

3.2.1 *Modern periodic table*

In 1913, Henry Moseley proved that the atomic number is the fundamental property rather than its atomic mass. Modern Periodic Law: Properties of elements are a periodic function of their atomic numbers. The periodic table, based on the Modern Periodic Law is called the Modern Periodic Table as shown in Table 3.2.

Table 3.2: Modern Periodic table of element



1 IA 1A	2 IIA 2A	3 IIIB 3B	4 IVB 4B	5 VB 5B	6 VIB 6B	7 VIIB 7B	8	9 VIII 8	10	11 IB 1B	12 IIB 2B	13 IIIA 3A	14 IVA 4A	15 VA 5A	16 VIA 6A	17 VIIA 7A	18 VIIIA 8A
1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57-71	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89-103	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Fl	115 Uup	116 Lv	117 Uus	118 Uuo

Lanthanide Series	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
Actinide Series	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

Modern Periodic Table

Periods

The horizontal rows in the Modern Periodic Table are called periods.

The Modern Periodic Table consists of seven periods which are numbered from 1 to 7.

In each period, a new shell starts filling up. The period number is also the number of shell which starts filling up.

The elements in a period have consecutive atomic numbers, and the number of elements in each period is given below

First period contains 2 elements and is called a very short period.

Second and third periods contain 8 elements and are called as short periods.

Fourth and fifth periods are long periods and contain 18 elements each.

Sixth period is the longest and contains 32 elements.

Seventh period is an incomplete period.

Groups

i. The vertical columns are called groups and consist of eighteen groups numbered from 1 to 18.

ii. Elements having the same number of valence electrons are present in the same group.

iii. Elements present in the same group show the same chemical properties.

iv. Group 1 contains alkali metals and these elements contain 1 electron in their outermost shell.

v. Group 2 contains alkaline earth metals and these elements contain 2 electrons in their outermost shell.

vi. Groups 3 to 12 have their two outermost shells incomplete.

vii. Groups 13 to 18 group contain 3 to 8 electrons in their outermost shell.

viii. Group 18 elements have complete outermost shells. So they are called noble elements or noble gases.

ix. The element hydrogen has been placed at the top of group 1 because its electronic configuration is similar to alkali metals.

Self Assessment Exercise:

Classify and discuss the periodic table of elements

4.0 Conclusion

The classic model of an atom (the Bohr model) has protons and neutrons in the middle, surrounded by circling electrons. Electrons are stuck in their orbits unless something pushes them out. These orbits are called “shells,” or “energy levels”. Each shell can only have certain a certain number of electrons.

5.0 Summary

Element contains microscopic particles called *atoms*. The *atoms* of an element always have as many *electron* as *protons*. Atoms usually have about as many *neutrons* as *protons*. Different kinds of atoms make different elements. Elements are organized in the Periodic Table. Printed on a Periodic Table are several important pieces of information, which described the number of protons and electron of the particular element.

6.0. Tutor-Marked Assignment

a) Describe the atomic structure of an element.
b) Classify and discuss the periodic table of elements.

7.0 References/Further Readings

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UNIT 2: INTRODUCTION TO GAS KINETICS

CONTENTS

1.0 Introduction

The Kinetic theory of gases attempts to explain all of the concepts of classical thermodynamics, such as temperature and pressure, in terms of an underlying microscopic theory based on atoms and molecules. One of the most fundamental properties of any macroscopic system is the so-called equation of state. This is the equation that specifies the exact relation between pressures P, volume V, and temperature T for a substance. The equation of state for a gas is very different to the equation of state of a liquid.

2.0 Objective

At the end of this unit student should be able to

- Understand the basic concepts of atomic and molecular movement of gas during reactions.
- Explain the dynamics of motion in gas and its equation of state.
- Understand the characteristics and behaviour ideal gas due to heat exchange.

3.0 Main Content

3.1 Kinetic Theory of Gases

The kinetic theory of gas provides connection between the microscopic and macroscopic behaviour of gases. In this note we shall encounter an explanation for the fact that heat always flows spontaneously from a hot object to a cold object and never in the reverse. This fact is encapsulated in the second law of thermodynamics.

3.1.1 A Microscopic View of a Gas

The history of studies into the nature of gases is very long. We know from experiments too numerous to go into here (and the theory we shall present in this note) that an ordinary volume of gas is comprised of billions of molecules. At normal temperatures the molecules move in all directions at high speeds. We infer that the motion of any one molecule, if it could be followed, would be quite random. At any instant the molecules do not all move at the same speed, but with a range or distribution of speeds, some faster some slower. The speed distribution of a typical gas (N2) at 20 °C has been studied in detail; it is drawn in Figure 3.10.

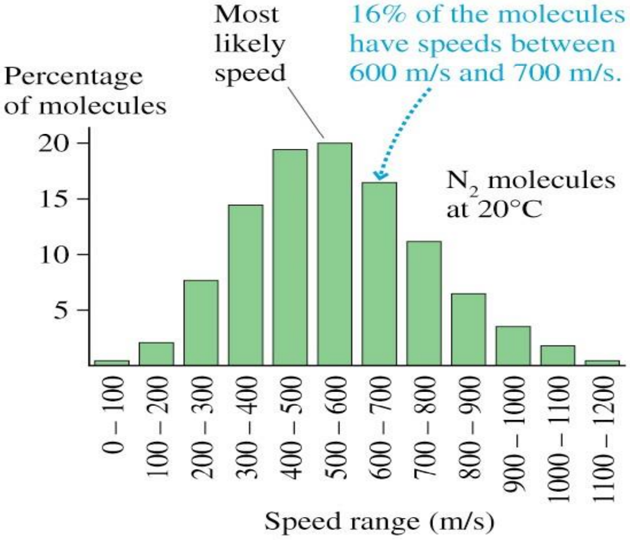

Figure 3.10: The distribution of molecular speeds in a volume of nitrogen gas at a temperature of 20 ˚C as determined by experiment

Figure 3.10 indicates that although we cannot state what speed a particular molecule moves at (because, in fact, we cannot follow its movements), we can state what is its most likely or probable speed. You can see from the figure that 20% of the molecules have a speed in the range 500 – 600 ms^{-1}. Some, of the order of 0.5%, are moving very slowly, in the range 0 – 100 ms^{-1}. Others, of the order of 0.5%, are moving very quickly, in the range 1100 -1200 ms^{-1}.

3.1.2 Assumption in kinetic energy

The bulk properties of an ideal gas are described exactly by the equation of state for ideal gas:

$$PV = nRT \qquad (3.34)$$

Eqn. 34 is known as the ideal gas law. To simplify our task of describing a gas theoretically we shall make these additional assumptions about it:

1. It consists of many identical monatomic molecules of mass *m*.

2. Its molecules move randomly within the gas and collide elastically with the walls of the container.=

We may also write the equation of state in the form;

$$f(P, V.T) = 0 \qquad (3.35)$$

where the function f depends on the particular system. The simplest system is an ideal gas. In this case, the equation of state is:

$$PV = NRT \qquad (3.36)$$

where N = Number of particles, and K is the Boltzmann constant. For Vander Waal's gas, the equation of state is:

$$\left(P + \frac{a}{V^2}\right)\left(\overline{V} - b\right) = RT \qquad (3.37)$$

where a and b are constants

Note that R = $N_A K$, $\overline{V} = \frac{v}{n}$, n = $\frac{N}{N_A}$, where n = number of moles, N= number of molecules, NA = Avogadro's number = $6.02\text{x}10^{23}$ Mol^{-1}

The equation of state of a Vander Waal's gas reduces to that of an ideal gas for a = b = 0.

The equation of state can be represented on a graph of pressure Vs. Volume, often called a PV diagram.

Example1: Derive a formula for the work done by any gas (ideal or not) which expands isobarically.

Solution: If P is a constant then,

$$W = \int_{vi}^{vf} PdV$$

$$= P\int_{vi}^{vf} dV$$

$$= [V]_{vi}^{vf}$$

$$= (V_f - V_i)$$

$= P\Delta V$

Self Assessment Exercise:

State the kinetic theory of gas and discuss its application.

3.2 Maxwell's Distribution

The Maxwell distribution of velocities describes the thermal motions of distribution of classical particles.

Consider a one-dimensional (along x-axis say) system of ideal particles of mass m. The distribution function $f(x, v_x, t)$, is such that $f(x, v_x, t)\, dxdv_x$, is the probability of finding a particle between x and $x+dx$ with velocity between v_x and $v_x + dv_x$.

The total probability must be equal to unity so we have:

$$\iint_{-Lx/2}^{Lx/2}{}_{-\infty}^{\infty} dv_x f(x, v_x, t) = 1 \qquad (3.38)$$

where is the length of the system.

