COURSE GUIDE

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INTRODUCTION

AEA 303: Agricultural Production Economics is a one semester two (2) credit units course designed for 300 level students. The course is designed for the undergraduate students in the school of Agricultural Sciences. The course will expose you to an understanding of many concepts in Agricultural Production Economics. The knowledge gained in this course will assist you to advise farmers and policy makers on the most profitable level of farm production.

The course consists of two major parts, i.e. The Course Guide and The Study Guide. The Study Guide consists of five modules and eighteen units. The modules and the units under them are listed below:

Module 1 Nature of Agricultural Production Economics

Unit 1 Meaning and Scope of Agricultural Economics
Unit 2 Meaning and Scope of Agricultural Production Economics
Unit 3 Concepts in Agricultural Production Economics
Unit 4 Characteristics Features of Agricultural Production

Module 2 Theory of Production Economics

Unit 1 Meaning and Uses of Production Economics
Unit 2 Expression of Production Function
Unit 3 Functional Forms of Production Function
Unit 4 Time Periods in the Production Process

Module 3 Factor-Product Relationship

Unit 1 Laws of Returns
Unit 2 Classical Production Function
Unit 3 Output and Profit Maximization under One Variable Input
Unit 4 Resource Allocation Involving More Than One Variable Input

Module 4 Factor-Factor and Product-Product Relationships

Unit 1 Profit Maximization in Factor-Factor Relationship
Unit 2 Important Concepts in Factor-Factor Relationship
Unit 3 Product-Product Relationship

Module 5 Production Costs

Unit 1 Meaning and Types of Cost

Unit 2 Farm Cost Functions
Unit 3 Cost Functions and Production Function

This course guide tells you briefly on what the course is all about, what course materials you will be using and how you can work your way through these materials with minimum assistance. It suggests some general guidelines for the amount of time you might spend in order to successfully complete each unit of the course. It also gives you some guidance on your Tutor Marked Assignment (TMA). Details of this TMA will be made available in the assignment file. There are regular tutorial classes that linked to the course. You are therefore advised to attend to this session regularly.

WHAT YOU WILL LEARN IN THIS COURSE

AEA 303: Agricultural Production Economics consist of five major components arranged in modules:

- Nature of Agricultural Production Economics
- Theory of Production Economics
- Factor-Product Relationship
- Factor-Factor and Product-Product Relationship
- Production Costs

The first part which is on the nature of agricultural production economics will introduce you into some concepts and background of agricultural production economics. Issues discussed in unit one includes: meaning and scope of agricultural economics and uses of economics in agriculture. In unit two, you will look at the meaning and scope of agricultural production economics. Unit three of the first part of the study guide discussed some important concepts vital to the understanding of this course. Some of the concepts include: production, efficiency, variables, slope, coefficients, e.t.c. The last unit of the first part focused attention on the characteristic features of agricultural production.

Theory of production economics forms the major part of our discussion in the second module. Unit 1 of that part will be devoted to the meaning and uses of production economics. The second unit will cover the expression of production function. Functional forms of production function will occupy unit three of this module and the fourth unit will discuss the time periods in the production process.

The third part of this course, which is on factor-factor relationship, is divided into four units. Unit 1 will look at the laws of returns while unit two will focus on the classical production function. Unit three of this third part of the course will explain step by step how to calculate output

and profit maximization under one variable input. The last unit of this part (unit 4) will explain resource allocation involving more than one variable input.

The fourth part of the course will focus on factor-factor and product-product relationships. This part will be discussed under three units. Unit one will focus on profit maximization under factor-factor relationship. The second unit of the module will explain some important concepts in factor-factor relationship. Unit three of this part will discuss all the various aspects of product-product relationship.

The last part of the course will focus on the production cost. This part will be in three units. Unit one will look at the meaning and types of costs. Unit two will look at the various aspects of farm cost functions. Such aspects will include: total cost, fixed cost, variable cost and marginal cost. Unit three which rounded up the course will look at the relationship between cost functions and production functions.

COURSE AIMS

The aim of this course is to give understanding of the meaning of various concepts of agricultural production economics. This aim will be achieved by trying to:

- explain the nature of agricultural production economics
- describe the theory of production economics
- outline the relationships between factor and product
- explain the relationships between two factors in production process
- explain the relationships between two products in production process
- describe the concept of production cost

OBJECTIVES

In order to achieve the aims of this course, there are sets of overall objectives. Each unit also has specific objectives. The unit objectives are always included in the beginning of the unit. You need to read them before you start working through the unit. You may also need to refer to them during your study of the unit to check your progress. You should always look at the unit objectives after completing a unit. In doing so, you will be sure that you have followed the instruction in the unit.

Below are the wider objectives of the course as a whole. By meeting these objectives you should have achieve the aims of the course as a whole. On successful completion of the course, you should be able to:

- define agricultural economics and agricultural production economics
- give the various types of production function
- identify time periods in production process
- calculate the output and profit maximization under one variable input
- explain the basic concepts involved in factor-factor relationship
- calculate profit maximization under factor-factor relationship
- describe the concept of product-product relationship
- identify the various types of relationships between two products
- define farm cost and identify the various types of farm cost
- explain the various concepts of farm cost functions
- explain the relationship between production function and cost functions

COURSE REQUIREMENTS

To complete this course you are required to read the study units, read suggested books and other materials that will help you achieve the stated objectives. Each unit contains Tutor Marked Assignment (TMA) and at intervals as you progress in the course, you are required to submit assignment for assessment purpose. There will be a final examination at the end of the course.

During the first reading, you are expected to spend a minimum of two hours on each unit of this course. During the period of two hours, you are also answering the self assessment exercises and questions. As a two credit course, it is expected that the lecture contact hours will be eight (8).

In addition to eight (8) hours of lectures with the course facilitator, tutorial classes will also be organised for students to discuss the technical areas of this course. In addition to tutorial classes, I will also advice that you form discussion group with your course mates to discuss some of these questions. Discussion group of between three to five people will be ideal.

COURSE MATERIAL

You will be provided with following materials for this course:

Course Guide
The material you are reading now is called course guide, which introduce you to this course

Study Guide
The textbook prepared for this course by National Open university of Nigeria is called Study Guide. You will be given a copy of the book for your personal use.

Text Books
At the end of each unit, there is a list of recommended textbooks which though not compulsory for you to acquire or read, are necessary as supplements to the course materials

Other Materials
In addition to the above materials it is very essential for you to collect your assignment file.

STUDY UNITS

There are eighteen (18) study units in this course divided into five modules as follows:

Module 1 Nature of Agricultural Production Economics

Unit 1 Meaning and Uses of Agricultural Economics
Unit 2 Meaning and Scope of Agricultural Production Economics
Unit 3 Concepts in Agricultural Production Economics
Unit 4 Characteristics/Features of Agricultural Production

Module 2 Theory of Production Economics

Unit 1 Meaning and Uses of Production Function
Unit 2 Expression of Production Function
Unit 3 Types/Forms of Production Function
Unit 4 Time Periods in the Production Process

Module 3 Factor-Product Relationship

Unit 1 Laws of Return
Unit 2 Classical Production Function
Unit 3 Output and profit maximization under one variable input
Unit 4 Resource Allocation involving more than one Variable Inputs

Module 4 Factor – Factor and Product-Product Relationships

Unit 1 Profit Maximization Factor-Factor Relationships
Unit 2 Important Concepts in Factor-Factor Relationships
Unit 3 Types of Product-Product Relationships

Module 5 Production Costs

Unit 1 Meaning and Types of Cost
Unit 2 Farm Cost Functions
Unit 3 Cost Functions and Production Function

Each unit in the study guide consists of a table of contents arranged in the following order:

1.0 Introduction
1.0 Objectives
2.0 Main contents (Reading Materials)
3.0 Conclusion
4.0 Summaries of key issues and ideas
5.0 Tutor Marked Assignments
6.0 References/Further Reading

At intervals in each unit, you will be provided with a number of exercises or self-assessment questions. These are to help you test yourself on the materials you have just covered or to apply it in some way. The value of these self-test is to help you evaluate your progress and to re-enforce your understanding of the material. At least one tutor-marked assignment will be provided at the end of each unit. The exercise and the tutor-marked assignment will help you in achieving the stated objectives of the individual unit and that of the entire course.

TEXTBOOKS AND REFERENCES

For detailed information about the areas covered in this course, you are advised to consult more recent edition of the following recommended books:

Abbot, J.C. and J.P. Makeham (1980). Agricultural Economics and Marketing in the Tropics. London, Longman Publishers.

Adegeye, A.J. and J.S. Dittoh (1985). Essentials of Agricultural Economics. Ibadan, Impact Publishers.

Doll, J.P. and Orazen, P. (1978). Production Economics: Theory with Application. USA, John Willey and Sons Publishers.

Ibitoye, S.J. and Idoko, D. (2009). Principles of Agricultural Economics. Kaduna. Euneeks and Associates Publishers. Pp 82-92.

Nweze, N.J. (2002). Agricultural Production Economics: An Introductory Text. Nsukka. AP Express Publishers.

Olayide, S.O. and Heady E.O. (1982). Introduction to Agricultural Production Economics. Ibadan. University Press Ltd.

Olukosi, J.O. and A.O. Ogungbile (1989). Introduction to Agricultural Production Economics: Principles and Applications. Zaria. AGTAB Publishers Ltd.

Marshall A.C. (1998). Modern Farm Management Techniques. Owerri. Alphabet Nigeria Publishers.

Reddy, S.S., P.R. Ram, T.V. Sastry and I.B. Devi (2004). Agricultural Economics. New Delhi. Oxford and Ibh Publishers Ltd.

ASSESSMENT

There are two components of assessment for this course.

1. Tutor-Marked Assignment
2. End of Course Examination

TUTOR-MARKED ASSIGNMENT

The TMA is the continuous assignment component of this course. It account for 30 percent of the total score. You will be given about six TMAs to answer. At least four of them must be answered from where the facilitator will pick the best three for you. You must submit all your TMAs before you are allowed to sit for the end of course examination. The TMAs would be given to you by your facilitator and return to him or her after you have done the assignments.

FINAL EXAMINATION AND GRADING

This examination concludes the assessment for the course. It constitutes 70 percent of the whole course. You will be informed of the time for the examination through your study centre manager.

SUMMARY

AEA 303: Agricultural Production Economics is designed to provide background information on agricultural production economics for students of school of Agricultural Sciences. By the time you complete studying this course, you will be able to answer the following questions:

1. (a) Give a concise definition of agricultural economics
   (b) What is the role of economics in agricultural production?

(c) Which of Mankind's activities are studied in agricultural economics?

2. (a) Explain the meaning of the followings:
   (i) Production
   (ii) Production Economics
   (iii) Agricultural Production Economics
   (b) What are the goals of agricultural production economics?
3. Give the meaning of the following agricultural production economics concept:
   (a) Variable
   (b) Coefficient
   (c) Efficiency
   (d) Resources
   (e) Slope
4. (a) List ten (10) features of agricultural production that distinguish it from industrial production and discuss any five (5) of them.
5. (a) Define production function
   i. Explain five usefulness of production function
   ii. List four data that can be generated for production function
   iii. List three goals of production function
6. (a) List and explain five ways of expressing production function.
7. (a) List five types of production functions commonly used in agricultural production economics
   (b) State their algebraic forms for two variable inputs
   (c) With examples, give full description of any three of them
8. Explain with examples, the concept of short run and long run period of time in the production process
9. With the aid of tables, graphs and algebra differentiate between the laws of increasing, constant and decreasing returns.
10. Explain the meaning of the following concepts in production function:
    - Average Physical Product (APP)
    - Marginal Physical Product (MPP)
    - Law of diminishing returns
    - Rational Production Stage
    - Irrational Production
11. Consider the production function of a farmer below:
    $Y = 10 + 200X - 2X^2$ and the price of input = ₦10 and price of output = ₦ 50.
    Calculate the optimum profit and output of this function.
12. Find the marginal physical product (MPP) of the following functional forms:
    - $Y = a + b_1X_1 + b_2X_2 + b_3X_1^2 + b_4X_2^2$

- $Y = aX_1^{b1}X_2^{b2}$
- $Y = a - b_1X_1 - b_2X_2 + b_3X_1^{.5} + b_4X_2^{.5} + b_5X_1^{.5}X_2^{.5}$

13. Consider the production function of a yam farmer using fertilizer (X_1) and Yam Seed (X_2) as variable inputs: $Y = 20X_1 + 4X_2 - 2X_1X_2$
    Find
    - The optimum level of yam output (Y)
    - Levels of X_1 and X_2 required to produce this optimum level of Y
14. Explain the meaning of the following concepts:
    - Isoquant
    - Marginal Rate of Technical Substitution
    - Elasticity of Input Substitution
    - Isocost line
    - Expansion Path
15. Explain with illustrations the following concepts:
    - Production Possibility Curve
    - Iso-revenue line
    - Output Expansion Path
    - Competitive Product
    - Complementary Product
16. (a) What is Agricultural Cost?
    (b) What is the implication of cost to a farmer?
17. Discuss the classical measures of the following farm cost functions: Total Cost, Fixed Cost, Variable Cost and Marginal Cost.

MAIN COURSE

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MODULE 1 NATURE OF AGRICULTURAL PRODUCTION ECONOMICS

Unit 1 Meaning and Scope of Agricultural Economics
Unit 2 Meaning and Scope of Agricultural Production Economics
Unit 3 Concepts in Agricultural Production Economics
Unit 4 Characteristics/Features of Agricultural Production

UNIT 1 MEANING AND SCOPE OF AGRICULTURAL ECONOMICS

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4.0 Conclusion
5.0 Summary
6.0 Tutor-Marked Assignment
7.0 References/Further Reading

1.0 INTRODUCTION

This is the first unit under the nature of Agricultural Production Economics. In this unit you are going to study the meaning, scope and uses of economics in agriculture. Agricultural production economics is a branch of Agricultural Economics and therefore you need to know something about Agricultural Economics. Agricultural Economics is also a specialized branch of Agriculture. Agriculture remains the economic heart of most developing countries. In Africa, agriculture provides about two-thirds of employment, generates over one-third of the National Income and over half of export earnings. Given the large contribution of this sector to the overall economy, agricultural production can then be regarded as the key component of growth and development.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the meaning of agricultural economics
- describe the scope of agricultural economics
- state the importance of agricultural economics

3.0 MAIN CONTENT

3.1 Meaning of Agricultural Economics

Agricultural economics is regarded as an arm of general economics. Olukosi and Ogungbile (1989) viewed agricultural economics as an applied branch of general economics which deals with the allocation of scarce resources which include land, labour, capital and management among different types of crops, livestock and other enterprises to produce goods and services which satisfies human wants. They further stressed that agricultural economics also involves the study of the relationship between agriculture and the general economy, because all economic relationships are interdependent.

Similarly, Nweze (2002) defined agricultural economics as an applied branch of general economics that deals with the application of techniques and principles of economics to agricultural problems. In the application of economic techniques and principles, agricultural economists strive to increase efficiency of resource use in agriculture.
Other definitions of agricultural economics by different authors also exist.

Reddy *et al.* (2009) defined agricultural economics as an applied field of economics in which the principles of choice are applied in the use of scarce resources such as land, labour, capital and management in farming and allied activities. It deals with the principles that help the farmer in the efficient use of land, labour, and capital. Its role is evident in offering practicable solutions in using scarce resources of the farmers for maximization of income.

In the opinion of Olayide and Heady (1982), Agricultural economics is an applied social science dealing with how humans choose to use technical knowledge and the scarce productive resources such as land, labour, capital and management to produce food and fibre and to distribute it for consumption to various members of the society over time.

From these definitions of agricultural economics one can conclude that the field of agricultural economics involves the use of economic principles for the purpose of solving practical problems in agriculture.

3.2 Scope of Agricultural Economics

The scope of agricultural economics is as wide as the scope of economics itself, because there is hardly any aspect of economics that is not relevant to agriculture. According to Olukosi and Ogungbile (1989), there are wide areas of specialization in agricultural economics and these areas include: farm management, production economics, agribusiness, agricultural marketing, price analysis, resource development and land economics. Other areas include; Agricultural policy, agricultural fiancé, international agriculture, agricultural cooperatives, and project evaluation and planning.

3.3 Uses of Economics in Agriculture

Economics is very relevant in the field of agriculture. Some of the uses of economics in agriculture are highlighted below:

i. Economics helps in deciding the level of production that will be more profitable to the farmer. In order to achieve this goal, economist advice the farmer on what type of crop to grow or animal to rear and at what scale of operation.
ii. Economics will also assist in explaining the market situation for this product and the general distribution.
iii. Economics is very useful in the area of formulating agricultural policies as well as implementing agricultural policies and its interpretation.
iv. Economics also borders on financing of agricultural projects, formation of agricultural cooperatives and efficient management of the finance.
v. Other areas of usefulness of economics in agricultural production include: the study of availability of farm inputs and their costs. For example, farm land and the rent paid, type of capital and the interest rate etc.
vi. There are also study of farm organization and allocation of the resources to achieve the optimum level of production.

SELF-ASSESSMENT EXERCISE

i. List three productive resources used by farmers
ii. Itemize the areas of specialization in agricultural economics
iii. Discuss five uses of economics in agriculture

4.0 CONCLUSION

In this unit we have learnt about the meaning of agricultural economics, scope of agricultural economics and uses of economics in agricultural sector. We can conclude here that agricultural economics covers all the

sector of the economy. There is no part of human endeavour that is not covered by this branch of economics.

5.0 SUMMARY

In this unit you have learnt

(i) The various definitions of agricultural economics by different authors. Agricultural economics involves the use of economic principles in solving practical problems in agriculture.
(ii) We also learnt that agricultural economics covers a wide range of areas like farm management, production economics, agricultural finance, agricultural marketing etc.
(iii) Finally, in this unit we learnt about the usefulness of economics in the field of agriculture. Economics helps in deciding the level of production, explain market situation, formulation of agricultural policies, financing of agricultural projects, formation of agricultural cooperatives and resource allocation.

6.0 TUTOR-MARKED ASSIGNMENT

1. Give a concise definition of agricultural economics
2. What is the role of economics in agricultural production?
3. Which of Mankind's activities are studied in agricultural economics?

7.0 REFERENCES/FURTHER READING

Abbot, J.C. and J.P. Makeham (1980). Agricultural Economics and Marketing in the Tropics. London, Longman Publishers.

Adegeye, A.J. and J.S. Dittoh (1985). Essentials of Agricultural Economics. Ibadan, Impact Publishers.

Nweze, N.J. (2002). Agricultural Production Economics: An Introductory Text. Nsukka. AP Express Publishers.

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Marshall A.C. (1998). Modern Farm Management Techniques. Owerri. Alphabet Nigeria Publishers.

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UNIT 2 MEANING AND SCOPE OF AGRICULTURAL PRODUCTION ECONOMICS

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  3.2 Meaning of Agricultural Production Economics
  3.3 Scope of Agricultural Production Economics
4.0 Conclusion
5.0 Summary
6.0 Tutor-Marked Assignment
7.0 References/Further Reading

1.0 INTRODUCTION

In unit 1 we learnt the various approaches to the definition of agricultural economics by different authors. We also learnt from those definitions that the ultimate aim of agricultural economics is to assist farmers optimize their farm resources. We further learnt that agricultural economics has wide scope, as wide as the economics itself. We finally identified the areas in which economics are useful to agricultural production.

In this unit 2 of module 1, you will learn about the meaning of production economics, meaning of agricultural production economics and the scope of agricultural production economics.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define production economics
- outline the four major areas of production economics
- state the meaning of agricultural production economics

3.0 MAIN CONTENT

3.1 Meaning of Production Economics

We already explained the meaning of agricultural economics and the uses of economics in agriculture; we also need to know the meaning of

production before we can attempt the definition of production economics.

Production can be defined as the transformation of one thing into another. For example, when one puts flour, sugar etc together to make cakes, this is called production. In strict economic sense, production is more than putting things together. It is concerned with the whole process of making goods and services available to customers i.e. it is the creation of utility to satisfy human wants.

There are basically three types of production viz: primary production, secondary production and tertiary production.
In the process of producing agricultural commodities, resources (inputs) which are not only limited in both quantity and quality but also have alternatives and often conflicting uses, are employed. The main focus of production economics therefore, is the management of resources (land, labour, capital and entrepreneur) in the process of producing commodities. Included in this goal of resource management are choice and decision making among the alternative uses and alternative end (product/output). The two major goals of production economics are:

a. Provision of guidance to individual farmers for efficiency in resource use in production, and
b. Provision of guidance to customers for efficiency in resource use, consumption and process.

3.2 Meaning of Agricultural Production Economics

Agricultural production economics is an applied field of economic science which is essentially concerned with the application of the principles of choice to the utilisation of capital, labour, land, water and management resources in the farming industry (Olayide and Heady, 1982). This definition shows that as a study of resource efficiency, agricultural production economics is specifically concerned with the conditions under which the ends of objectives of farm operators/managers, farm families and the consumers can be attained to the greatest degree possible. The definition also implies an involvement of technical science in the specification of the physical relationships between resources and product. It connotes that the problem of choice involved should be one of economics just as is the problem of how resources have to be employed to maximize the profit of the farm-firm.

3.3 Scope of Agricultural Production Economics

Agricultural production economics is a branch of applied economics where economic principles are applied in the use of land, labour, capital and management on farms and in agricultural industry. The basic

concept of the theory of firm and the principles of resource allocation are the core of agricultural production economics. Production economics variables, unlike those of consumption are real and can be measured in tangible physical terms. Measurement of variables in this branch of economics is therefore more exact than other branches of economics. Research can therefore be conducted in a controlled manner as in the case of physical sciences.

Agricultural production economics is based on the principles of optimization i.e. maximization and minimization. It is concerned with the conditions which are necessary to be fulfilled if a producer has to satisfy his objectives such as profit maximization or wants to produce a given level of output with minimum cost or resources.

Although the main concern of agricultural production economist is to attain economic efficiency in the use of resources, he has to be knowledgeable and familiar with the physical production information, factors of production, products, marketing conditions, government policies and administration. He should be concerned with the factors relating to economic efficiency in the use of agricultural resources in different locations and regions around him. It is the task of agricultural production economist to provide guidance and advice to farm families and agricultural industry on how to use their resources including time, most efficiently in production in order to achieve their objectives and welfare.

According to Olayide and Heady (1982), the field of agricultural economics involves four main issues:

a. Maximization of some objective functions such as net revenue and gross margin and minimization of the cost of production;
b. Choice in terms of resource allocation;
c. The role of choice indicator i.e. yardstick for comparing alternatives;
d. The economic implications of each of the three facets of the field of economics.

SELF-ASSESSMENT EXERCISE

i. Differentiate between agricultural economics and production economics
ii. List four areas of production economics

4.0 CONCLUSION

This unit has introduced you to the meaning of production economics and agricultural production economics. Essentially we can conclude that production is one of the major economic activities which consist of production, consumption and distribution.

5.0 SUMMARY

The main points in this unit are:

a. Production is the process of changing goods and services into different ones.
b. The purpose of the production process is to produce goods that have more utility or value to the society than the goods used in the production process.
c. Production economics represent the sets of rules that must be mastered before embarking on a production process
d. The field of agricultural production economics involves four main goals:
   - Maximization or minimization of some objective functions
   - Choice in terms of resource allocation
   - Yardstick for comparing alternatives, and
   - Economic implications of the above objectives.

6.0 TUTOR-MARKED ASSIGNMENT

1 (a) Explain the meaning of the followings:

(iv) Production
(v) Production Economics
(vi) Agricultural Production Economics

(b) What are the goals of agricultural production economics?

7.0 REFERENCES/FURTHER READING

Abbot, J.C. and J.P. Makeham (1980). Agricultural Economics and Marketing in the Tropics. London, Longman Publishers.

Adegeye, A.J. and J.S. Dittoh (1985). Essentials of Agricultural Economics. Ibadan, Impact Publishers.

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Olayide, S.O. and Heady E.O. (1982). Introduction to Agricultural Production Economics. Ibadan. University Press Ltd.

Olukosi, J.O. and A.O. Ogungbile (1989). Introduction to Agricultural Production Economics: Principles and Applications. Zaria. AGTAB Publishers Ltd.

Reddy, S.S., P.R. Ram, T.V. Sastry and I.B. Devi (2004). Agricultural Economics. New Delhi. Oxford and Ibh Publishers Ltd.

UNIT 3 CONCEPTS IN AGRICULTURAL PRODUCTION ECONOMICS

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7.0 References/Further Reading

1.0 INTRODUCTION

By now you must be familiar with the meaning of agricultural economics, production economics and agricultural production economics. We have already discussed the scope of this specialized branch of economics. During the study of this course, you will come across some terms and concepts in which you need to know their meanings. This unit is devoted to the explanation of these concepts. Some important concepts that we will be discussing in unit 3 includes: Production, Types of production, Factors of production and efficiency.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define production
- explain the three types of production
- explain the four factors of production
- give the meaning of at least two other common terms used in agricultural production economics

3.0 MAIN CONTENT

3.1 Concept of Production

3.1.1 Meaning of Production

Production is the process whereby some goods and services are transformed into other goods. The transformed goods are known as inputs, factors or resources and the newly created goods are called products, outputs or yield in the case of crops.

3.1.2 Types of Production

There are basically three types of production namely:

a. Primary production
b. Secondary production
c. Tertiary production

a. Primary Production
This includes all branches of production that may not be easily consumed at the initial stage but used for further production. For example, production of cassava, mining and quarrying are all classified as primary production.

b. Secondary Production
This production comprises of all kinds of manufacturing and constructing works i.e. turning the new materials produced in primary production into finished goods.

c. Tertiary Production
This type of production involves the provision of direct services such as the distribution of goods and services at each level of production to the final consumers.

