NATIONAL OPEN UNIVERSITY OF NIGERIA

FACULTY OF MANAGEMENT SCIENCES

COURSE CODE: BUS 800

COURSE TITLE: QUANTITATIVE ANALYSIS

COURSE GUIDE

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1.0 INTRODUCTION

BUS 800: Quantitative Analysis is a two credit course for students offering M.Sc. Business Administration in the School of Management Science.

The course will consist of sixteen (16) units, that is, four modules of four (4) units for each module. The material has been developed to suit undergraduate students in Business Administration at the National Open University of Nigeria (NOUN) by using an approach that treats Quantitative Analysis.

A student who successfully completes the course will surely be in a better position to manage operations of organizations in both private and public organizations.

The course guide tells you briefly what the course is about, what course materials you will be using and how you can work your way through these materials. It suggests some general guidelines for the amount of time you are likely to spend on each unit of the course in order to complete it successfully. It also gives you some guidance on your tutor-marked assignments. Detailed information on tutor-marked assignment is found in the separate assignment file which will be available in due course.

2.0 WHAT YOU WILL LEARN IN THIS COURSE

This course will introduce you to some fundamental aspects of Quantitative Analysis, set theory; basic concepts in probability; probability distributions; decision theory; forecasting models and techniques, linear programming (graphs and simplex methods); introduction to operations research; network models, simulation, Game Theory, Project Management, Inventory Control, Sequencing, Modelling in OR and Cases for OR Analysis

3.0 COURSE AIMS

The course aims, among others, are to give you an understanding of the intricacies of Quantitative Analysis and how to apply such knowledge in making real life decisions, and managing production and operations units in both private and public enterprises.

The Course will help you to appreciate Rationale behind Quantitative Analysis, Elements of Decision Analysis, Types of Decision Situations, Decision Trees, Operational Research, Approach to Decision Analysis, System Analysis, Modelling in OR, Simulation, Cases for OR Analysis, Mathematical Programming, Game Theory, Inventory Control and Sequencing.

The aims of the course will be achieved by your ability to:

- Identify and explain Quantitative Analysis;

- Identify and use various criteria for solving problems in different decision situations;
- discuss the decision tree and solve problems involving the general decision tree and the secretary problem;
- Trace the history and evolution of operation research OR;
- Explain the different approaches to decision analysis;
- discuss the concept of system analysis and identify the various categories of systems;
- Describe model and analyse the different types of models;
- Defined simulation and highlight the various types of simulation models;
- Solve different types of problems involving Linear Programming;
- Apply various techniques in solving gaming problems.
- Identify and solve problems using the sequencing techniques.

4.0 COURSE OBJECTIVES

By the end of this course, you should be able to:

- Identify and explain Quantitative Analysis;
- Identify and use various criteria for solving problems in different decision situations;
- discuss the decision tree and solve problems involving the general decision tree and the secretary problem;
- Trace the history and evolution of operation research OR;
- Explain the different approaches to decision analysis;
- discuss the concept of system analysis and identify the various categories of systems;
- Describe model and analyse the different types of models;
- Defined simulation and highlight the various types of simulation models;
- Solve different types of problems involving Linear Programming;
- Apply various techniques in solving gaming problems.
- Solving inventory problem
- Identify and solve problems using the sequencing techniques.

5.0 WORKING THROUGH THIS COURSE

To complete this course, you are required to read all study units, attempt all the tutor marked assignments at the end of each study unit. You would need to study carefully, the principles and concepts guiding each of the topics in the course in this material Quantitative Analysis provided by the National Open University of Nigeria (NOUN). You will also need to undertake practical exercises for which you need access to a personal computer running a suitable Windows for which you have command over. Each unit contains self-assessment exercises, and at certain points during the course, you will be expected to submit assignments. At the end of the course is a final examination. The course should take you about a total of 20 weeks to complete. Below are the components of the course, what you have to do, and how you should allocate your time to each unit in order to complete the course successfully on time.

6.0 COURSE MATERIALS

Major components of the course are:

- Course Guide
- Study Units
- Textbooks
- Assignment file

7.0 STUDY UNITS

The study units in this course are as follows:

BUS 800: QUANTITATIVE ANALYSIS

MODULE ONE

Unit 1 Sets & Subsets

Unit 2 Basic Set Operations

Unit 3 Set of Numbers

Unit 4 Functions

MODULE TWO

Unit 1: Probability Theory and Applications contents

Unit 2: Decision Theory

Unit 3: Types of Decision Situations

Unit 4: Decision Trees

MODULE THREE

Unit 1 : Operations Research (OR)

Unit 2: Modeling In Operations Research

Unit 3: Simulation

Unit 4: Systems Analysis

MODULE 4

Unit 1: Sequencing

Unit 2: Games Theory

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Unit 4: Case Analysis

MODULE ONE

Unit 1 Sets & Subsets

Unit 2 Basic Set Operations

Unit 3 Set of Numbers

Unit 4 Functions

UNIT 1: SETS & SUBSETS

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1.0 INTRODUCTION

The theory of sets lies at the foundation of mathematics. It is a concept that rears its head in almost all fields of mathematics; pure and applied.

This unit aims at introducing basic concepts that would be explained further in subsequent units. There will be definition of terms and lots of examples and exercises to help you as you go along.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- Identify sets from some given statements
- Rewrite sets in the different set notation
- Identify the different kinds of sets with examples

3.0 MAIN BODY

3.1 SETS

As mentioned in the introduction, a fundamental concept in all a branch of mathematics is that of set. Here is a definition **"A set is any well-defined list, collection or class of objects".**

The objects in sets, as we shall see from examples, can be anything: But for clarity, we now list ten particular examples of sets:

Example 1.1 The numbers 0, 2, 4, 6, 8

Example 1.2 The solutions of the equation $x^2+ 2x+1 = 0$

Example 1.3 The vowels of the alphabet: a, e, i, o, u

Example 1.4 The people living on earth

Example 1.5 The students Tom, Dick and Harry

Example 1.6 The students who are absent from school

Example 1.7 The countries England, France and Denmark

Example 1.8 The capital cities of Nigeria

Example 1.9 The number 1, 3, 7, and 10

Example 1.10 The Rivers in Nigeria

Note that the sets in the odd numbered examples are defined, that is, presented, by actually listing its members; and the sets in the even numbered examples are defined by stating properties that is, rules, which decide whether or not a particular object is a member of the set.

3.1.1 Notation

Sets will usually be denoted by capital letters;

A, B, X, Y,......

Lower case letters will usually represent the elements in our sets:

Lets take as an example; if we define a particular set by actually listing its members, for example, let A consist of numbers 1,3,7, and 10, then we write

A={1,3,7,10}

That is, the elements are separated by commas and enclosed in brackets { }.

We call this the tabular form of a set

Now, try your hand on this

Exercise 1.1

State in words and then write in tabular form

1. $A = \{x\} \mid x^2 = 4^2\}$

2. $B = \{x\} \mid x - 2 = 5\}$

3. $C = \{x\} \mid$ x is positive, x is negative}

4. $D = \{x\} \mid$ x is a letter in the word “correct”}

Solution:

1. It reads “A is the set of x such that x squared equals four”. The only numbers which when squared give four are 2 and -2. Hence $A = \{2, -2\}$

2. It reads “B is the set of x such that x minus 2 equals 5”. The only solution is 7; hence $B = \{7\}$

3. It read “C is the set of x such that x is positive and x is negative”. There is no number which is both positive and negative; hence C is empty, that is, $C = \emptyset$

4. It reads “D is the set of x such that x is letter in the work ‘correct’. The indicated letters a re c,o,r,e and t; thus $D = \{c,o,r,e,t\}$

But if we define a particular set by stating properties which its elements must satisfy, for example, let B be the set of all even numbers, then we use a letter, usually x, to represent an arbitrary element and we write:

$B = \{x \mid x \text{ is even}\}$

Which reads “B is the set of numbers x such that x is even”. We call this the set builders form of a set. Notice that the vertical line “ | ” is read “such as”.

In order to illustrate the use of the above notations, we rewrite the sets in examples 1.1-1.10. We denote the sets by A_1, A_2, ... A_{10} respectively.

Example 2.1: $A_1 = \{0, 2, 4, 6, 8\}$

Example 2.2: $A_2 = \{x \mid x^2+2x + 1 = 0\}$

Example 2.3: $A_3 = \{a, e, i, o, u\}$

Example 2.4: $A_4 = \{x \mid x$ is a person living on the earth$\}$

Example 2.5: $A_5 = \{$Tom, Dick, Harry$\}$

Example 2.6: $A_6 = \{x \mid x$ is a student and x is absent from school$\}$

Example 2.7: $A_7 = \{$England, France, Denmark$\}$

Example 2.8: $A_8 = \{x \mid x$ is a capital city and x is in Nigeria$\}$

Example 2.9: $A_9 = \{1, 3, 7, 10\}$

Example 2.10: $A_{10} = \{ x \mid x$ is a river and x is in Nigeria$\}$ It is easy as that!

Exercise 1.2

Write These Sets in a Set-Builder Form

1. Let A consist of the letters a, b, c, d and e
2. Let B = {2, 4, 6, 8........}
3. Let C consist of the countries in the United Nations
4. Let D = {3}
5. Let E be the Heads of State Obasanjo, Yaradua and Jonathan

Solution

1. A={x │x appears before f in the alphabet} = {xx is one of the first letters in the alphabet}
2. B = {x │x is even and positive}

3. C = {x | x is a country, x is in the United Nations}

4. D = {x | x – 2 = 1} = {x | 2x = 6}

5. E = {x | x was Head of state after Abacha}

If an object x is a member of a set A, i.e., A contains x as one of its\elements, then we write: x∈A

Which can be read "x belongs to A" or 'x is in A". If, on the otherhand, an object x is not a member of a set A, i.e A does not contain x as one of its elements, then we write; x∉A

It is a common custom in mathematics to put a vertical line " | " or " \ " through a symbol to indicate the opposite or negative meaning of the symbol.