The Maxwell distribution applies to a thermal distribution, at a temperature T. It is stationary (independent of t) and homogeneous (independent of x) with the specific form as:

$$F(v_x) = \frac{1}{L_x}\left(\frac{m}{2\pi KT}\right)^2 exp\left[\frac{-mv_x{}^2}{2KT}\right] \qquad (3.39)$$

Now, the standard integral:

$$\int_{-\infty}^{\infty} dx e^{-x^2/2} = (2\pi)^{1/2} \qquad (3.40)$$

For a particle in 3 - D, equation (3.38) becomes

$$\int d^3 \times d^3 v f(x, v, t) = 1 \qquad (3.41)$$

with $d^3xd^3v = dx\, dy\, dz\, dv_x\, dv_y\, dv_z$ as Cartesians coordinates

$= r^2 drd\cos\theta d\emptyset v^2 dvd\cos\theta_v\, d\emptyset_v$ (3.42) in spherical polar coordinates

Similarly, in 3 – D, equation (3.39) becomes,

$$f(v) = \frac{1}{V}\left(\frac{m}{2\pi KT}\right)^{3/2} exp\left[\frac{-mv^2}{2KT}\right] \qquad (3.43)$$

A Maxwellian distribution corresponds to a distribution of the form: *exp* [-*ε*/*KT*] where $\varepsilon = \frac{1}{2}mv^2$ is the Kinetic Energy of the particle. This form generalizes a canonical distribution for the state with energy ε of any system. We are only concerned (at the present) with the case of a classical ideal gas.

Self Assessment Exercise:

State Maxwell's distribution of kinetic energy of gas particles in motion

3.3 Thermodynamics and Equations of State for Ideal gas

3.3.1 Thermodynamics of an ideal gas

The basic properties of an ideal gas can be calculated using the Maxwellian distribution function. Two important quantities are the internal energy and the pressure for a given *N*, *V* & *T*. The internal energy of an ideal gas consisting of *N* particle in a volume *V* at a temperature *T* may be evaluated as follows: The energy of each individual particle is $\varepsilon = \frac{1}{2}mv^2$, and the total energy is found by summing oven all the particles in the system. The total energy for a statistical distribution is N times the mean energy where,

Mean energy$=< \varepsilon > \int d^3 \times d^3\, v\varepsilon f(v)$ (3.44)

i.e., $U = N< \varepsilon >$

Evaluating the mean energy of the particles by averaging over the Maxwellian distribution (equation 3.43) gives;

$$U = \frac{3}{2}NKT \qquad (3.45)$$

Which is the internal energy of an ideal gas. If the gas consists of particles (such as molecules) that can rotate or oscillate, then Equation (3.45) generalizes to $U = \frac{g}{2}NKT$ where *g* is the number of degrees of freedom. The specific form of equation (3.45) corresponds to *g* =3 degrees of freedom for a structure-less particle with these being its motion in the *x*, *y*, *z* directions for example.

3.3.2 The equation of state for an ideal gas

Another property of an ideal gas is the equation of state. The pressure may be calculated by noting that it is the force per unit area on the surface of the system.

Consider a system which is a cube of sides L, so that its volume is $V=L^3$. Consider the force on the surface in the $x - y$ plane. Each particle that reflects from the surface has change sign. This corresponds to an impulse, $2mv_x$. The problem reduces to a one-dimensional problem because the v_y and v_z components are unaffected.

The number of particles between v_x and $v_x + dv_x$ reflecting from the plane per unit time is $v_x f(v_x) dv_x/L$. The force exerted is;

$$\frac{1}{L}\int_{-\infty}^{\infty} dv_x 2mv^2 f(v_x) \qquad (3.46)$$

Evaluating this for a Maxwellian distribution, and dividing by the area L^2 to get the force per unit area gives.

$$\frac{1}{L^3}\int_{-\infty}^{\infty} dv_x 2mv^2 f(v_x) = \frac{NKT}{V} \qquad (3.47)$$

Equation (3.47) is the perfect gas Law.

3.3.3 The Entropy of an ideal gas

The entropy determines whether or not a change can occur spontaneously: a change can occur only if the entropy does not increase. The thermodynamics of an ideal gas can be determined given the equation of state, $PV = NKT$, and the assumption that C_v is independent of temperature. In particular, one may then derive explicit expressions for the entropy and for all the state functions. from $dU = TdS - PdV$, with $U = c_v T$ and $PV = NKT$, we have,

$$dS = c_v \frac{dT}{T} + NK\frac{dV}{V} \qquad (3.48)$$

$$\int dS = \int c_v \frac{dT}{T} + \int NK\frac{dV}{V} \qquad (3.49)$$

$$S = S_0 + c_v \ln\left(\frac{T}{T_0}\right) + NK \ln\left(\frac{V}{V_0}\right) \qquad (3.50)$$

where S_0, T_0 and V_0 are constants of integration.

Now, for a monatomic gas, with $c_v = \frac{3}{2}NK$,

$$S = S_0 + NK \ln\left(\frac{T^{3/2}V}{T_0{}^{3/2}V_0}\right) \qquad (3.51)$$

Combining all the constants in (3.50) and (3.51) into a single constant, and with $c_v/NK = 3/2$, the entropy can then be written as:

$$S = constant + NK \ln(T^{cv/NK} V) \qquad (3.52)$$

3.3.4 Adiabatic equation of state

Consider changes at constant entropy, called adiabatic changes. An adiabatic change is reversible. For an ideal gas, $dS = 0$ in (3.48) implies that;

$$c_v \frac{dT}{T} + NK \frac{dV}{V} = 0 \qquad (3.53)$$

Integrating (3.53), we find that,

$$\ln(T^{cv} V^{NK}) = constant \qquad (3.54)$$

Eliminating T using $PV = NKT$, this relation may be rewritten in the form

$$PV^{\Gamma} = constant, \Gamma = \frac{c_v + NK}{c_v} \qquad (3.55)$$

where Γ is called the adiabatic index.

For a monatomic gas, $c_v = 3/2$, then $\Gamma = 5/3$

3.3.5 The Entropy of mixing

A simple example of the increase in entropy is when two gases mix. Consider a system at a fixed temperature, in which initially there are N_1 particle in a volume V_1 and N_2 particles in a volume V_2, and that these are allowed to mix so that there $N_1 + N_2$ particles in a volume V_1+V_2 . The change in entropy is:

$$\Delta S = (N_1 + N_2)K \ln(V_1 + V_2) - N_1 K ln V_1 - N_2 K ln V_2$$

$$= K ln\left(\frac{(V_1+V_2)^{N_1+N_2}}{V_1{}^{N_1} V_2{}^{N_2}}\right) \qquad (3.56)$$

In the particular case when, $V_1 = V_2 = \frac{V}{2}$, and $N_1 = N_2 = \frac{N}{2}$, then Eqn. (3.56) reduces to

$$\Delta S = NK ln 2 \qquad (3.57)$$

It follows that the entropy increases in this case and it is not difficult to see that the entropy change as a result of mixing is always positive.

Remark: We have not said whether the gases are identical or not. If they are identical then the change in entropy must be zero, and yet the calculation seems to suggest that there is a change in entropy. This is referred to as the Gibb's Paradox. There is no simple physical resolution of the Gibbs Paradox within the framework of classical statistical mechanics.

Self Assessment Exercise:

Discuss the enthalpy and entropy of an ideal gas with respect to the equation of state.

4.0 Conclusion

The kinetic theory of gas can be used to determine the thermal energy and specific heats of various systems. The Maxwell distribution of velocities describes the thermal motions of distribution of classical particles. The thermodynamics of an ideal gas can be determined when its equation of state is known. The entropy determines whether or not a change can occur spontaneously: a change can occur only if the entropy does not increase.

5.0 Summary

The units revealed that kinetic theory of gases explains the concepts of classical thermodynamics, such as temperature and pressure, in terms of an underlying microscopic theory based on atoms and molecules. It further developed the equation of state, which is one of the most fundamental properties of any macroscopic system. This is the equation that specifies the exact relation between pressures P, volume V, and temperature T for a substance. The equation of state for a gas is very different to the equation of state of a liquid.

6.0. Tutor-Marked Assignment

a) State the kinetic theory and discuss its application.
b) State Maxwell's distribution of kinetic energy of gas particles in motion
c) Discuss the enthalpy and entropy of an ideal gas with respect to the equation of state.

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MODULE FOUR

NUCLEAR CHEMISTRY, LATTICE STRUCTURE AND EXTRACTION OF METALS

UNIT 1: INTRODUCTION TO NUCLEAR CHEMISTRY

CONTENTS

1.0 Introduction

When an element has two or more species of atoms, each with the same number of protons but a different number of neutrons, the different species are called *isotopes*. Different isotopes of the same element have the same *atomic number*, but they have a different *mass number*, which is the sum of the numbers of protons and neutrons in the nucleus. In the context of nuclear science, protons and neutrons are called *nucleons*, because they reside in the nucleus. The atom's mass number is often called the *nucleon number*, and a particular type of nucleus, characterized by a specific atomic number and nucleon number, is called a *nuclide*.

2.0 Objective

At the end of this unit student should be able to

- Know the nuclear model and composition of atom.
- Explain what is responsible for the stability of atom.
- Understand nuclear reaction and radioactivity of elements.