SELF-ASSESSMENT EXERCISE

Discuss the three types of production

3.2 Factors of Production

The resources used for the production of a product are known as factors of production. Factors of production are termed inputs, which may mean the use of the services of land, labour, capital and organization in the process of production.

The term output refers to the commodity produced by various inputs.

a. **Land**

The term land is used in the widest sense to include all kinds of natural resources, farmland, mineral wealth such as coal and metal ores and fishing-grounds. Perhaps the main services of land are the provision of a site where production can take place. Land differs fundamentally from other factors in three ways;

i. It is fixed in supply
ii. It has no cost of production
iii. It varies in quantity

b. **Capital**

Capital comprises of buildings, machinery, raw materials, partly finished goods and means of transport i.e. capital is considered as a stock of producers' goods used to assist in production of other goods.

c. Labour

Labour is the human effort employed in production. It is indispensible to all forms of production. The supply of labour services can be varied either by a change in the number of hours or days worked in a given period of time. The supply of a labour in a country depends on these three factors:

i. The total population of the country
ii. The proportion of the population available for employment, and
iii. The number of hours worked by each person per year.

d. Entrepreneur

Entrepreneur describes the managerial ability of the owner of the firm or its manager. The entrepreneur is responsible not only for deciding what method of production shall be adopted but also for organizing the work of others, he has to make many other important decisions such as what to produce and how much to produce. Perhaps the primary function of the entrepreneur is to bear the risk and uncertainty of production.

3.3 Other Concepts in Agricultural Production Economics

The concepts discussed below are adopted from Reddy *et al.* (2009):

3.3.1 Resources

Anything that aids in production is called a resource. They physically enter the production process to transform into output. For example- seeds, fertilizers, feeds, veterinary medicines etc.
Resources can be classified into the followings:

i. **Fixed Resources:** Resources which remain unchanged irrespective of the level of production are fixed resources. These resources exist only in the short run. The costs associated with these resources are called fixed costs. Farmer has little control over the use of these resources. For example; land, buildings, machinery implements etc.
ii. **Variable Resources:** Resources which change with the level of production are called variable resources. The higher the level of production, the greater the use of these resources. The costs which are associated with these resources are called variable costs. These resources exist in the short run as well as long run. Farmer can exercise greater control over the use of these resources. Examples are: seeds, fertilizers, plant protection chemicals, feed etc. The distinctions between fixed and variable resources cease to exist in the long run. In the long run all resources are varied.
iii. **Flow Resources:** The resources which cannot be stored and should be used as and when they are available. For instance, if the services of a labourer available on a particular day are not used, then they are lost forever, similarly, the services of machinery and farm buildings etc.
iv. Stock Resources: Stock resources are those which facilitate for their storage when they are not used in one production period. Examples are: seeds, fertilizers, feed etc.

3.3.2 Productivity

Productivity denotes the efficiency with which various inputs are converted into products. It signifies the relationship between output and inputs. In simple terms, output per unit of input is called productivity. For example productivity can be expressed as 10kg of output/ha.

3.3.3 Efficiency

Efficiency means absence of wastage or using resources as effectively as possible to satisfy the farmer's need and goals.

Efficiency can be expressed in the following ways:

i. Technical Efficiency: It is the ratio of output to input
ii. Economic Efficiency: It is the expression of technical efficiency in monetary value by attaching prices. In other words, the ratio of value of output to value of input is called economic efficiency. It is the maximization of profit per unit of input.
iii. Allocative Efficiency: It occurs when no possible organization of production can make any one better off without making someone else worse off. It refers to resource use efficiency. It is an ideal situation in which costs are minimum and profits are maximum.

3.3.4 Variable

Any quantity which can have different values in the production process. Other concepts associated with variable are:

i. Independent Variable: it is a variable whose value does not depend on other variables. Such variables influence the dependent variable. Examples are: land, labour, liquid money, fertilizer etc.
ii. Dependent Variable: A variable that is governed by another variable. Example is crop output.
iii. Constant: A quantity that does not change its value in a general relation between variables.
iv. Coefficient: When rate per unit is calculated we use the term coefficient, a multiplying factor. For example;
    (a) The regression coefficient of an input to production function denotes response of output per unit of input
    (b) Elasticity coefficient of input gives the percentage change in crop output per one cent increase in input level.
    (c) Technical coefficient refers to requirements of inputs per unit of land or per unit of crop output.

3.3.5 Slope

Slope of a line represent the rate of change in one variable that occurs when another changes i.e. it is the rate of change in the variable on the vertical axis per unit of change in the variable on the horizontal axis. Slope is always expressed as a number. Slope varies at different points on a curve but remains the same on all points of a given line.

SELF-ASSESSMENT EXERCISE

i. Itemize the four factors of production
ii. Explain the following concepts:
- Resources
- Productivity
- Efficiency
- Variable
- Slope

4.0 CONCLUSION

This unit has exposed you to some basic concepts used in agricultural production economics. Concepts like production, factors of production, efficiency, variables, productivity, coefficient, slope etc are very essential in understanding agricultural production economics.

5.0 SUMMARY

The main points in this unit include the followings:

i. Production means transformation of input into output
ii. Production can be classified into three: primary, secondary and tertiary production
iii. Resources used for production are called factors of production
iv. Factors of production can be grouped into four-land, labour, capital and entrepreneur
v. Resources are anything that aids in production and can be classified into variable resources, fixed resources, flow resources and stock resources.
vi. Productivity means efficiency with which inputs are converted into output
vii. Efficiency also means absence of wastage or using resources as effectively as possible to satisfy the farmer's goals
viii. Efficiency can be classified into- technical efficiency, economic efficiency and allocative efficiency.
ix. Variables are any quantity which can have different values in the production process. Variables can be dependent or independent
x. In agricultural production economics we can identify the following types of coefficients: regression coefficient, elasticity coefficient and technical coefficient
xi. Slope of a line represent the rate of change in one variable that occurs when another changes.

6.0 TUTOR-MARKED ASSIGNMENT

1. Give the meaning of the following agricultural production economics concepts:

    a. Variable
    b. Coefficient
    c. Efficiency
    d. Resources
    e. Slope

7.0 REFERENCES/FURTHER READING

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UNIT 4 CHARACTERISTIC FEATURES OF AGRICULTURAL PRODUCTION

CONTENTS

1.0 Introduction
2.0 Objectives
3.0 Main Content
3.1 Characteristic Features of Agricultural Production
4.0 Conclusion
5.0 Summary
6.0 Tutor-Marked Assignment
7.0 References/Further Reading

1.0 INTRODUCTION

In unit 3 of this module 1, you learnt about the various concepts of agricultural production economics. Some of the concepts explained in that unit include: primary, secondary and tertiary production, factors of production, resources, productivity, efficiency, variable, coefficient and slope.

In this unit 4, you will learn about the various features that distinguish agricultural production from other forms of production.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- list ten features that distinguish agricultural production from industrial production.
- explain five characteristic features of agricultural production.

3.0 MAIN CONTENT

3.1 Characteristic Features of Agricultural Production

Agricultural production is a specialized sector of the economy. The conditions under which it operates and the nature of its products differ from all other non-agricultural sectors of the economy. Reddy *et al.* (2009) identified some of these features to include the following: farming as a way of life, dependence on weather, seasonality of production, perishable nature of agricultural products, joint products, bulkiness and problems of standardization. Other features include: time

lag in the production of agricultural products, large proportion of land, law of diminishing returns, nature of demand, efficiency of capital and low shares of producer in consumer's payment.

3.1.1 Farming as a Way of Life

Most farmers in Nigerian regard farming as a way of life rather than a business concern. This is to say that profit maximization is not their ultimate goal. They are mostly concerned with satisfying the immediate family needs. In line with this assertion, most of them operate small farm size with multiple cropping and scattered plots of farmland. The main goal of non-agricultural or industrial sector is profit maximization. Unlike agricultural production, industrial production is more organized towards achieving this goal.

3.1.2 Dependence on weather

Nigerian agriculture is mostly dependent on natural rainfall. So, Nigeria agriculture is at the mercy of natural weather conditions. Rainfall may be too much leading to flooding of farmland and in some years, rainfall may be too small to sustain the crops leading to drought. All these erratic rainfall conditions can lead to total crop failure. Other weather conditions like temperature, humidity and wind also influence significantly some farming activities. Weather in most cases has no serious effect on industrial production. In industry the production activity can take place under the control of the entrepreneur. The entrepreneur can plan and control the entire production process. The entrepreneur can decide at any point in time to decrease or increase the level of production as dictated by the market situation.

3.1.3 Seasonality of Production

The production of most agricultural commodities in Nigeria depend entirely on weather conditions especially rainfall. Since rainfall is seasonal, most of the agricultural commodities can only be produced during the rainy season. Rainfall dictates most of the farming activities like the time of cultivation, planting and even harvesting. In the case of non-agricultural or industrial production, as long as the raw materials are available, the production can go on throughout the year.

3.1.4 Perishable Nature of Agricultural Products

Unlike industrial products, some agricultural commodities are perishable within a period of one week if they are not consumed. Most fruits and vegetables belong to this category. The perishability of these products coupled with the influence of weather results in price variation of these

commodities. Most industrial products are durable and are not subjected to frequent price changes.

3.1.5 Joint Products

Some of the agricultural products are jointly produced. For example, cassava flour and starch, cotton lint and cotton seed, palm oil and palm kernel etc. Since the products pass through the same production process it will be difficult if not impossible to isolate their production costs. In this case both the costs of the main product and their by-products are calculated together. In industrial production, it is possible to separate the cost of production of several products that are produced in the same plant.

3.1.6 Bulkiness of Agricultural Products

Most of agricultural commodities harvested raw from the farm are bulky in nature. The implication of this is that the cost of transporting them from the farm to the market will be high. Similarly, the space and cost of storing them will equally be high. These high costs of storage and transportation imposed some limitations on the movement of these commodities from surplus or production centres to other areas. In contrast, industrial products are neatly packaged and pose no problem of storage and transportation. Industrial products can easily be made available in any part of the country.

3.1.7 Problems of Standardization

There are variations in the farm products with regards to size, shape, appearance, colour etc. This is due to the availability of a large number of varieties of these crops. The implication of this is that, it will be difficult if not impossible to have uniform standard for measuring and grading of the products. In industrial sector, machines can be employed to produce products of the same grade, size and quality.

3.1.8 Price Fluctuation

Agricultural commodities are subjected to price fluctuations due to time lag in their productions. Weather and other factors impose limitations between the period of decision to produce and actual realization of the output. Due to uncertainties surrounding agricultural production, this time lag in the production may upset the plans of the farmer.

Farmers have no control over the weather and marked situations. Between the period of planting and harvesting, price of the product can fall. The price fluctuations of agricultural commodities cause variations

in farm incomes. This type of situation does not occur in industrial sectors. Entrepreneur determines the prices of their products due to the current situation on ground.

3.1.9 Required Large Proportion of Land

Most farmers in Nigeria practiced subsistence farming with at least two to four plots of land located in different areas. These scattered farmlands require large proportion of farmland which is not the case in non-agricultural sector. Most industries required relatively small proportion of land and are located in one place.

3.1.10 Law of Diminishing Returns

The law of diminishing returns is applicable to both agriculture and industry, but the difference is that it sets in earlier in agriculture than industry. The obvious reasons are the dependence of agriculture on weather conditions, exhaustion and variations in soil fertility and limited scope of division of labour.

3.1.11 Efficiency of Capital

We earlier define efficiency to mean absence of wastage or using resources as effectively as possible to satisfy the farmer's need. The rate of profit maximization to satisfy farmer's goal is very slow in agriculture compared to industry. This is because farm business takes relatively larger time to return the investment through income. Therefore industrial sector is more efficient in resource utilization than the agricultural sector of production.

3.1.12 Nature of Demand

Most agricultural commodities belong to the necessity of life, and therefore, their demand are relatively inelastic. This implies that demand for agricultural products are relatively steady irrespective of the price. Most industrial products are elastic in demand. Increase or decrease in the prices of industrial products may have significant influence on their demand.

3.1.13 Low Producer's Profit Margin

Another characteristic features that distinguish agricultural production is the general low profit margin accruing to farmers. Agricultural marketing is characterized by the existence of too many middlemen. Middlemen in agricultural sector unlike their counterparts in the

industrial sector require no formality in the handling of the products and can fix any price acceptable to them.

Middlemen in industrial sector are guided by rules and regulations guiding the handling of their products.

SELF-ASSESSMENT EXERCISE

What are the features of agricultural production?

4.0 CONCLUSION

This unit identified some major characteristic features of agricultural production that distinguish it from industrial production. It is quite evident in the discussion that agricultural sector has some peculiar features that distinguished it from other non-agricultural sectors.

5.0 SUMMARY

The main points in this unit include the following:

a) That agricultural sector is distinct from the industrial sector because of the following peculiar features of agricultural production:
   - Farming as a way of life
   - Dependence on weather
   - Seasonality of production
   - Perishable nature of Agricultural Production
   - Joint Products
   - Bulkiness of agricultural products
   - Problems of standardization
   - Price fluctuation
   - Required larger proportion of land
   - Law of Diminishing returns
   - Efficiency of Capital
   - Nature of demand, and
   - Low producer's profit margin

6.0 TUTOR-MARKED ASSIGNMENT

List ten (10) features of agricultural production that distinguish it from industrial production and discuss any five (5) of them.

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MODULE 2 THEORY OF PRODUCTION ECONOMICS

Unit 1 Meaning and Uses of Production Function
Unit 2 Expression of Production Function
Unit 3 Types / Forms of Production Function
Unit 4 Time Period in the Production Process

UNIT 1 MEANING AND USES OF PRODUCTION FUNCTION

CONTENTS

1.0 Introduction
2.0 Objectives
3.0 Main Content
3.1 Meaning of Production Function
3.2 Uses of Production Function
4.0 Conclusion
5.0 Summary
6.0 Tutor Marked Assignment
7.0 References/Further Reading

1.0 INTRODUCTION

In module 1, we discussed the nature of agricultural economics. We specifically discussed the meaning and scope of agricultural production economics, some concepts in agricultural production economics and the features of agricultural production. By going through the units, we believe that you now have enough background information that you will come across in the later part of this curse. Unit 1 of this module 2 is designed to give you further insight into the understanding of production economics. The unit will give the meaning and uses of production function.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the meaning of production function
- discuss four uses of production function

3.0 MAIN CONTENT

3.1 Meaning of Production Function

The production function expresses a functional relationship between quantities of inputs and outputs. It shows how and to what extent, output changes with variations in inputs during a specified period of time.

Basically, production function is a schedule or table showing the amount of output obtained from various combinations of inputs given the state of technology.

Algebraically it can be expressed as
$Y = f(X_1, X_2, X_3, X_4)$
Where:
Y = farm output per unit of time
X_1, X_2, X_3, X_4 = inputs such as land, labour, capital and entrepreneurial ability.

The production function is determined by technical conditions of production and may be rigid or flexible. In the short run, the technical conditions of production are rigid so that the various input resources used to produce a given output are fixed. This in the actual sense is a rare situation in the theory of production. Even in the short run, it is possible to increase the quantities of one input keeping the quantities of other inputs constant in order to have more output. It should be noted that production is a flow and therefore the transformation of factor inputs into output must be expressed in many units per period of time.

3.2 Uses of Production Function

Production function was first developed by the physical and biological scientist in the experimental laboratory to find out the quantity of certain variable that will produce maximum output level. In doing this, they are transforming resources into output.

In the case of physical and biological scientists, simple linear relationship was enough to express the discrete relationship between input and output. For example, in artificial insemination, what they need to know is just the little quantity that will be enough to fertilize the ovum.

The physical and biological scientists still needed to know the basis for profitability of their products that is, the economic effects. For example,

they needed to know the optimum level of the dosage that will achieve the expected desired goal. It was at this point that the economists found it as a useful tool of analysis to establish the production function as we have it today.

Olayide and Heady (1982) itemize some of the usefulness of production function as follows:

a) Production function enables us to derive how national product is produced from the various resources. Production function expresses the relationship between national product and the available resources used in producing it.
b) Production function is also used to assess inter-regional or international trade balances. Coefficient is used in the inter-regional and international trade to apportion goods to various countries. The coefficients derived from the production function serve as the base for determining optimum patterns of intra-state, inter-state, inter-regional and international trade.

   This optimization of trade derived from production function is the natural consequence of optimum output at minimum cost that is based on comparative advantage for attainment of maximum net revenue. It is also the consequence of regional specialization.
c) Production function is equally useful in the allocation of total output or national income. In other words production function is a useful tool in the marginal production theory of distribution.
d) Production function served as a useful tool in the maximization of profit of a farm. In this aspect, production function provide the major data that is needed to determined or specify the use of resources and the pattern of outputs which maximize farm-firm profits.
e) Production function is useful in the algebraic function of the theory of supply. The algebraic nature of supply-functions rests in large part on the nature of the production function.

For the production function to actualize the usefulness above, production economist obtains the following data – experimental, cross-sectional, time series and engineering data to achieve the following goals:

i. the of point maximum output and input relationships
ii. the point of economic optimum and
iii. the quantity of resources to use in producing a given maximum either physical or economic maximum.

SELF-ASSESSMENT EXERCISE

i. Define a production function
ii. State the explicit form of a production function
iii. Itemize the usefulness of production function

4.0 CONCLUSION

Unit 1 of module 2 explains the meaning of production function and highlighted some of the usefulness of production function. We can conclude form this unit that production function is useful in all the sectors of the economy.

5.0 SUMMARY

The main points in this unit are as follows:

a. Production function is defined as the physical relationship between the output and inputs used in the production of the product
b. Production function is useful in the estimation of balance of trade
c. Production function enable us to estimate the nature of production
d. Production function provides guide on the allocation of total output or national income.
e. Production function is also useful in the maximization of profit.
f. Production function is useful in the theory of supply
g. Data needed in production function are obtained from experiment, time serves, cross-sectional and engineering sector.
h. Production function data are needed to achieve the following goals.
i. point of maximum output and input relationship
i. point of economic optimum
ii. quantities of resources to use in producing a given maximum.

6.0 TUTOR-MARKED ASSIGNMENT

1. Define production function
2. Explain five usefulness of production function
3. List four data that can be generated for production function
4. List three goals of production function.

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UNIT 2 EXPRESSION OF PRODUCTION FUNCTION

CONTENTS

1.0 Introduction
2.0 Objectives
3.0 Main Content
    3.1 Introduction
    3.2 Functional Notation
    3.3 Tabular Presentation
    3.4 Graphical Presentation
    3.5 Mathematical Presentation
    3.6 Written Word
4.0 Conclusion
5.0 Summary
6.0 Tutor-Marked Assignment
7.0 References/Further Reading

1.0 INTRODUCTION

In unit 1 of this module, we discussed the meaning and usefulness of production function. We defined production function as an expression of the technical or physical relationship which connects the number of units of inputs that are fed into a production process and the corresponding units of output that emerge. We also noted that production function is useful in almost all aspects of economy ranging from the balance of trade, national income to maximization of profit. In this unit, we shall look into the various ways of expressing production function.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- list five different ways of expressing production function
- explain three means of expressing production function.

3.0 MAIN CONTENTS

3.1 Introduction

Production function can be represented using different approaches. The most common approaches include: written form, functional notations, tabular expression, graphical expression and mathematical expression.

3.2 Functional Notation

Production function can be expressed using symbols. The most popular symbols commonly used in agricultural production economics is stated below:

$Y = f(X_1, X_2, X_3, \text{------}X_n)$
Where:
Y = the quantity of output
Xs = the quantities of inputs used in production
f = stands for the forms of the relationship that transforms inputs (X_s) into output (Y).

Functional notation can also provide information on which of the inputs are varied and which are fixed. This can be expressed by using vertical line to separate the varied inputs from the fixed inputs. This can be done as follows:

$Y = f(X_1,/X_2, X_3)$
The above functional notation implies that the quantity of output (Y) is a function of variable input X_1 given the quantity of other inputs X_2 and X_3. This means that inputs X_1 is the variable input while inputs X_2 and X_3 are the fixed inputs.

The major shortcoming of functional notation means of expressing production function is that it does not provide information on the quantity of output expected when X_1, X_2 and X_3 are combined as inputs. Information on the relationship between output and input as a whole is very important to the farmer and other organs of government.

SELF-ASSESSMENT EXERCISE

Identify the output and input(s) in the following functional notation: $Y=f(X_1, X_2, X_3.....X_n)$

3.3 Tabular Presentation

Production function can be expressed in form of table: this can be illustrated by showing the various quantity of yam in kg obtained from various quantities of fertilizer application.

Table1: Yam output from varying levels of fertilizer

Yam output (Kg)	Fertilizer Application (Kg)
10	0
20	2
30	4
40	6
50	8

Table 1 showed the relationship between yam output and the various quantities of fertilizer applied when other farm inputs like labour, farm size etc are fixed. For example 2 kg of fertilizer is required to produce 20 kg of yam while 8 kg of fertilizer is also required to produce 50 kg of yam.

Nweze (2002) summarize the usefulness of tabular presentation of production function thus:

i. So much information is contained in a small space
ii. It may be computationally easier to work directly with the tabulated production data especially when it involves one or more variable inputs.
iii. The quantitative relationships are better appreciated at a glance.

3.4 Graphical Expression

The relationship between yam output and fertilizer application tabulated above can be expressed in graphical form. This can be done by plotting the values of fertilizer application against the corresponding values of yam output. Each point on the curve will correspond to specific level of input and a corresponding level of output.

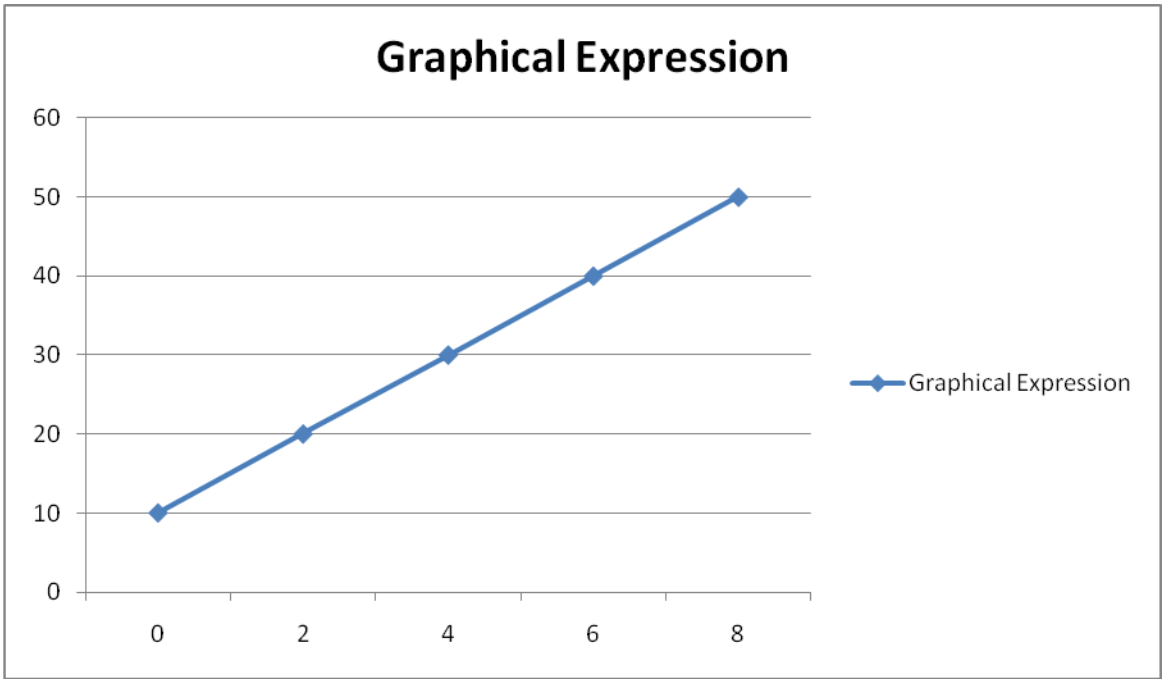


Fig 1: Typical production function

3.5 Mathematical Presentation

Mathematical expression is more explicit than ordinary functional notation. There are many ways of expressing production function mathematically. For example, production function can be expressed explicitly in linear form as presented below:

$Y = a + \beta x$
Where
Y = quantity of output
X = quantity of input used
a = a constant
β = coefficient of X

If a and β takes on specific values like 10 and 0.5 respectively, the above can then be expressed as follows:

$Y = 10 + 0.5X$
Unlike functional notation, this mathematical expression illustrated above has the same content as in the graphical expression. It is however more meaningful than the graphical approach because it allow one to obtain the intermediate values of the variables.

3.6 Written Word

The relationships between dependant and independent variables in production function can be describe or enumerated in words without resulting in to mathematical, graphical or tabular expression: this is a very weak way of showing the relationship between two variables because the magnitude of the relationship cannot be precisely stated by ordinary word. Secondly, it will be difficult to comprehend the statement at a glance. However, expression of production function in word form is necessary to complement other forms of expression.

SELF-ASSESSMENT EXERCISE

Explain five ways of expressing production function

4.0 CONCLUSION

We discussed the various ways by which production function can be expressed. The various ways identified in the unit include; written word, tabular form, graphical form, symbolic form and mathematical form. We can conclude here that combinations of these ways of expressing production function are necessary to adequately specify the relationships between two or more variables.

5.0 SUMMARY

The main points discussed in this unit are as follows:

(i) The following methods can be used to express production function; written word, table, graph, mathematics and symbols.
(ii) Words expression is needed to complement other methods
(iii) In tabular form, we can quickly appreciate the relationship at a glance
(iv) We can mathematically expressed the relationship as follows $Y = a + \beta x$
(v) We can also use symbol like $Y = f(X_1, X_2, X_3)$ to represent relationship between Y and X in production function
(vi) We can also plot the graph of Y against X to express this relationship.
(vii) We need combinations of these methods to satisfactorily explain the relationship in production functions.