Example 3:1: Let A = {a, e, i o, u}. Then a∈A, b∉A, f∉A.

Example 3.2: [redacted]B, 6∈B, 11∉B,14∈B

3.1.2 Finite & Infinite Sets

Sets can be finite or infinite. Intuitively, a set is finite if it consists of a specific number of different elements, i.e. if in counting the different members of the set the counting process can come to an end. Otherwise a set is infinite. Lets look at some examples.

Example 4:1: Let M be the set of the days of the week. The M is finite

Example 4:2: Let N = {0[redacted],6,8........}. Then N is infinite

Example 4:3: Let P = {x x is a river on the earth}. Although it maybe difficult to count the number of rivers in the world, P is still a finite set.

Exercise 1.3: Which sets are finite?

1. The months of the year
2. {1, 2, 3, ... 99, 100}
3. The people living on the earth
4. {x | x is even}
5. {1, 2, 3,........}

Solution:

The first three sets are finite. Although physically it might be impossible to count the number of people on the earth, the set is still finite. The last two sets are infinite. If we ever try to count the even numbers, we would never come to the end.

3.1.3 Equality of Sets

Set A is equal to set B if they both have the same members, i.e if every element which belongs to A also belongs to B and if every element which belongs to B also belongs to A. We denote the equality of sets A and B by:

$$A = B$$

Example 5.1 Let A = {1, 2, 3, 4} and B = {3, 1, 4, 2}. Then A = B,

that is {1,2,3,4} = {3,1,4,2}, since each of the elements

1,2,3 and 4 of A belongs to B and each of the elements

3,1,4 and 2 of B belongs to A. Note therefore that a set does not change if its elements are rearranged.

Example 5.3 Let E={x | x^2–3x = -2}, F={2,1} and G ={1,2,2, 1},

Then E= F= G

3.1.4 Null Set

It is convenient to introduce the concept of the empty set, that is, a set which contains no elements. This set is sometimes called the null set.

We say that such a set is void or empty, and we denote its symbol $\emptyset$

Example 6.1: Let A be the set of people in the world who are older than 200 years. According to known statistics A is the null set.

Example 6.2: Let B = $\{x \mid x^2 = 4, x \text{ is odd}\}$, Then B is the empty set.

3.2 SUBSETS

If every element in a set A is also a member of a set B, then A is called subset of B.

More specifically, A is a subset of B if $x \in A$ implies $x \in B$. We denote this relationship by writing; $A \subset B$, which can also be read "A is contained in B".

Example 7.1

The set C = {1,3,5} is a subset of D = {5,4,3,2,1}, since each number 1, 3 and 5 belonging to C also belongs to D.

Example 7.2

The set E = {2,4,6} is a subset of F = {6,2,4}, since each number 2,4, and 6 belonging to E also belongs to F. Note, in particular, that E = F. In a similar manner it can be shown that every set is a subset of itself.

Example 7.3

Let G = {x | x is even}, i.e. G = {2,4,6}, and let F = {x | x is a positive power of 2}, i.e. let F = {2,4,8,16.....} Then $F \subset G$, i.e. F is contained in G.

With the above definition of a subset, we are able to restate the definition of the equality of two sets.

Two set A and B are equal, i.e. $A = B$, if an only if $A \subset B$ and $B \subset A$. If A is a subset of B, then we can also write:

$$B \supset A$$

Which reads "B is a superset of A" or "B contains A". Furthermore, we write:

$$A \not\subset B$$

if A is not a subset of B.

Conclusively, we state:

1. The null set $\emptyset$ is considered to be a subset of every set
2. If A is not a subset of B, that is, if $A \not\subset B$, then there is at least one element in A that is not a member of B.

3.2.1 Proper Subsets

Since every set A is a subset of itself, we call B a proper subset of A if, first, is a subset of A and secondly, if B is not equal to A. More briefly, B is a proper subset of A if:

$B \subset A$ and $B \neq A$

In some books "B is a subset of A" is denoted by

$B \subseteq A$

and B" is a proper subset of A" is denoted by

$B \subset A$

We will continue to use the previous notation in which we do not distinguished between a subset and a proper subset.

3.2.2 Comparability

Two sets A and B are said to be comparable if:

$A \subset B$ or $B \subset A$;

That is, if one of the sets is a subset of the other set. Moreover, two sets A and B are said to be not comparable if:

$A \not\subset B$ and $B \not\subset A$

Note that if A is not comparable to B then there is an element in A.which is not in B and ... also, there is an element in B which is not in A.

Example 8.1: Let $A = \{a,b\}$ and $B = \{a,b,c\}$. The A is comparable to B, since A is a subset of B.

Example 8.2: Let R – $\{a,b\}$ and $S = \{b,c,d\}$. Then R and S are not comparable, since $a \in R$ and $a \notin S$ and $c \notin R$

In mathematics, many statements can be proven to be true by the use of previous assumptions and definitions. In fact, the essence of mathematics consists of theorems and their proofs. We now proof our first

Theorem 1.1: If A is a subset of B and B is a subset of C then A is a subset of C, that is,

$$A \subset B \text{ and } B \subset C \text{ implies } A \subset B$$

Proof: (Notice that we must show that any element in A is also an element in C).
Let x be an element of A, that is, let $x \in A$. Since A is a subset of B, x also belongs to B, that is, $x \in B$. But by hypothesis, $B \subset C$; hence every element of B, which includes x, is a number of C. We have shown that $x \in A$ implies $x \in C$. Accordingly, by definition, $A \subset C$.

3.2.3 Sets of Sets

It sometimes will happen that the object of a set are sets themselves; for example, the set of all subsets of A. In order to avoid saying "set of sets", it is common practice to say "family of sets" or "class of sets". Under the circumstances, and in order to avoid confusion, we sometimes will let script letters A, B,..........

Denote families, or classes, of sets since capital letters already denote their elements.

Example 9.1: In geometry we usually say "a family of lines" or "a family of curves" since lines and curves are themselves sets of points.

Example 9.2: The set $\{\{2,3\}, \{2\}, \{5,6\}\}$ is a family of sets. Its members are the sets $\{2,3\}$, $\{2\}$ and $\{5,6\}$.

Theoretically, it is possible that a set has some members, which are sets themselves and some members which are not sets, although in any application of the theory of sets this case arises infrequently.

Example 9.3: Let $A = \{2, \{1,3\}, 4, \{2,5\}\}$. Then A is not a family of sets; here some elements of A are sets and some are not.

3.2.4 Universal Set

In any application of the theory of sets, all the sets under investigation will likely be subsets of a fixed set. We call this set the universal set or universe of discourse. We denote this set by U.

Example 10.1: In plane geometry, the universal set consists of all the points in the plane.

Example 10.2: In human population studies, the universal set consists of all the people in the world.

3.2.5 Power Set

The family of all the subsets of any set S is called the power set of S.

We denote the power set of S by: 2^S

Example 11.1: Let $M = \{a,b\}$ Then $2^M = \{\{a, b\}, \{a\}, \{b\}, \emptyset\}$

Example 11.2: Let $T = \{4,7,8\}$ then $2^T = \{T, \{4,7\}, \{4,8\}, \{7,8\}, \{4\}, \{7\}, \{8\}, \emptyset\}$

If a set S is finite, say S has n elements, then the power set of S can be shown to have 2^n elements. This is one reason why the class of subsets of S is called the power set of S and is denoted by 2^S.

3.2.6 Disjoint Sets

If sets A and B have no elements in common, i.e if no element of A is in B and no element of B is in A, then we say that A and B are disjoint

Example 12.1: Let A = {1,3,7,8} and B = {2,4,7,9}, Then A and B are not disjoint since 7 is in both sets, i.e 7∈A and 7∈B

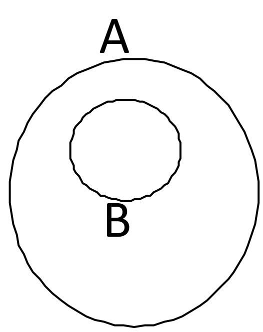
Example 12.2: Let A be the positive numbers and let B be the negative numbers. Then A and B are disjoint since no number is both positive and negative.

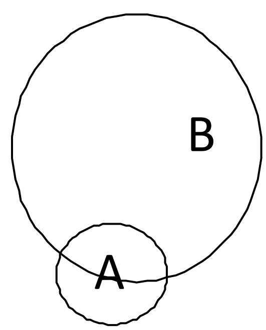
Example 12.3: Let E = {x, y, z} and F = {r, s, t}, Then E and F are disjoint.

3.3 VENN-EULER DIAGRAMS

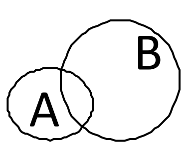
A simple and instructive way of illustrating the relationships between sets is in the use of the so-called Ven-Euler diagrams or, simply, Venn diagrams. Here we represent a set by a simple plane area, usually bounded by a circle.

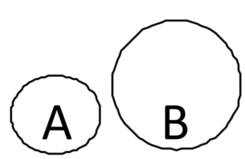
Example 13.1: Suppose A⊂B and, say, A≠B, then A and B can be described by either diagram

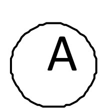




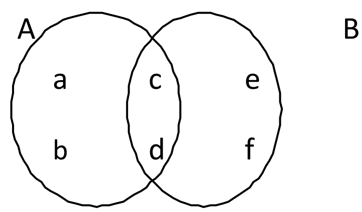
Example 13.2: Suppose A and B are not comparable. Then A and B can be represented by the diagram on the right if they are disjoint, or the diagram on the let if they are not disjoint.