3.0 Main Content

3.1 The Nucleus and Radioactivity

3.1.1 The nucleus of an atom

Atoms of each element are not necessarily identical; they can differ with respect to the number of neutrons in their nuclei. When an element has two or more species of atoms, each with the same number of protons but a different number of neutrons, the different species are called *isotopes*. Different isotopes of the same element have the same atomic number, but they have a different *mass number*, which is the *sum of the numbers of protons and neutrons in the nucleus*. In the context of nuclear science, protons and neutrons are called *nucleons*, because they reside in the nucleus. The *atom's mass number* is often called the *nucleon number*, and a particular type of nucleus, characterized by a specific atomic number and nucleon number, is called a *nuclide*. Nuclides are represented in chemical notation by a subscript atomic number (Z) and superscript nucleon number (A) on the left side of the element's symbol (X) as shown in Figure 4.1:

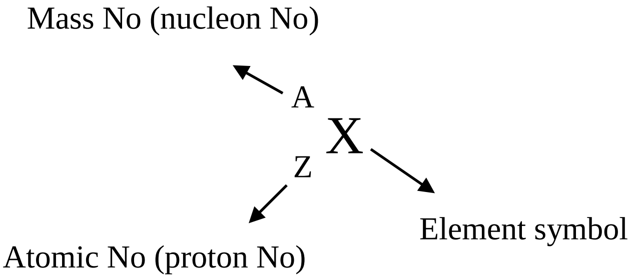

Figure 4.1: Chemical notation of a nuclide or Nuclide symbol

For example, the most abundant nuclide of uranium has 92 protons and 146 neutrons, so its atomic number is 92, its nucleon number is 238 (92 + 146), and its symbol is $^{238}_{92}\mathrm{U}$. Often, the atomic number is left off of the symbol. Nuclides can also be described with the name of the element followed by the nucleon number. Therefore, $^{238}_{92}\mathrm{U}$ is commonly described as $^{238}\mathrm{U}$ or uranium-238.

Example 1

A nuclide that has 26 protons and 33 neutrons is used to study blood chemistry. Write its nuclide symbol in the form of $^{A}_{Z}\mathrm{X}$. Write two other ways to represent this nuclide.

Solution

Because this nuclide has 26 protons, its atomic number, Z, is 26, identifying the element as iron, Fe. This nuclide of iron has 59 total nucleons (26 protons + 33 neutrons), so its nucleon number, A, is 59. Therefore, its nuclide symbol is written as; $^{59}_{26}\mathrm{Fe}$ or $^{59}\mathrm{Fe}$ or iron-59

3.1.2 Radioactivity of an atom

The spontaneous decay of an unstable nucleus with accompanying emission of radiation is termed radioactivity. As the atomic number increases, more neutrons are needed to help bind the nucleus together, so there is a high neutron: proton ratio. Many heavy elements undergo several sequential emissions before forming a more stable nuclei.

Self Assessment Exercise:
Explain the nucleus of an atom and its radioactivity

3.2 Nuclear Stability

Two forces act upon the particles within the nucleus to produce the nuclear structure. One, called the ***electrostatic force*** (or ***electromagnetic force***), is the force that causes opposite electrical charges to attract each other and like charges to repel each other. The positively charged protons in the nucleus of an atom have an electrostatic force pushing them apart. The other force within the nucleus, called the ***strong force***, holds nucleons (protons and neutrons) together.

If one proton were to encounter another, the electrostatic force pushing them apart would be greater than the strong force pulling them together, and the two protons would fly in separate directions. Therefore, nuclei that contain more than one proton and no neutrons do not exist. Neutrons can be described as the nuclear glue that allows protons to stay together in the nucleus. Because neutrons are uncharged, there are no electrostatic repulsions between them and other particles. At the same time, each neutron in the nucleus of an atom is attracted to other neutrons and to protons by the strong force. Therefore, adding neutrons to a nucleus increases the attractive forces holding the particles of the nucleus together without increasing the amount of repulsion between those particles. As a result, although a nucleus that consists of only two protons is unstable, a helium nucleus that consists of two protons and two neutrons is very stable. The increased stability is reflected in the significant amount of energy released when two protons and two neutrons combine to form a helium nucleus as shown in Figure 4.2.

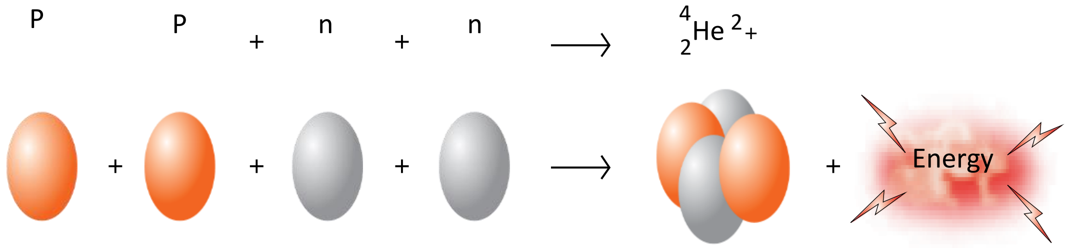

Figure 4.2: Formation of helium nucleus

For many of the lighter elements, the possession of an equal number of protons and neutrons leads to stable atoms. For example, carbon-12 atoms, $^{12}_{6}C$, with six protons and six neutrons, and oxygen-16 atoms, $^{16}_{8}O$, with eight protons and eight neutrons, are both very stable. Larger atoms with more protons in their nuclei require a higher ratio of neutrons to protons to balance the increased electrostatic repulsion between protons. Table 4.1 shows the steady increase in the neutron-to-proton ratios of the most abundant isotopes of the elements in group 15 on the periodic table.

Table 4.1: Neutron-to-Proton Ratio for the Most Abundant Isotopes of the Group 15 Elements

Element	No of neutrons	No of protons	Neutrons-to-Proton ratio
Nitrogen, N	7	7	1 to 1

Phosphorus, P	16	15	1.07 to 1
Arsenic, As	42	33	1.27 to 1
Antimony, Sb	70	51	1.37 to 1
Bismuth, Bi	146	83	1.52 to 1

There are 264 stable nuclides found in nature. The graph in Figure 4.1 shows the neutron-to-proton ratios of these stable nuclides. Collectively, these nuclides fall within what is known as the band of stability.

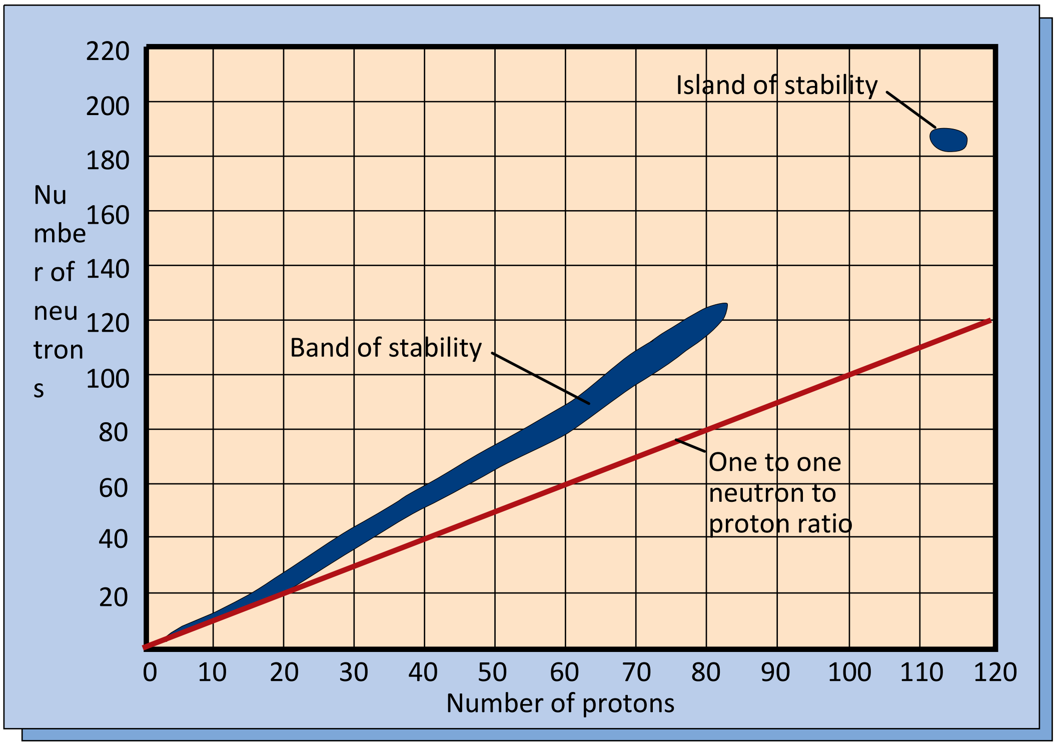

Figure 4.1: The band of stability

A nuclide containing numbers of protons and neutrons that place it outside this band of stability will be unstable until it undergoes one or more nuclear reactions that take it into the band of stability. We call these unstable atoms ***radioactive nuclides***, and the changes they undergo to reach stability are called ***radioactive decay***. Note that the band of stability stops at 83 protons. All of the known nuclides with more than 83 protons are radioactive, but scientists have postulated that there should be a small island of stability around the point representing 114 protons and 184 neutrons. The relative stability of the heaviest atoms that have so far been synthesized in the laboratory suggests that this is true.

Self Assessment Exercise:

Discuss the bands of nuclear stability in atoms.