6.0 TUTOR-MARKED ASSIGNMENT

List and explain five ways of expressing production function.

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UNIT 3 TYPES/FORMS OF PRODUCTION FUNCTION

CONTENTS

1.0 Introduction
2.0 Objectives
3.0 Main Content
    3.1 Introduction
    3.2 Linear Function
    3.3 Quadratic Function
    3.4 Cobb-Douglas Power Function
    3.5 Square Root Function
    3.6 Semi-Log Function
4.0 Conclusion
5.0 Summary
6.0 Tutor-Marked Assignment
7.0 References/Further Reading

1.0 INTRODUCTION

In the last unit we looked at the various ways we can express production function. Some of the ways we discussed in that unit include: written word, Table, Graph, Symbol and mathematics. We also discovered that the use of combinations of these methods are necessary to adequately describe the relationship between output and input. In this unit, we shall go a step further to assess the various types of production functions used in agricultural production economics. You are strongly advised to carefully study this unit so that you can follow and understand the subsequent units of this course.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- identify and list the five major types of production function used in agricultural production economics.
- write the mathematical expression of these production functions
- explain the application of at least three of these functions
- state the major difference between these functions in agricultural production economics.

3.0 MAIN CONTENTS

3.1 Introduction

In the estimation of production function parameters, many equation forms have been fitted. Five of such equations that are commonly used in agricultural production economics will be examine here. Those examined include: linear, quadratic, Cobb-Douglas, square-root and semi-log functions.

3.2 Linear Function

(a) Algebraic Form

The algebraic form of this equation can be expressed for the single variables case as: $Y = a + \beta X$
Where Y = is the output and X is the variables input
a = intercept shown n the Y = axis, and
β = slope or gradient of the curve
The special features of linear function include

i. The curve is a straight line
ii. The slope of the curve must be constant
iii. The marginal product (MP) is also a straight horizontal line

(b) Marginal Product of Linear of Function

$Y = a + bX$……1

$\frac{\delta Y}{\delta X} = b$ (MPP)……2

(c) Elasticity of Production

If elasticity of production is given by this equation

$\frac{\Delta Y}{\Delta X} \cdot \frac{X}{Y}$……..3

Similarly, b which is the slope of the line gives the way in which Y is changing for a unit change in the input X.
Therefore, if we make b the subject of equation 1, we then have

$B = \frac{Y - a}{X}$ ----------4

Substituting b in equation 3, we have elasticity $= \frac{(Y - a)}{X} \times \frac{X}{Y}$

$= (Y - a)$……..5

From the above equation of elasticity we can deduce that:

i. if a = 0, then elasticity = 1
ii. If a is greater than 0, then elasticity is less than 1
iii. if a is less than 0, then elasticity is greater than I

SELF-ASSESSMENT EXERCISE

Try the above equation with two variable inputs: $Y = a + bx, + CX_2$

3.3 Quadratic Function

(a) Algebraic Form

For Single variables input we have:
$Y = a + bX - Cx^2$
Where Y = output and X = variable input
a = the constant
b and c = the coefficients
Quadratic function allows diminishing total product. The coefficients of X^2 must have negative signs which implies diminishing marginal returns

(b) Marginal Product of Quadratic Function

$Y = a + bx - cx^2$ ……..6

$\frac{\delta Y}{\delta X} = b - 2cx$ ……..7

Note that marginal product of quadratic function declines by a constant absolute amount, secondly, marginal product curve is linear and thirdly, quadratic function allows negative marginal product

(c) Elasticity in Quadratic Function

$Ep = \frac{\Delta y}{\Delta x} \cdot \frac{X}{Y}$

Using equation 6 above we have

$Ep = \frac{bx - 2cX^2}{a + bx - cX^2}$

Note here that:

i. Elasticity in quadratic function declines with input magnitude

SELF-ASSESSMENT EXERCISE

Find the marginal products and elasticities of quadratic function involving two variable inputs: $y = a + b_1X_1 + b_2X_2 – b_3X_1^2 – b_4 X_2^2 + b_5 X_1 X_2$

3.4 COBB – Douglas Power Function

(a) Algebraic Form

The single variable input of this function is presented as follows

$Y = aX^b$

Where Y = output, X = variable input,

a = constant and b = elasticity of production

The Cobb – Douglas function is easy to estimate in its logarithmic form. The above general form can be written in log form as follows

$\log Y = \log a + b \log X$

(b) Marginal Product

$Y = aX^b$ ……..…9

$\frac{\delta Y}{\delta X} = baX^{b-1}$ (MPP) …….10

Note:

i) Cobb-Douglas power function allows constant, increasing or decreasing marginal productivity

ii) Cobb – Douglas allows any of the three above but not all the three

iii) Marginal product declines if all other inputs are held constant.

(c) Elasticity of Production

$Ep = \frac{\Delta Y}{\Delta X} \frac{X}{Y}$……..11

$\frac{\Delta Y}{\Delta X}$ in equation 10 $= baX^{b-1}$ ….12

and $Y = aX^b$

Substituting the two equations (12 and 13) in equation 11
We have

$Ep = \frac{baX^b}{X} \cdot \frac{X}{aX^b} = b$……..……14

Note the following features

i. The linearized log function of Cobb-Douglas power function is easier to fit

ii. The coefficients of the function are the direct elasticities, i.e. the partial elasticity are equal to each of the parameters
iii. When coefficient b = 1, we have a case of constant returns to scale.
iv. When coefficient is greater than 1, we have a case of increasing returns to scale
v. When coefficient is less than 1, we have a case of decreasing returns to scale

Olukosi and Ogungbile (1989) identified some major shortcomings of using this function as follows:

i. Cobb-Douglas function assumes a constant elasticity of production over the entire output –input curve and therefore, the function cannot be used for data which indicates both increasing and decreasing marginal productivity.
ii. Similarly, it cannot be used for data with both positive and negative marginal products
iii. It normally exhibits a non-linear relationship and it does not give a defined maximum response at all input levels

SELF-ASSESSMENT EXERCISE

Find the marginal products and elasticities of this Cobb-Douglas power function: $Y = aX_1^{b1}X^{b2}_2$

3.5 Square Root Function

(a) Algebraic Form

A single variable algebraic form of square root function is illustrated below:
$Y = a + bX + cX^{0.5}$

Where Y = dependent variable (output), X = independent variable (input), a = constant, b and c are the coefficients.
Just as in the case of quadratic function, square root function also allows diminishing total product.

(b) Marginal Product

$Y = a – bX + cX^{0.5}$----------15

$\frac{\delta Y}{\delta X} = -b + 0.5cX^{-0.5}$ (MPP)---------16

The marginal product of this function declines at diminishing rate.

(c) Elasticity of Production

$Y = a + bX + cX^{0.5}$

$Ep = \frac{\Delta Y}{\Delta X} \cdot \frac{X}{Y}$

But $\frac{\Delta Y}{\Delta X}$ in equation $16 = b + 0.5cX^{-0.5}$

Therefore

$$Ep = 0.5cX^{-0.5} - b \cdot \frac{X}{Y}$$

$$= \frac{[0.5cX^{-0.5} - b]X}{Y} \quad \text{----------17}$$

Elasticity in this function declines at high level of input and output. This situation can happen under certain biological conditions.

SELF-ASSESSMENT EXERCISE

Determine the marginal products, elasticities and Rate of Products Substitution for this function:

$$Y = a - b_1X_1 - b_2X_2 + b_3X_1^{0.5} + b_4X_2^{0.5} + b_5X_1^{0.5}X_2^{0.5}$$

3.6 Semi-Log Function

(a) Algebraic Function

$Y = a + b \log X$

This function is very useful in aggregate production function analysis.

(b) Marginal Product

$Y = a + b \log X$ ……..18

$\frac{\delta Y}{\delta X} = \frac{b}{X}$ (MPP)……..19

The marginal product declines with increase in variable input and vice versa

(c) Elasticity of Production

$Y = a + b \log X$

$Ep = \frac{\Delta Y}{\Delta X} \cdot \frac{X}{Y}$

From equation 19, MPP $= \frac{b}{X}$

$EP = \frac{b}{X} \cdot \frac{X}{Y} = \frac{b}{Y}$ …….20

While marginal product varies with input, elasticity of production varies with output.

SELF-ASSESSMENT EXERCISE

Determine the MPP, RTS and EP for the Semi-log function of the two variable inputs below:

$Y = a + b_1 \log X_1 + b_2 \log X_2$

4.0 CONCLUSION

In this unit we have examined five types of functional forms commonly used in estimating parameters in agricultural production economics. The functional forms examined include: linear function, quadratic function, Cobb-Douglas power function, square-root function and Semi-log function. We can conclude here that the choice of these functions will depend on situation at hand.

5.0 SUMMARY

The main points in this unit include the followings:

i. Linear functions is of the form $Y = a + bX$ for single input and $Y = a + bX_1 + cX_2 + dX_3$ for three variable inputs
ii. The algebraic form of quadratic function is $Y = a + bX_1 - cX_1^2 + dX_2 - eX^2_2 + fX_3 - gX^2_3$ for three variable inputs
iii. Cobb – Douglas power function can be presented as $Y = aX^b_1X^c_2X_3^d$ for three variable inputs
iv. Square root function can also be expressed as:
$Y = a + bx_1 + cX_1^{0.5} + dX_2 + eX_2^{0.5} fX_3 + gX_3^{0.5}$ for three variable inputs
v. Semi-log function can be expressed as follows $Y = a + \log X_1 + b_2 \log X_2 + b_3 \log X_3$ for three variables inputs
vi. Cobb- Douglas function can be linearized into logarithmic form as follows:

$Y = aX_1^bX^c_2X^d_3$ linearized as $\text{Log } Y = \log a + b \log X_1 + c \log X_2 + d \log X_3$

6.0 TUTOR-MARKED ASSIGNMENT

1. List five types of production functions commonly used in agricultural production economics
2. State their algebraic forms for two variable inputs
3. With examples, give full description of any three of them.

7.0 REFFERENCES/FURTHER READING

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UNIT 4 TIME PERIODS IN THE PRODUCTION PROCESS

CONTENTS

1.0 Introduction
2.0 Objectives
3.0 Main Contents
    3.1 Meaning
    3.2 Short Run Production
    3.3 Long Run Production Process
4.0 Conclusion
5.0 Summary
6.0 Tutor-Marked Assignment
7.0 References/Further Reading

1.0 INTRODUCTION

This unit 4 is the end of the units in module 2: in the last unit, we discovered the various types of production functions commonly used in agricultural production economics. These include: linear function, quadratic function, Cobb-Douglas function, square-root function and semi-log function. In this unit we shall go a step further to examine the time periods in the production process.

2.0 OBJECTIVES

At the end of this unit, you should be able to explain the meaning of:

- time periods in the production process
- explain short run period of production
- describe long run period of production

3.0 MAIN CONTENTS

3.1 Meaning

The time periods in the production process refers to the period of time required for resources to adjust from its present status to new production positions. In a production process, before a farmer can increase his output, at least one of the resources must increase. Farm inputs like land, labour and capital cannot increase at the same rate. Labour can be adjusted quickly in response to increase demand for farm products, but capital like building and farm equipment may take longer time. This

phenomena in the production process enables us to distinguish between short run and long run production periods.

3.2 Short Run Period

The short run production period is defined as a period of time over which at least one input can be varied while keeping other inputs fixed. During this period farmer cannot increase or decrease their land and capital resources.

However, some resources like labour can vary within short notice in other to adjust to new level of output. Short run production period varies according to the type of farm enterprise.

3.3 Long Run Period

The long run period of production process is defined as the period of time long enough that all inputs can be varied. During this period of time farm output can be increased with increase in all the farm inputs. At this period of production process both farm size, farm building, farm equipments can be varied.

The conditions for output and profit maximization in the short run production process differ from that of long run period. In discussing resource allocations, consideration must be given to the time period of production process.

SELF-ASSESSMENT EXERCISE

Explain the time periods in production process

4.0 CONCLUSION

In this unit, we examined the meaning and types of time period of production process. We identified two major types in the time period. The short run and the long run period time in production. We can conclude here, that the consideration of time is very important in output and profit maximization.

5.0 SUMMARY

The main points in this unit include the followings:

i. Time period in the production process consider the period needed for inputs to adjust to change in output.

ii. The period that allow one input to vary while other inputs are fixed is called short run period of production
iii. The period long enough for all factors of production to vary is called long run period of production
iv. Time period in the production process is an important factor in the determination of maximum output and profit.

6.0 TUTOR-MARKED ASSINGMENT

Explain with examples, the concept of short run and long run period of time in the production process.

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MODULE 3 FACTOR – PRODUCT RELATIONSHIP

Unit 1 Laws of Returns
Unit 2 Classical Production Function
Unit 3 Output and Profit Maximization Under One Variable Input
Unit 4 Resource Allocation Involving More Than One Variable Input

UNIT 1 LAWS OF RETURNS

CONTENTS

1.0 Introduction
2.0 Objectives
3.0 Main Contents
  3.1 Introduction
  3.2 Law of Increasing Returns
  3.3 Law of Constant Returns
  3.4 Law of Decreasing Returns
4.0 Conclusion
5.0 Summary
6.0 Tutor-Marked Assignment
7.0 References/Further Reading

1.0 INTRODUCTION

You are welcome to the first unit of Module 3. In Module 2, we discussed the theory of production economics. In that module, we discussed the meaning and uses of production function. Other things discussed include: methods of expressing production function, types of production function and time periods in the production process. In this unit, we shall focus our attention on the laws of returns.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- identify the three types of returns we have in production relationships.
- explain the laws of returns
- use table, graph and mathematics to illustrate the three laws of returns.

3.0 MAIN CONTENTS

3.1 Introduction

In attempt to either maximize output or profit, farmers combine varying levels of input factors in the production process. This various levels of input usage by farmers allow the operations of the laws of returns.
The usefulness of this concept lies in its role as a classifying device. It provides a convenient way of classifying particular types of technological condition.

In a resource allocation involving one variable input factor while keeping the other factors fixed, three different types of relationships can be identified: increasing returns, constant returns and decreasing returns. These relationships can be discussed using written words, tables, graphs and mathematics.

3.2 Law Of Increasing Returns

Law of increasing returns is a situation in the production process in which each successive unit of variable input adds more and more to the output. That is, every addition to variable input adds more and more to the output. In other words, every addition of variable input to the fixed factors in the production process yield more to the total output than the previous unit of output. This situation in the production process can also be illustrated in a tabular form.

Table 2: Hypothetical Example of increasing Return

Variable Input X	Total Output Y	Change in Input △X	Change in Output △Y	Marginal Return △Y/ △X
1	5	-	-	-
2	10	1	5	5
3	20	1	10	10
4	35	1	15	15
5	55	1	20	20

From Table 2, addition of one unit of variable factor to 1 unit of input, yielded marginal returns of 5. Addition of one unit to the second variable factor yielded marginal returns of 10 and from 10 to 15 and 15 to marginal returns of 20. Every additions of one unit of variable factor yielded more than the previous marginal returns.

This relationship can also be expressed graphically thus:

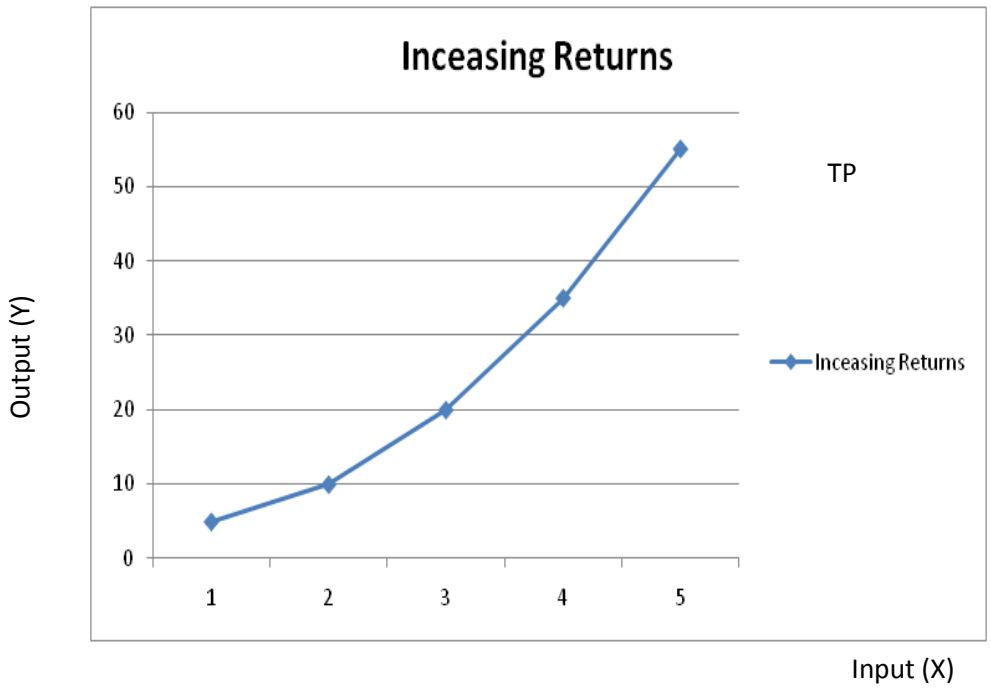


Fig. 2: Graphical Illustration of Increasing Returns

This relationship can also be expressed in mathematical form thus:

$$\frac{\Delta Y_1}{\Delta X_1} < \frac{\Delta Y_2}{\Delta X_2} < \frac{\Delta Y_3}{\Delta X_3} < \frac{\Delta Y_4}{\Delta X_4} < \frac{\Delta Y_5}{\Delta X_5}$$

This shows that the marginal return in out Y_1 and variable input X_1 is less than the marginal returns involving output Y_2 and variable input X_2 and so on.

3.3 Law of Constant Returns

The law of constant returns states that without varying the proportions in which the factor of production area combined, there is an increase in output proportionate to the increase in the total quantity of factors employed. In other words, the addition of each successive unit of the variable factor to the fixed factors adds the same to the output. For example if all the quantities of all the inputs used in producing a given

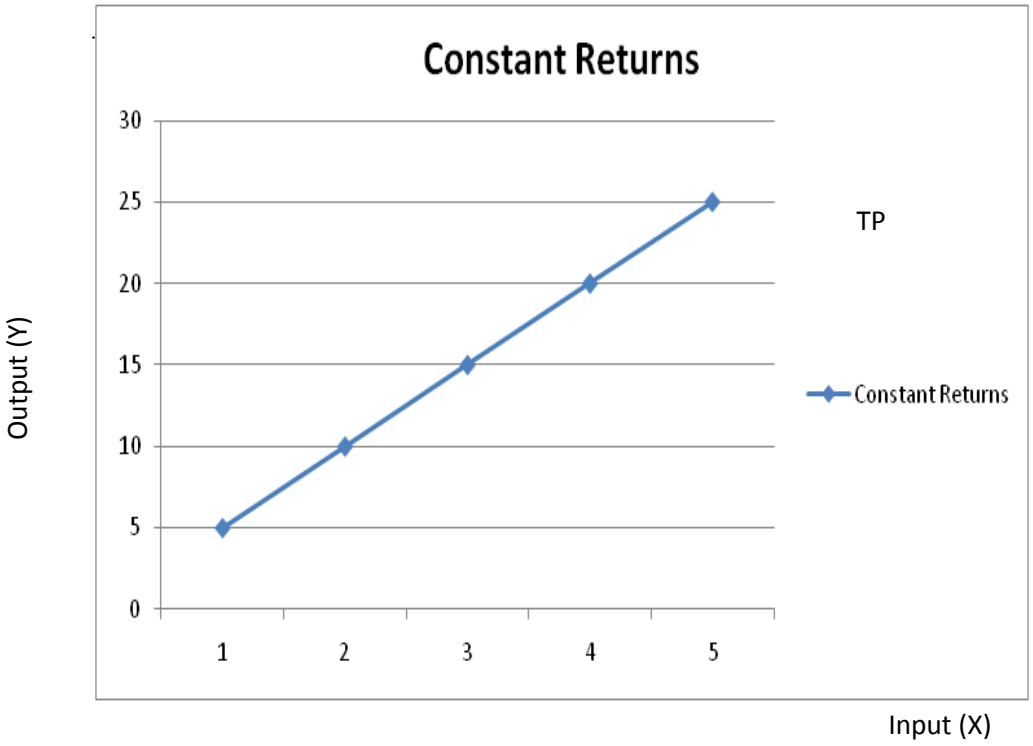
output is increased by 10 percent and then output increased by the same 10 percent, then the returns is said to be constant.
Each successive unit of variable input added results in an equal quantity of additional output.

This relationship can be expressed in a tabular form.

Table 3: Hypothetical Illustration of Constant Returns.

Variable Input (X)	Total Output (Y)	Change in Input (ΔX)	Change in Output (ΔY)	Marginal returns ΔY/ΔX
1	10	-	-	-
2	15	1	5	5
3	20	1	5	5
4	25	1	5	5
5	30	1	5	5

Table 3 showed that addition of one unit of variable input factor continuously yield marginal returns of 5, i.e every addition of one unit of variable input factor yielded the same marginal returns of 5 units.
This relationship can also be expressed in a graphical form by plotting output against the corresponding values of variable input factors.



Fg.3: Graphical Illustration of constant Return.

The graph showed a linear relationship producing a straight line curve. The relationship can also be expressed in a mathematical form using algebraic equation

$$\frac{\Delta Y_1}{\Delta X_1} = \frac{\Delta Y_2}{\Delta X_2} = \frac{\Delta Y_3}{\Delta X_3} = \frac{\Delta Y_4}{\Delta X_4} = \frac{\Delta Y_5}{\Delta X_5}$$

This shows that the marginal returns in output Y_1 and variable input X_1 is exactly the same as in other relationships.

The condition of constant returns is unlikely to be met in agricultural business.

3.4 Law of Decreasing Returns

The law of decreasing returns states that the addition of each successive unit of the variable input to the fixed inputs in the production process, adds less and less to the total output than the previous unit. Alternatively, it can be stated that for each addition to the variable factor the addition to the total output declines. This law can further be illustrated thus: suppose that the quantities of all the inputs used in producing a given output are increased for example by 10 percent and if output increases by smaller proportion e.g a percent or less, then return to scale is said to be decreasing. Decreasing returns can also be expressed in a tabular form

Table 4: Hypothetical Illustration of Decreasing Returns

Variable Input (X)	**Total output (Y)**	**Change in input** (X) △	**Change in Output** (Y) △	**Marginal Returns** Y/X △ △
1	10	-	-	-
2	35	1	25	25
3	45	1	20	20
4	60	1	15	15
5	70	1	10	10

From Table 4, addition of 1 unit of X to the initial unit of X, yielded marginal returns of 25 units. Addition of 1 unit of X to the second variable input yielded less marginal returns of 20. Every subsequent additions of one unit of variable factors yielded less than the previous marginal returns.

This relationship can equally be represented in a graphical form.

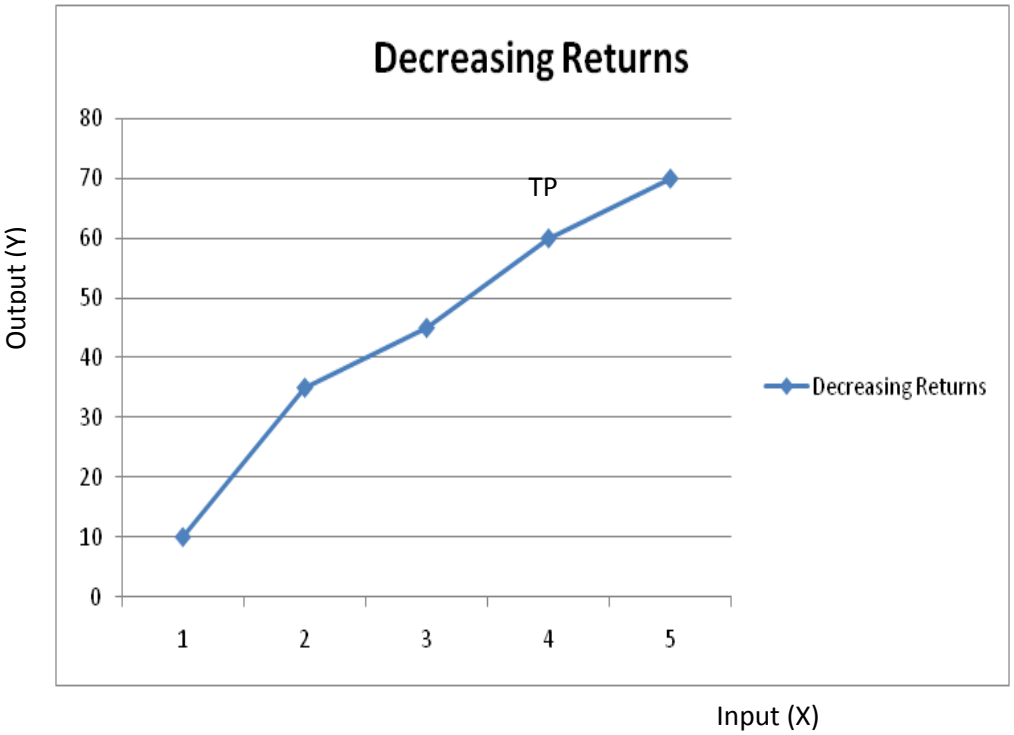


Fig. 4: Graphical Representation of Decreasing Returns

The graph depicts a concave relationship to the x-axis. We can also express this relationship between variable input factor and output in algebra form as follows:

$$\frac{\Delta Y_1}{\Delta X_1} > \frac{\Delta Y_2}{\Delta X_2} > \frac{\Delta Y_3}{\Delta X_3} > \frac{\Delta Y_4}{\Delta X_4} > \frac{\Delta Y_5}{\Delta X_5}$$

Mathematically, this relationship shows that the marginal returns in output Y_1 and variable input X_1 is more than the marginal returns involving output Y_2 and input X_2 and son on.