Example 13.3: Let A = {a, b, c, d} and B= {c, d, e, f}. Then we illustrate these sets a Venn diagram of the form:



3.4 AXIOMATIC DEVELOPMENT OF SET THEORY

In an axiomatic development of a branch of mathematics, one begins with:

1. Undefined terms

2. Undefined relations

3. Axioms relating the undefined terms and undefined relations.

Then, one develops theorems based upon the axioms and definitions

Example 14:1: In an axiomatic development of Plane Euclidean geometry

1. "Points" and "lines" are undefined terms

2. "Points on a line" or, equivalent, "line contain a point" is an undefined relation

3. Two of the axioms are:

Axiom 1: Two different points are on one and only one line

Axiom 2: Two different lines cannot contain more than one point in common

In an axiomatic development of set theory:

1. “Element’ and “set” are undefined terms

2. “Element belongs to a set” is undefined relation

3. Two of the axioms are

Axiom of Extension: Two sets A and B are equal if and only if every element in A belongs to B and every element in B belongs to A.

Axiom of Specification: Let P(x) be any statement and let A be any set. Then there exists a set:

$B = \{a \mid a \in A, P(a) \text{ is true}\}$

Here, P(x) is a sentence in one variable for which P(a) is true or false for any $a \in A$. for example P(x) could be the sentence “$x^2 = 4$” or “x is a member of the United Nations”

4.0 CONCLUSION

You have been introduced to basic concepts of sets, set notation e.t.c that will be built upon in other units. If you have not mastered them by now you will notice you have to come back to this unit from time to time.

5.0 SUMMARY

A summary of the basic concept of set theory is as follows:

- A set is any well-defined list, collection, or class of objects.

- Given a set A with elements 1,3,5,7 the tabular form of representing this set is $A = \{1, 3, 5, 7\}$.
- The set-builder form of the same set is $A = \{x | x = 2n + 1, 0 \leq n \leq 3\}$
- Given the set $N = \{2,4,6,8,....\}$ then N is said to be infinite, since the counting process of its elements will never come to an end, otherwise it is finite
- Two sets of A and B are said to be equal if they both have the same elements, written $A = B$
- The null set, $\emptyset$, contains no elements and is a subset of every set
- The set A is a subset of another set B, written $A \subset B$, if every element of A is also an element of B, i.e. for every $x \in A$ then $x \in B$
- If $B \subset A$ and $B \neq A$, then B is a proper subset of A
- Two sets A and B are comparable if $A \subset B$ and $B \subset A$
- The power set 2^2 of any set S is the family of all the subsets of S
- Two sets A and B are said to be disjoint if they do not have any element in common, i.e their intersection is a null set.

6.0 TUTOR-MARKED ASSIGNMENTS

1. Rewrite the following statement using set notation:

1. x does not belong to A.
2. R is a superset of S
3. d is a member of E

4. F is not a subset of G

5. H does not included D

2. Which of these sets are equal: {r,t,s}, {s,t,r,s}, {t,s,t,r}, {s,r,s,t}?

3. Which sets are finite?

    1. The months of the year
    2. {1,2,3,......99, 100}
    3. The people living on the earth
    4. {x | x is even}
    5. {1,2,3........)

The first three set are finite. Although physically it might be impossible to count the number of people on the earth, the set is still finite. The last two set are infinite. If we ever try to count the even numbers we would never come to the end.

4. Which word is different from each other, and why: (1) empty, (2) void, (3) zero, (4) null?

7.0 REFERENCE AND FURTHER READING

Seymour, L.S. (1964). Outline Series: Theory and Problems of Set Theory and related topics, pp. 1 – 133.

Sunday, O.I. (1998). Introduction to Real Analysis (Real-valued functions of a real variable, Vol. 1)

UNIT 2: BASIC SET OPERATIONS

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7.0 References and Further Readings

1.0 INTRODUCTION

In this unit, we shall see operations performed on sets as in simple arithmetic. This operations simply give sets a language of their own.

You will notice in subsequent units that you cannot talk of sets without reference, sort of, to these operations.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- Compare two sets and/or assign to them another set depending on their comparability.

- Represent these relationships on the Venn diagram.

3.0 MAIN BODY

3.1 SET OPERATIONS

In arithmetic, we learn to add, subtract and multiply, that is, we assign to each pair of numbers x and y a number x + y called the sum of x and y, a number x – y called the difference of x and y, and a number xy called the product of x and y. These assignments are called the operations of addition, subtraction and multiplication of numbers. In this unit, we define the operation Union, Intersection and difference of sets, that is, we will assign new pairs of sets A and B. In a later unit, we will see that these set operations behave in a manner some what similar to the above operations on numbers.

3.1.1 Union

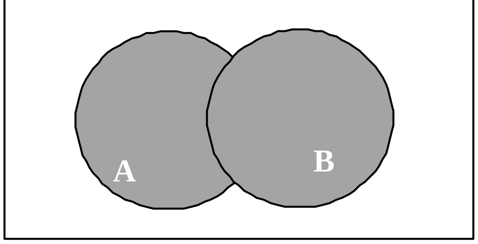
The union of sets A and B is the set of all elements which belong to A or to B or to both. We denote the union of A and B by;

$A \cup B$

Which is usually read "A union B"

Example 1.1: In the Venn diagram in fig 2-1, we have shaded $A \cup B$,

i.e. the area of A and the area of B.



A∪B is shaded

Fig 2.1

Example 1.2: Let S = {a, b. c. d} and T = {f, b, d, g}.

Then S T = {a, b, c, d, f, g}.

Example 1.3: Let P be the set of positive real numbers and let Q be the set of negative real numbers. The P∪Q, the union of P and Q, consist of all the real numbers except zero. The union of A and B may also be defined concisely by:

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Remark 2.1: It follows directly from the definition of the union of two sets that
A∪B and B∪A are the same set, i.e.,

$$A \cup B = B \cup A$$

Remark 2.2: Both A and B are always subsets of A and B that is,

$A \subset (A \cup B)$ and $B \subset (A \cup B)$

In some books, the union of A and B is denoted by A + B and is called the set-
theoretic sum of A and B or, simply, A plus B

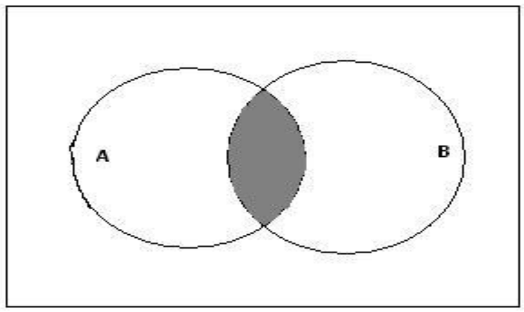
3.1.2 Intersection

The Intersection of sets A and B is the set of elements which are common to A and B, that is, those elements which belong to A and which belong to B. We denote the intersection of A and B by:

$A \cap B$

Which is read "A intersection B".

Example 2.1: In the Venn diagram in fig 2.2, we have shaded $A \cap B$, the area that is common to both A and B



$A \cap B$ is shaded

Fig 2.2

Example 2.2: Let S = {a, b, c, d} and T = {f, b, d, g}. Then $S \cap T = \{b, d\}$

Example 2.3: Let V = 2, 3, 6, ...} i.e. the multiples of 2; and

Let W = {3, 6, 9,....} i.e. the multiples of 3. Then

$V \cap W = \{6, 12, 18......\}$

The intersection of A and B may also be defined concisely by

$A \cap B = \{x \in A, x \in B\}$

Here, the comma has the same meaning as "and".

Remark 2.3: It follows directly from the definition of the intersection of two sets that;

$A \cap B = B \cap A$

Remark 2.4:

Each of the sets A and B contains A∩B as a subset, i.e.,

$(A\cap B)\subset A$ and $(A\cap B) \subset B$

Remark 2.5: If sets A and B have no elements in common, i.e. if A and B are disjoint, then the intersection of A and B is the null set, i.e. $A\cap B = \emptyset$

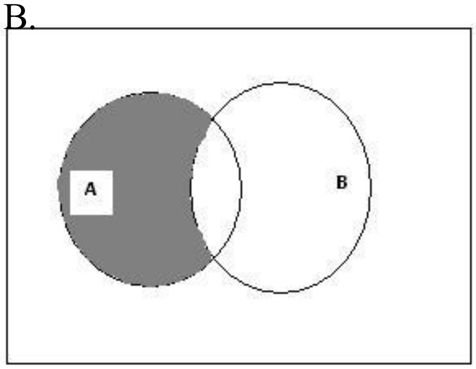
In some books, especially on probability, the intersection of A and B is denoted by AB and is called the set-theoretic product of A and B or, simply, A times B.

3.1.3 DIFFERENCE

The difference of sets A and B is the set of elements which belong to A but which do not belong to B. We denote the difference of A and B by A – B

Which is read "A difference B" or, simply, "A minus B".

Example 3.1: In the Venn diagram in Fig 2.3, we have shaded A – B, the area in A which is not part of B.



A – B is shaded

Fig 2.3

Example 3.2: Let R be the set of real numbers and let Q be the set of rational numbers. Then R – Q consists of the irrational numbers.

The difference of A and B may also be defined concisely by

$A - B = \{x \mid x \in A, x \notin B\}$

Remark 2.6: Set A contains A – B as a subset, i.e.,

$(A - B) \subset A$

Remark 2.7: The sets (A – B), A∩B and (B – A) are mutually disjoint, that is, the intersection of any two is the null set.

The difference of A and B is sometimes denoted by A/B or A ~ B

3.1.4 Complement

The complement of a set A is the set of elements that do not belong to A, that is, the difference of the universal set U and A. We denote the complement of A by **A′**

Example 4.1: In the Venn diagram in Fig 2.4, we shaded the complement of A, i.e. the area outside A. Here we assume that the universal set U consists of the area in the rectangle.