3.3 Types of Radioactive Emissions

One of the ways that nuclides with more than 83 protons change to reach the band of stability is to release two protons and two neutrons in the form of a helium nucleus, which in this context is called an ***alpha particle.*** Natural uranium, which is found in many rock formations on earth, has three isotopes that all experience alpha emission, the release of alpha particles. The isotope composition of natural uranium is 99.27% uranium-238, 0.72% uranium-235, and a trace of uranium-234. The nuclear equation for the alpha emission of uranium-238, the most abundant isotope, is:

$$^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He} \qquad (4.1)$$

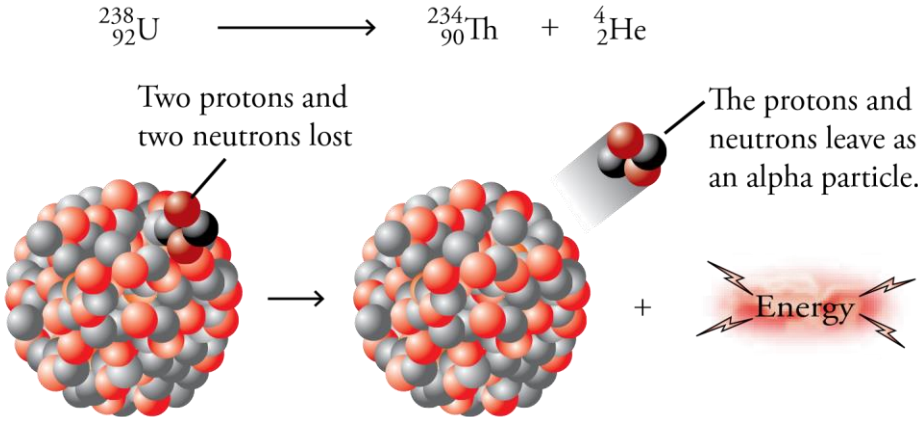

Figure 4.2: Alpha emission in natural uranium

In nuclear equations for ***alpha emission***, the alpha particle is written as either α or $^{4}_{2}$He. Note that in alpha emission, the radioactive nuclide changes into a different element, with an atomic number that is lower by 2 and a mass number that is lower by 4.

Some radioactive nuclides have a neutron-to-proton ratio that is too high, placing them above the band of stability. To reach a more stable state they undergo ***beta emission*** (β^-). In this process, a neutron becomes a proton and an electron. The proton stays in the nucleus, and the electron, which is called a beta particle in this context, is ejected from the atom.

$n \rightarrow p + e^-$

In nuclear equations for beta emission, the electron is written as either β, β^-, or ${}^{0}_{-1}e$. Iodine -131, which has several medical uses, including the measurement of iodine uptake by the thyroid, is a *beta* emitter Figure 4.3:

$$ {}^{131}_{55}\mathrm{I} \rightarrow {}^{131}_{54}\mathrm{Xe} + {}^{0}_{-1}\mathrm{e} \qquad (4.2) $$

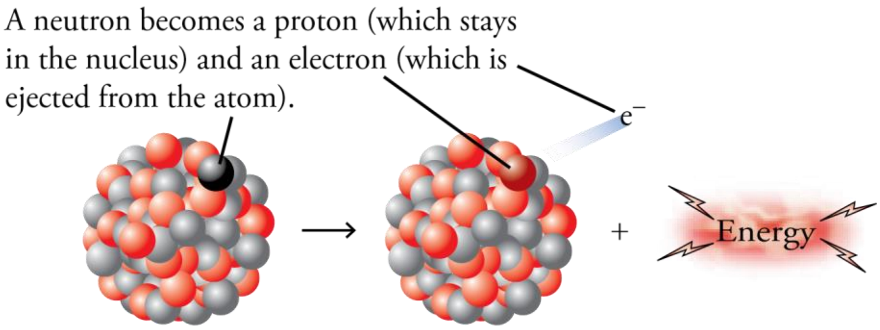

Figure 4.3: Beta emission/radiation

Note that in beta emission, the radioactive nuclide changes into a different element, with an atomic number that is higher by 1 but the same mass number.

If a radioactive nuclide has a neutron-to-proton ratio that is too low, placing it below the band of stability, it can move toward stability in one of two ways, positron emission or electron capture. ***Positron emission*** (β^+) is similar to beta emission, but in this case, a proton becomes a neutron and an anti-matter electron, or anti-electron. The latter is also called a ***positron*** because, although it resembles an electron in most ways, it has a positive charge. The neutron stays in the nucleus, and the positron speeds out of the nucleus at high velocity.

$p \rightarrow n + e^+$

In nuclear equations for positron emission, the electron is written as either β^+, ${}^{0}_{-1}e$ or ${}^{0}_{1}e$. Potassium-40, which is important in geologic dating, undergoes positron emission:

$$ {}^{40}_{19}\mathrm{K} \rightarrow {}^{40}_{18}\mathrm{Ar} + {}^{0}_{+1}\mathrm{e} \qquad (4.3) $$

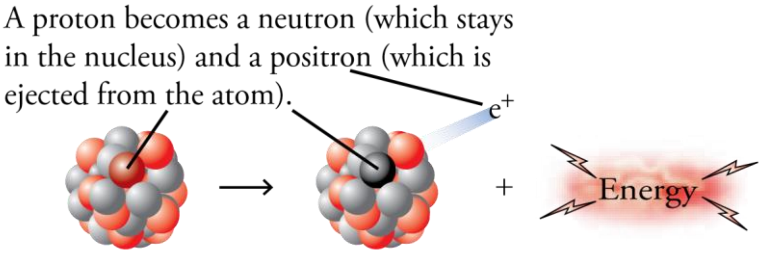

Figure 4.4: Positron emission from potassium

Note that in positron emission, the radioactive nuclide changes into a different element, with an atomic number that is lower by 1 but the same mass number.

The second way that an atom with an excessively low neutron-to-proton ratio can reach a more stable state is for a proton in its nucleus to capture one of the atom's electrons. In this process, called electron capture, the electron combines with the proton to form a neutron.

$e^- + p \rightarrow n$

Iodine-125, which is used to determine blood hormone levels, moves toward stability through electron capture. Like positron emission, electron capture causes the radioactive nuclide to change to a new element, with an atomic number that is lower by 1 but with the same mass number.

$$ {}^{0}_{-1}e + {}^{125}_{53}I \rightarrow {}^{125}_{52}Te \qquad (4.4) $$

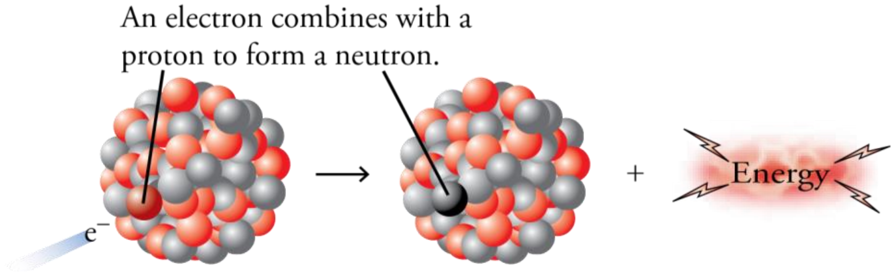

Figure 4.5: Electron capture process

Because radioactive decay leads to more stable products, it always releases energy. Some of this energy is released in the form of kinetic energy, adding to the motion of the product particles, but often some of it is given off as the form of radiant energy called ***gamma rays***. Gamma rays can be viewed as streams of high energy photons. For example, cesium-137 is a beta emitter that also releases gamma radiation. The energy released in the beta emission leaves the product element, barium-137, in an excited state. When the barium-137 descends to its ground state, it gives off photons in the gamma ray region of the radiant energy spectrum, Figure 4.6.

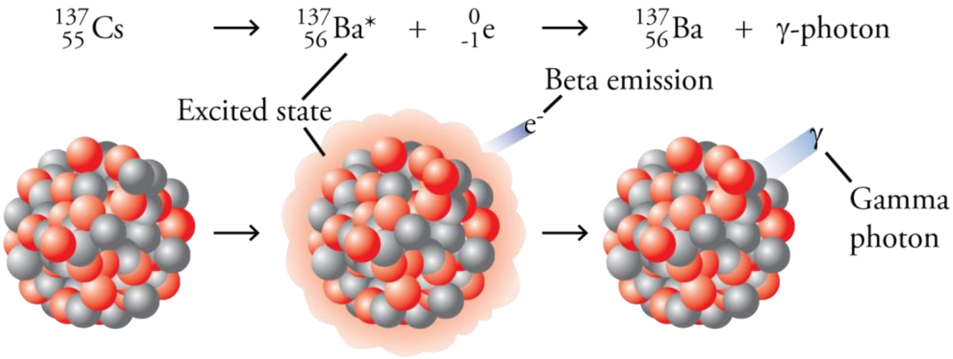

Figure 4.6: Gamma rays radiation

Self Assessment Exercise:

Describe the types of radioactive emissions.

3.4 Nuclear Reactions and Nuclear Equations

3.4.2 Nuclear reactions

During nuclear reaction, matter is converted to energy and huge amounts of energy is released. The energy thus generated, can be harvested, transmitted and distributed for use thereafter. Nuclear reactions involve a specific isotope of an element, and different isotopes of an element may undergo different nuclear reactions.

3.4.2 Differences between chemical reactions and nuclear reactions

1. Atomic number of nuclei may change (elements are converted to other elements or an element can be converted to an isotope of that element).
2. Nuclear reactions involve changes in the nucleus, whereas chemical reactions involve the loss, gain, and sharing of electrons.
3. Different isotopes of the same element may undergo very different nuclear reactions, even though an element’s isotopes all share the same chemical characteristics.
4. Unlike chemical reactions, the rates of nuclear reactions are unaffected by temperature, pressure, and the presence of other atoms to which the radioactive atom may be bonded.
5. Nuclear reactions, in general, give off much more energy than chemical reactions.