SELF-ASSESSMENT EXERCISE

List and explain the three types of returns in production relationships

4.0 CONCLUSION

This unit focused on the laws of returns. In this unit we identified three major types of returns in production process. These returns can be increasing, constant or decreasing. Knowledge of these relationships is very important in agricultural production because it provides a convenient way of classifying particular types of technological condition.

5.0 SUMMARY

The main points discussed in this unit include the followings:

i. Relationships between variable input factors and output can be classified into three: increasing returns, constant returns and decreasing returns

ii. In the stage of increasing returns each successive unit of variable input adds more and more to the output

iii. Increasing returns can be expressed algebraically as:

$$\frac{\Delta Y_1}{\Delta X_1} < \frac{\Delta Y_2}{\Delta X_2} \text{----------} \frac{\Delta Y_n}{\Delta X_n}$$

iv. In constant returns, addition of each successive unit of variable factor to the fixed factors adds the same to the output.

v. Constant returns can be expressed algebraically as follows:

$$\frac{\Delta Y_1}{\Delta X_1} = \frac{\Delta Y_2}{\Delta X_2} \text{----------} \frac{\Delta Y_n}{\Delta X_n}$$

vi. In decreasing returns, each addition to the variable factor, the addition to the total output declines.

vii. Decreasing returns can be expressed algebraically as follows:

$$\frac{\Delta Y_1}{\Delta X_1} > \frac{\Delta Y_2}{\Delta X_2} \text{----------} \frac{\Delta Y_n}{\Delta X_n}$$

viii. The three classifications can be expressed in written words, tabular form, graphical form and algebraic form.

6.0 TUTOR-MARKED ASSIGNMENT

With the aid of tables, graphs and algebra differentiate between the laws of increasing, constant and decreasing returns.

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UNIT 2 CLASSICAL PRODUCTION FUNCTION

CONTENTS

1.0 Introduction
2.0 Objectives
3.0 Main Contents
    3.1 Relationship Between TPP, APP and MPP
    3.2 Law of Diminishing Returns
    3.3 Stages of Production Function
    3.4 Reasons for Increasing, Decreasing and Negative Returns
4.0 Conclusion
5.0 Summary
6.0 Tutor-Marked Assignment
7.0 References/Further Reading

1.0 INTRODUCTION

In unit 1 of this module, we discussed the laws of returns. We classified return to scale in production process into three – increasing returns, constant returns and decreasing returns. We differentiated these three stages through written words, tabular form, graphical form and algebraic form. In this unit 2, we shall go a step further to look at classical production function in a case of one variable input.

Under the classical production function we shall discuss the relationship between Total Physical Product (TPP), Average Physical Product (APP) and Marginal Physical Product (MPP). We shall also use these relationships to establish the law of diminishing returns and classify production function into three stages.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the meaning of total physical product, average physical product and marginal physical product.
- plot the graph of a classical production function
- establish the relationship between TPP and APP, TPP and MPP and APP and MPP.
- explain the law of diminishing returns
- differentiate the three stages of production function
- give one reason for increasing, decreasing and negative returns.

3.0 MAIN CONTENTS

3.1 Relationship between Total, Average and Marginal Products

In other to study the relationship between Total Physical Product (TPP), Average Physical Product (APP) and Marginal Physical Product (MPP) in agricultural production process, we shall employ the technique of marginal analysis.

In using this technique in solving problems of resource allocation in agricultural production, certain assumptions must hold: that

i. Farmers purchase their inputs and sell their products in a purely competitive markets
ii. Farmers want to maximize profits from the variable inputs
iii. Prices of input and output and their relationships are known with certainty.

The relationship between total product, average product and marginal product can be illustrated in both tabular and graphical forms. The tabular illustration is presented in Table 5.

Table 5: Tabular Expression of the Relationship Between TPP, APP and MPP.

Input (X)	Output (Y)	APP Y/X	Change in Input (ΔX)	Change in Output(ΔY)	MPP (ΔY/ΔX)
0	0	-	-	-	-
4	24	6.0	4	24	6.0
8	48	6.0	4	24	6.0
12	80	6.7	4	32	8.0
16	120	7.5	4	40	10.0
20	150	7.5	4	30	7.5
24	170	7.1	4	20	5.0
28	180	6.4	4	10	2.5
32	180	5.6	4	0	0.0
36	170	4.7	4	-10	-2.5

This table showed a production function involving one variable input (X) and one farm product (Y). The table showed that the application of 0 units to fixed factors resulted in output of 8 units. Addition of 4 units of input (X) to the fixed factor resulted in 24 units of output (Y) etc.
The Average Physical Product (APP) is the ratio of Total Physical Product (TPP) to the quantity of input used in producing that amount of output. It is the amount of product obtained per unit of input at a particular level of production or level of input.

$$\text{Thus; Average Product} = \frac{\text{Total Product (Y)}}{\text{Quantity of Input Used (X)}}$$

Marginal Product or Marginal Productivity or Marginal Physical Product measures the rate at which the input is transformed into the output. It is the addition to total product due to the addition of one unit of variable input. Marginal product is therefore that rate of change in total product as the quantity of input increases.

$$\text{Marginal Product} = \frac{\text{Change in output } (\Delta Y)}{\text{Change in Input } (\Delta X)}$$

The relationship between TPP, APP and MPP shown in the table can also be expressed in a graphical form.

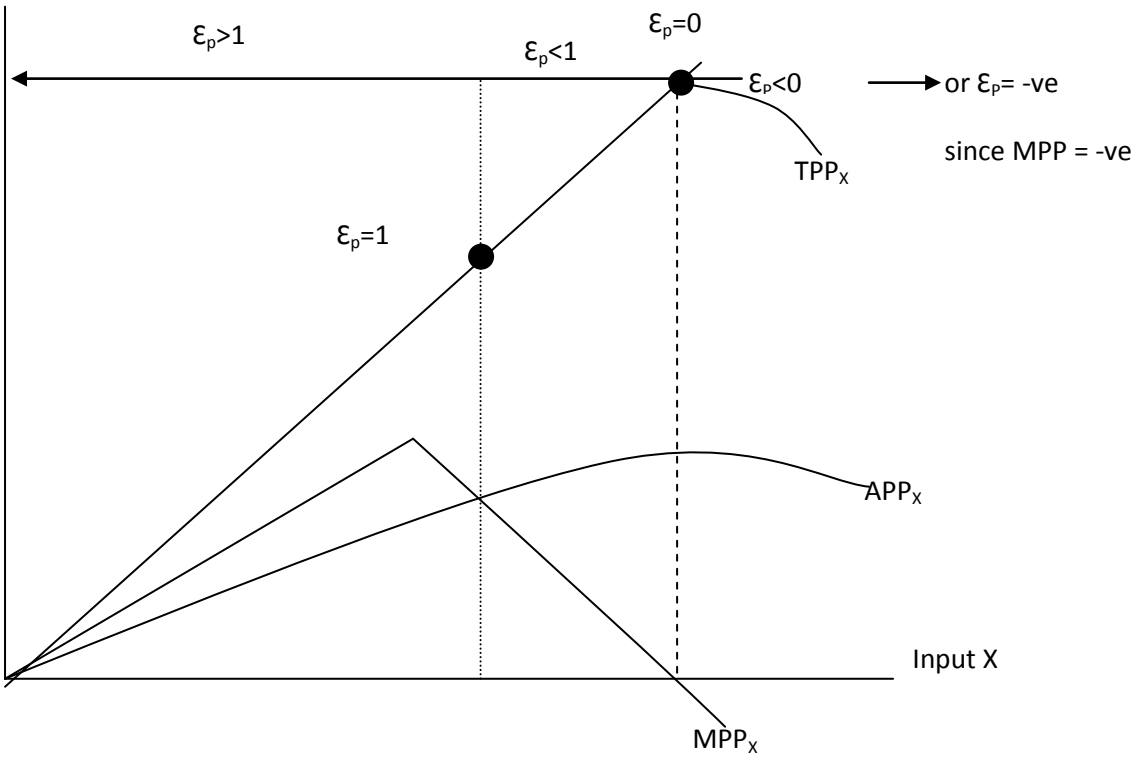


Fig. 4: Graphical Representation of the Relationship between TP, AP and MP

Table 5 and Fig. 4 will give us a clearer understanding on the relationship between production function variables (TP, AP and MP). We can divide these relationships into three categories:

i. Relationship between TPP and APP

From fig. 4, we can see that APP curve is similar in shape to TPP, this is because APP is purely derived from TPP.

Where TPP is increasing at an increasing rate, APP curve is also increasing.

ii. Relationship between TPP and MPP

- ❖ When TPP curve is increasing at an increasing rate, the MPP curve is also increasing.
- ❖ When TPP curve is increasing at a decreasing rate, the MPP curve is decreasing.
- ❖ The point at which MPP is at its maximum corresponds to the inflection point on the TPP curve.
- ❖ Beyond the inflection point, TPP continues to increase at a decreasing rate but MPP begins to decrease.
- ❖ Where TPP reaches its peak and has zero slope, MPP becomes zero.
- ❖ At the point where TPP curve begins to decline (point of intensive margin), MPP is negative.

iii. The relationship between MPP and APP

- ❖ When APP is increasing, MPP curve is above APP curve [MPP > APP].
- ❖ When APP is decreasing, MPP curve is below the APP curve [MPP<APP]
- ❖ Where APP curve is at its peak, both APP and MPP are equal (MPP = APP)
- ❖ Where MPP curve is zero, APP curve is still decreasing but positive.
- ❖ At the point where MPP is negative, APP curve is decreasing but still positive.

SELF-ASSESSMENT EXERCISE

Complete the table by calculating the Average Product and Marginal Product.

Variable Unit Of Labour (kg)	Fixed Factor (Land)	Total Product (Kg)	Average Product (kg)	Marginal Product
1	3	8		
2	3	18		
3	3	36		
4	3	48		
5	3	55		
6	3	60		
7	3	60		
8	3	56		

3.2 Law of Deminishing Returns

Diminishing returns is the most observed phenomena in production for most inputs especially at normal production levels. The law of diminishing returns states that if the quantity of one variable factor is increased by equal amount while the quantity of other factors are kept constant, the marginal product and average product of the variable factor will eventually decrease. According to Olukosi and Ogungbile (1989), law of diminishing returns can be stated in terms of the TPP, APP and MPP.

In terms of TPP, the law states that "Given a set of fixed input, if we apply successive equal increments of a variable input, the total physical output first increases at an increasing rate, then increases at a decreasing rate, reaches a maximum and then declines".

In terms of APP, the law states that "Given a set of fixed inputs, if we apply successive equal increments of a variable input, the APP increases, reaches a maximum (when it equals to MPP) and decreases henceforth".

When stated in terms of MPP, the law states that "Given a set of fixed inputs, if we apply successive equal increments of a variable input, the MPP first increases, reaches a maximum, decreases to zero and then become negative". It is therefore important when stating this law to specify whether it is in terms of TPP; APP or MPP. The law is also called law of variable proportions because various amounts of the variable inputs are applied to a fixed quantity of input factors.

This law is important because it enables us to identify the right amount of variable input to be combined with the fixed input. The fixed factors like farm size imposes a limit to the amount of additional output that can

be obtained from the addition of more variable input. This implies that the law cannot stand if there is no fixed factor in the production process.

3.3 Stages of Production Function

A typical production function such as the type illustrated in Table 5 and Fig 4 can be divided into three distinct stages. Fig. 4 showing TPP, APP and MPP curves can be reproduced to show the three stages in production process.

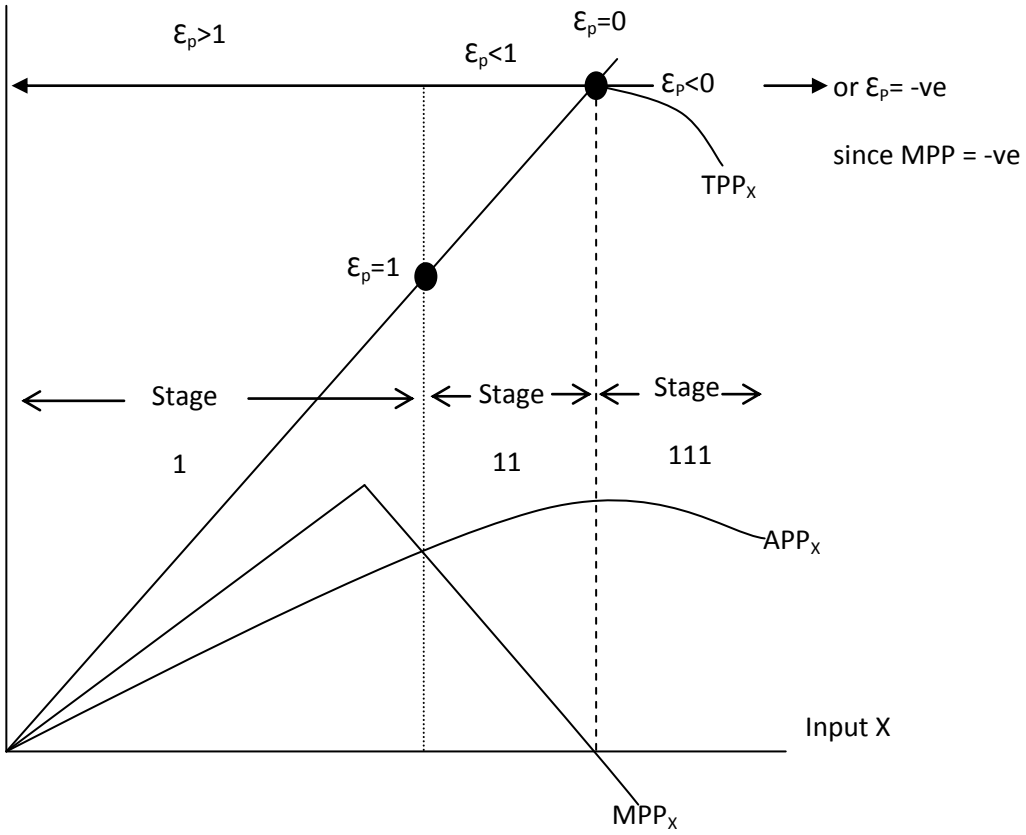


Fig. 5: Illustration of the Three Stages of Production

Characteristic Features of Stage 1

- ❖ Stage 1 ends with the extensive margin where APP equals MPP
- ❖ TPP first increases at an increasing rate and then from the point of inflection begins to increase at a decreasing rate.
- ❖ MPP increases and reaches a peak and begins to decline.
- ❖ APP also continues to increase and reaches its peak where stage 1 terminates.
- ❖ At this stage MPP is greater than APP
- ❖ The average rate at which input X is being converted to output is still increasing.

- ❖ In this stage, it is advisable for farmers to continue to add more variable input.
- ❖ It is an irrational stage of production
- ❖ All the input resources are increasing.
- ❖ The technical efficiency of variable input decreases as indicated by the decrease in APP.
- ❖ The technical efficiency of fixed inputs increases as indicated by increase in TPP.

Characteristic Features of Stage 2

- ❖ Stage 2 of production starts where APP is equal to MPP.
- ❖ After this point APP is greater than MPP
- ❖ At the end of stage 2 MPP is equal to zero
- ❖ In stage 2, MPP is equal to or less than APP but equal to or greater than zero.
- ❖ This stage represents the region of rational production where profit could be maximized.
- ❖ Elasticity of production is less than one, but equal to zero at the end of the stage.

Characteristic Features of Stage 3

- ❖ Stage 3 of production starts where MPP is equal to zero
- ❖ In stage 3, MPP becomes negative
- ❖ Both TPP and APP continue to decrease
- ❖ Elasticity of production is less than zero
- ❖ Variable resource is in excess relative to fixed resources.
- ❖ The technical efficiency of variable resource and fixed resources declines
- ❖ Stage 3 is therefore irrational stage of production

Summary of Three Stages of Production Function:

S.No.	Stage 1	Stage II	Stage III
1.	Starts from the origin And ends where MPP = APP	Starts from where APP is maximum and ends where MPP is zero.	Starts from where MPP is zero or TPP is maximum.
2.	TPP increases at Increasing rate up to the point of reflection.	TPP increase at decreasing rate	TPP decreases at increasing rate
3.	MPP > APP	MPP < APP	-

The characteristic features of these three stages of production showed that stage 2 should be the area of operation for a rational farmer. In stage

2, the ratio of the variable resource to fixed resources is higher to the extent that adjustment between the two extremes is possible. Based on physical considerations, production is limited to stage 2. However, the determination of the exact point of maximum net revenue required additional information of the prices of input and output.

3.4 Reasons for Increasing, Decreasing and Negative Returns

Reddy *et al* (2004) gave reasons for the observation of increasing, decreasing and negative returns in production process as follows:

In stage 1, the surplus fixed inputs are not put to efficient utilization due to insufficient availability of variable input. Under this situation application of more quantities of variable input makes the excess fixed input relative to variable input to be more efficiently utilized. This situation leads to increasing returns. As we continue to apply more of the variable input to fixed input a time will come when we reach maximum marginal physical product. Beyond this point any further addition of variable input to fixed inputs will result into less addition to output. This is the point of decreasing returns.

As we noticed the situation of decreasing returns, if more and more of variable input is added to the fixed factors, the variable resource becomes too large to fixed resources. At this stage the unused variable resource will impede production process instead of enhancing it. This leads to the situation of negative returns.

4.0 CONCLUSION

In this unit, we discussed the relationship between total, average and marginal physical products in a production function. We used that relationship to establish the law of diminishing returns as well as classifying production process into three stages. We finally in this unit give reasons why we observed increasing, decreasing and negative returns in a production process. The interaction between the concepts of TPP, APP and MPP made us to identify the rational and irrational regions of production.

5.0 SUMMARY

The main points in this unit include the followings:

i. There are assumptions that underlie the use of the technique of marginal analysis in resource allocation.

ii. APP = TPP (Y)

iii. $MPP = \frac{\text{Change in TPP } (\Delta Y)}{\text{Change in Input Used } (\Delta X)}$

iv. The relationship between TPP, APP and MPP can be expressed both in tabular and graphical form

v. APP and MPP curves are similar in shape to TPP because both of them are produced from TPP.

vi. Law of diminishing returns can be defined from the perspective of TPP, APP and MPP.

vii. Three stages or regions can be identified in production process.

viii. Stages 1 and 3 represent irrational regions in the production process because maximum net return can never be achieved in the two regions.

ix. Stage 2 represents the rational region of production where we can obtain maximum profit.

x. Availability of both variable and fixed inputs in resource allocation will determine whether a production process will operates increasing, decreasing or negative returns.

6.0 TUTOR-MARKED ASSIGNMENT

1. Explain the meaning of the following concepts in production function:

   (a) Average Physical Product (APP)
   (b) Marginal Physical Product (MPP)
   (c) Law of diminishing returns
   (d) Rational Production Stage
   (e) Irrational Production

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UNIT 3 OUTPUT AND PROFIT MAXIMIZATION UNDER ONE VARIABLE INPUT

CONTENTS

1.0 Introduction
2.0 Objectives
3.0 Main contents
    3.1 Determination of optimum output
    3.2 Determination of optimum profit
    3.3 Input Demand Curve
    3.4 Elasticity of production
4.0 Conclusion
5.0 Summary
6.0 Tutor-Marked Assignment
7.0 References/Further Reading

1.0 INTRODUCTION

You are welcome to another unit under module 3. In unit 2, we discussed the classical production function. Under the unit, we explained the relationship between TPP, APP and MPP in a production function. We further used this relationship to establish the law of diminishing returns. We also used that relationship to categorized production process into three stages, we identified the distinguishing features of each stage. We then proceeded to find reasons for increasing, decreasing and negative returns in production process. In this unit we shall focus our attention on output and profit maximization involving variable input.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

i. explain how optimum output is determined
ii. describe how optimum profit is determined using production function approach
iii. formulate the determination of optimum profit using profit function approach.
iv. explain how input demand curve is obtained from production function
v. formulate output elasticity

3.0 MAIN CONTENTS

3.1 Determination of Optimum Output

In other to determine optimum level of production in a resource allocation involving one variable input and one product, certain assumptions must hold:

i. Consideration is given to the effect of varying only one input.
ii. That only one product is produced
iii. That the objective of farm is to maximize profits
iv. Both the fixed inputs and variable input together with the product are divisible.

In line with the above assumptions, if the objective is to maximize output, the condition is that farm input should be applied until its marginal product equals zero. This expression can be presented in algebraic form using quadratic equation.

$Y = a + bX - cX^2$……...i

Where:

Y = Output
X = Variable input
a = constant term (intercept)
b & c = coefficients

To obtain the maximum output, we need to equate marginal physical product (MPP) to zero by finding the derivative of equation i

$\frac{\Delta Y}{\Delta X} = b - 2cX$ (MPP)……ii

To determine the amount of input X that maximizes output Y, we need to equate MPP to zero. Thus in this equation ii- MPP = b-2cX.

Therefore $b - 2cX = 0$…….iii

From equation iii, if we know the values of b and c, we can calculate the amount of input X at which TPP reaches its maximum.

Any addition to variable input beyond this level will result in inefficient allocation of resources. If the variable input level is less than this optimum amount, it will still result into inefficient allocation of resources.

Therefore, maximum output under this condition is the highest amount of output that could be technically produced. It is the highest feasible output and does not consider the prices of input and output in its computation. That is why the value of optimum output is usually higher than the economic optimum output.

Working Example

1. Consider the production function of maize output as follows: $Y = 100 + 400X - 2X^2$
   Where Y = maize output (kg) and X = fertilizer application (kg)

Calculate: (a) the level of input that will maximize maize output.
(b) The optimum quantity of maize that could be produced.

Solution to the problem

(a) Consider the equation: $Y = 100 + 400X - 2X^2$
Optimum level is achieved when MPP = 0

Therefore $\frac{\triangle Y}{\triangle X} = 400 - 4X = 0$

$4X = 400$ and $X = \frac{400}{4} = 100$

Answer = The level of input that will maximize maize Output = 100kg of fertilizer

(b) Optimum quantity of maize that could be produced

This can be obtained by substituting the value of input X into the formula

$$\begin{aligned} Y &= 100 + 400X - 2X^2 \\ &= 100 + 400(100) - 2(100)^2 \\ &= 100 + 40{,}000 - 2(10{,}000) \\ &= 40{,}100 - 20{,}000 = 20{,}100\text{kg} \end{aligned}$$

Optimum Output of maize produced = 20,100kg

SELF-ASSESSMENT EXERCISE

1. Find the level of input and the optimum product that could be produced from the production function below:

$$Q = 300X - 2X^3$$

3.2 Determination of Optimum Profit

In a factor – product relationship involving one variable input and one product, if the objective is to maximize profit (i.e economic optimum), then prices of the input and output in addition to the quantities of input and output are required.

We can approach profit maximization through any of the following methods:

(a) Production function Approach
(b) Total Profit function Approach

a. Production Function Approach

Note that the condition for profit maximization under one variable input requires that input be applied until the value of its marginal product is equal to the price of the input i.e. VMPx = Px……iii

Consider the following: Let

P_x = Price of input
P_y = Price of output
$\triangle X$ = Change in input
$\triangle Y$ = Change in output
$P_X \Delta X$ = Additional unit cost of input
$Py\ \Delta Y$ = Additional unit value of output

From equation III, $MPx = \frac{\Delta Y}{\Delta X}$ and V = Py

Therefore $VMPx = Py\ (\frac{\Delta Y}{\Delta X})$

At optimum profit VMPx = Px

$$Py\ .\ (\frac{\Delta Y}{\Delta X}) = Px \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots iv$$

Where
Py = Price of output Y
Px = Price of input X
$\frac{\Delta Y}{\Delta X}$ = Marginal product of X

We can rewrite equation iv thus:

$Py\ \Delta Y = P_X\ \Delta X$……..v

Equation v shows that $Py\ \Delta Y$ = Marginal Revenue and $P_X\ \Delta X$ = Marginal Cost.

We can therefore states that profit is maximized when marginal Revenue (MR) equals Marginal Cost (MC).

Let us consider equation iv in another way

$$Py \cdot \frac{\Delta Y}{\Delta X} = P_X$$

We can also rewrite this equation as follows:

$$\frac{\Delta Y}{\Delta X} = \frac{P_X}{P_y} \quad \text{……vi}$$

Where ΔY = Marginal product and $\frac{P_y}{P_X}$ = input – output ratio

Therefore, profit maximization can also be achieved by equating marginal product to input – output price ratio. The only condition that can make equation ii under output maximization equals equation vi under profit maximization is that, price of input must be zero i.e. if inputs are supplied free of charge which violate one of our assumptions.

b. Total Profit Function Approach

We can also determine profit maximization level using total profit function:

Profit (π) can be calculated by deducting Total Cost (TC) from Total Revenue (TR).

Thus $\pi = TR - TC$

$TR = P_yY$

TC = Fixed cost (F) + Variable Cost (P_XX)

Therefore $\pi = P_yY - P_xX + F$ ……..vii OR

$\pi = P_yY - P_xX - F$

Under the equation vii, profit is maximized when marginal profit = O

Therefore marginal profit in equation Vii

$$\frac{\Delta \pi}{\Delta X} = Py \cdot \frac{\Delta Y}{\Delta X} - Px = 0$$

Therefore $Py\ (\Delta Y/\ \Delta X)$ = Px ($VMP_X = P_X$)

and $\frac{\Delta Y}{\Delta X} = \frac{Px}{Py}$ = (MP = Input – Output price ratio)

Or Py ΔY = Px ΔX = (Marginal Revenue = Marginal Cost)
All these indicate the best point to produce. Profit maximization level is the point where the use of additional input yield exactly the same amount of additional return. At this point it neither pays to increase nor decrease the use of the input in production.