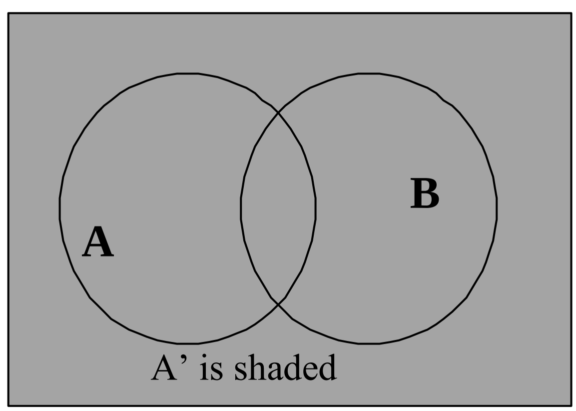


Fig. 2.4

Example 4.2: Let the Universal set U be the English alphabet and let T = {a, b, c}. Then;

T' = {d, e, f, ..., y, z}

Example 4.3:

Let E = {2, 4, 6, ...}, that is, the even numbers.

Then E′ = {1, 3, 5, ...}, the odd numbers. Here we assume that the universal set is the natural numbers, 1, 2, 3,.....

The complement of A may also be defined concisely by;

$$A' = \{x \mid x \in U, x \notin A\}$$ or, simply,

$$A' = \{x \mid x \notin A\}$$

We state some facts about sets, which follow directly from the definition of the complement of a set.

Remark 2.8: The union of any set A and its complement A′ is the universal set, i.e.,

$$A \cup A' = U$$

Furthermore, set A and its complement A′ are disjoint, i.e.,

$$A \cap A' = \emptyset$$

Remark 2.9: The complement of the universal set U is the null set ∅, and vice versa, that is,

$U' = \emptyset$ and $\emptyset' = U$

Remark 2.10: The complement of the complement of set A is the set A itself. More briefly,

$(A')' = A$

Our next remark shows how the difference of two sets can be defined in terms of the complement of a set and the intersection of two sets. More specifically, we have the following basic relationship:

Remark 2.11: The difference of A and B is equal to the intersection of A and the complement of B, that is,

$A - B = A \cap B'$

The proof of Remark 2.11 follows directly from definitions:

$A - B = \{x \mid x \in A, x \notin B\} = \{x \mid x \in A, x \notin B'\} = A \cap B'$

3.2 OPERATIONS ON COMPARABLE SETS

The operations of union, intersection, difference and complement have simple properties when the sets under investigation are comparable. The following theorems can be proved.

Theorem 2.1: Let A be a subset of B. Then the union intersection of A and B is precisely A, that is,

$A \subset B$ implies $A \cap B = A$

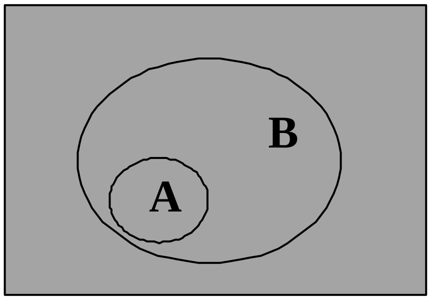
Theorem 2.2: Let A be a subset of B. Then the of A and B is precisely B, that is,

$A \subset B$ implies $A \cup B = B$

Theorem 2.3: Let A be a subset of B. Then B' is a subset of A', that is,

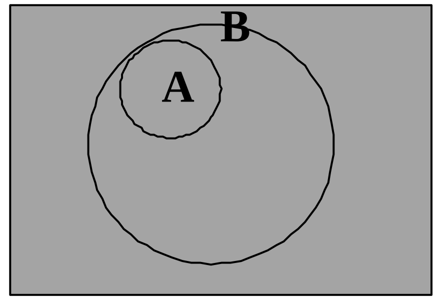
$A \subset B$ implies $B' \subset A'$

We illustrate Theorem 2.3 by the Venn diagrams in Fig 2-5 and 2-6. Notice how the area of B' is included in the area of A'.



B′ is shaded

Fig 2.5



A′ is shaded

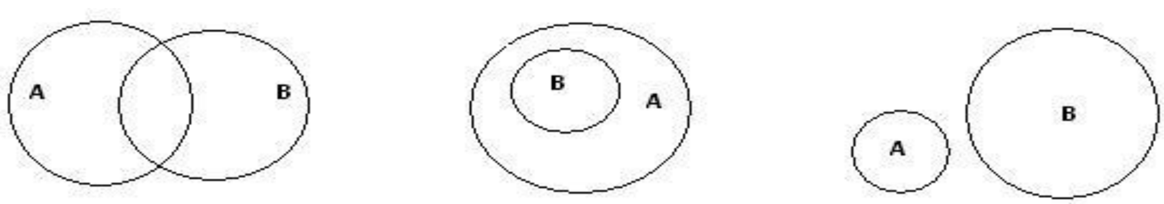
Fig 2.6

Theorem 2.4: Let A be a subset of B. Then the Union of A and (B – A) is precisely B, that is,

A⊂B implies A∪(B – A) = B

Exercises

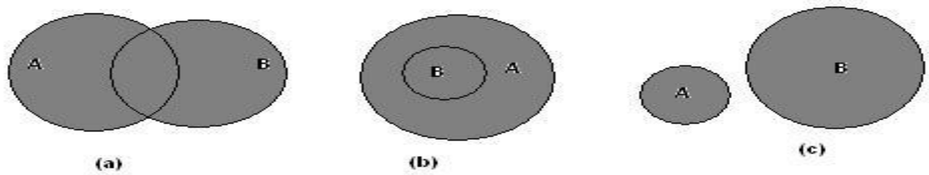
1. In the Venn diagram below, shade A Union B, that is, A∪B:



(a) (b) (c)

Solution:

The union of A and B is the set of all elements that belong to A and to B or to both. We therefore shade the area in A and B as follows:



2. Let A = {1,2,3,4}, B = {2,4,6,8} and C = {3,4,5,6}. Find

(a) A∪B, (b) A∪C, (c) B∪C, (d) B∪B

Solution:

To form the union of A and B we put all the elements from A together with the
elements of B Accordingly,

A∪B = {1,2,3,4,6,8}
A∪C = { 1,2,3,4,5,6}
B∪C | = {2,4,6,8,3,5}
B∪B = {2,4,6,8}

Notice that B∪B is precisely B.

3. Let A, B and C be the sets in Problem 2. Find (1) (A∪B)∪C,(2) A∪(B∪C).

Solution:

1. We first find (A∪B) = {1,2,3,4,6,8}. Then the union of {A∪B} and C is

   (A∪B)∪C = {1,2,3,4,6,8,5}

2. We first find (B∪C) = {2,4,6,8,3,5}. Then the union of A and

   (B∪C) is A∪(B∪C) = {1,2,3,4,6,8,5}.

Notice that (A∪B)∪C = A∪(B∪C)

4.0 CONCLUSION

You have seen how the basic operations of Union, Intersection, Difference and Complement on sets work like the operations on numbers. These are also the basic symbols associated with set theory.

5.0 SUMMARY

The basic set operations are Union, Intersection, Difference and Complement defined as:

- The Union of sets A and B, denoted by A∪B, is the set of all elements, which belong to A or to B or to both.
- The intersection of sets A and B, denoted by A∩B, is the set of elements, which are common to A and B. If A and B are disjoint then their intersection is the Null set ∅
- The difference of sets A and B, denoted by A – B, is the set of elements which belong to A but which do not belong to B.
- The complement of a set A, denoted by A', is the set of elements, which do not belong to A, that is, the difference of the universal set U and A.

6.0 TUTOR – MARKED ASSIGNMENTS

1. Let X = {Tom, Dick, Harry}, Y = {Tom, Marc, Eric} and Z = {Marc, Eric, Edward}. Find (a) X∪Y, (b) Y∪Z(c) X∪Z
2. Prove: A∩∅=∅.
3. Prove Remark 2.6: (A – B)⊂A.

4. Let U = {1,2,3,...,8,9}, A = {1,2,3,4}, B = {2,4,6,8} and C = {3,4,5,6}. Find (a) A', (b) B', (c) $(A \cap C)'$, (d) $(A \cup B)'$, (e) (A')', (f) (B – C)'

5. Prove: B – A is a subset of A'

7.0 REFERENCES AND FURTHER READINGS

Seymour, L.S. (1964). Outline Series: Theory and Problems of Set Theory and related topics, pp. 1 – 133.

Sunday, O.I. (1998). Introduction to Real Analysis (Real – valued functions of a real variable), Vol. 1

UNIT 3: SET OF NUMBERS

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3.0 Main Body

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1.0 INTRODUCTION

Although, the theory of sets is very general, important sets, which we meet in elementary mathematics, are sets of numbers. Of particular importance, especially in analysis, is the set of ***real numbers,*** which we denote by R

In fact, we assume in this unit, unless otherwise stated, that the set of real numbers R is out universal set. We first review some elementary properties of real numbers before applying our elementary principles of set theory to sets of numbers. The set of real numbers and its properties is called the ***real number system.***

2.0 OBJECTIVES

After studying this unit, you should be able to do the following:

- Represent the set of numbers on the real line
- Perform the basic set operations on intervals

3.0 MAIN BODY

3.1 REAL NUMBERS, R

One of the most important properties of the real numbers is that points on a straight line that can represent them. As in Fig 3.1, we choose a point, called the origin, to represent 0 and another point, usually to the right, to represent 1. Then there is a natural way to pair off the points on the line and the real numbers, that is, each point will represent a unique real number and each real number will be represented by a unique point. We refer to this line as the ***real line.*** Accordingly, we can use the words point and number interchangeably.

Those numbers to the right of 0, i.e. on the same side as 1, are called the *positive numbers* and those numbers to the left of 0 are called the *negative numbers.* The number 0 itself is neither positive nor negative.