3.4.3 Nuclear equations

The equations that describe nuclear reactions are different from those that describe chemical reactions because in nuclear equations charge is disregarded. If you study the nuclear changes for alpha, beta, and positron emission already described in this section, you will see that the products must be charged. For example, when an alpha particle is released from a uranium-238 nucleus, two positively charged protons are lost. Assuming that the uranium atom was uncharged initially, the thorium atom formed would have a -2 charge. Because the alpha particle is composed of two positively charged protons and two uncharged neutrons (and no electrons), it has a +2 overall charge.

$$^{238}_{92}U \rightarrow ^{234}_{90}Th^{2-} + ^{4}_{2}He^{2+} \qquad (4.5)$$

The ions lose their charges quickly by exchanging electrons with other particles. Because we are usually not concerned about charges for nuclear reactions, and because these charges do not last very long, they are not usually mentioned in nuclear equations. Therefore, nuclear equations must clearly show the changes in the atomic numbers of the nuclides (the number of protons) and the changes in their mass numbers (the sum of the numbers of protons and neutrons). Note that in each of the following equations, the sum of the superscripts (mass numbers, A) for the reactants is equal to the sum of the superscripts for the products. Likewise, the sum of the subscripts (atomic numbers, Z) for the reactants is equal to the sum of the subscripts for the products.

Self Assessment Exercise:

Briefly explain nuclear reactions and nuclear equations.

4.0 Conclusion

Certain nuclides are more stable than we might have expected them to be if the trend for the change in binding energy per nucleon were smooth. During nuclear reaction, matter is converted to energy and huge amounts of energy is released. Nuclear reactions involve a specific isotope of an element, and different isotopes of an element may undergo different nuclear reactions.

5.0 Summary

A nuclide containing numbers of protons and neutrons that place it outside the band of stability will be unstable until it undergoes one or more nuclear reactions that take it into the band of stability. We call these unstable atoms radioactive nuclides, and the changes they undergo to reach stability are called radioactive decay. The band of stability stops at 83 protons. All of the known nuclides with more than 83 protons are radioactive. Although, scientists have postulated that there should be a small island of stability around the point representing 114 protons and 184 neutrons.

6.0. Tutor-Marked Assignment

a) Explain the nucleus of an atom and its radioactivity.
b) Discuss the bands of nuclear stability in atoms
c) Describe the types of radioactive emissions.
d) Differentiate between chemical reaction and nuclear reaction.
e) Briefly explain nuclear reactions and nuclear equations.

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UNIT 2: SOLIDS AND LATTICE STRUCTURES

CONTENTS

1.0 Introduction

Solid can be categorized into either crystalline or non-crystalline solids. A crystal is a solid in which the constituent atoms, molecules, or ions are packed in a regularly ordered, repeated pattern extending in all three spatial dimensions. Crystals are structures found in crystalline solids. Solids are made up of crystals, while crystals are made up of *lattice* and *basis*. The lattice is the underlying pattern of the crystal. A crystal structure consists of identical copies of the same physical units called the *basis* assigned to all the points of the lattice. The crystal is being described by the lattice that can contain more than one atom/ion assigned to each point of the lattice, usually referred to as a *basis*.

2.0 Objective

At the end of this unit student should be able to

- Understand the nature and structure of solids.
- Know the specification of unit cells and primitive cells.
- Understand the planes and directions of basis and lattice in crystals.

3.0 Main Content

3.1 The Nature and Structure of Crystalline Solids

3.1.1 The nature of solids

In an assembly of atoms or molecules a solid phase is formed whenever the interatomic (intermolecular) attractive forces significantly exceed the disruptive thermal forces and thus restrict the mobility of atoms, forcing them into more-or-less fixed positions. From energy considerations it is evident (as discussed in previous units) that in such solids the atoms or molecules will always attempt to assume highly ordered structures which are characterized by symmetry. Depending on the nature of the acting interatomic forces, all solids may be subdivided into:

(a) Ionic solids (NaCl)

(b) Covalent solids (Diamond)

(c) Metallic solids (Fe, Ni, etc.)

(d) Van der Waals solids (ice, solid He)

Solids as we encounter them in nature may or may not reflect the internally ordered arrangement in their appearance. We find, for example, well-formed quartz crystals, garnets, diamonds and snowflakes which are all characterized by flat bounding planes which intersect at characteristic angles. On the other hand, we also observe rounded stones and man-made cast solid objects with no external evidence of internal order Figure 4.7.

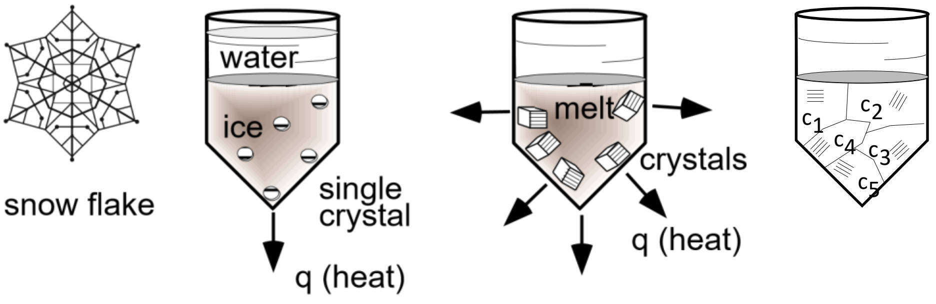

Figure 4.7: Crystals, internal order and external appearance

To understand the external appearance of the solid state it is necessary to consider the formation of solids from different phases. Solids, for example, are formed upon cooling of liquids (melts) - by freezing or solidification; this solidification process normally proceeds in total confinement and the resulting “cast” structure will have an external appearance which reflects in detail the confining geometry (and not the internal order). Moreover, depending on solidification conditions, the solid body may be either a single crystal or polycrystalline. Polycrystalline solids (in excess of 95% of the solid state encountered) may be thought of as an assembly of microscopic single crystals with random orientation held together like a maze structure by the interwoven irregular shapes of the individual crystals.

A typical example of an “unconfined” phase transformation is the formation of snowflakes where the external boundaries of the solid have assumed crystalline appearance, reflecting in detail the internally ordered molecular (H_2O) arrangement.

Another unconfined formation of a solid is precipitation from solution (sugar crystals, CuSO4 and the like). Similarly, the formation of crystals from the vapour phase leads to bodies which externally reflect elements of internal order.

3.1.2 The structure of crystal

From the earlier discussion it should be apparent that, when strong interatomic forces exist, atoms tend to pack closely together - the closeness of packing being particularly pronounced in the solid state. In this case atoms can be regarded as hard spheres and the problem of close packing can be treated as one in which the whole assembly has a tendency toward efficient packing. A little thought or a few simple experiments with ping-pong balls quickly convince us that regular arrangements of the spheres generally lead to more compact assemblies than irregular arrangements, Figure 4.8. The same principle applies to arrangement of atoms in the solid state. Where strong attractive forces are exerted we find that the atoms or molecules concerned arrange themselves in a regular three-dimensional pattern.

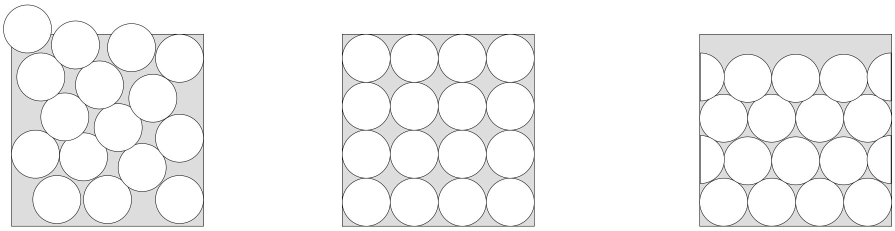

Figure 4.8: The packing density of ordered solid systems

It is this regularity which is the basis of crystallinity in materials: i.e., a crystal structure is nothing more than an orderly array of atoms or molecules. This definition of a crystal is distinct from the popular concept based on observation of external symmetry of crystals, often seen during the study of elementary chemistry, in which some crystals appear cubic, others needle-shaped and so on. The regular external shape is obtained only when the conditions of crystallization are favourable to development of flat, geometric faces. In most instances, particularly with metals, these conditions are absent, and the crystals have irregular surfaces even though the internal arrangement is perfectly geometric.

3.1.3 Space Lattice

Atomic arrays in crystals are conveniently described with respect to a three-dimensional net of straight lines. Consider a lattice of lines, as in Figure 4.9, dividing space into equal-sized prisms which stand side-by-side with all faces in contact, thereby filling all space and leaving no voids. The intersections of these lines are points of a space lattice, i.e., a geometrical abstraction which is useful as a reference in describing and correlating symmetry of actual crystals. These lattice points are of fundamental importance in describing crystals for they may be the positions occupied by individual atoms in crystals or they may be points about which several atoms are clustered. Since prisms of many different shapes can be drawn through the points of a space lattice to partition it into cells, the manner in which the network of reference lines is drawn is arbitrary. It is not necessary that the lines be drawn so that atoms lie only at corners of unit prisms.