3.3 Input Demand Curve

We have already discovered that the rational stage of production where profit can be maximized is stage 2. In this stage, we noticed a downward sloping of marginal product. Between the beginning and end of stage 2, we can derive an input demand curve for the purpose of determining an optimum input level that will maximize profit.

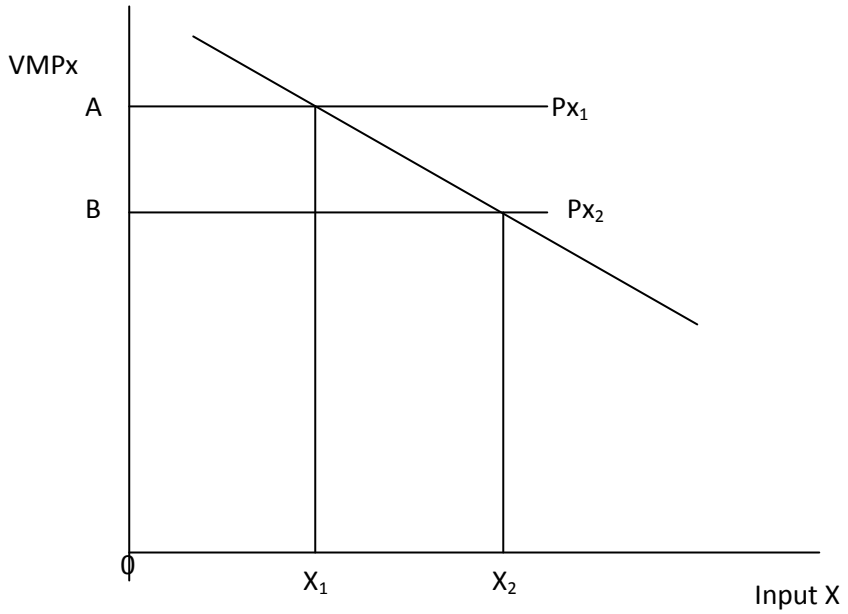


Fig. 7: Derived Demand Curve for profit-maximizing input level

The condition for profit maximization is that the value of marginal product should be equal to the input price i.e. VMPx = Px. Fig.6 showed VMPx on the vertical line and X on the horizontal line. The implication of this relationship is that where VMPx meet X on the input demand curve, profit is maximized.

From fig. 6, when the input price is Px, then the input quantity that will be required to maximize profit is X_1 (point A). Similarly, if the input price is reduced to P_{X2}, the quantity of input X required to maximize profit will be X_2 (point B). We can therefore say that input demand is the quantity of input X that a farmer will require to maximize profit.

3.4 Elasticity of Production

Elasticity of production or output elasticity measures the degree of responsiveness of output to changes in the variable input. It can also be defined as measures of the percentage change in output resulting from a given percentage change in the input.

Mathematically expressed as

$$Ep = \frac{Y_1 - Y_2}{Y_1} \div \frac{X_1 - X_2}{X_1}$$

$$= \frac{\triangle Y}{Y} \div \frac{\triangle X}{X}$$

$$= \frac{\triangle Y}{\triangle X} \times \frac{X}{Y}$$

Note that $\frac{\Delta Y}{\Delta X}$ = MPP and $\frac{Y}{X}$ = APP

Remember that $\frac{X}{Y}$ is the reciprocal of APP.

Therefore, we can express the relationship thus:

$$Ep = MPP.\frac{1}{APP} = \frac{MPP}{APP}$$

This end product of output elasticity will guide us to determine elasticity of production in the three stages of production function.

- ❖ In stage 1 where MPP is greater than APP, elasticity of production will be greater than one.
- ❖ At the end of stage 1 where MPP equals APP, elasticity of production will be one
- ❖ In stage 2 where APP is greater than MPP output elasticity will be less than one, but greater than zero.
- ❖ At the end of stage 2 where MPP is zero output elasticity is also zero
- ❖ In stage 3 of production output elasticity will be negative.

We can also use this relationship between MPP and APP to determine output elasticity in Cobb-Douglas power function.
Note that Ep = MPP/APP

Consider power function involving one variable input and one output:

$Y = aX^b$

Remember that we have already estimated the MPP of this function under the types of production function as:

$$MPP = \frac{baX^b}{X} \text{ and } APP = \frac{Y}{X}$$

This relationship between MPP and APP can show that Cobb-Douglas power function assumed a constant output elasticity throughout the entire production function.

$$Ep = \frac{MPP}{APP} = baX^b/X \div X/Y$$
$$= baX^b/Y$$

Remember that Y in the equation = ax^b

$$\text{Therefore } Ep = \frac{baX^b}{ax^b} = b$$

b which is the coefficient of the function is also the elasticity of production. If b = 1, we have constant returns, if b<1 we have the case of decreasing rate and if b>1 we have the case of decreasing returns.

SELF-ASSESSMENT EXERCISE

i. List four assumptions for determining the optimum level of output
ii. State the condition for output maximization
iii. The condition for profit maximization using one variable input X and producing one output Y, can be expressed as ----------
iv. The only way in which the profit maximizing input level will be the same as the output maximizing input level is ----------

4.0 CONCLUSION

In this unit, we discussed output and profit maximization under one variable input and one output. Under the unit, we showed how we can determine output maximization level. We also showed how we can determine profit maximization level using either production function or total profit function approach. Furthermore, we also showed graphically how we can determine the optimum level of input for profit maximization and on the final note; we showed how to calculate elasticity of production. This unit conclude factor –product relationship involving one variable input and one output.

5.0 SUMMARY

The main points in this unit include the followings:

i. To determine optimum levels of output and profit, certain assumptions must hold.
ii. Output maximization requires that input be applied until MPP equals zero (MPP = 0)
iii. Profit maximization condition requires that input be applied until MPP is equal to the price of the input (VMPx = Px).
iv. The condition that can make the objectives of output and profit maximization equal is when input price is free (Px = 0)
v. We can use production function or total profit function to determine optimum profit
vi. We can use the downward slope of MPP in stage 2 of production function to determine the optimum input level that will maximize profit and this approach is called input demand curve
vii. Output elasticity or elasticity of production measures the degree of responsiveness of output to changes in the variable input.
viii. Output elasticity varies as we move along the slope of production function.
ix. Output elasticity in Cobb-Douglas power function is the same with the values of its coefficients.

6.0 TUTOR-MARKED ASSIGNMENT

Consider the production function of a farmer below:
$Y = 10 + 200X - 2X^2$ and the price of input = ₦10 and price of output = ₦ 50.

Calculate the optimum profit and output of this function.

7.0 REFERENCES/FURTHER READING

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UNIT4 RESOURCE ALLOCATION INVOLVING MORE THAN ONE VARIABLE INPUT

CONTENTS

1.0 Introduction
2.0 Objectives
3.0 Main Contents
3.1 Production Function Involving Two Variable Inputs
3.2 Profit Maximizing Condition Involving more than one variable input
3.3 Estimation of Marginal Physical Product (MPP) in a two variable inputs.
4.0 Conclusion
5.0 Summary
6.0 Tutor-Marked Assignment
7.0 References/Further Reading

1.0 INTRODUTION

You are welcome to the last unit of this module 3. This module handled resource allocation involving factor- product relationship. In unit 3 of the module we discussed output and profit maximization involving one variable input and one output. We showed how we can graphically estimate the optimum level of input that can maximize profit. We also showed how one can use MPP and APP to calculate output elasticity. In this last unit of this module, we want to go a step further to see what happen to profit maximization in the case of more than one variable input.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the interaction between two variable inputs to produce output
- describe the least-cost input combination that will result into optimum profit
- use production function to determine optimum profit of two variable inputs
- calculate the Marginal Physical Products (MPP) of two variable inputs using different functional forms.

3.0 MAIN CONTENTS

3.1 Production Function Involving Two Variable Inputs

We have already considered a production function in which only one variable input is used to produce an output. In farming, this situation may be very difficult as we can hardly increase just one variable without adjusting one or two other ones. For instance increase in farm size may require additional labour, increase in fertilizer application may also require additional labour, increase in seed planted will require additional farm land etc. It is therefore necessary to further investigate what happens to the variation of two variable inputs on one output.

This investigation will require some assumptions: that

i. farmers operates in a purely competitive market where they have no control over the prices of inputs and output.
ii. farmers purchased their farm inputs and sell all products.
iii. at least one of the farm inputs must be fixed.

This could be expressed functionally as

$$Y = f(X_1, X_2/X_3 \text{ ----} X_n)$$

The relationship of the effect of two variable inputs on one output can be represented in a tabular form

Table 6: The effect of variable inputs X_1 and X_2 on an output

Variable Input X_1	Variable Input X_2 1	2	3	4
5	60	55	45	30
10	88	80	70	60
15	108	100	80	60
20	118	100	75	75
25	103	95	75	66

This situation will require a three dimensional graph to show the effects of X_1 and X_2 on output.
You are advised to consult Nweze (2002), Olukosi and Ogungbile (1989) and Olayide and Heady (1982) on how to construct this graph.
In this illustration we can see the various combinations of variable inputs X_1 and X_2 that will result into the production of an output (Y).

The table showed that variable input X_1 ranges from 5 units to 25 units, while variable input X_2 ranges from 1 unit to 5 units.

The various amount of output obtained in response to the various combinations of input X_1 and X_2 are also shown in the table. For example combination of 10 units of input X_1 and 2 units of input X_2 will produce an output of 80.

The major concern here is, which of these various combinations will give us the optimum profit desired?

3.2 Profit Maximization Condition Involving More Than One Variable Input

We have already determined profit maximization under one variable input and one output. The condition for profit maximization under more than one variable input require the expansion of the earlier condition for one variable input and then use it to examine the relationships between the variable inputs.

Be reminded that the condition for profit maximization under one variable input is that the value of marginal product of input should be equal to the price of the input.

Thus $VMP_X = Px$ ----------i

What we need to do in the case of more than one variable input is to apply this condition to each of the variable input.
For variable input X_1, it will be stated thus:

$$VMPX_1 = P_{x1}$$ ----------ii.

Remember that V= Price of output (P_y), and

$$MPX_1 = \frac{\Delta Y}{\Delta X_1}$$

We can rewrite equation ii as
$Py\ (\Delta Y/\Delta X_1) = Px_1$----------iii
Or $Py\ (\Delta Y/\Delta X_1)/Px_1 = 1$----------iv

Similarly, for variable input X_2, it will be

$$VMPx_2 = Px_2 \text{ ----------v}$$

Following the same process above, we have

$Py\ (\Delta Y/\Delta X_2) = Px_2$

Or $Py\ (\Delta Y/\Delta X_2)/Px_2 = 1$----------vi

For variable input X_n, we have $VMPx_n = Px_n$ ----------vii

Following the example of variable input X_1

We can rewrite equation vii as:

$Py\ (\Delta Y/\Delta X_n) = Px_n$----------viii

Or $Py\ (\Delta Y/\Delta X_n)/Px_n = 1$----------ix

If we consider equations iv, vi and ix, we discovered that all the equations are equal to 1.

We can therefore state that:

$Py\ (\Delta Y/\Delta X_1)/Px_1 = Py\ (\Delta Y/\Delta X_2)/Px_2 = Py(\Delta Y/\ \Delta X_n)/Px_n$

Since Py cut across the three equations, we can therefore divide through by Py.

$(\Delta Y/\Delta X_1)/Px_1 = (\Delta Y/\Delta X_2)/Px_2 = (\Delta Y/\ \Delta X_n)/Px_n$----------x

Or $\frac{MPPx_1}{Px_1} = \frac{MPPx_2}{Px_2} = \frac{MPPx_n}{Px_n}$ ----------xi

Equation xi can be interpreted to mean that for a product to be produced at least-cost using 'n' variable inputs, the resources must be allocated in such a way that the last amount spent on each resource yields the same marginal product. In other words, to produce a given level of output of product with 'n' variable inputs at a minimum cost, the ratio of the marginal products of the variable inputs must be equal to their respective prices.

Note that just as we have in the case of profit maximization of one variable input under total profit function, we can equally use total profit function to arrive at the above conclusion. What we need to do is to equate marginal revenue (MR) to marginal cost (MC) for each of the input.

3.3 Estimation of MPP in A Two Variable Inputs

The key factor in profit maximization condition is marginal physical product. It is therefore essential for you to be familiar with how to calculate the MP using different functional forms. We have already presented some of these functional forms; we shall use some of them here.

(a). Quadratic function

$Y= a+ b_1X_1 + C_2 X_2 – dX^2_1 - eX^2_2 + fX_1X_2$

To obtain the MPP of this equation requires the partial derivates of the function with respect to X_1 and X_2

$$\frac{\Delta Y}{\Delta X_1} = b- 2dX_1 + fX_2 = MPPX_1$$

$$\frac{\Delta Y}{\Delta X_2} = C- 2eX_2 + fX_1 = MPPX_2$$

If the prices of X_1, X_2 and output Y are known, we can use this function to calculate optimum profit for the equation ($VMPPx_1 = Px_1$ and $VMPPx_2 = Px_2$)

(b). Cobb- Douglas Power Function

$Y= aX_1^bX_2^c$

Taking the partial derivative of this equation with respect to X_1 and X_2, MPP will give us:

$$\frac{\Delta Y}{\Delta X_1} = \frac{baX_1^bX^c_2}{X_1} = MPPx_1$$

$$\frac{\Delta Y}{\Delta X_2} = \frac{caX_1^bX^c_2}{X_2} = MPPx_2$$

If the prices f variable inputs X_1 and X_2 and that of the output Y are given, we can estimate the profit-maximizing condition for this function.

(c) Semi –log function

Consider variables X_1 and X_2 in the semi-log function below:
$Y= a+b \log X_1 +c \log X_2$

The MPP of this function will be obtained by taking the partial derivatives of the function with respect to X_1 and X_2

$$\frac{\Delta Y}{} = \frac{b}{} = MPPx_1$$

$$\frac{\Delta X_1}{\Delta X_2} = \frac{c}{X_2} = MPPx_2$$

For maximum profit and output, prices of X_1 , X_2 and Y are required.

(d). Square Root Function

Square root function involving two variable inputs can be written as follows:

$$Y = a - bX_1 - cX_2 + dX_1^{.5} + eX_2^{.5} + fX_1^{.5}X_2^{.5}$$

To calculate the MPP of this functional form, take the partial derivatives of the function with respect to X_1 and X_2.

$$\frac{\Delta Y}{\Delta X_1} = -b + .5dX_1^{-.5} + .5fX_1^{-.5}X_2^{.5} = MMPx_1$$

$$\frac{\Delta Y}{\Delta X_2} = -c + .5eX_2^{-.5} + .5fX_1^{.5}X_2^{-.5} = MMPx_2$$

SELF-ASSESSMENT EXERCISE

Find the marginal physical product of this function $Y = a\,e^{bx}{}_1 + cX_2$

4.0 CONCLUSION

In this last unit of this module, we learnt about resource allocation involving more than one variable input. We also looked at the condition for profit maximization for more than one variable input and specifically worked with profit maximization involving two variable inputs. We finally used different functional forms to determine MPP of two variable inputs. We can then conclude here that profit maximization involving two or more variable inputs followed the same pattern with that of profit maximization involving one variable input.

5.0 SUMMARY

The main points in this unit include the followings:

i. There are assumptions underlining the application of profit-maximization condition for more than one variable input
ii. There are various outcomes of adjustments or variations between inputs X_1 and X_2.
iii. Graphical expression of the relationship between two variable inputs and one requires a three dimensional graph.

iv. Profit maximizing condition for more than one variable input follow the same step as in one variable input (VMPx = Px).

v. Profit maximization under more than one variable input requires that

$$\frac{MPPx_1}{Px_1} = \frac{MPPx_2}{Px_2} = \frac{MPPx_n}{Px_n}$$

vi. Marginal Physical Product (MPP) can be estimated using different types of functional forms e.g. quadratic function, Cobb-Douglas power function, semi-log function and square root function.

6.0 TUTOR-MARKED ASSIGNMENT

Find the marginal physical product (MPP) of the following functional forms:

(a) $Y = a + b_1X_1 + b_2X_2 + b_3X_1^2 + b_4X_2^2$

(b) $Y = aX_1^{b1}X_2^{b2}$

(c) $Y = a - b_1X_1 - b_2X_2 + b_3X_1^{.5} + b_4X_2^{.5} + b_5X_1^{.5}X_2^{.5}$

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MODULE 4 FACTOR – FACTOR AND PRODUCT-PRODUCT RELATIONSHIPS

Unit 1 Profit Maximization in Factor-Factor Relationships
Unit 2 Important Concepts in Factor-Factor Relationships
Unit 3 Product-Product Relationships

UNIT 1 PROFIT MAXIMIZATION IN FACTOR-FACTOR RELATIONSHIPS

CONTENTS

1.0 Introduction
2.0 Objectives
3.0 Main Contents
  3.1 Determination of Optimum Profit Using Production Function Approach
  3.2 Determination of Optimum Profit Using Total Profit Function Approach
  3.3 Constrained Profit Maximization
4.0 Conclusion
5.0 Summary
6.0 Tutor-Marked Assignment
7.0 References/Further Reading

1.0 INTRODUCTION

This is the first unit of module 4. The focus of the module is the relationships between factor-factor and product-product. In this unit, we shall focus our attention on profit maximization in factor-factor relationships. Our major task in this unit will be determination of optimum profit using either production function or total profit function approach.

We shall also look at the effect of imposing constrain on optimum profit

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- determine the profit maximizing input levels using production function approach.
- determine the profit maximizing input levels using total profit function approach.
- calculate the effect of imposing constrain on optimum profit.

3.0 MAIN CONTENTS

3.1 Determination of Optimum Profit Using Production Function Approach

We have already discussed that profit maximization involving more than one variable input is also regarded as a factor-factor relationship. This is because in both cases, we looked at the effect of two or more variable inputs on one product.

Here we shall focus our attention on the effect of varying two inputs on one product. For example, we can examine the effect varying fertilizer and farm size on the output of maize.

Here we are still going to follow our steps under profit maximization involving more than one variable input. Here, let us consider the implicit form of the model again.

$Y=F(X_{1,} X_{2\,/}X_3\ldots\ldots X_n)$
Where,

Y = Farm Output e.g. maize
X_1 and X_2 – variable inputs e.g. fertilizer, farmland
X_3-------X_n = fixed inputs e.g. labour, capital etc.

In this model, two variable inputs are combined to produce a given level of output. For us to determine the profit-maximizing level of the equation $Y = F(X_1, X_2)$ we take the partial derivates with respect to X_1 and X_2

Remember that VMPx = Px at optimum profit.
Therefore at optimum profit level

$$\frac{\Delta Y}{\Delta X_1} Py = Px_1 \text{ and}$$

$$\frac{\Delta Y}{\Delta X_2} Py = Px_2$$

Or

$$\frac{\Delta Y}{\Delta X_1} = \frac{Px_1}{Py} \quad \text{i.e. } MPPx_1 = Px_1/Py$$

$$\frac{\Delta Y}{\Delta X_2} = \frac{Px_2}{Py} \quad \text{i.e.} \quad MPPx_2 = Px_2/Py$$

We can separately solve for inputs X_1 and X_2 to arrive at profit maximization if the two inputs does not interfere with each other. The major concern here is that as output (Y) is held constant, we want to look at the possibilities of substituting X_1 for X_2 or vice versa.

3.2 Determination of Optimum Profit Using Total Profit Function Approach

Here we shall adopt similar approach when determining optimum profit for one variable input using total profit function.

You will remember that under that approach we used the formula: $\prod$ = TR – TC

Where,

$\prod$ = Total Profit
TR = Total Revenue (RyY)
TC = Total Cost [(PxX) + F]

We can expand this formular to cover two variable inputs as follows:

$\prod = PyY - Px_1 X_1 - Px_2 X_2 - F$ ----------ii

To get the maximum we can differentiate this function with respect to X_1 and X_2

$\frac{\Delta \prod}{\Delta X_1} = Py(\Delta Y/\Delta X_1) - Px_1 = 0$ ----------iiii

$\frac{\Delta \prod}{\Delta X_2} = Py(\Delta Y/\Delta X_2) - Px_2 = 0$ ----------iv

We can rewrite equation iii thus;
$Py(\Delta Y/\Delta X_1) = Px_1$

$\Delta Y/\Delta X_1 = Px_1 / Py$ ----------v
Similarly, we can rewrite equation v thus:
$Py(\Delta Y/\Delta X_2) = Px_2$

$\Delta Y/\Delta X_2 = Px_2 / Py$ ----------vi

Thus arriving at the same conclusion when we used production function approach.

SELF-ASSESSMENT EXERCISE

i. Use the profit function $\prod = PyY - Px_1X_1 - Px_2X_2 - F$ to proof that $VMPx_1/Px_1 = VMPx_2/Px_2 = 1$

ii. Determine the minimum cost combination of inputs X_1 and X_2 for an output level of 250 units in the table below (Price of input X_1 ₦50 and that of X_2 = ₦100):

Unit of X_1	Unit of X_2	Cost of X_1	Cost of X_2	Total outlay
1	30			
2	25			
3	20			
4	15			
5	10			
6	5			

3.3 Constrained Optimization

In all the output and profit maximization discussed so far, we have not imposed any condition limiting the extent to which inputs can be combined. Under that condition we are operating unconstrained output or profit maximization.

What this then means is that farmers can achieve their objective function without any constraint- no cash hindrance, no farmland limitation etc.

In reality, farmers may face some limitations in try to achieve their objective function of profit maximization especially cash. Cash outlay set a limit to which farmers can go in optimizing their profit function.
In other to impose limitation into the objective function of profit maximization we need to subject the profit function to a constraint by using Lagrange multiplier (ʎ). The product of this function can then be added to the original profit function to form the new Lagrange function
Note the objective function:

$\prod = PyY - (Px_1 X_2 + Px_2X_2 + F)$----------i
The budget constraint function or Lagrange multiplier
$C = Px_1X_1 + Px_2 X_2$ ----------ii

Set equation ii to zero
$Px_1 X_1 + Px_2 X_2 - C = 0$ ----------iii

We can now add equations i and iii together

$\prod = PyY - Px_1X_1 - Px_2X_2 - F - (Px_1X_1 + Px_2X_2 - C)$----------iv
Multiply through by Lagrange multiplier (ʎ) to obtain Lagrange function as
$\prod = PyY - Px_1X_1 - Px_2X_2 - F - ʎ(Px_1X_1 + Px_2X_2 - C)$------V

We can now obtain our constrained optimum profit by taking the first-order partial derative of the equation V with respect to X_1, X_2 and ʎ.

For detailed study of constrained profit maximization you can consult Nweze (2002), Olayide and Heady (1982), Olukosi and Ogungbile (1989), Marshall (1998) and other advanced books on Agricultural production economics

4.0 CONCLUSION

In this unit, we discussed profit maximization in a factor-factor relationship. In the unit, we showed how to determine optimum profit using both production function and total profit function methods. We went further to show how profit optimization can be achieved when constraint is imposed on the objective function. We can conclude here that, there is difference in unconstrained and constrained profit maximization objective of farmers.

5.0 SUMMARY

The main points learnt in this unit include the followings:

1. Profit maximizing condition for factor-product involving more than one variable input is the same with factor-factor relationships
2. The same outcome were obtained when profit maximization in a factor-factor relationship is determined using either production function or total profit function approach.
3. Profit is maximized in a factor-factor relationship when $MPPx_1 = Px_1/Py$ and $MPPx_2 = Px_2 / Py$
4. Langrage function combines objective function with Lagrange multiplier function in a constraint profit optimization.

6.0 TUTOR-MARKED ASSINGEMENT

1. Consider the production function of a yam farmer using fertilizer (X_1) and Yam Seed (X_2) as variable inputs: $Y = 20X_1 + 4X_2 – 2X_1X_2$
   Find
   a. The optimum level of yam output (Y)

b. Levels of X_1 and X_2 required to produce this optimum level of Y

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UNIT 2 IMPORTANT CONCEPTS IN FACTOR – FACTOR RELATIONSHIP

CONTENTS

1.0 Introduction
2.0 Objectives
3.0 Main Contents
    3.1 Isoquant and Marginal Rate of Technical Substitution (MRTS)
    3.2 Elasticity of Input Substitution and Isocost Line
    3.3 Determination of Least Cost Combination
    3.4 Isocline, Ridge line and Input Expansion Path
4.0 Conclusion
5.0 Summary
6.0 Tutor-Marked Assignment
7.0 Reference Further Reading

1.0 INTRODUCTION

In unit 1 of this module, we discussed profit maximization in factor-factor relationship. We illustrated this using two different approaches. The first method involved the use of production function approach to determine optimum profit. The second method involved the use of total profit function approach. We equally determined profit maximization conditions when constrain is imposed.

In this unit 2 of the module, we shall go a step further to consider some important concepts related to factor-factor relationships. Notable among these concepts are: isoquant, marginal rate of technical substitution (MRTS), Elasticity of input substitution, isocost and determination of least cost combinations. Other concepts are: Isocline, Ridgeline and Expansion path.

2.0 OBJECTIVES

At the end of the unit, you should be able to:

- define Isoquant
- explain the relationship between Isoquant and MRTS
- calculate the elasticity of input substitution
- plot the graphs and explain the meanings of Isocline, ridge line and expansion path.

3.0 MAIN CONTENTS

3.1 Isoquant and Marginal Rate of Technical Substitution (MRTS)

3.1.1 Isoquant

The word Isoquant is derived from two Latin words: Iso and Quant. Iso means equal while quant means quantity. Therefore the literary meaning of isoquant is equal quantity. Instead of Isoquant some authors used other terms like Iso-Product curve or equal product curve or product indifference curve. In the theory of consumption, we have indifference curve while in the theory of production we have product indifference curve.

Isoquant can be described as a curve which shows the combinations of two variable inputs X_1 and X_2 required to produce a given quantity of a particular product. Isoquants are lines joining all combinations of factors of production which yield equal products.