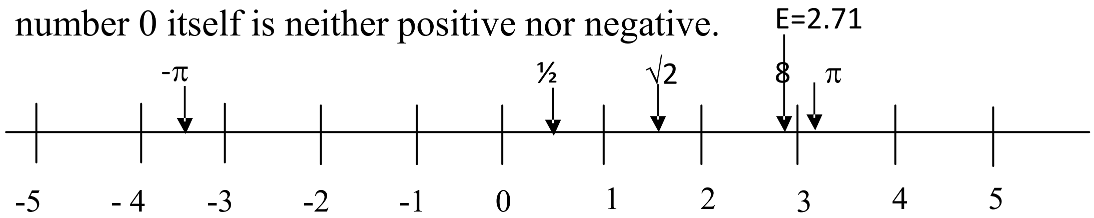


Fig 3.1

3.1.2 Integers, Z

The integers are those real numbers

…, -3, -2, -1, 0, 1, 2, 3,…

We denote the integers by Z; hence we can write

Z = { …, -2, - 1, 0, 1, 2,…}

The integers are also referred to as the "whole" numbers.

One important property of the integers is that they are "closed" under the operations of addition, multiplication and subtraction; that is, the sum, product and difference of two integers is again in integer. Notice that the quotient of two integers, e.g. 3 and 7, need not be an integer; hence the integers are not closed under the operation of division.

3.1.3 Rational Numbers, Q

The *rational numbers* are those real numbers, which can be expressed as the ratio of two integers. We denote the set of rational numbers by Q. Accordingly,

$Q = \{x \mid x = p\ q \text{ where } p \in z, q \in z\}$

Notice that each integer is also a rational number since, for example, 5 = 5/1; hence Z is a subset of Q.

The rational numbers are closed not only under the operations of addition, multiplication and subtraction but also under the operation of division (except by 0). In other words, the sum, product, difference and quotient (except by 0) of two rational numbers is again a rational number.

3.1.4 Natural Numbers, N

The *natural numbers* are the positive integers. We denote the set of natural numbers by N; hence N = {1,2,3…..}

The natural numbers were the first number system developed and were used primarily, at one time, for counting. Notice the following relationship between the above numbers systems:

$N \subset Z \subset Q \subset R$

The natural numbers are closed only under the operation of addition and multiplication. The difference and quotient of two natural numbers needed not be a natural number.

The *prime numbers* are those natural numbers p, excluding 1, which are only divisible 1 and p itself. We list the first few prime numbers: 2,3,5,7,11,13,17,19…

3.1.5 Irrational Numbers, Q'

The irrational numbers are those real numbers which are not rational, that is, the
set of irrational numbers is the complement of the set of rational numbers Q in the
real numbers R; hence Q' denote the irrational numbers. Examples of irrational numbers are √3, π, √2, etc.

3.1.6 Line Diagram of the Number Systems

Fig 3.2 below is a line diagram of the various sets of number, which we have investigated. (For completeness, the diagram include the sets of complex numbers, number of the form a + bi where a and b are real. Notice that the set of complex numbers is superset of the set of real numbers.)

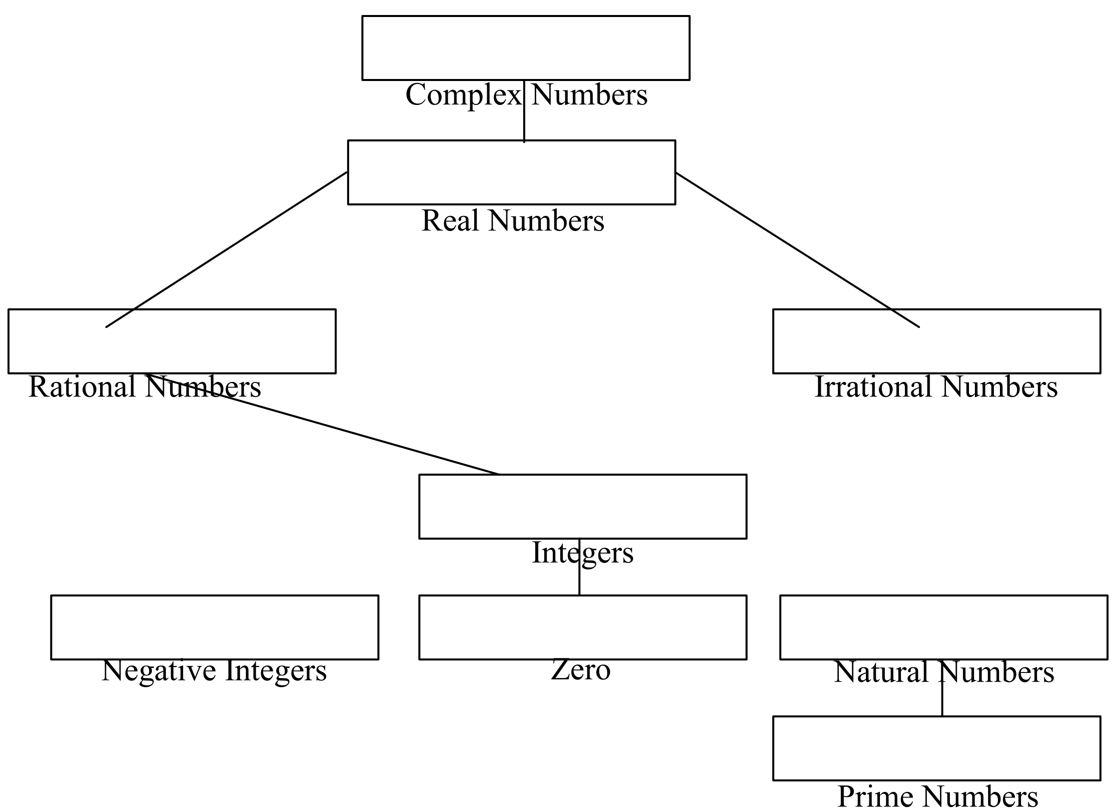


Fig 3.2

3.2 DECIMALS AND REAL NUMBERS

Every real number can be represented by a "non-terminating decimal". The decimal representation of a rational number p/q can be found by "dividing the denominator q into the numerator p". If the indicated division terminates, as for

3/8 = .375

We write 3/8 = .375000

Or 3/8 = .374999…

If we indicated division of q into p does not terminate, then it is known that a block of digits will continually be repeated; for example, 2/11 = .181818…

We now state the basic fact connecting decimals and real numbers. The rational numbers correspond precisely to those decimals in which a block of digits is continually repeated, and the irrational numbers correspond to the other non-terminating decimals.

3.3 INEQUALITIES

The concept of "order" is introduced in the real number system by the

Definition: The real number a is less than the real number b,

written $a < b$

If $b - a$ is a positive number.

The following properties of the relation $a < b$ can be proven. Let a, b and c be real numbers; then:

P_1: Either $a < b$, $a = b$ or $b < a$.

P_2: If $a < b$ and $b < c$, then $a < c$.

P_3: If $a < b$, then $a + c < b + c$

P_4: If $a < b$ and c is positive, then $ac < bc$

P_5: If $a < b$ and c is negative, then $bc < ac$.

Geometrically, if $a < b$ then the point a on the real line lies to the left of the point b.

We also denote $a < b$ by $b > a$

Which reads “b is *greater then* a”. Furthermore, we write

$a < b$ or $b > a$

if $a < b$ or $a = b$, that is, if a is not greater than b.

Example 1.1: $2 < 5$; $-6 < -3$ and $4 < 4$; $5 > -8$

Example 1.2: The notation $x < 5$ means that x is a real number which is less than 5; hence x lies to the left of 5 on the real line.

The notation $2 < x < 7$; means $2 < x$ and also $x < 7$; hence x will lie between 2 and 7 on the real line.

Remark 3.1: Notice that the concept of order, i.e. the relation $a < b$, is defined in terms of the concept of positive numbers. The fundamental property of the positive numbers which is used to prove properties of the relation $a < b$ is that the positive numbers are closed under the operations of addition and multiplication. Moreover, this fact is intimately connected with the fact that the natural numbers are also closed under the operations of addition and multiplication.

Remark 3.2: The following statements are true when a, b, c are any real numbers:

1. $a < a$

2. if $a < b$ and $b < a$ then $a = b$.

3. if $a < b$ and $b < c$ then $a < c$.

3.4 ABSOLUTE VALUE

The absolute value of a real number x, denoted by ÷ x÷ is defined by the formula

$$|x| = \begin{cases} x \text{ if } x > 0 \\ -x \text{ if } x < 0 \end{cases}$$

that is, if x is positive or zero then $|x|$ equals x, and if x is negative then $|x|$ equals – x. Consequently, the absolute value of any number is always non-negative, i.e. $|x| > 0$ for every $x \in R$.

Geometrically speaking, the absolute value of x is the distance between the point x on the real line and the origin, i.e. the point 0. Moreover, the distance between any two points, i.e. real numbers, a and b is $|a - b| = |b - a|$.

Example 2.1: $|-2| = 2$, $|7| = 7$. $|-p| = p$

Example 2.2: The statement $|x| < 5$ can be interpreted to mean that the distance between x and the origin is less than 5, i.e. x must lies between -5 and 5 on the real line. In other words,

$|x| < 5$ and $-5 < x < 5$

have identical meaning. Similarly,

$|x| < 5$ and $-5 < x < 5$

have identical meaning.