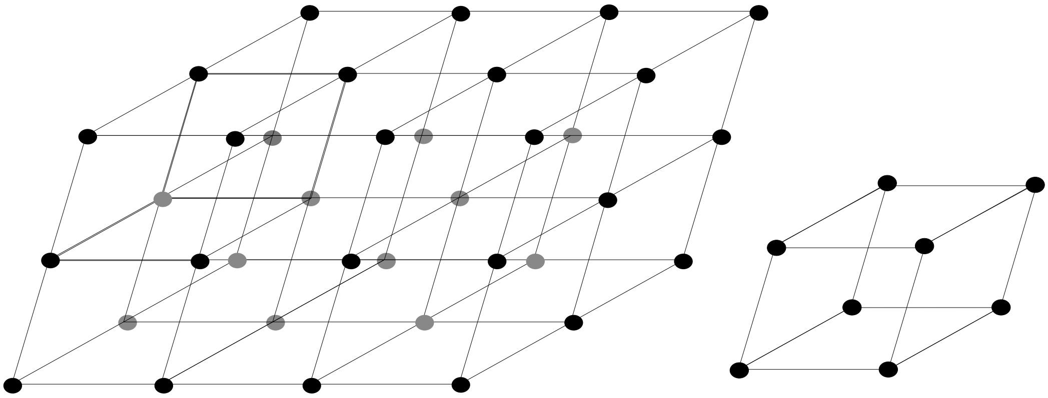

Figure 4.9: Space lattice prism with atoms lying at centres and corners

3.1.4 Crystal system and bravais lattice

There are fourteen space lattices (fig. 4). That is, no more than fourteen ways can be found in which points can be arranged in space so that each point has identical surroundings. Of course, there are many more than fourteen ways in which atoms can be arranged in actual crystals; thus there are a great number of crystal structures. Too often the term “lattice” is loosely used as a synonym for “structure”, an incorrect practice which is frequently confusing. The distinction can be clearly seen if we remember that a space lattice is an array of points in space. It is a geometrical abstraction which is useful only as a reference in describing and correlating symmetry of actual crystals. A crystal structure, however, is the arrangement of atoms or molecules which actually exists in a crystal. It is a dynamic, rather than a static, arrangement and is subject to many imperfections.

Name **7 Crystal systems** **14 Bravais Lattices**

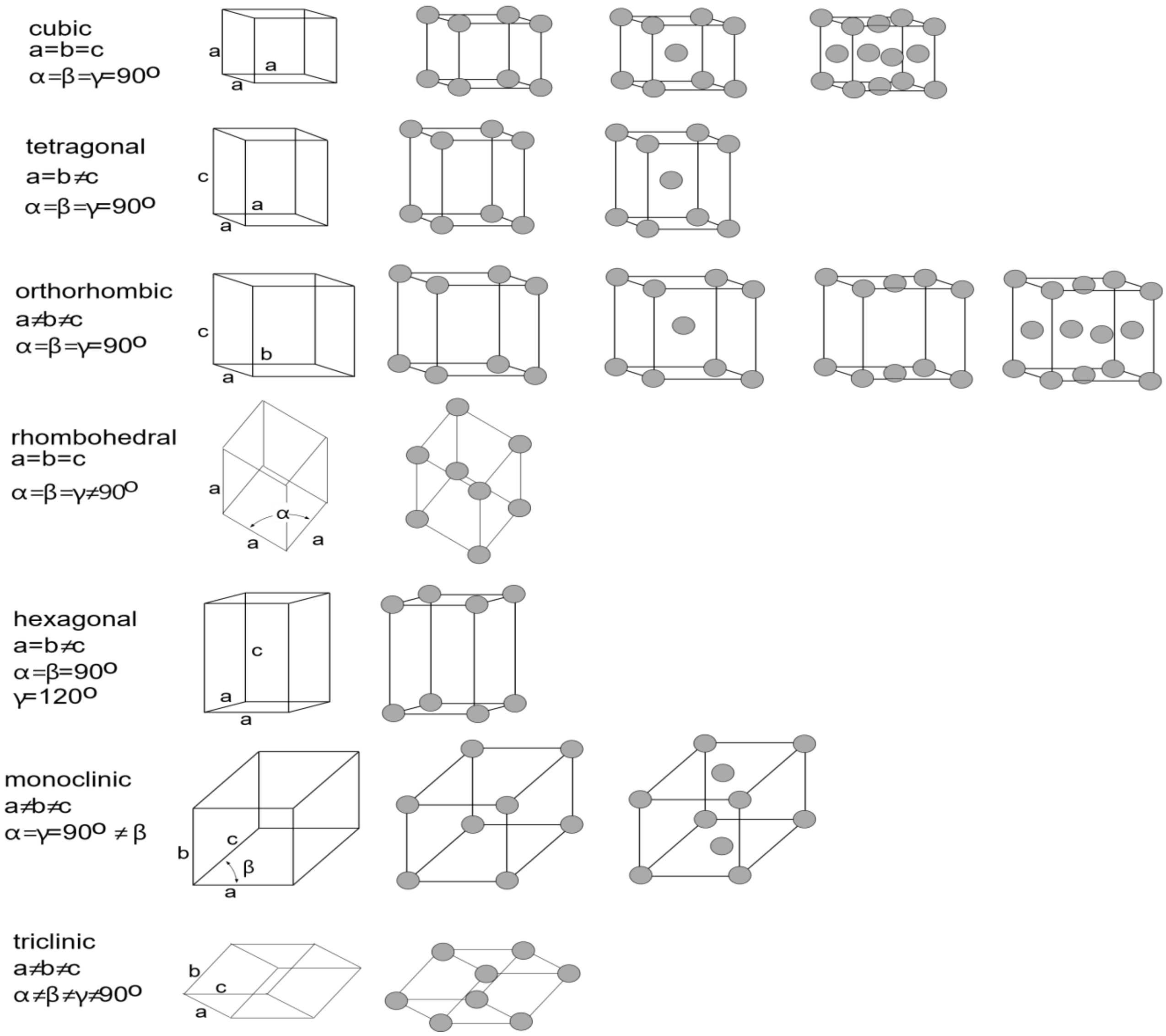

Figure 4.10: The 7 crystal systems and the 14 Bravais lattices

Self Assessment Exercise:

Briefly describe the nature and structure of crystalline solids

3.2 Specification of unit cells and primitive cells

3.2.1 Specifying unit cells and primitive cells

To specify a given arrangement of points in a space lattice, it is customary to identify a unit cell with a set of coordinate axes, chosen to have an origin at one of the lattice points (Figure 4.11). In a cubic lattice, for example, we choose three axes of equal length that are mutually perpendicular and form three edges of a cube. Each space lattice has some convenient set of axes, but they are not necessarily equal in length or orthogonal. Seven different systems of axes are used in crystallography, each possessing certain characteristics as to equality of angles and equality of lengths. These seven crystal systems are tabulated in Table 4.1 (to be considered in conjunction with Figure 4.10.

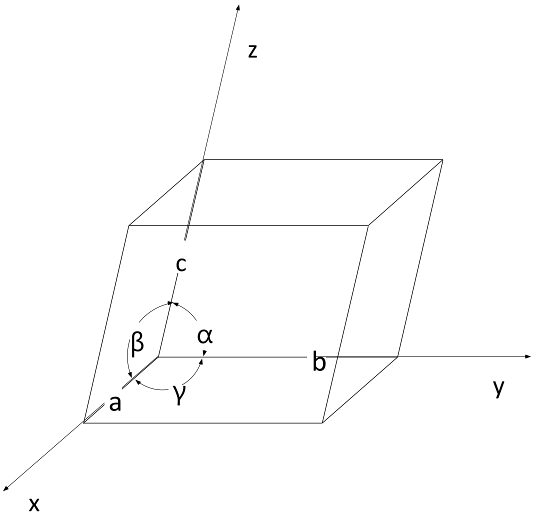

Figure 4.11: Unit cell parameters

Table 4.1: The seven crystal systems

System	Parameters	Interaxial angles
Triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma$
Monoclinic	$a \neq b \neq c$	$\alpha = \gamma = 90° \ \beta$
Orthorhombic	$a \neq b \neq c$	$\alpha = \beta = \gamma$
Tetragonal	$a = b \neq c$	$\alpha = \beta = \gamma$
Cubic	$a = b = c$	$\alpha = \beta = \gamma = 90°$
Hexagonal	$a = b \neq c$	$\alpha = \beta = 90°, \gamma = 120°$
Rhombohedral	$a = b = c$	$\alpha = \beta = \gamma \ 90°$

3.2.2 Unit cells vs Primitive cells

In the literature we often find reference to unit cells and to primitive cells. The *primitive cell* may be defined as a geometrical shape which, when repeated indefinitely in three dimensions, will fill all space and is the equivalent of one atom. The unit cell differs from the primitive cell in that it is not restricted to being the equivalent of one atom. In some cases the two coincide. For instance, in Figure 4.10 all fourteen space lattices are shown by their unit cells. Of these fourteen, only seven (which are those?) are also primitive cells.

Primitive cells are drawn with lattice points at all corners, and each primitive cell contains the equivalent of one atom. For instance, a simple cubic unit cell has an atom at each corner. However, at any of these given corners, this atom must be shared with seven other identical cubes which fill the volume surrounding this point. Thus there is effectively only 1/8 of the atom which can be assigned to that particular *unit cell*. Since there are eight corners in a cube, there is the equivalent of one atom, and thus the *primitive cell* and unit cell coincide.