This definition of Isoquant assumes that it is possible to obtain the same output using various combinations of variable factors. The aim of farmer here will then be to look for a particular combination of the factor that will require the least cost.

This relationship between two variable inputs to produce the same output can be expressed in algebraic form as well as tabular and graphical forms.

Algebraic form can be expressed as:

$$Q = f(X_1, X_2)$$

Where Q = constant output and X_1 and X_2 = inputs

In a tabular form Isoquant can also be illustrated as follows:

Table 7: Combinations of X_1 and X_2 producing 100 units of output

Variable input (X_1)	Variable input (X_2)	Total Output (Q)
1	37	100 units
2	28	100 units
3	21	100 units
4	15	100 units
5	10	100 units
6	6	100 units

7	3	100 units
8	1	100 units

The relationship between factors X_1 and X_2 to produce 100 units of Q can also be expressed in graphical form

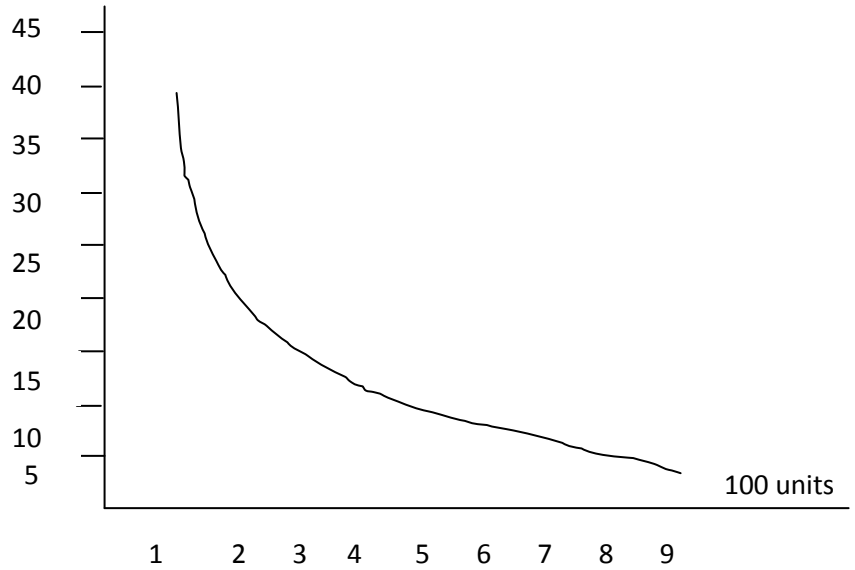


Fig. 8: Isoquant Curve of inputs X_1 and X_2

The combination of inputs X_1 and X_2 could be seed and fertilizer, fertilizer and labour, tractor and labour etc. while the output could be maize, yam, rice etc. If we are interested in input combination which yielded 100 units of the output, we need to draw contour lines through the points representing the given quantity (100 units) such as the points between 1 and 37, 2 and 28, 3 and 21, 4 and 15 up to 8 and 1. This line represents the isoquant or Iso-product curve.

Iso-product curve or Isoquant can be shown for any level of output such as at 100 units, 150 units, 200 units etc. As such, several Iso-product curves can be shown for various level of output with different levels of input combinations. When several numbers of these curves are drawn in one graph, it is called Isoquant map or Iso-product contour.

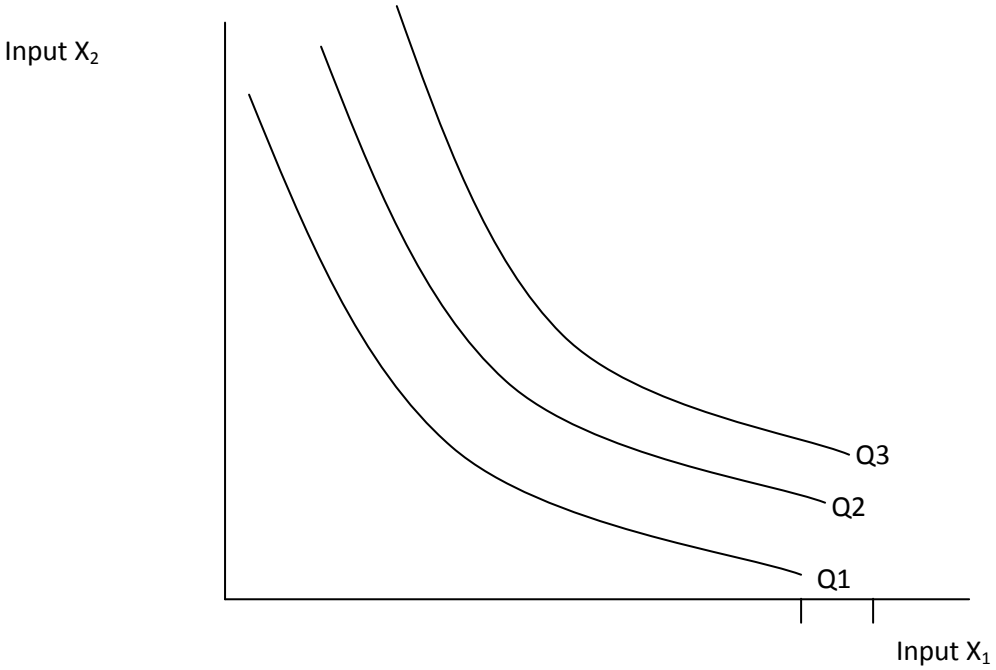


Fig. 9: Iso-product contour or Isoquant Map

In Isoquant map Q3 > Q2 > Q1, therefore Q3 can be 200 units, while Q2 could be 150 units and Q1 can be 100 units.

Characteristic Features of Isoquant

(a) Isoquants slope downwards from left to right; this means that Isoquant have negative slope. In order words, if we reduce the quantity of input X_1, we must increase the quantity of input X_2 in order to maintain the same level of output.

(b) Isoquants are convex to the origin: The amount by which we have to increase input X_1 divided by the amount by which we have to reduce input X_2 is referred to as the rate of technical substitution. The absolute slope of Isoquant decrease as we move from left to the right of the curve indicating diminishing rate of technical substitution. As a result of diminishing marginal rate of technical substitution each one unit of input added replaces less and less than the previous unit.

(c) The Curves are Smooth and Continuous
This means that both inputs are infinitely divisible. Therefore we can easily take any fractional value of them. In other words, we are not forced to use input X_1 and X_2 in a fixed proportion.

(d) Output Levels Increase as we Move away from the Origin
Isoquant representing higher level of output are placed further away from the origin and vice versa.

(e) Isoquants are non-intersecting
Two Isoquants cannot intersect each other because the same combination of two inputs cannot produce two different levels of output.

3.1.2 Marginal Rate of Technical Substitution (MRTS)

The slope of Isoquant measured the marginal rate of technical substitution. It is the rate of exchange between two productive resources which are equally preferred. In order words, it is the rate at which inputs substitute for each other to maintain a constraint output. MRTS indicates the absolute amount by which one input is decreased or given up in order to gain a unit of another input by one unit in the process of substitution.

Marginal Rate of Technical Substitution (MRTS) also called Marginal Rate of Input Substitution (MRIS) can be computed using the following equation.

$$MRTS = \frac{\text{Quantity of input sacrificed}}{\text{Quantity of input gained}} = \frac{\Delta X_2}{\Delta X_1}$$

Where
X_1 = Added input, and
X_2 = Replaced input

This showed the amount by which X_2 must be decreased to maintain the same level of output when X_1 is increased by one unit. The characteristic features of MRTS include the followings:

i. MRTS is the slope of Isoquant
ii. Isoquant slope downward from left to the right and therefore MRTS is negative
iii. MRTS is equal to the ratio of the marginal products of the inputs

i.e $\frac{\Delta X_1}{\Delta X_2} = \frac{MPPx_2}{MPPx_1}$

This relationship can lead to three different forms of input substitution:

a. Constant rate of input substitution
b. Diminishing Rate of input substitution, and
c. Increasing Rate of Input substitution

Table 8: Tabular Illustration of MRTS

Input X_1	Input X_2	Change in $X_1(\Delta X_1)$	Change in X_2 (ΔX_2)	MRTS $\Delta X_2/\Delta X_1$
A. Constant MRTS				
1	50	-	-	-
2	45	1	5	5
3	40	1	5	5
4	35	1	5	5
B. Decreasing MRTS				
1	50	-	-	-
2	30	1	20	20
3	20	1	10	10
4	15	1	5	5
C. Increasing MRTS				
1	50	-	-	-
2	45	1	5	5
3	35	1	10	10
4	15	1	20	20

These three forms of input substitution can also be illustrated graphically as shown below:

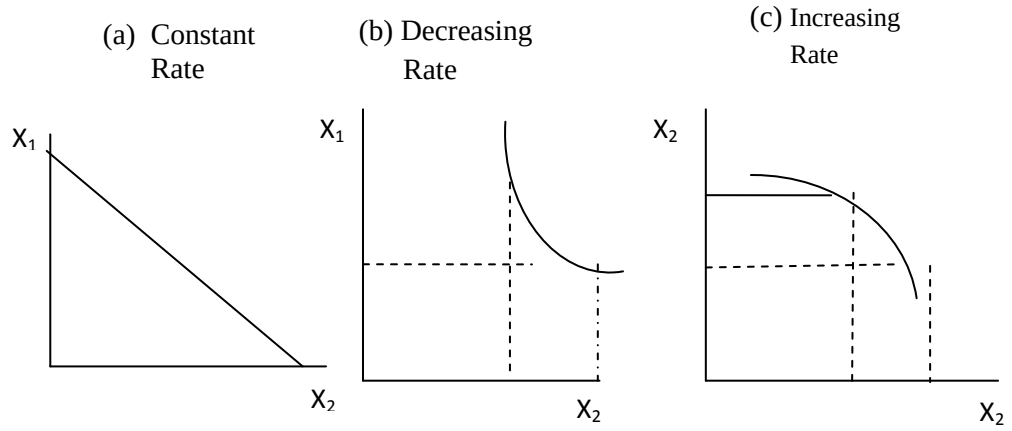


Fig. 10: Types of Marginal Rate of Input Substitution

In a constant marginal rate input substitution the amount by which X_2 must be changed to offset a change in the amount of X_1 in other to maintain output at a given level is constant. For example, if input X_1 increased by one unit and 5 units of input X_2 must be sacrificed to remain at the given output level, the MRTS ($\Delta X_2 / \Delta X_1$) =5/1 =5. This MRTS 5 must be maintained throughout the production process. This implies that the slope of the Isoquant is constant and the Isoquant is a straight line. This occurrence is possible in agriculture. For example constant MRTS is possible between labour and capital, farm yard manure and fertilizer etc. The inputs that substitute at constant rate are called perfect substitutes or competitive substitutes because they can be used in place of each other.

Diminishing Marginal Rate of Input Substitution (DMRIS) occurs when lesser and lesser quantity of one input (X_1) is reduced so as to gain another input (X_2) by one unit i.e when X_2 is increased, X_1 progressively decreased. Since diminishing returns occurs in agriculture, the phenomenon of DMRIS is also observed in agricultural production. For example between labour and farm machinery, labour and capital, bullocks and human labour etc.

Increasing Marginal Rate of Input Substitution (IMRIS) occurs when the input being increased substitutes for successively larger quantities of the input being replaced. For example if at onset additional one unit of X_1 required a decrease of 5 units of X_2 and the addition of another one unit of X_1 required a decrease of 10 units of X_2 and so on, that phenomenon is an increasing marginal rate of input substitution.

3.2 Elasticity of Input Substitution and Isocost Line

3.2.1 Elasticity of Input Substitution

We have already discussed elasticity of production under factor-product relationship Elasticity of input substitution under factor-factor relationship is similar to that of elasticity of production. Instead of looking at input X and output Y, we will now focus on input X_1 and input X_2.

Therefore elasticity of input substitution (Es) can be defined as:

$$Es = \frac{\text{Percentage Change in Input } X_1}{\text{Percentage Change is Input } X_2}$$

i.e $\frac{\Delta X_1}{X_1} \div \frac{\Delta X_2}{X2_2} = \frac{\Delta X_1}{\Delta X_2} \times \frac{X_2}{X_1}$

Or

$MRSx_2$ for X_1 . X_2 / X_1

In a technical input substitution the slope of Isoquant is negative and therefore the elasticity of input substitution under that condition is also negative. This is because as one input is increasing the other one is decreasing and vice versa.

3.2.2 Isocost Line

Isocost line is also known as Budget line or price line or Iso-outlay line. It is the locus of all combination of two inputs that can be purchased with a given outlay of funds.

For Isocost line to be drawn we must determine the total outlay and the prices per unit of X_1 and X_2.

Let us consider two variable inputs X_1 and X_2 if a farmer has a fund of ₦200 to spend on the two inputs and the price per unit of X_1 is ₦10 while the price per unit of X_2 is ₦5. If the farmer spend the whole fund on input X_1, he can purchased 20 units (₦200 / ₦10 = 20) and if the farmer spend the whole of the fund on input X_2 alone he can purchase 40 units (₦200 / ₦5 = 40). In between these two points we can plot the Isocost line showing the various combinations of X_1 and X_2 at a total outlay of ₦200

The characteristic features of Isocost lines are:

i. When the total outlay increase, the Isocost line shift away from the origin i.e shift upward from the left to the right
ii. The slope of Isocost line shows the inverse price ratio of the two variable inputs i.e only changes in the unit price of either input factors or both can change the slope of the Isocost line

3.2 Determination of Least Cost Combinations

In a factor- factor relationships there are various possible combinations of input factors that can produce a given level of output. The major concern here is to determine the particular combination that will require the least cost i.e. the point of minimum cost.

The following methods can be used to determine the least cost combination of resources:

i. Tabular method
ii. Graphical method
iii. Algebraic method

Tabular Method
This method involved the computation of the total amount of fund when the various units of resources and their unit prices are given. However, this method can only be used if few combinations are involved. When the total costs are computed for the various costs combinations, we can then select the least cost combination.

Table 9: Least Cost Combination of Inputs

Units of Input X_1	Units of Input X_2	Cost of Input X_1 @ ₦ 10	Cost of Input X_2 @ ₦ 5	Total Amount (₦)
5	20	50	100	150
10	25	100	125	225
15	30	150	150	300
20	35	200	175	375
25	40	250	200	450

From the table, it is advisable to choose the combination of 5 units of X_1 and 20 units of X_2 which gives the least cost combination of ₦ 150.

Graphical Method

In our discussions on isoquant and isocost we already established that the slope of isoquant indicates the MRTS, while the slope of isocost indicates inverse price ratio. The least cost combination occurs where MRTS equals the input price ratio. This point is met at the point of tangency of isoquant curve and iso-cost line. At this point MRTS is equated with the price ratio.

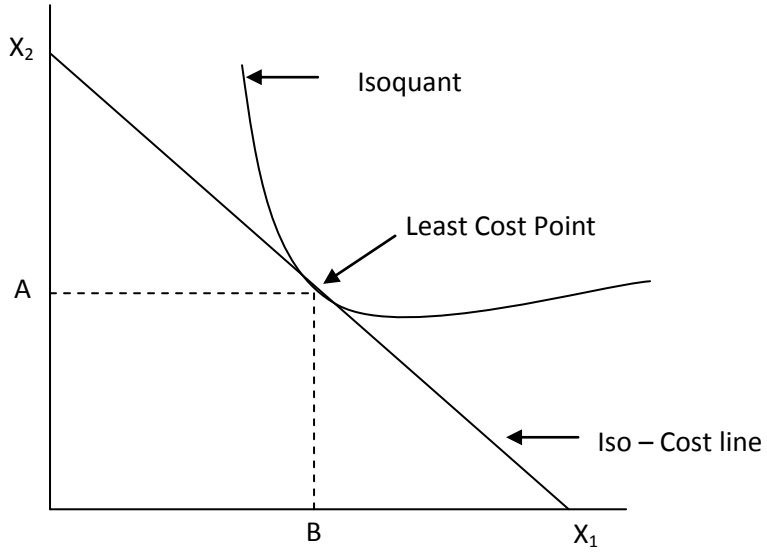


Fig. 11: Least Cost Combination

Algebraic Method

We have already demonstrated under the graphical method that the least combination occurs when the marginal rate of technical substitution is equated with the price ratio of the inputs. Mathematically, this relationship can also be computed thus:

$$MRTS_{x_1x_2} = \frac{\Delta X_2}{\Delta X_1} \text{ when we substitute } X_1 \text{ for } X_2$$

and

$$MRTS_{x_2x_1} = \frac{\Delta X_1}{\Delta X_2} \text{ when we substitute } X_2 \text{ for } X_1$$

Similarly, the inverse Price Ratio (PR) can also be computed thus:

$$PR = \frac{Px_1}{Px_2} \text{ when we substitute } X_1 \text{ for } X_2$$

and

$$PR = \frac{Px_2}{Px_1} \text{ when we substitute } X_2 \text{ for } X_1$$

Algebraically, least-cost input combination is obtained when the producer employs inputs in such a way as to equate the marginal rate of technical substitution to the ratio of their prices.

3.4 Isocline, Ridgeline and Expansion Path

3.4.1 Isocline

Isocline is defined as a line which connects points of equal slope on a production surface. This definition implies that isoclines pass through points of equal MRTS on an isoquant map. This also means that it is possible to have more than one isocline on a production surface as there are different MRTS on an isoquant. However, the isoclines for each MRS is constant.

3.4.2 Input Expansion Path

Expansion path can be described as an isocline that connects the points of least cost combination for different values of output. Expansion path is a line or curve on which the slope of isoquant (MRTS) equals the slope of iso-cost line (price ratio). The expansion path indicates the best way of producing different levels of output given the prices of inputs. Since expansion path correspond to the point where MRTS equate price ratio, we can use this relationship to determine the point of maximum profit and the corresponding levels of output and input factors that will maximize profit on an expansion path.

That is, where $VMPx_1 = Px_1$ and $VMPx_2 = Px_2$.

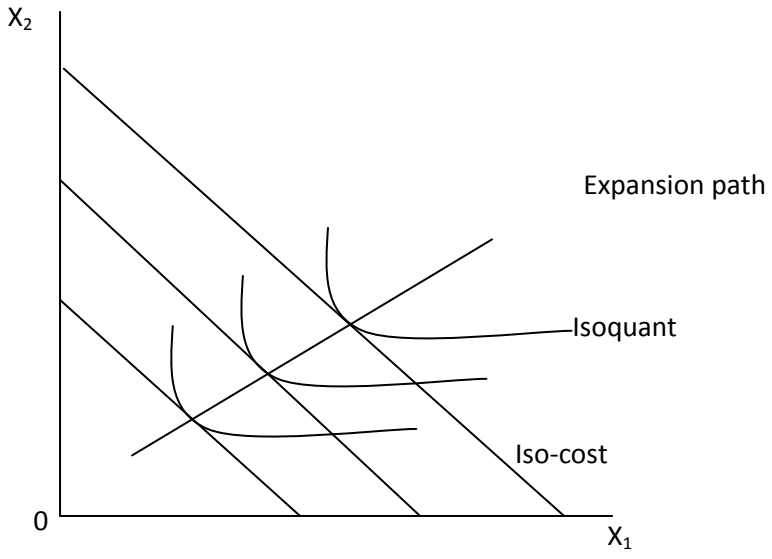


Fig. 12: Expansion Path

3.4.3 Ridgeline

Ridgeline is a line which connects points of zero slope on the successive isoquants. Ridgeline shows the boundaries for the stages of production function in the factor-factor relationship where the MPP and MRTS for each input = 0. It marks the limit of the efficiency of resource use. Ridge line represents the point of maximum output for each input given a fixed quantity of other inputs. Ridge line represents the limit of input substitution. Beyond the line, it is not possible to substitute inputs. Beyond the lower ridgeline and above the upper ridgeline MPP is negative.

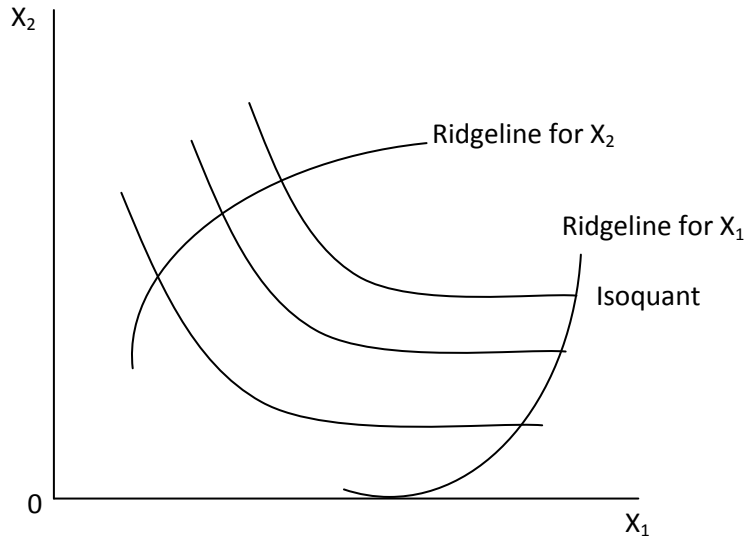


Fig. 13: Ridge lines

SELF-ASSESSMENT EXERCISE

i. Define the following terms: Isoquant; Isocost line; Expansion path; and Marginal Rate of Technical Substitution.

ii. Determine the minimum cost combination of inputs X_1 and X_2 for an output level of 200 units given the following table:

Units of X_1	Units of X_2	Cost of X_1	Cost of X_2	Total Amount
4	24			
6	18			
8	16			
10	15			
12	17			
14	14			
16	10			
18	12			

20 14
Px_1 = ₦50 and Px_2 = ₦100

4.0 CONCLUSION

We discussed in this unit some important concepts in factor-factor relationship. The unit shed light on the meaning of concepts like isoquant, marginal rate of technical substitution, elasticity of input substitution, iso-cost, iso-cline, ridgeline and expansion path. This unit also discussed the determination of least cost combination of input factors.

5.0 SUMMARY

The main points learnt in this unit include the followings:

i. Isoquant is a curve which shows the combination of two inputs required to produce a given quantity of a particular product
ii. Isoquant slope downward
iii. Isoquant are convex to the origin
iv. Isoquant curves are smooth and continuous
v. Isoquant are non intersecting
vi. MRTS is the slope of isoquant
vii. $\Delta X_1/\Delta X_2 = MPPx_2/MPPx_1$
viii. Marginal Rate of Input Substitution can be constant, decreasing or increasing
ix. Elasticity of Input Substitution is computed as $\Delta X_1/\Delta X_2 \, . \, X_2/$ or $MRSx_2$ for $X_1 \, . \, X_2/X_1$
x. Iso-cost line is also known as budget line or price line or iso-outlay line
xi. Least cost combination can be determined using tabular method, graphic method and algebraic method.
xii. \Isocline passes through points of equal MRTS on an isoquant map
xiii. Expansion path is also an isocline that passes through the points of least cost combination for different levels of output
xiv. Ridgeline shows the limit of maximum output for each input given a fixed quantity of other inputs.

6.0 TUTOR-MARKED ASSIGNMENT

1. Explain the meaning of the following concepts:

    i. Isoquant
    ii. Marginal Rate of Technical Substitution
    iii. Elasticity of Input Substitution
    iv. Isocost line

v. Expansion Path

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UNIT 3 PRODUCT- PRODUCT RELATIONSHIPS

CONTENTS

1.0 Introduction
2.0 Objectives
3.0 Main Contents
    3.1 Types of product- product relationships
    3.2 Production possibility curve (PPC)
    3.3 Iso-Revenue and Output Expansion Path
    3.4 Determination of Optimum Revenue Combination
    3.5 Profit maximization under product-product relationship
4.0 Conclusion
5.0 Summary
6.0 Tutor-Marked Assignment
7.0 References/Further Reading

1.0 INTRODUCTION

In unit 2 of this module, we discussed some important concepts in factor-factor relationship. Among the concepts we explain the meaning of Isoquant and Marginal Rate of Technical Substitution we also explain the meaning of Iso-cost line and elasticity of input substitution. In that unit, we determined the least cost combination using tabular method, graphic method and algebraic method. Finally, we also looked at the relationship between the concepts of Isocline, expansion path and ridgeline. In this unit 3 of this module we shall identify the various types of product – product relationship, plot the graph of production possibility curve and explain the concepts of Iso-revenue and output expansion path. In the concluding part of this unit we will explain how to determine the optimum revenue combination.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- identify and explain the meaning of the four types of product-product relationships
- explain the concept of production possibility curve
- describe the concepts of Iso-revenue and output expansion path
- determine the optimum revenue combination
- explain the concept of production possibility curve involving more than one variable input.

3.0 MAIN CONTENTS

3.1 Types of Product-Product Relationships

Product-product relationship is a production relationship involving two different enterprises with a fixed amount of input factor. It involves the consideration of a fixed amount of input factor and the various combinations of two products that can be obtained from it. It involves the determination of the optimum combination of two enterprises y_1 and y_2 that can be produced given the fixed amount of resource x. In other words, to produce more hectares of crop Y_1 with a fixed hectares of land will imply the production of less hectares of crop Y_2 and vice versa.

The implicit production function of a product- product relationship can be written as:

$$X = f\ (Y_1, Y_2)$$

Where

X = Fixed input (e.g. Land)
Y_1 = Enterprise 1 (e.g. Maize)
Y_2 = Enterprise 2 (e.g. Cowpea)

Agricultural products bear several relationships with one another on the farm some of these relationships that exist among farm enterprises include the followings:

a. Competitive Products
b. Joint Products
c. Complementary Products, and
d. Supplementary Products

3.1.1 Competitive Products

Competitive products are among the types of relationships that exist in product-product relationships. Two products are said to be competitive when increase in the level of production of one results in decrease in the level of production of another and vice versa, given the fixed amount of resources.