3.5 INTERVALS

Consider the following set of numbers;

$A_1 = \{x \mid 2 < x < 5\}$

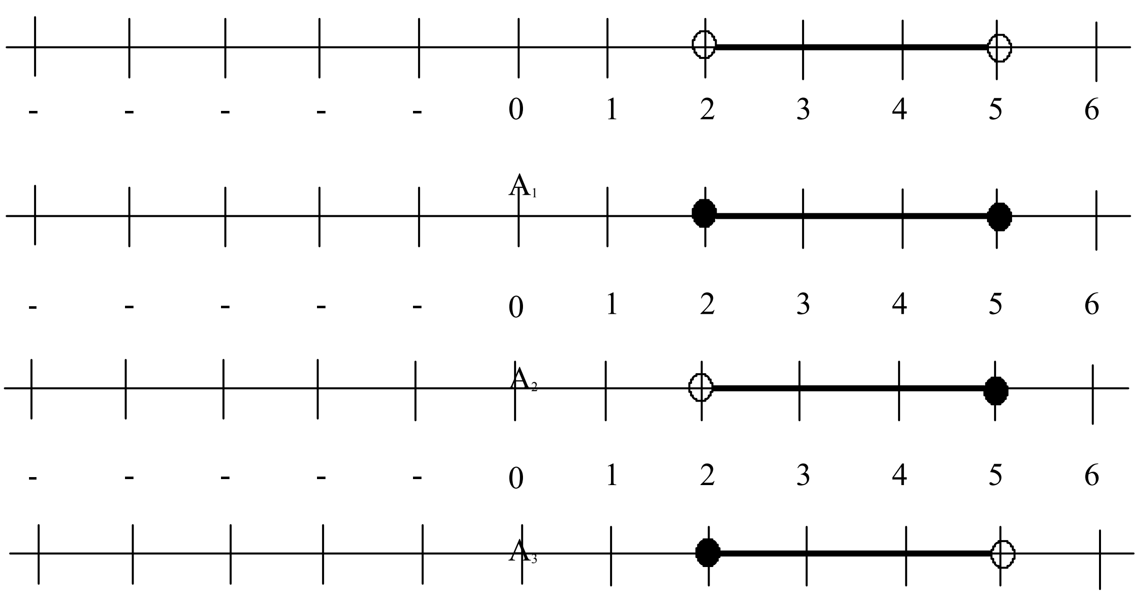
$A_2 = \{x \mid 2 \leq x \leq 5\}$

$A_3 = \{x \mid 2 < x \leq 5\}$

$A_4 = \{x \mid 2 \leq x < 5\}$

Notice, that the four sets contain only the points that lie between 2 and 5 with the possible exceptions of 2 and/or 5. We call these sets intervals, the numbers 2 and 5 being the endpoints of each interval. Moreover, A_1 is an ***open interval*** as it does not contain either end point: A_2 is a ***closed interval*** as it contains bother endpoints; A_3 and A_4 are ***open-closed*** and ***closed-open*** respectively.

We display, i.e. graph, these sets on the real line as follows.



-        -   -   -   -   0   1   2   3   4   5   6

A_4

Notice that in each diagram we circle the endpoints 2 and 5 and thicken (or shade) the line segment between the points. If an interval includes an endpoint, then this is denoted by shading the circle about the endpoint.

Since intervals appear very often in mathematics, a shorter notation is frequently
used to designated intervals, Specifically, the above intervals are sometimes denoted by;

$A_1 = (2, 5)$

$A_2 = [2, 5]$

$A_3 = (2, 5)$

$A_4 = [2, 5)$

Notice that a parenthesis is used to designate an open endpoint, i.e. an endpoint that is not in the interval, and a bracket is used to designate a closed endpoint.

3.5.1 Properties of Intervals

Let $\Im$ be the family of all intervals on the real line. We include in $\Im$ the null set $\varnothing$ and single points $a = [a, a]$. Then the intervals have the following properties:

1. The intersection of two intervals is an interval, that is, $A \in \Im$, $B \in \Im$ implies $A \cap B \in \Im$

2.The union of two non-disjoint intervals is an interval, that is, $A \in \Im$, $B \in \Im$, $A \cap B \neq \varnothing$ ipmlies $A \cup B \in \Im$

3. The difference of two non-comparable intervals is an interval, that is, $A \in \Im$, $B \in \Im$, $A \not\subset B$, $B \not\subset A$ implies $A - B \in \Im$

Example 3.1: Let A = {2, 4), B = (3, 8). Then

A Ç B = (3, 4), A È B = [2, 8)

A – B = [2, 3], B – A = [4, 8)

3.5.2 Infinite Intervals

Sets of the form

$A = \{x \mid x > 1\}$

$B = \{x \mid \underline{x > 2\ }$

$C = \{x \mid x < 3\}$

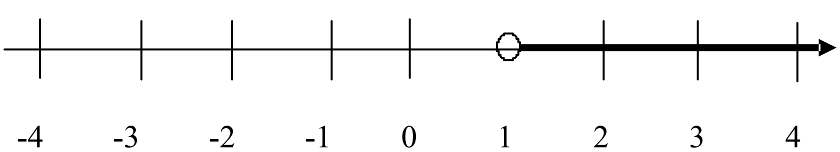
$D = \{x \mid x \leq 4\}$

$E = \{x \mid x \in \Re\}$

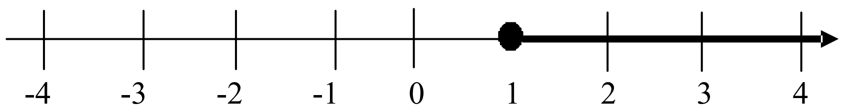
Are called infinite intervals and are also denoted by

$A = (1, \infty)$, $B = [2, \infty)$, $C = (-\infty, 3)$, $D = (-\infty, 4]$, $E = (-\infty, \infty)$

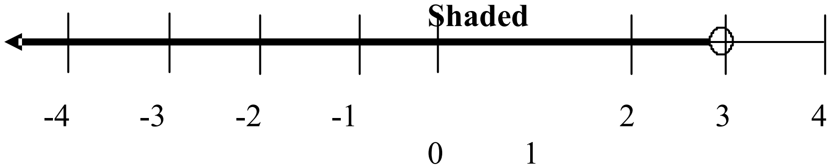
We plot these intervals on the real line as follows:



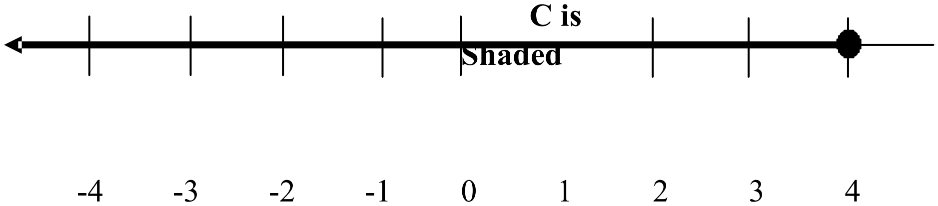
A is Shaded



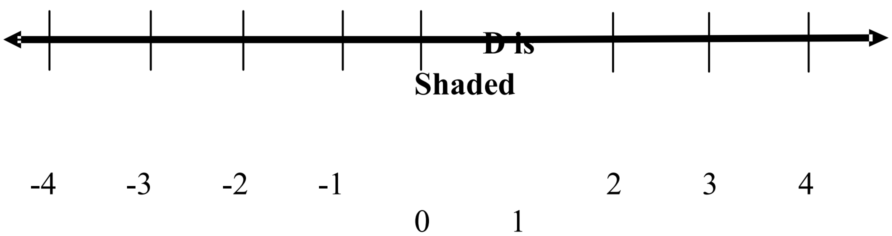
B is Shaded



C is Shaded



D is Shaded



E is Shaded

3.6 BOUNDED AND UNBOUNDED SETS

Let A be a set of numbers, then A is called ***bounded*** set if A is the subset of a finite interval. An equivalent definition of boundedness is;

Definition 3.1: Set A is ***bounded*** if there exists a positive number M such that $|x| < M$.

for all $x \in A$. A set is called ***unbounded*** if it is not bounded

Notice then, that A is a subset of the finite interval [-M, M].

Example 4.1: Let A = {1, ½, 1/3…}. Then A is bounded since A is certainly a subset of the closed interval [0, 1].

Example 4.2: Let A = {2, 4, 6,…..}. Then A is an unbounded set.

Example 4.3: Let A = {7, 350, -473, 2322, 42}. Then A is bounded

Remark 3.3: If a set A is finite then, it is necessarily bounded.

If a set is infinite then it can be either bounded as in example 4.1 or unbounded as in example 4.2

4.0 CONCLUSION

The set of real numbers is of utmost importance in analysis. All (except the set of complex numbers) other sets of numbers are subsets of the set of real numbers as can be seen from the line diagram of the number system.

5.0 SUMMARY

In this unit, you have been introduced to the sets of numbers. The set of real numbers, $\Re$, contains the set of integers, Z, Rational numbers, Q, Natural numbers, N, and Irrational numbers, Q'.

Intervals on the real line are open, closed, open-closed or closed-open depending on the nature of the endpoints.

6.0 TUTOR-MARKED ASSIGNMENTS

1. Prove: If $a < b$ and $B < c$, then $a < c$

2. Under what conditions will the union of two disjoint interval be an interval?

3. If two sets R and S are bounded, what can be said about the union and Intersection of these sets?

7.0 REFERENCES AND FURTHER READINGS

Seymour, L.S. (1964). Outline Series: Theory and Problems of Set Theory and related topics, pp. 1 – 133.

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UNIT 4: FUNCTIONS

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3.1 Definition

3.2 Mappings, Operators, Transformations

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1.0 INTRODUCTION

In this unit, you will be introduced to the concept of functions, mappings and transformations. You will also be given instructive and typical examples of functions.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- Identify functions from statements or diagrams
- State whether a function is one-one or onto
- Find composition function of two or more functions

3.0 MAIN BODY

3.1 DEFINITION

Suppose that to each element in a set A there is assigned by some manner or other, a unique element of a set$\Re$. We call such assignment of ***function.*** If we let ¦ denote these assignments, we write;

$$f: A \longrightarrow B$$

which reads "f is a function of A onto B". The set A is called the ***domain*** of the function f, and B is called the ***co-domain*** of f. Further, if a $\in$ A the element in B which is assigned to a is called the ***image*** of a and is denoted by; *f*(a) which reads "f of a".

We list a number of instructive examples of functions.