Continuing, consider the body-centred cubic (BCC) lattice. In this case there is one atom at the centre of the cube and one atom contributed by the eight corners. This cell, then, has two atoms and, to avoid confusion, should be termed a unit cell. In the face-centred cubic lattice there are six face atoms, but each face atom is shared by two cells. Consequently, each face contributes 1/2 an atom. The faces thus contribute three atoms and the corners one, for a total of four atoms in the unit cell. The face-centred cubic structure (FCC) can also be considered as four interpenetrating simple cubic cells.

In the study of crystals the primitive cell has limited use because the unit cell more clearly demonstrates the symmetrical features of a lattice. In other words, the unit cell can usually be visualized readily whereas the primitive cell cannot. For example, the cubic nature of the face-centred cubic lattice is immediately apparent in the unit cell, but it is not nearly so obvious in the rhombohedral primitive cell.

Self Assessment Exercise:

Briefly discuss on specification of unit cell and primitive cells and their differences.

3.3 Lattice Plane and Directions

It is desirable to have a system of notation for planes within a crystal or space lattice such that the system specifies orientation without giving position in space. Miller indices are used for this purpose. These indices are based on the intercepts of a plane with the three crystal axes – i.e., the three edges of the unit cell. The intercepts are measured in terms of the edge lengths or dimensions of the unit cell which are the unit distances from the origin along the three axes. For instance, the plane that cuts the x-axis at a distance from the origin equal to one-half the x-dimension of the cell is said to have an x-intercept equal to 1/2, and if it cuts the y-axis at 1/2 the y-dimension of the cell, the y-intercept is 1/2, regardless of the relative magnitudes of the x- and y-dimensions. If a plane is parallel to an axis, it intercepts the axis at infinity. To determine Miller indices (hkl) of a plane, we take the following steps:

1. Find the intercepts on the three axes in multiples or fractions of the edge lengths along each axis.

2. Determine the reciprocals of these numbers.

3. Reduce the reciprocals to the three smallest integers having the same ratio as the reciprocals.

4. Enclose these three integral numbers in parentheses, e.g., (hkl).

A cube has six equivalent faces. If we have a definite orientation and wish to discuss one specific plane of these six, it is possible to specify this plane by using the proper Miller indices. Parentheses are used around the Miller indices to signify a specific plane. On the other hand, it is often advantageous to talk about planes of a “form” – i.e., a family of equivalent planes such as the six faces of a cube. To do this it is customary to use the Miller indices, but to enclose them in curly brackets (braces).

Self Assessment Exercise:
Explain the planes and direction of lattice in crystalline solids.

4.0 Conclusion

Solids are made up of crystals and crystals are made up of lattice. The lattice is the underlying pattern of the crystal. A crystal structure consists of identical copies of the same physical units called the *basis* assigned to all the points of the lattice. The crystal is being described by the lattice that can contain more than one atom/ion assigned to each point of the lattice, usually referred to as a *basis*.

5.0 Summary

The units deals with the structures found in crystalline solids i.e. crystals. Solid can be categorized into either crystalline or non-crystalline solids. A crystal is a solid in which the constituent atoms, molecules, or ions are packed in a regularly ordered, repeated pattern extending in all three spatial dimensions x, y, and z.

6.0. Tutor-Marked Assignment

a) Briefly describe the nature and structure of crystalline solids.
b) Differentiate between unit cells and primitive cells.
c) Discuss the specification of unit cells and primitive cells.
d) Explain the planes and direction of lattice in crystalline solids.

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UNIT 3: GENERAL PRINCIPLES OF EXTRACTION OF METALS

CONTENTS

1.0 Introduction

Metal extraction process is the combination of similar and unique kind of steps known as *Unit Processes/Unit Operations*. Unit operations are physical operations, while unit processes are chemical processes, both of which are combined to extract metals from the ores. Some metal oxides get reduced easily while others are very difficult to be reduced (reduction means electron gain or electronation). In any case, heating is required. To understand the variation in the temperature requirement for thermal reductions (pyrometallurgy) and to predict which element will suit as the reducing agent for a given metal oxide (MxOy),

2.0 Objective

At the end of this unit student should be able to

- Know the various techniques used in extraction of metals.
- Understand the processes involved in extraction of metals.
- Understand the model of selecting the correct process routes, reactors, optimize and control metallic extraction.

3.0 Main Content

3.1 The Principle of Extractive Metallurgy

3.1.1 What is extractive metallurgy?

Extractive metallurgy deals with extraction of metals from its naturally existing ore/minerals and refining them. These minerals include inorganic compounds with more than one metals in association with non-metals like S, O, N etc. Naturally existing minerals include sulphides, oxides, halides like: Hematite (Fe_2O_3), Magnetite (Fe_3O_4), Chalcopyrite ($CuFeS_2$), Dolomite ($CaCO_3.MgCO_3$) etc.

3.1.2 Sources of metals

Sources of metals include:

Earth Crust (Aluminium: 8.1%, Iron 5.1%, Calcium: 3.6%, Sodium: 2.8%, Potassium: 2.6%, Magnesium: 2.1%, Titanium: 2.1%, Manganese: 0.10%).

Ocean water (Na: 10500 g/ton, Mg: 1270 g/ton, Ca: 400 g/ton, K: 380 g/ton);

Ocean nodules (Mn: 23.86%, Mg 1.66%, Al 2.86%, Fe 13.80 %..)

Recycled scrap (at the end of metals' life)

The mineral map of the word is shown in Figure 4.12.

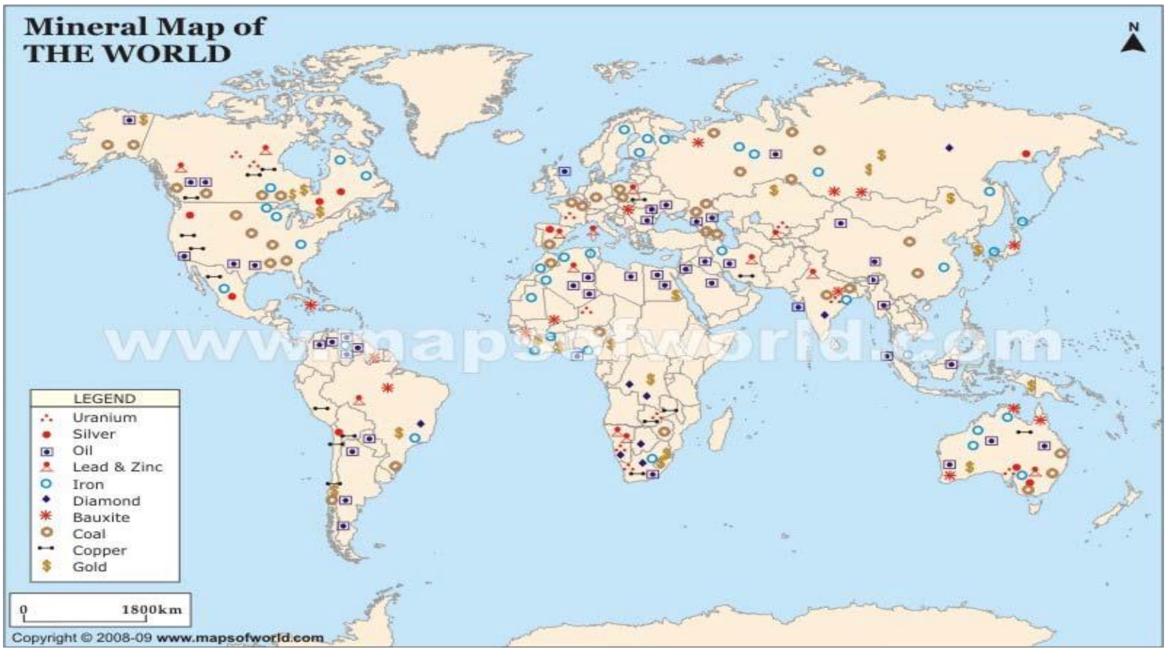

Figure 4.12: The mineral map of the world (www.mapsofworld.com)

3.1.3 Types of ores

1. *Oxide ores*: Examples: Fe_2O_3, Fe_3O_4

Apart from Fe, other heavy metals which are produced from oxide ores are: Manganese, Chromium, Titanium, Tungsten, Uranium and Tin.

2. *Sulphide ores*: Copper ore (CuFeS2, Chalcopyrite), Sphalerite (Zn,Fe)S, Galena PbS, Pyrite FeS_2. Others sulphide ores include: Nickel, Zinc, Mercury and Molybdenum

3. *Halide ores*: Rock salts of Sodium, Magnesium chloride in sea water

3.1.4 Metal extraction processes

Metal extraction process is the combination of similar and unique kind of steps known as Unit Processes/unit operations.

- i. *Unit operations*: Physical operations like crushing, grinding, sizing, mixing through agitation, filtration, distillation, and comminution.
- ii. *Unit processes*: Chemical processes like leaching, smelting, roasting, Electrolysis, decarburization, dephosphorization, degassing, deoxidation etc.

Combination of all unit steps/processes results in a chart known as the extraction *Flow-Sheets*.