The marginal rate of product substitution in competitive product is negative. This means that increasing the output of one product will lead to decreasing the output of the other product. Competitive products require the same set of inputs and one product can only be increased by

diverting resources the quantity of resources and output of the other product. Example of competitive products in agriculture include: maize and sorghum, Cassava and Yam, Cocoa and Rubber, Sheep and Goat etc.

3.1.2 Joint Products

Joint products are two products which must be produce together by a given production process. In joint products, the two products are combined in fixed proportions and it is impossible to produce one without the other. However, on the long run, technology can make the manipulation of the proportions of the two products possible. Almost all agricultural products are produce in conjunction with other products.

Examples of joint products in agricultural production are: Mutton and wool, cotton lint and cotton seed, palm oil and palm kernels, beef and hides etc. the production possibility curve for joint products is represented by a point which could be as many as the levels of resources.

3.1.3 Complementary Products

Complementary products occur when an increase in the output of one product results in an increase in the output of the other product with productive resources held constant in amount. In other words, a shift of resources form product Y_1 to product Y_2 will increase the two products. The marginal rate of product substitution in complementary products is greater than zero (positive). This means that increasing the production of output Y_1 by diverting the use of more resources to its production will lead to increase in the production of product Y_2

The two products do not compete for the resources. One of the products contributes an element of production required by another thereby helping each other in production. This occurrence is a common phenomena in agricultural production. Examples of such occurrence in agriculture include the followings: legume crop and Maize crop, the legume fixes nitrogen thereby improving the soil fertility for the maize production. Livestock and sorghum, sorghum provides straws for the livestock to feed and livestock in turn provides farm yard manure for the sorghum.

3.1.4 Supplementary Products

Two products are said to be supplementary if the quantify of one product can be increase without increasing or decreasing the quantity of the other product. The supplementary products occurs when enterprises

can be produced only during a distinct and limited period of the year and the resources employed in the production give off a flow of services throughout the period.

Examples of supplementary products in agriculture include the followings:

i. The use of family labour for the production of broilers and tomatoes
ii. The use of tractor for ploughing cassava plot at the beginning of rainy season and later use for ploughing in dry season vegetable gardening

In general since crop production is seasonal most resources are slack during the off season, farmers can then make the best use of this off season period to introduce supplementary enterprise that will make use of these slack resources.

3.2 Production Possibility Curve (PPC)

Production Possibility Curve (PPC) is a curve which shows how one output Y_1 can be transformed into another output Y_2 by reducing output Y_1 and transferring the resources thus saved into production of Y_2. It is based on the assumption that resources on the economy are fixed in total and therefore the alternative combinations of Y_1 and Y_2 are technically feasible PPC represents all possible combinations of two products Y_1 and Y_2 that could be produced with given amount of resources. The main purpose of PPC is to determine the most profitable combination of enterprise given the resource limitation facing the farmer.

The characteristic features of PPC include the followings:

a. PPC is concave to the origin
b. A change in the level of input resource will shift the curve
c. The slope of PPC indicates the Marginal Rate of Product Substitution (MRPS).

PPC is also called Iso-revenue curve or Iso-factor curve because each point to the curve represents combinations of outputs from equal amounts of input. PPC is also known as opportunity curve because the curve presents to the farmer all the production opportunities between products Y_1 and Y_2 with a given amount of resources

The slope of the PPC indicates the rate at which one product is transformed into another product and therefore called transformation

curve. In conclusion, PPC is also known as Iso-resource curve or transformation curve.
PPC can be illustrated in a tabular form as follows:

Table 10: Production Possibility Schedule for Products Y_1 and Y_2

Input X (Farm Size)	Output Y_1 (Maize)	Output Y_2 (Cowpea)	MPP of Y_1	MPP of Y_2
0	0	0	-	-
1	13	16	13	16
2	20	26	7	10
3	26	34	6	8
4	31	40	5	6
5	35	44	4	4

The table shows that the farmer has a limited input of 5 hectares of farm land and has two alternatives of using the land for the production of maize (Y_1) and cowpea (Y_2). The alternatives are using the entire 5 hectares of land for the production of Maize (Y_1) alone or using it to produce cowpea (Y_2) alone. In between these two extremes, there are other available options like using 1 hectare for one crop and the remaining 4 hectares for the other, 2 hectares for one and 3 hectares for the other. These opportunities can be present in another table

Table 11: Production Possibilities for 5 hectares of land

Input X Hectares of Land	Output Y_1 (Maize)	Input X Hectares	Output Y_2 (Cowpea)
0	0	5	44
1	13	4	40
2	20	3	34
3	26	2	26
4	31	1	16
5	35	0	0

The different levels of farm land (X) and the corresponding levels of maize output (Y_1) and cowpea output (Y_2) represent two production functions.

PPC is a convenient method of showing the two production functions in a single graph.

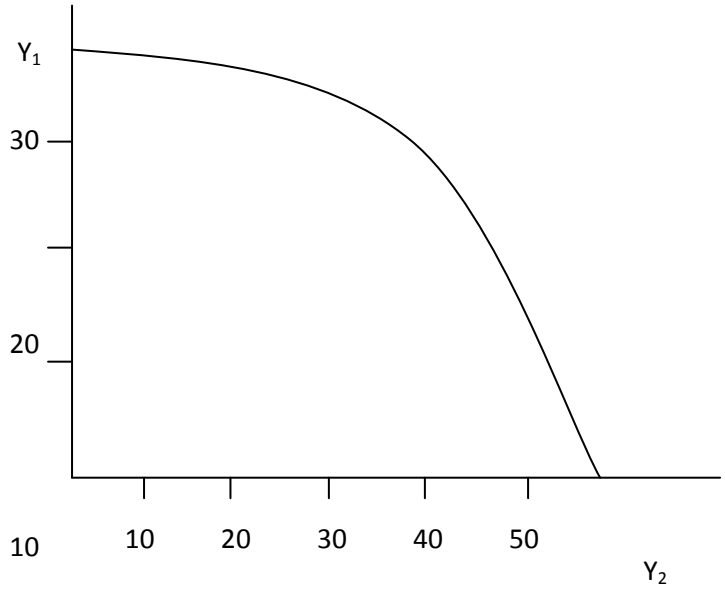


Fig. 14: Production Possibility Curve with 5 Hectares of Farm Land

The PPC as plotted on the graph is used to compare two production functions on the same graph subject to the limit of the available input. Let us consider the table and the graph above; if the farmer has only 5 hectares of farmland to produce Maize (Y_1) cowpea (Y_2), by using all the 5 hectares of land for the production of Maize (Y_1), he will produce 35 units of Maize and 0 unit of cowpea. Alternatively, by using all the 5 hectares of land for the production of cowpea, he will produce 44 units of cowpea and 0 unit of maize. There are then other possibilities apart from these two extremes. For example if 3 hectares of land is used for maize production he will obtain 26 units and the remaining 2 hectares available for cowpea will also yield 26 units of cowpea. One hectare applied to maize and four hectares to cowpea will results in 13 units of maize and 40 units of cowpea.

3.3 Iso-Revenue Line and Output Expansion Path

3.3.1 Iso-Revenue Line

Iso-revenue line is a line which defines all possible combinations of two products which would yield equal revenue. In other words, it is the locus of pints of various combinations of Y_1 and Y_2 whose sales yield the same revenue.

Let us assumed that the total revenue for two products
Y_1 and Y_2 to be:

$TR = P_1 Y_1 + P_2 Y_2$
Where P_1 = price per unit of product Y_1
P_2 = Price per unit of product Y_2

Let us assumed also that the total revenue from the two products Y_1 and Y_2 = ₦200 and the unit price of product Y_1 is ₦2.00 and unit price of product is ₦5.00 . Which means that 100 units of Y_1 could be obtained if only Y_1, is produced (ie. ₦200/₦2=100). Alternatively if only Y_2 is produced we can also obtain 40units (₦200/₦5=40). In between 100units of Y_1 and 40units of Y_2, several other combinations could also earn ₦200. For example 50units of Y_1 and 20units of Y_2 will also earn ₦200. We can then join all these various points together to form Iso-revenue line.

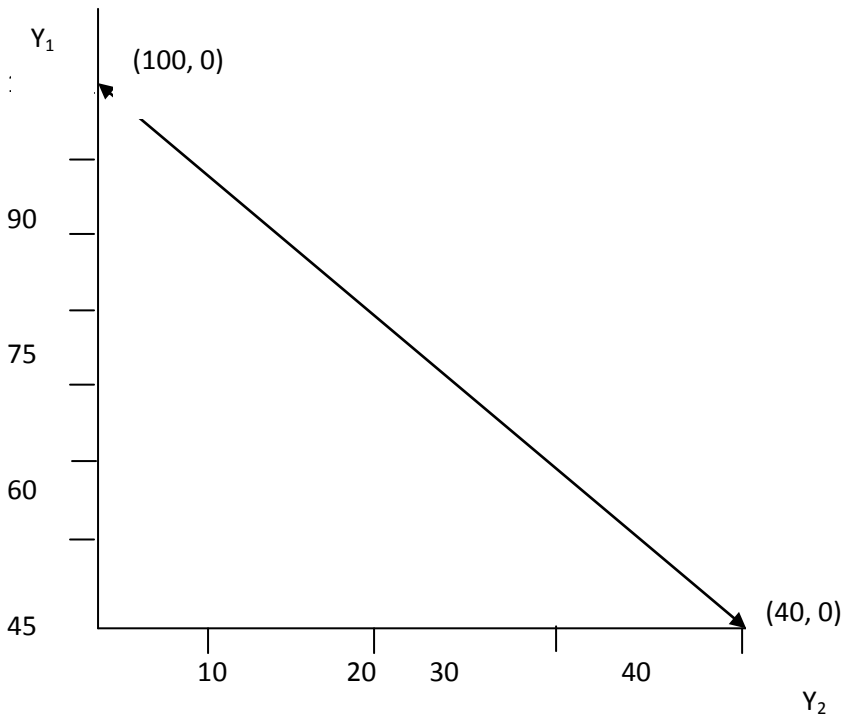


Fig. 15: 1so-revenue line for total revenue of ₦200

The ratio of the unit price of the two products determines the slope of the ISO-revenue line. The characteristic features of the ISO-revenue line include the followings;

a. ISO-revenue lines are parallel to each other
b. The slope of the ISO-revenue line indicates the inverse price ratio of the products
c. ISO-revenue line is a straight line
d. ISO-revenue line shifts upward away from the origin as total revenue increases

SELF-ASSESSMENT EXERCISE

Apart from the combinations of 100 and 0, 0 and 40 and 50 and 20 units obtained for total revenue of ₦200 with price of Y_1=₦2 and price of Y_2=₦5, find 5 other possible combinations between Y_1 and Y_2.

3.3.2 Output Expansion Path

Output expansion path is the line which connects the points of optimal product combinations on the successive production possibility curves. It is possible to have different levels of total revenue target and each level produce different ISO-revenue lines. Similarly, different total combinations of two products will also produce different production possibility curves. It is then possible to present the production possibility curves for these different units with their respective ISO-revenue lines of equal gradient on the same graph. The line connecting the maximum revenue points is the output expansion path.

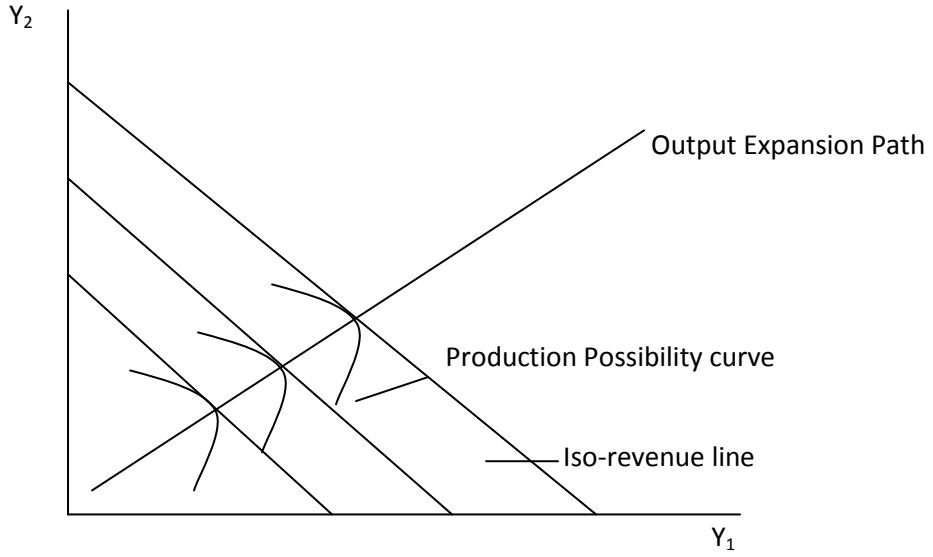


Fig. 16: PPC and Iso-revenue for different units of Y_1 and Y_2

3.4 Determination of Optimum Resource Combination

In other to obtain the point of optimum resource combination we need to know the combinations of the two products that will results into this optimum net revenue. The following methods can be used to determine the optimum product combinations to maximize net revenue:

a. Tabular method
b. Marginal Rate of Product Substitution (Algebraic Method)
c. Graphical Method

3.4.1 Tabular Method

In other to determine the maximum revenue combination from production possibility data, we need to know the unit price for each product to compute the total revenue of each output combination. The output combination which yields the maximum revenue will be chosen as the maximum revenue combination.

Let us consider our production possibility data where the price of maize (Y_1)=₦2.0 and the price of cowpea (Y_2)=₦5.0

Table 12: Tabular Computation of Optimum Output Combination

Maize (Y_1)	**Cowpea (Y_2)**	**Revenue from maize (P_1Y_2)**	**Revenue from Cowpea (P_2Y_2)**	**Total Revenue ($P_1Y_2 + P_2Y_2$)**
0	44	0	220	220
13	40	26	200	226
20	34	40	170	210
26	26	52	130	182
31	16	62	80	142
35	0	70	0	70

From the table, the combination of 13units of maize (Y_1) output and 40 units of cowpea (Y_2) output gives the maximum income of N226

3.4.2 Marginal Rate of Product Substitution (Algebraic Method)

Marginal Rate of Product substitution (MRPS) is the absolute amount by which one product is decreased in other to gain another product by one unit. In case of two products y_1 and y_2, MRPS of y_1 for y_2 implies that the amount of Y_2 to be given up in order to gain y_1 by one unit while input use remain constant.

Mathematically written as

$$MRPS = \frac{\text{No of units of replaced product}}{\text{No of units of replacing product}} = \frac{\Delta Y_2}{\Delta Y_1}$$

In this case, Y_2 is the replaced product and y_1 is the replacing product. Just as we have in the case of factor-factor, product-product can also substitute at constant rate, decreasing rate and increasing rate. We have already established earlier that optimum revenue combination obtained where MRPS is equated to the price ratio of the two products.
Therefore, the price ratio (PR) can be computed as

$$PR = \frac{\text{Price per unit of replacing product}}{\text{Price per unit of replaced product}} = \frac{Py_1}{Py_2}$$

Therefore optimum combination of product is obtained as follows:

$$\frac{Y_2\Delta}{Y_1\Delta} = \frac{Py_1}{Py_2} \quad \text{OR} \quad \frac{Y_1}{Y_2} = \frac{\Delta Py_2}{\Delta Py_1} \quad __$$

3.4.3 Graphical Method

In order to determine the optimum revenue combination of two products using graphical approach, we need production possibility data and the Iso-revenue data. Just as we demonstrated under the output expansion path, both PPC and Iso-revenue can be plotted on the same graph.

We have already shown under the algebraic method that optimum revenue is obtained where the MRPS equates their Price Ratio (PR). Similarly, under the graphical approach, the slope of the PPC indicates the MRPS and that of 1so-revenue line represent the PR. This then means that at the point where Iso-revenue line is tangent to PPC is the point of maximum combination of the two products. At that point both PPC and Iso-revenue are equal.

3.4 Profit Maximization Under Product-Product Relationship

We have already established that profit is maximized under product-product relationship when MRPS is equated to the Price Ratio (PR).

Given the data of product substitution and the respective unit price of each product, we can determine the profit maximizing combination of the two products.
For example: Consider the table below where 5 hectares of farmland is used to produce Maize (Y_1) and cowpea (Y_2). The price per unit of Maize is ₦3.00 while the price per unit of cowpea is ₦9.00

Farm land (Ha) Cowpea Output X (Y_2)	**Maize Output (Y_1)**	**Farm Land (Ha) (X)**
0	0	5	41
1	10	4	37
2	15	3	31
3	19	2	23
4	22	1	14
5	25	0	0

The price ratio for the two products is

$$\frac{Py_1}{Py_2} = \frac{₦3}{₦9} \quad OR \quad \frac{Py_2}{Py_1} = \frac{₦9}{₦3}$$

The MRPS can be calculated by first computing the changes that occur in Y_1 and Y_2.

Since MRPS is either $Y1/Y_2$ or Y_2/Y_1, we then use the changes in y1 and y2 to compute our MRPS

Maize Output (Y_1)	**Cowpea Output (Y_2)**	**Y_1**	**ΔY_2**	**$\Delta Y_1/\Delta Y_2$**	**$\Delta Y_2/\Delta Y_2$**
0	41	-	-	-	-
10	37	10	-4	-10/2	-2/10
15	31	5	-6	-5/4	-4/5
19	23	4	-8	-4/6	-6/4
22	14	3	-9	-3/9	-9/3
25	0	3	-14	-3/14	-14/3

From above illustration, MRPS is equal to price ratio at output combination where Y_1=22units and Y_2=14units. At that point 4 hectares of land is devoted to the cultivation of Y_1 (Maize) and 1 hectare for the cultivation of Y_1 (cowpea). Therefore, the point of optimum combination of Y_1 and Y_2 that maximize revenue is at where Y_1 is 22 and Y_2 is 14.

We can also determine the total revenue at the point since

$TR=Py_1Y_1+Py_2Y2$
Since Py_1 =₦3 and Y_1=22 units and
Py_2=₦9 and Y_2=14 units

Therefore, TR=22(₦3) +14(₦9)
= ₦ 66 + ₦ 126
=₦192

SELF-ASSESSMENT EXERCISE

Compute the Marginal Rate of Product Substitution for the following two products:

Product Y_1 (Maize)	**Product Y_2 (Cowpea)**
0	44
13	40
20	34
26	26
31	16
35	0

What is the optimum profit if the price of Y_1 is ₦2 and price of Y_2 is ₦6

4.0 CONCLUSION

We have learnt some important concepts under the product-product relationship. In this unit, we identified the various types of product-product relationships and the production possibility curve. We also learnt about the relationship that exists between the productions possibility curve, Iso-revenue line and output expansion path. We equally learnt in this unit, how to determine the optimum revenue combination and profit maximization under the product-product relationship. We can conclude here that the relationship between two products follow similar pattern with the relationship between two input factors.

5.0 SUMMARY

We have learnt in this unit, the followings;

i. Two products can relate together as joint, competitive, complementary or supplementary
ii. All the four types of product relationships are observed in agricultural production.
iii. Products are competitive when increase in production of one will reduce the output of the other one
iv. Two products are joint if increase in the production of one will increase the output of the other one
v. Two products are said to be complementary if increase or decrease in the output of one will not affect the output of the other one
vi. If one input can be used to produce used to produce two products at different production periods, the two products are said to be supplementary
vii. If input is fixed, it is possible to have some various combinations of output from two products. When these combinations are plotted it is called PPC
viii. It is possible to fix two revenue and then have various combinations of output of two products. The line joining the two extremes is called iso-revenue line
ix. Optimum combinations is obtained when MRPS is equated to PR

6.0 TUTOR-MARKED ASSIGNMENT

1. Explain with illustrations the following concepts:

    a. Production Possibility Curve
    b. Iso-revenue line
    c. Output Expansion Path
    d. Competitive Product
    e. Complementary Product

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MODULE 5 PRODUCTION COSTS

Unit 1 Meaning and Types of Cost
Unit 2 Farm Cost Functions
Unit 3 Cost Functions and Production Function

UNIT 1 MEANING AND TYPES OF COST

CONTENTS

1.0 Introduction
2.0 Objectives
3.0 Main Content
    3.1 Meaning of Agricultural Cost
    3.2 Types of Cost
    3.3 Importance of Cost in Agricultural Production
4.0 Conclusion
5.0 Summary
6.0 Tutor-Marked Assignment
7.0 References/Further Reading

1.0 INTRODUCTION

You are welcome to the last module of this study guide. You will recall that module 4 was on resource allocation involving both factor-factor and product-product relationships. We discussed various concepts under these topics. You can still remember concepts like isoquant, isocost line, isoclines, ridgeline, expansion path, production possibility curve, iso-revenue line etc. In this last module, we shall discuss production cost and in this unit of the module we want to first explain the meaning of cost, identify the various types of cost and discuss the usefulness of cost.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define agricultural farm cost
- list and explain the meaning of at least four different types of cost
- discuss the importance of cost in agricultural production.

3.0 MAIN CONTENT

3.1 Meaning of Agricultural Cost

Agricultural cost according to Ibitoye and Idoko (2009) is referred to as the value of agricultural output. That is, it is the expenses incurred by farmers in the process of producing a farm product such as yam, maize, eggs, goats etc. It is also known as expenses incurred in producing a particular amount f product in a particular period. Hence one can talk of the cost of producing 10 tons of yam in a season or 100 litres of milk per week or 100 crates of eggs per month.

Cost of production can also be referred to as accounting cost, cost of materials used in the production process such as labour costs, feed costs, fertilizer costs, maintenance and repair costs, selling and administrative costs, depreciation costs, taxes and interest payment on money borrowed.

Agricultural economist view costs of production as embracing a considerable alternative costs or opportunity costs. For instance, if a farmer produces only commodity A and no B, the cost of producing A is the accounting cost plus the forgone return on B known as economic costs.

3.2 Types of Cost

In considering costs involved in agricultural production, Abbot and Makeham (1980) indentified five main farm costs. These are variable costs, overhead costs, financial costs, capital costs and personal costs

3.2.1 Variable Costs

These costs are also known as direct costs. They are the costs which vary as the size and level of output of farm activities varies. For example, if the area under maize production is increased by 50%, then seed, fertilizer, labour inputs etc will also increase but may not be necessarily at the same rate. It is even possible for input like labour not to be affected by this increase. Some farm production may have direct influence on some of their variable costs. For example, if you double the cattle numbers, variable costs such as feed and veterinary fees will also double.

Variable costs are directly associated with the level of intensity of each activity. In a crop production activities for example, crop yield can be

influenced by the amount of money invested on variable costs like fertilizer, seeds, weeding etc.
It is essential for farmers to identify the variable costs of his activities on the farm such as to give him an idea of the size of the change in cost which will occur if he expands or contract one or more of his farm activities. His knowledge of variable costs and probably the gross income will guide the farmer in making decision on the merit of making the change on those activities.
Examples of farm variable costs include: seed, fertilizer, animal feed, agro-chemicals, livestock, seasonal labour, electricity bill, fuel and oil, machines maintenance and repairs etc.

3.2.2 Overhead Costs

Overhead cost is also known as fixed cost of production. These costs do not vary with the level of output, which means that they may be incurred even when production is not undertaking. Thus a small increase or decrease in the area of crop or number of farm animals is not likely going to affect the overhead costs of the farm. Examples of farm overhead costs include: depreciation of plant and equipment, interest on loan, rental payments for machinery, wages and salaries of permanent staff, land maintenance and rent etc.

Overhead costs can be either of the following types: Total Overhead Costs, Operational Costs or Activity Overheads. The total overhead costs are the unavoidable costs which must be met every year. The total gross margin is normally the only source other than borrowing, from which the overhead cost can be met. The essential components of total overhead costs include the followings: living expenses of the farmer, wages for permanent staff, loan interest and repayments, taxes, general repairs, insurance premium, replacement of capital items such as plants, machinery, building etc, travel and other business expenses.

The operational costs are used in calculating the true profit in accounting sense. They are overheads associated with annual operational expenses of the farm. Therefore, such overhead expenses like repayment of loans, living expenses, interest on loans and income tax are normally excluded in operational costs. All other costs considered under total overhead costs are part of operational costs. Some of the costs include: wages of permanent workers, depreciation charges, operator's allowance, general repairs, insurance charges, telephone and other business expenses.

The third type of overhead cost is the Activity Overheads. This type of overheads are those costs which would not be incurred if the business operation was terminated. Such cost includes depreciation charges on equipment.

3.2.3 Finance Costs

These costs are associated with loan repayment and insurance costs and interest paid on loan.

3.2.4 Capital Costs

These costs are usually associated with costs incurred in the process of providing capital assets used in farm production. Examples of capital costs include costs on land clearing, land purchase, machinery, building, plantation and livestock establishment.

3.2.5 Personal Costs

The living or personal costs of the farmer are normally included in the total overhead costs. Personal costs are one of the most important and unavoidable items in the total farm overheads. Personal costs on the farm include costs of food items, clothing, medical expenses, school fees, costs of traveling etc. some of these personal costs items are directly related to the level of output of the farmer.

3.3 Importance of Cost in Agricultural Production

One of the major motives of any farmer is profit maximization. Profit is obtained by subtracting total cost from total revenue ($\pi = TR - TC$). It is important therefore, for farmers to understand the nature and structure of production costs and how they affect the decision making process.

Understanding of cost functions also help farmers to determine the most profitable level of production as well as the level of output in which production process depends. Cost functions also help to determine the least cost combination (cost minimization) that will determine maximum profit.

Cost functions also help the farmer to determine how much variable factor to be employed in combination with fixed factors in the production of an output for maximum benefit.

SELF-ASSESSMENT EXERCISE

i. Differentiate between variable, overhead, personal, finance and capital costs. Give examples in each case.
ii. Itemize three relevance of cost to a farmer.

4.0 CONCLUSION

In this unit, we discussed the meaning and the different types of farm cost. We learnt about variable cost, overhead costs, financial costs, capital costs and personal costs. We also concluded that without the knowledge of farm costs it will be impossible to determine farm profit.