Example 1.1: Let f assign to each real number its square, that is, for every real number x let $f(x) = x^2$. The domain and co-domain of f are both the real numbers, so we can write: f: $\Re$**>** $\Re$

The image of -3 is 9; hence we can also write f (-3) = 9

or f : 3→ 9

Example 1.2: Let f assign to each country in the world its capital city. Here, the domain of f is the set of countries in the world; the co-domain of f is the list of capital cities in the world. The image of France is Paris, that is, f(France) = Paris

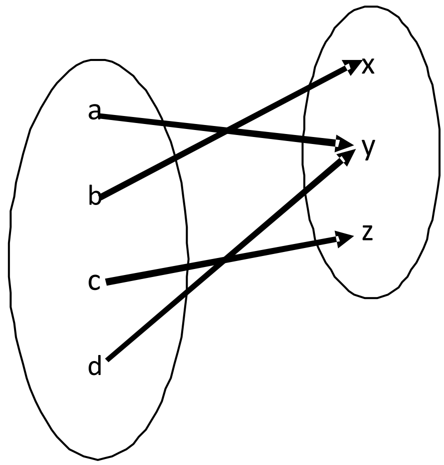
Example 1.3: Let A = {a, b, c, d} and B = {a, b, c}. Define a function f of A into B by the correspondence f(a) = b, f(b) = c, f(c) = c and f(d) = b. By this definition, the image, for example, of b is c.

Example 1.4: Let A = {-1, 1}. Let f assign to each rational number in $\Re$ the number 1, and to each irrational number in $\Re$ the number -1. Then f: $\Re$→A, and f can be defined concisely by

$$\begin{cases} f(x) = 1 \text{ if x is rational} \\ -1 \text{ if a is irrational} \end{cases}$$

Example 1.5: Let A = {a, b, c, d} and B = {x, y, z}. Let f:

A → B be defined by the diagram:



Notice that the functions in examples 1.1 and 1.4 are defined by specific formulas.
But this need not always be the case, as is indicated by the other examples. The rules of correspondence which define functions can be diagrams as in example 1.5, can be geographical as in example 1.2, or, when the domain is finite, the correspondence can be listed for each element in the domain as in example 1.4.

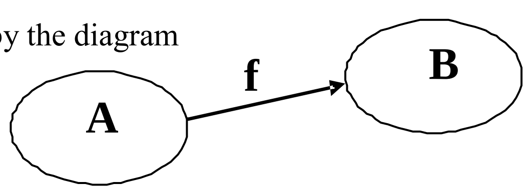
3.2 MAPPINGS, OPERATORS, TRANSFORMATIONS

If A and B are sets in general, not necessarily sets of numbers, then a function f of A into B is frequently called a mapping of A into B; and the notation

f: A ---> B is then read "f maps A into B". We can also denote a mapping, or function, f of A into B by

$$A \xrightarrow{f} B$$

Or by the diagram



If the domain and co-domain of a function are both the same set, say

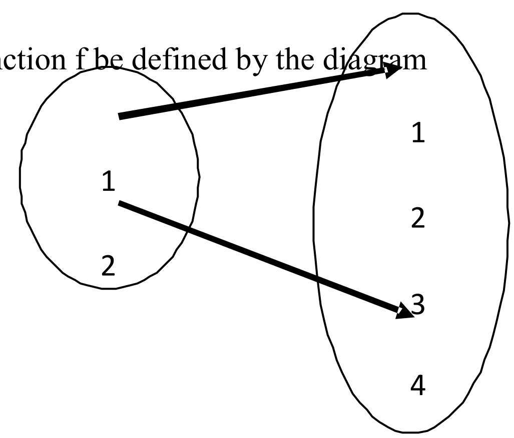
f: A ---> A then f is frequently called an ***operator*** or ***transformation*** on A. As we will see later operators are important special cases of functions.

3.3 EQUAL FUNCTIONS

If f and g are functions defined on the same domain D and if f (a) = g (a) for every $a \in D$, then the functions f and g are equal and we write f = g

Example 2.1: Let f (x) = x^2 where x is a real number. Let g(x) = x^2 where x is a complex number. Then the function f is not equal to g since they have different domains.

Example 2.2: Let the function f be defined by the diagram



Let a function g be defined by the formula g(x) = x^2 where the domain of g is the set {1, 2}. Then f = g since they both have the same domain and since f and g assign the same image to each element in the domain.

Example 2.3: Let f: $\Re \rightarrow \Re$ and g:$\Re \rightarrow \Re$. Suppose f is defined by f(x) = x^2 and g by g(y) = y^2. Then f and g are equal functions, that is, f = g. Notice that x and y are merely dummy variable in the formulas defining the functions.

3.4 RANGE OF A FUNCTION

Let f be the mapping of A into B, that is, let f: A→ B. Each element in B need not appear as the image of an element in A. We define the range of f to consist precisely of those elements in B which appear and the image of at least one element in A. We denote the range of f: A →

By f(A)

f(A)

Notice that f(A) is a subset of B. i.e. f(A)

Example 3.1: Let the function f: $\Re \rightarrow \Re$ be defined by the formula f(x) = x^2. Then the range of f consists of the positive real numbers and zero.

Example 3.2: Let f: A→ B be the function in Example 1.3. Then f(A) = {b, c}

3.5 ONE – ONE (INJECTIVE) FUNCTIONS

Let f map A into B. Then f is called a ***one-one or Injective function*** if different elements in B are assigned to different elements in A, that is, if no two different elements in A have the same image. More briefly, f: A → B is one-one if f(a) = f(a') implies a = a` or, equivalently, a = a` implies f(a) ≠ f(a`)

Example 4.1: Let the function f: $\Re \rightarrow \Re$ be defined by the formula $f(x) = x^2$. Then f is not a one-one function since f(2) = f(-2) = 4, that is, since the image of two
different real numbers, 2 and -2, is the same number, 4.

Example 4.2: Let the function f: $\Re \rightarrow \Re$ be defined by the formula $f(x) = x^3$. Then f is a one-one mapping since the cubes of the different real numbers are themselves different.

Example 4.3: The function f which assigns to each country in the world, its capital city is one-one since different countries have a different capital that is no city is the capital of two different countries.

3.6 ONTO (SUBJECTIVE) FUNCTION

Let f be a function of A into B. Then the range f(A) of the function f is a subset of B, that is, $f(A) \subset B$. If f(A) = B, that is, if every member of B appears as the image of at least one element of A, then we say "f is a function of A onto B", or "f maps A onto B", of "f is an ***onto or Subjective function".***

Example 5.1: Let the function f: $\Re \rightarrow \Re$ be defined by the formula $f(x) = x^2$. Then f is not an onto function since the negative numbers do not appear in the range of f, that is no negative number is the square of a real number.

Example 5.2: Let f: A→ B be the function in Example 1.3. Notice that f(A) = {b, c}. Since B = {a, b, c} the range of f does not equal co-domain, i.e. is not onto.

Example 5.3: Let f: A → B be the function in example 1.5: Notice that

f(A) = {x, y, z} = B that is, the range of f is equal to the co-domain B. Thus f maps A onto B, i.e. f is an onto mapping.

3.7 IDENTITY FUNCTION

Let A be any set. Let the function f: A→ A be defined b y the formula f(x) = x, that is, let f assign to each element in A the element itself. Then f is called the identity function or the identity transformation on A. We denote this function by 1 or by

1_A.

3.8 CONSTANT FUNCTIONS

A function f of A onto B is called a ***constant function*** if the same element of b∈ B is assigned to every element in A. In other words, f: A → B is a constant function if the range of f consists of only one element.

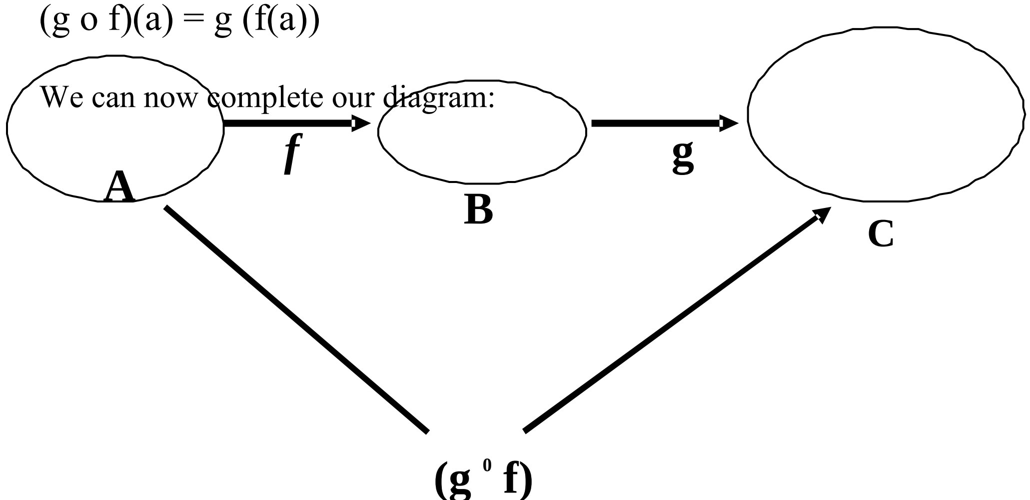
3.9 PRODUCT FUNCTION

Let F: A → B and g: B→C be two functions then the product of functions f and g is denoted

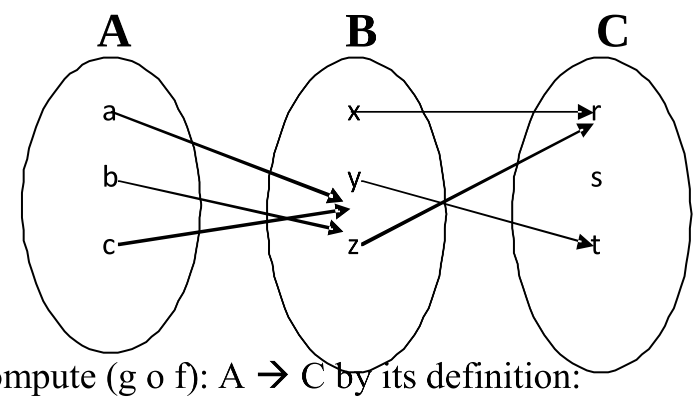
(g o f) : A → C defined by

(g o f)(a) = g (f(a))

We can now complete our diagram:



Example 7.1: Let f: A → B and g: B → C be defined by the diagrams



We compute (g o f): A → C by its definition:

(g o f)(a) = g(f(a)) = g(y) = t

(g o f)(b) = g(f(a)) = g(z) = r

(g o f)(c) = g(f(a)) = g(y) = t

Notice that the function (g o f) is equivalent to "following the arrows" from A to C in the diagrams of the functions f and g.