3.1.5 Classification of unit processes/operation by different criteria

a) *According to phases involved*:

i. Gas-Solid: Roasting, Gas reduction
ii. Gas-liquid: steelmaking blowing/refining, Distillation
iii. Liquid-Liquid: Slag metal reactions
iv. Solid-solid: Leaching, precipitation etc.

b) *According to equipment involved*:

i. Fixed bed: Sintering, percolation leaching
ii. Fluidized bed: Fluidized roasting and reduction
iii. Shaft furnace: Iron blast furnace, lime calcination kiln
iv. Rotary kiln: Drying and calcination
v. Ritort: Coke open, carbothermic zinc production, Mg production by Pidgeon

vi. Reverberatory furnace: Matte smelting (Cu etc.), open hearth steelmaking
vii. Electric arc furnace: Steelmaking, matte smelting, ferro alloy production
viii. Cell for salt fuse electrolysis: Production and refining of aluminium
ix. Cell for aqueous electrolysis: Electrolytic reduction and refining

c) *According to chemical reaction*

i. Oxidation: Roasting, sintering, LD steelmaking
ii. Reduction: Blast furnace ironmaking
iii. Slag metal reactions: Steelmaking, matte smelting
iv. Chlorination: Titanium (converting to tetrachloride)
v. Electrolytic reduction: Zinc and Aluminium production
vi. Electrolyte refining: Refining of Copper and Nickel

d) *According to methods of metal extraction*

i. Physical separation/Mineral processing

The objective is to concentrate the metallic content in the ore, achieved by a series of comminution (crushing and grinding), screening and separation process

ii. *Pyrometallurgy*

It involves the smelting, converting and refining of metal concentrate.

iii. *Hydrometallurgy*

It involves the precipitation of metal in an aqueous solution.

iv. *Electrometallurgy*

Electrolysis process to extract metal. Electro-winning: Extraction of the metal from electrolyte; Electro-refining: Refining of impure metals in the form of an anode.

Majority of metals are extracted by pyrometallurgical route because it is fast, easily adaptable and is the cheapest of all.

Self Assessment Exercise:

What is extractive metallurgy and briefly state the sources of metals and types of ores.

3.2 Extraction of Crude Metal from Concentrated Ore

3.2.1 Calcination:

Calcination is a process of heating the ore strongly either in a limited supply of air or in the absence of air. During this process the following changes take place:

i. The volatile matter is removed.
ii. Moisture is removed.
iii. Water from the hydrated oxides is removed.
iv. The ore becomes porous.

Carbonates decompose to oxides in the form;

$$Fe_2O_3.xH_2O(s) \xrightarrow{\Delta} Fe_2O_3\ (s) + xH_2O(g)$$
$$ZnCO_3\ (s) \xrightarrow{\Delta} ZnO(s) + CO_2(g)$$
$$CaCO_3.MgCO_3(s) \xrightarrow{\Delta} CaO(s) + MgO(s\ ) + 2CO_2(g)$$

3.2.2 Roasting:

Roasting is the process of heating the ore strongly in the presence of excess or regular supply of air in a furnace at a temperature below the melting point of the metal.

Examples of Sulphide ores roasted include:

$$2ZnS + 3O_2 \rightarrow 2ZnO + 2SO_2$$
$$2PbS + 3O_2 \rightarrow 2PbO + 2SO_2$$
$$2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 2SO_2$$

3.2.2 Reduction of oxide to the metal:

Reduction of the metal oxide usually involves heating it with some other substance acting as reducing agent (C or CO or even another metal). The reducing agent (e.g., carbon) combines with the oxygen

$$M_xO_y + yC \rightarrow xM + y\ CO$$

During reduction, the oxide of a metal decomposes:

$$M_xO(s) \rightarrow xM \text{ (solid or liq) } + \frac{1}{2}O_2 \text{ (g)}$$

Some metal oxides get reduced easily while others are very difficult to be reduced (reduction means electron gain or electronation). In any case, heating is required. To understand the variation in the temperature requirement for thermal reductions (pyrometallurgy) and to predict which element will suit as the reducing agent for a given metal oxide (MxOy), Gibbs energy interpretations (not covered in this courseware) are made.

Self Assessment Exercise:

Briefly discuss the processes involved in the extraction of crude metal from concentrated ores.

3.3 Electrochemical Principles of Metallurgy

In the reduction of molten metal salt, electrolysis is done which is based on electrochemical principles following equation

$$\Delta G = -nFE^0 \qquad (4.6)$$

where n is the number of electrons and E^0 is the electrode potential of redox couple.

More reactive metals have large negative values of the electrode potential and are difficult to reduce. Figure 4.13 shows the electrolytic cell for the extraction of Aluminium. Na_3AlF_6 or CaF_2 is added to alumina to lower the melting point, and to increase the conductivity of the electrolyte. The overall reaction is

$$2Al_2O_3 + 3C \longrightarrow 4Al + 3CO_2$$

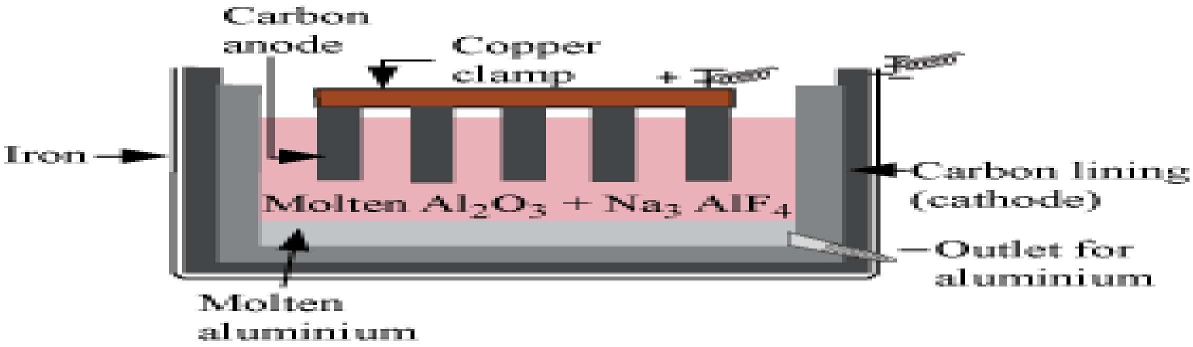

Figure 4.13: Electrolytic cell for the extraction of Aluminium

This electrolytic process is known as Hall−Heroult process. The oxygen liberated at the anode reacts with the carbon of the anode to produce CO and CO_2. The cell reactions are;

$$\begin{aligned}\text{Cathode : } & Al^{3+}_{(melt)} + 3e^{-} \longrightarrow Al_{(l)} \\ \text{Anode : } & C_{(s)} + O^{2-}_{(melt)} \longrightarrow CO_{(g)} + 2e^{-} \\ & C_{(s)} + 2O^{2-}_{(melt)} \longrightarrow CO_{2(g)} + 4e^{-}\end{aligned}$$

3.3.1 The role of cryolite in the metallurgy of aluminium

1. To decrease the melting point of the mixture from 2323 K to 1140 K.

2. To increase the electrical conductivity of Al2O3.

3.3.2 The role of graphite rod in the electrometallurgy of aluminium

In the electrometallurgy of aluminium, a fused mixture of purified alumina (Al_2O_3), cryolite (Na_3AlF_6) and fluorspar (CaF_2) is electrolysed. In this electrolysis, graphite is used as the anode and graphite-lined iron is used as the cathode. During the electrolysis, Al is liberated at the cathode, while CO and CO_2 are liberated at the anode, according to the following equation.

$$\begin{aligned}\text{Cathode : } & Al^{3+}_{(melt)} + 3e^{-} \longrightarrow Al_{(l)} \\ \text{Anode : } & C_{(s)} + O^{2-}_{(melt)} \longrightarrow CO_{(g)} + 2e^{-} \\ & C_{(s)} + 2O^{2-}_{(melt)} \longrightarrow CO_{2(g)} + 4e^{-}\end{aligned}$$

If a metal is used instead of graphite as the anode, then O_2 will be liberated. This will not only oxidise the metal of the electrode, but also convert some of the Al liberated at the cathode back into Al_2O_3. Hence, graphite is used for preventing the formation of O_2 at the anode. Moreover, graphite is cheaper than other metals.

Self Assessment Exercise:

Explain the basis of reduction of a molten metal salt.

4.0 Conclusion

Metal extraction process is the combination of similar and unique kind of steps known as Unit Processes/Unit Operations. Unit operations are physical operations, while unit processes are chemical processes, both of which are combined to extract metals from the ores. Combination of all unit steps/processes results in a chart known as the extraction Flow-Sheets. Some metal oxides get reduced easily while others are very difficult to be reduced (reduction means electron gain or electronation).

5.0 Summary

The units deals with the principles of extractive metallurgy. It also deals with extraction of crude metal from concentrated ores. It further expatiated on electrochemical principles of metallurgy. Extractive metallurgy deals with extraction of metals from its naturally existing ore/minerals and refining them. Reduction of the metal oxide usually involves heating it with some other substance acting as reducing agent (C or CO or even another metal).

6.0. Tutor-Marked Assignment

a) What is extractive metallurgy and briefly state the sources of metals and types of ores.

b) Briefly classify the unit processes/operation in metal extraction by different criteria

c) Briefly discuss the processes involved in extraction of crude metal from concentrated ores.

d) Explain the basis of reduction of a molten metal salt.

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