5.0 SUMMARY

The main points discussed in this unit include the followings:

i. agricultural costs are the value of inputs used on the farm
ii. there are five major categories of costs- variable, overhead, financial, capital and personal costs
iii. variable cost is also called direct costs and are the costs which vary according to the level of output
iv. overhead costs also called fixed costs are costs that must be incurred even when there is no production
v. overhead costs can be categorized into total overhead cost, operational costs and activity costs
vi. loan repayment, insurance premium and interest on loan constitute fiancé cost
vii. all costs incurred while providing capital assets are categorized as capital costs
viii. cost of living incurred outside the farm are classified as personal costs e.g. food, clothing, school fees, medicals, travels etc
ix. costs enable farmers to calculate their profit on the farm

6.0 TUTOR-MARKED ASSIGNMENT

1 (a). What is Agricultural Cost?
(b). What is the implication of cost to a farmer?

7.0 REFERENCES/FURTHER READING

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UNIT 2 FARM COST FUNCTIONS

CONTENTS

1.0 Introduction
2.0 Objectives
3.0 Main Contents
    3.1 Classical Measures of Cost
    3.2 Short Run Cost and Long Run Cost
    3.3 Agricultural Cost Functions
4.0 Conclusion
5.0 Tutor-Marked Assignment
6.0 References/Further Reading

1.0 INTRODUCTION

Unit 1 of this module focused attention on the meaning and types of farm cost. By now you should be familiar with the definition of farm cost, different types of cost associated with agricultural production and the importance of costs to a farmer. In this unit we shall explain some cost concepts like total cost, variable cost, fixed cost, average total cost, average variable cost, average fixed cost and marginal cost. We shall try to distinguish between short run and long run costs.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the meaning of Total Costs (TC)
- calculate Average Total Cost (ATC), Average variable Cost (AVC) and Average Fixed Cost (AFC)
- calculate the Marginal Cost (MC) of farm operations
- differentiate between short run cost and long run cost
- tabulate and plot the graph of agricultural cost functions

3.0 MAIN CONTENTS

3.1 Classical Measures of Cost

The classical approach to cost measurement include: Total Cost, Variable Cost, Fixed Cost and Marginal Cost Functions.

3.1.1 Total Cost (TC)

Total Cost (TC) of production is the total outlay on a given output level. Total cost is given as Fixed Cost (FC) plus Variable Cost (VC). i.e. TC = FC + VC

Average Total Cost (ATC) or Average Cost (AC) is obtained by dividing Total Cost by output.

$$ATC = \frac{\text{Total Costs (TC)}}{\text{Output (Y)}} = \frac{TC}{Y} = \frac{TFC}{Y} + \frac{TVC}{Y} = \frac{TFC + TVC}{Y}$$

Therefore, ATC = AFC + AVC

3.1.2 Fixed Costs (FC)

Fixed costs are the costs of production which remains constant irrespective of change in output level. In the short run FC remains the same irrespective of the level of production but in the long run there are no fixed costs. Fixed cost is also known as overhead cost, indirect cost or sunk cost. When plotted on the graph, it is a horizontal straight line parallel to the x-axis.

Average Fixed Cost (AFC) is measured by this formula:

$$AFC = \frac{\text{Total Fixed Cost}}{\text{Output}} = \frac{TFC}{Y}$$

It is the cost of fixed input required for producing one unit of output. Therefore, AFC decreases as output level increases.
Mathematically, since ATC = AFC + AVC
Therefore, AFC = ATC− AVC

3.1.3 Total Variable Cost (TVC)

Total Variable Costs (TVC) of production are those costs associated with production process which varies in size positively with variations in output level. This means that TVC increases with increase in output vice versa and is zero when there is no production at all.
When TFC is deducted from TC what left is TVC.
Average Variable Cost (AVC) is equal to TVC divided by output.
That is AVC = TVC/Y
It is the TVC per unit of output (Y).

AVC = ATC− AFC
The relationship between TVC and output shows that when small amount of output is produced, cost of variable input per unit of output becomes very high. As more of the variable inputs are used in the production process, productivity of the variable inputs will increase due to economies of scale.

3.1.4 Marginal Cost (MC)

The Marginal Cost is the increase in Total Cost (TC) resulting from increasing the output by one unit. It is also known as incremental cost.

$$MC = \frac{\text{Change in TC}}{\text{Change in output}} = \frac{\Delta TC}{\Delta Y}$$

The only component that changes in Total Cost is the TVC. We can therefore use either TC or TVC to compute the MC.

$$MC = \frac{\Delta TC}{\Delta Y} = \frac{\Delta FC}{\Delta Y} + \frac{\Delta VC}{\Delta Y}$$

But $\frac{\Delta FC}{\Delta Y} = 0$, therefore $MC = \frac{\Delta VC}{\Delta Y}$

The relationship between output (Y) and Total Cost (TC), Total variable Cost (TVC), Total Fixed Cost (TFC) and Marginal Cost (MC) can be presented in a tabular form.

Table 7: The Relationship between TC, TVC, TFC, MC and Output of Commodity

Output		TFC	TVC	TC	AFC	AVC	ATC	MC
0	100	0	100	-	-	-	-	
1	100	40	140	100	40	140	40	
2	100	64	164	50	32	82	24	
3	100	80	180	33.3	26.7	60	16	
4	100	88	188	25	22	47	8	
5	100	96	196	20	19.2	39.2	8	

3.2 Short Run and Long Run Costs

In the short run, we can classify Total Cost into fixed cost and variable cost. Therefore all the classical measures of costs considered above occur in the short run.

In the long run, all costs can be varied because all inputs can increase or decrease with the level of output. The implication of this in cost function is that in the long run the relevant cost concepts are the total cost and marginal cost. Therefore, marginal cost in the long run cost is the rate of

change in total cost with respect to changes in output. The issue of variable cost is no longer relevant here.
Algebraically,

$$\text{Long Run Average Cost (LAC)} = \frac{LTC}{Y} \text{ and } LMC = \frac{\Delta LTC}{\Delta Y}$$

The relationships which exist among the LTC, LAC and LMC are the same as in the short run TC, AC and MC. Some of these relationships will be considered under the agricultural cost functions.

3.3 Agricultural Cost Functions

Marshal (1998) defined agricultural cost function as the relationship between agricultural production costs and output. Agricultural cost functions are derived from production functions in terms of the number of variable inputs used in the production process. Thus it is possible to have agricultural cost functions with one, two or more variable inputs.

Our illustration here will be based on cost functions with just one variable input. Here we shall illustrate the production costs for various levels of output with the production function with one variable thus:

$$Y = f(X)$$

The cost function for the above implicit production function can be written as:

$$Y = f(P_xX)$$

Where Y = Variable cost of input X
X = Variable input
P_x = Price of the input X

Various cost functions exist within a production function with one variable input, such functions include: Total Cost, Fixed Cost, variable Cost and Marginal Cost. We have already considered the meaning and measures of these costs.

We shall first consider the relationships between Total Cost (TC), Total Fixed Cost (TFC) and Total Variable Cost (TVC).

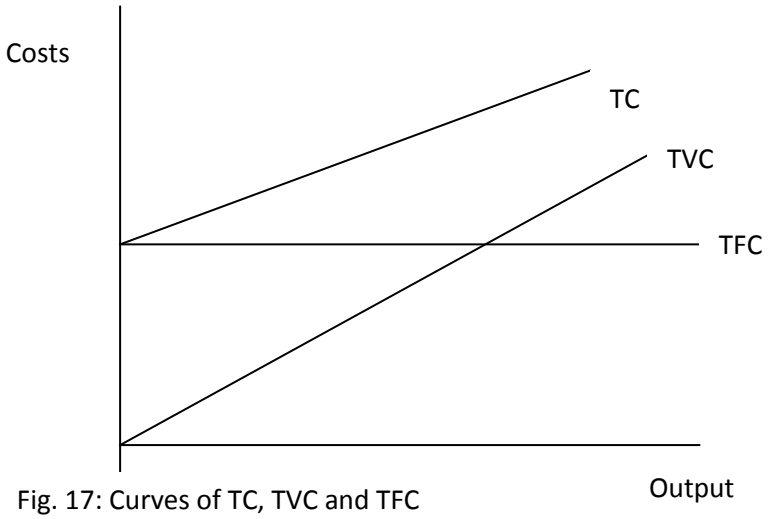


Fig. 17: Curves of TC, TVC and TFC

The graph showed that the TC axis is a summation of the TVC and TFC and that the shapes of TC and TVC curves reflect the total product function.

Another class of relationships exists between Average Total Cost (ATC), Average Fixed Cost (AFC), Average variable Cost (AVC) and Marginal Cost (MC). These relationships can be expressed graphically.

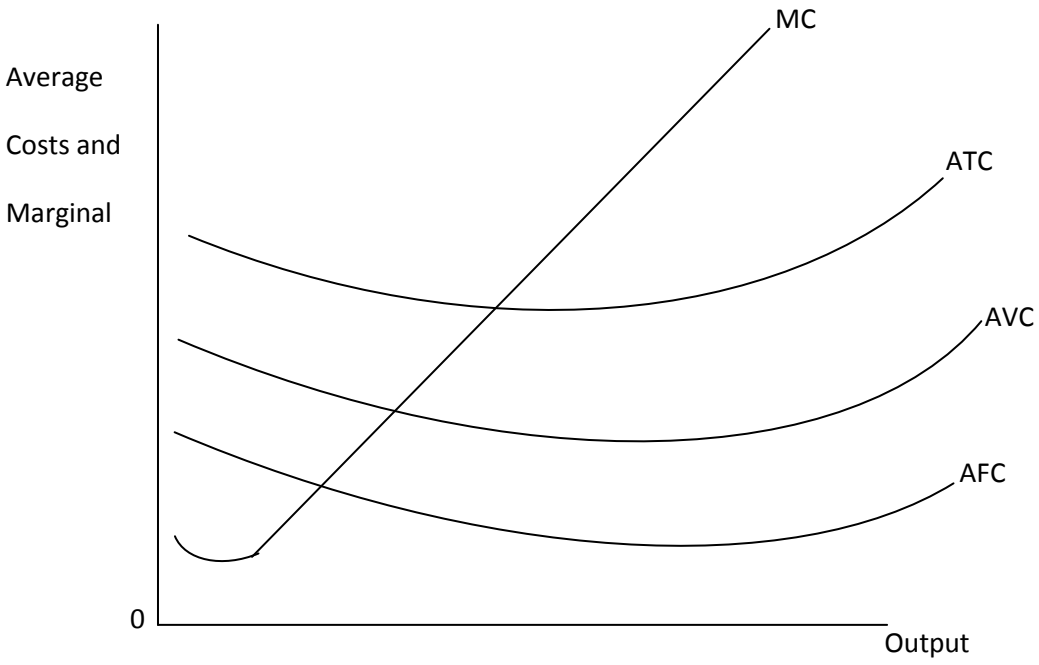


Fig. 18: ATC, AVC, AFC and MC curves

The graph represents the different shapes of agricultural cost functions in production involving one variable input. The following relationships between the different agricultural functions can be identified:

i. When ATC and AVC are at their maximum points, MC and ATC are equal. MC and AVC are also equal.
ii. MC and ATC are equal at an output level that is greater than the output level at which MC equals AVC
iii. AFC slopes downward from left to right and asymptotically approach the output axis as output increases
iv. The slope of ATC becomes zero at the minimum point of ATC and MC equals ATC at that point.

SELF-ASSESSMENT EXERCISE

Calculate the AFC, AVC, ATC and MC for each level of output from the following total cost data:

Input (X)	Output (Y)	Total Cost (₦)
0	0	100
1	4	200
2	9	300
3	15	400
4	21	500
5	26	600
6	32	700
7	35	800
8	36	900
9	35	1000
10	30	1100

4.0 CONCLUSION

In this unit, we focused our attention on farm cost functions. We discussed how to measure the different types of costs such as Total Cost, Fixed Cost, Variable Cost and Marginal Cost. We also differentiated between short run and long run costs. We concluded this unit by expressing the farm costs functions in tabular and graphical forms.

5.0 SUMMARY

The main points discussed in this unit include the followings:

i. Total Cost (TC) = Total Fixed Cost (TFC) + Total Variable Cost (TVC)
ii. Average Total Cost (ATC) = $\frac{\text{Total Cost (TC)}}{\text{Output (Y)}}$
iii. Fixed costs are the costs which do not vary with output
iv. Average Fixed Cost (AFC) = $\frac{\text{Total Fixed Cost (TFC)}}{\text{Output (Y)}}$

v. Total Variable Costs (TVC) are those costs which vary with output

vi. Average Variable Cost (AVC) $= \frac{\text{Total Variable Cost (TFC)}}{\text{Output (Y)}}$

vii. Marginal Cost (MC) $= \frac{\text{Change in TC}}{\text{Change in Y}} = \frac{\Delta TC}{\Delta Y}$

viii. Fixed cost and variable cost in short run costs are not relevant in long run costs.

ix. The measures of TC, AC and MC in the short run costs are the same as in LTC, LAC and LMC

x. When ATC and AVC are at the minimum points, MC and ATC and MC and AVC are equal.

6.0 TUTOR-MARKED ASSIGNMENT

Discuss the classical measures of the following farm cost functions: Total Cost, Fixed Cost, Variable Cost and Marginal Cost.

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UNIT 3 COST FUNCTIONS AND PRODUCTION FUNCTION

CONTENTS

1.0 Introduction
2.0 Objectives
3.0 Main contents
  3.1 Relationship Between Cost Curves and Production function
  3.2 Relationships Between APP and MPP Curves and AC and MC curves
  3.3 Total Revenue and Cost Functions
4.0 Conclusion
5.0 Summary
6.0 Tutor-Marked Assignment
7.0 Reference/Further Reading

1.0 INTRODUCTION

In the last unit we discussed farm functions. In the unit we looked at how we can measure total cost, fixed cost, variable cost and marginal cost. In that unit we differentiated between short run and long run costs and we graphically illustrated the relationships between ATC, AVC and MC curves. In this last unit we shall look at the relationships between cost functions and production function. This comparison will be done with the aid of tables and graphs.

2.0 OBJECTICVES

At the end of this unit, you should be able to:

- explain the relationship between cost curves and production function
- explain the relationships between APP and MPP curves and AC and MC curves
- express total value product in tabular and graphical forms

3.0 MAIN CONTENTS

3.1 Relationships between Cost Curves and Production Function

We have already discussed the components of cost functions under the short run. In the short run, Total cost comprised of Total Variable Cost and Total Fixed Cost.

The case of one variable input can be expressed as

$$TC=TVC+TFC$$
$$=P_XX+A$$

Where P_X= price of input

X=variable input
A=Total Fixed Cost

Similarly, production function (PF) expressed the relationship between input and output.

The case of one variable input and one output can be expressed as

$$Y=a+bX$$

Where

Y=output
X=input
b=coefficient
a=constant

The two functions as expressed above involved input (X).

The production function expressed output as a function of input and costs are also involved in this production. There can be no production without cost.

The level of cost determines the level of production. The knowledge of input price (P_X) allows the determination and analyzes of the relationship between the two functions. The graphical expression of cost functions and production function will give a clearer picture of the relationships.

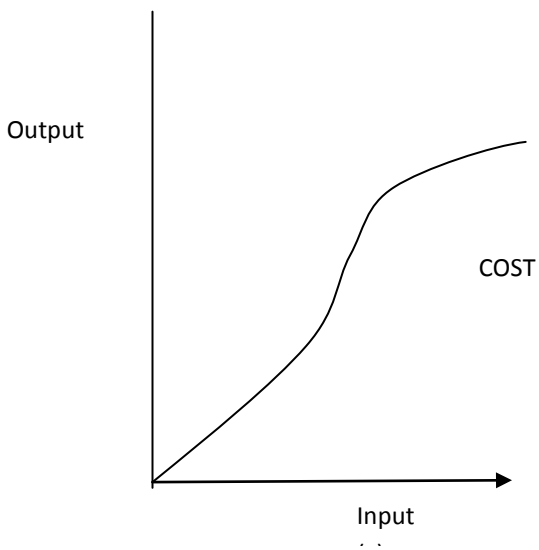


Fig. 19: Production Function

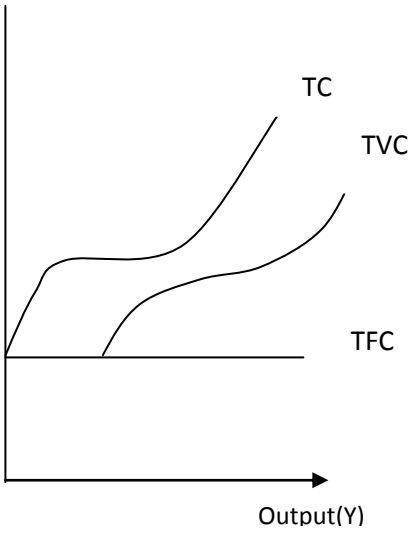


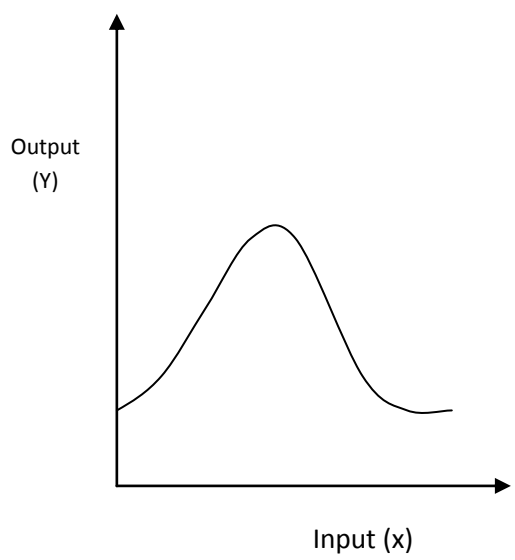
Fig. 20: Cost Functions

The total cost functions comprised of Total Cost (TC), Total Variable Cost (TVC) and Total Fixed Cost (TFC) curves. Output (Y) is expressed on the Y-axis of PF and on the X-axis of cost functions. The TC and TVC have the same shape and the gap between the two is a result of TFC. The graph shows an inverse relationship between PF and TC and TVC curves. This indicates that where Total Product (TP) is increasing at an increasing rate, the TC and TVC will increase at a decreasing rate and vice versa.

3.1 Relationship Between Average Product and Average Cost Curves and Marginal Product and Marginal Cost Curves

3.1.1 Average Product and Average Cost Curves

Average Product (AP) curve is derived from production function while Average Cost (AC) is derived from cost function. The relationship between these two functions can be expressed both graphically and algebraically just as in the case of production function and cost function.



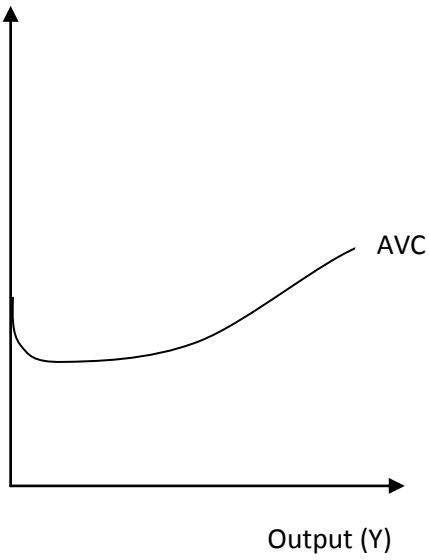


Fig. 21: Average Physical Product Curve Fig. 22: Average Variable Cost Curve

Just as we established in the case of PF and TC, there is also an inverse relationship between APP and AVC. As APP is increasing, AVC is decreasing. This relationship can also be expressed in algebraic form

$$APP = \frac{\text{Total Output}}{\text{Total Input}} = \frac{Y}{X} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

and

$$AVC = \frac{\text{Total Variable Cost}}{\text{Total Output}} = \frac{TVC}{Y} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

$TVC = P_x Y$

Therefore, $AVC = \frac{P_x Y}{Y}$ …….. (3)

$\frac{X}{Y}$ in AVC is an inverse of $\frac{Y}{X}$ in APP

We can therefore rewrite equation (3) as

$$AVC = P_X \cdot \frac{1}{APP} = \frac{}{APP} Px \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Equation (4) therefore confirms the inverse relationship between APP and AVC.

3.2.2 Marginal Product and Marginal Cost

Marginal product (MP) is derived from production function while marginal cost (MC) is derived from cost function. The relationship between the two functions can be expressed both graphically and algebraically as follows:

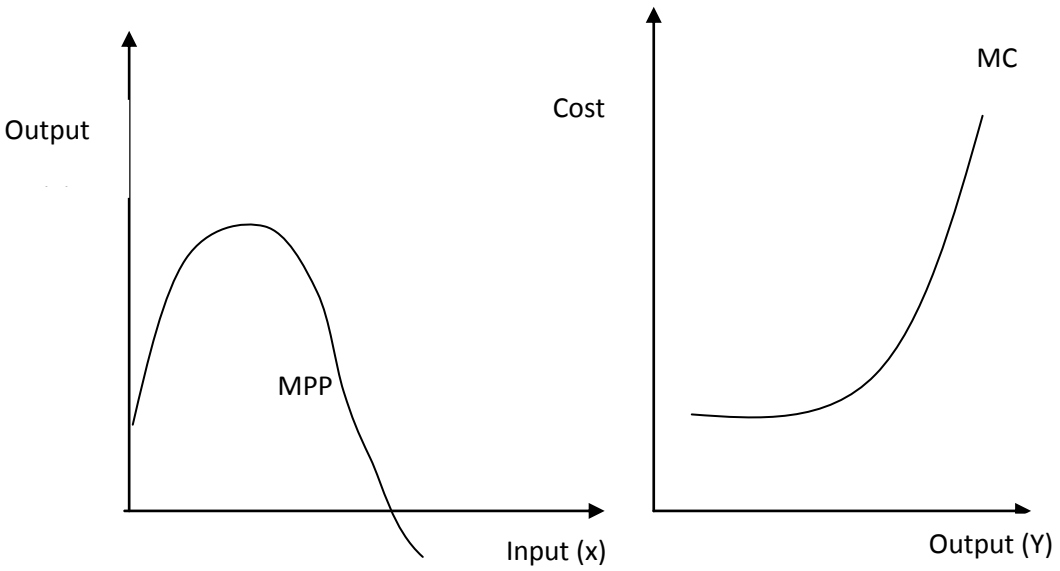


Fig. 23: Marginal Product Curve Fig. 24: Marginal Cost Curve

The graph of MP and MC showed that as long as the input prices are held constant, increasing MP implies decreasing MC. This expression can also be proved algebraically as follows:

$$MPP = \frac{\text{Change in Output}}{\text{Change in Input}} = \frac{\Delta Y}{\Delta X} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and

$$MC = \frac{\text{Change in Total Cost}}{\text{Change in Output}} = \frac{\Delta TC}{\Delta Y} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

But $\Delta TC = P_x, \Delta X$

$$MC = P_x \frac{\Delta X}{\Delta Y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$\frac{\Delta X}{\Delta Y}$ in MC is the reciprocal of $\frac{\Delta Y}{\Delta X}$ in MP

We can then rewrite equation (3) as follows:

$$MC = P_X \cdot \frac{1}{MMP} = \frac{P_X}{MPP} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

This equation (4) confirm our graphical expression that as long as the price of input is constant, increasing the MC will implies decreasing MP and vice versa.

3.3 Total Revenue and Cost Functions

Total Revenue (TR) or Total Value Product (TVP) is the monetary value of the output

i.e. $TR = P_y . Y$

Where

P_y = Price per unit of output

Y = Total Output.

We can deduce from here that

TR or TVP is related to output (P_y . Y) and we have already established that TC is related to input (P_x . X). We can then conclude that production function relates input to TR and also output to TC.

This relationship can be illustrated further in a tabular form:

Table 13: Revenue and Cost of a Farm

Output (Y) **1**	**TC (TVC + TFC) 2**	**Price or AR 3**	**TR (Py.Y) 4**	**ATC (3 x 1) 5**	**MC $\frac{\Delta in2}{\Delta in1}$ 6**	**MR $\frac{\Delta in4}{\Delta in1}$ 7**	**NR (4 – 2) 8**
0	10	20	0	-	-	-	10
1	122	20	20	12.0	2	20	8
2	14	20	40	7.0	2	20	26
3	16	20	60	5.3	2	20	34
4	18	20	80	4.5	2	20	62
5	20	20	100	4.0	2	20	80

SELF-ASSESSMENT EXERCISE

Differentiate between Average Product (AP) and Average Cost (AC) using graphical and algebraic expressions for both production and cost functions.

4.0 CONCLUSION

In this unit, we discussed the relationships between cost functions and production function. We showed the relationships between TC and TVC functions and total physical product function. In this unit we also discussed the relationships between average product and average cost curves and marginal product and marginal cost curves. We equally relate total revenue function to cost function. We can then conclude that production function serves as linkage between the cost functions and total revenue functions.

5.0 SUMMARY

The main points in this unit include the followings:

i. There are inverse relationships between production function and cost functions
ii. Where PF is increasing at an increasing rate, TC and TVC are increasing at a decreasing rate.
iii. Average Product (AP) is also inversely related to Average Variable Cost (AVC).

$$AVC = P_x/APP$$

iv. Similarly Marginal Product (MP) is inversely related to Marginal Cost (MC)

$$MC = \frac{P_x . 1}{MPP}$$

v. TR is related to output, TC is related to input and PF relates input to output. Therefore PF serves as linkage between TR and TC.

6.0 TUTOR-MARKED ASSIGNMENT

1. From the table below, find the values of Marginal Revenue and Marginal Cost

Output of Beans (Kg)	**Total Revenue (₦)**	**Total Cost ₦**
10	150	250
20	200	300
30	350	430
40	450	500
50	550	550

60	600	580
70	630	700

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