Example 7.2: To each real number let f assign its square, i.e. let $f(x) = x^2$. To each real number let g assign the number plus 3, i.e. let g(x) = x + 3. Then

$(g \text{ o } f)(x) = f(g(x)) = f(x+3) = (x+3)^2 = x^2 + 6x + 9$

$(g \text{ o } f)(x) = g(f(x)) = g(x^2) = x^2 + 3$

Remark 4.1: Let f: A → B. Then

I_Bo f = f and f ^{o}$_{1}$A = f

that is, the product of any function and identity is the function itself.

3.9.1 Associativity of Products of Functions

Let f: A → B, g: B → C and h: c → D. Then, as illustrated in Figure 4-1, we can form the production function g ^{o} f: A → C, and then the function h^{o} (g ^{o} f): A → D.

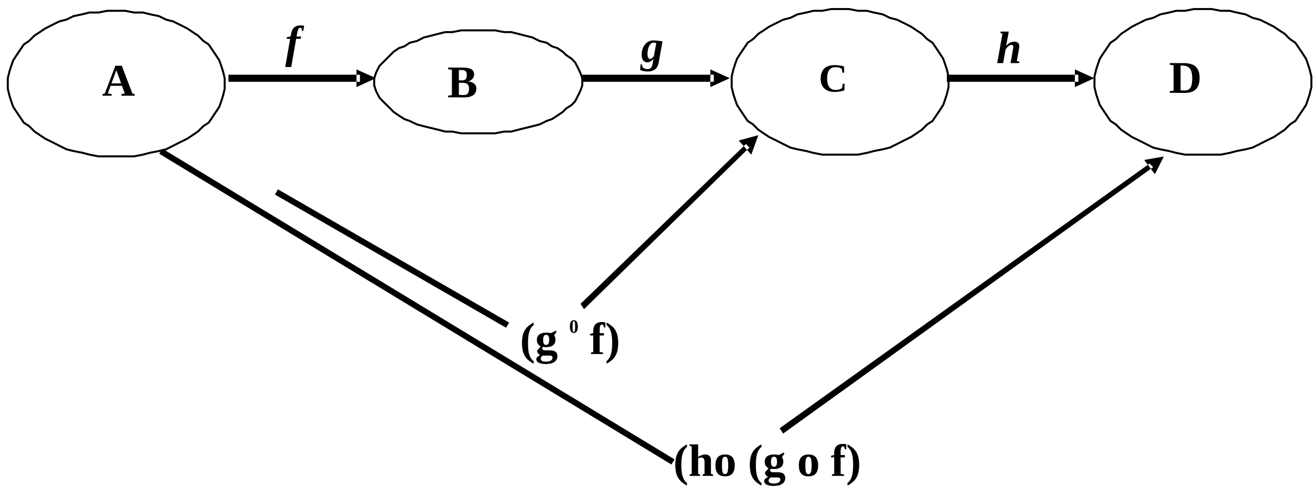


Fig. 4.1

Similarly, as illustrated in Figure 4-2, we can form the product function h ° g:

B → D and then the function (h o g) o f: A → D.

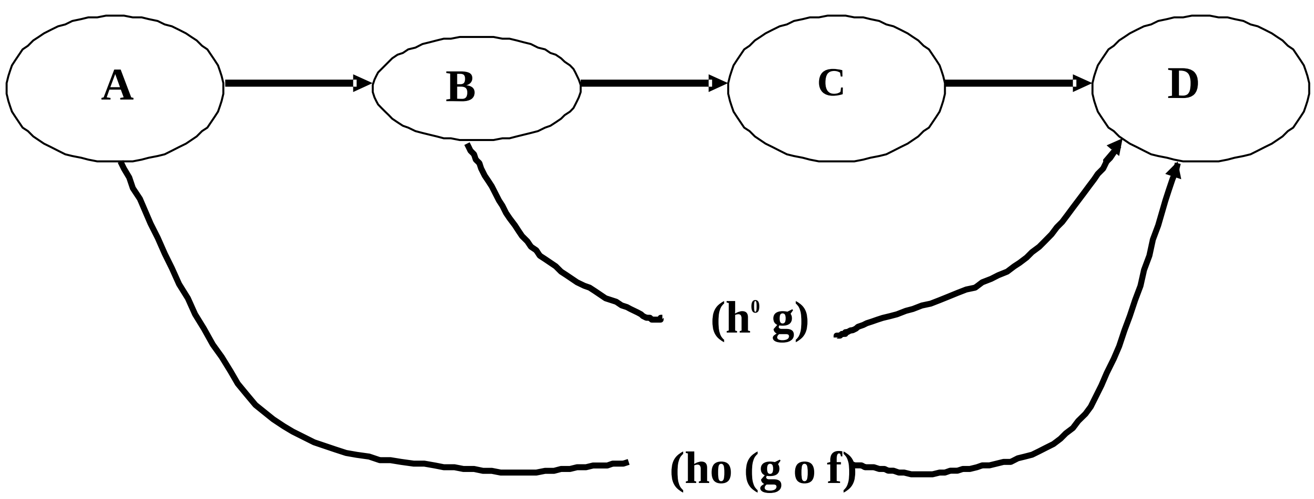


Fig 4.2

Both h ° (g ° f) and (h ° g) o f are function of A into D. A basic theorem on functions states that these functions are equal. Specifically,

Theorem 4.1: Let f: A →B, B → C and h: C →D. Then

(h o g) o f = h o (g o f)

In view of Theorem 4.1, we can write

$h^o\, g^o\, f: A \rightarrow D$ without any parenthesis.

3.10 INVERSE OF A FUNCTION

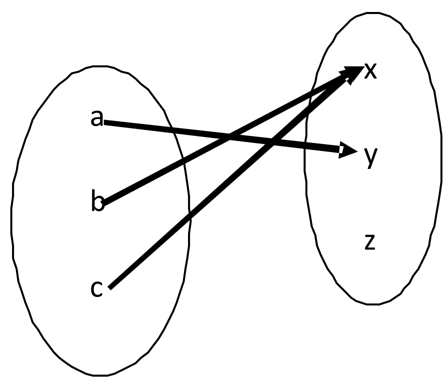
Let f be a function of A into B, and let $b \in B$. Then the ***inverse*** of b, denoted by f^1 (b)

Consist of those elements in A which are mapped onto b, that is, those element in A which have m as their image. More briefly, if $f: A \rightarrow B$ then

$f^1 (b) = \{x\}\ x \in A; f(x) = b\}$

Notice that f^{-1} (b) is always a subset of A. We read f^{-1} as "f inverse".

Example 8.1: Let the function $f: A \rightarrow B$ be defined by the diagram



Then $f^{-1}(x) = \{b, c\}$, since both b and c have x as their image point.

Also, $f^{-1}(y) = \{a\}$, as only a is mapped into y. The inverse of z, $f^{-1}(z)$, is the null set $\emptyset$, since no element of A is mapped into z.

Example 8.2: Let $f: \Re \rightarrow \Re$, the real numbers, be defined by the formula $f(x) = x^2$. Then $f^{-1}(4) = \{2, -2\}$, since 4 is the image of both 2 and -2 and there is no other real number whose square is four. Notice that $f^{-1}(-3) = \varnothing$, since there is no element in $\Re$ whose square is -3.

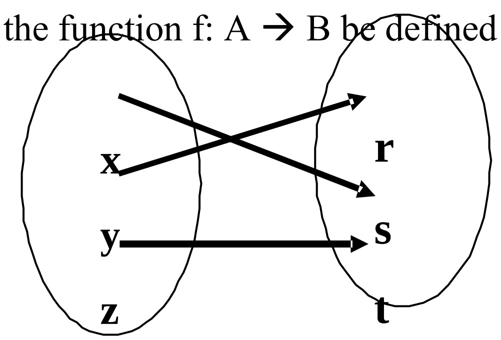
Example 8.3: Let f be a function of the complex numbers into the complex numbers, where f is defined by the formula $f(x) = x^2$. Then $f^{-1}(3) = \{\sqrt{3}i, -\sqrt{3}i\}$, as the square of each of these numbers is -3.

Notice that the function in Example 8.2 and 8.3 are different although they are defined by the same formula.

We now extend the definition of the inverse of a function. Let $f: A \rightarrow B$ and let D be a subset of B, that is, $D \subset B$. Then the inverse of D under the mapping f, denoted by $f^{-1}(D)$, consists of those elements in A which are mapped onto some element in D. More briefly,

$f^{1}(D) = \{x \mid x \in A, f(x) \in D\}$

Example 9.1: Let the function $f: A \rightarrow B$ be defined by the diagram



Then $f^{-1}(\{r, s\}) = \{y\}$, since only y is mapped into r or s. Also

$f^{1}(\{r, t\}) = \{x, y, z\} = A$, since each element in A as its image r or t.

Example 9.2: Let $f: \Re \rightarrow \Re$ be defined by $f(x) = x^2$, and let

$$D = [4, 9] = \{x \mid 4 \leq x < 9\}$$

Then $f^{-1}(D) = \{x \mid -3 \leq x \leq -2 \text{ or } 2 \leq x \leq 3\}$

Example 9.3: Let f: A → B be any function. Then $f^{-1}(B) = A$, since every element in A has its image in B. If f(A) denote the range of the function f, then $f^{-1}(f(A)) = A$

Further, if $b \in B$, then $f^{1}(b) = f^{-1}(\{b\})$

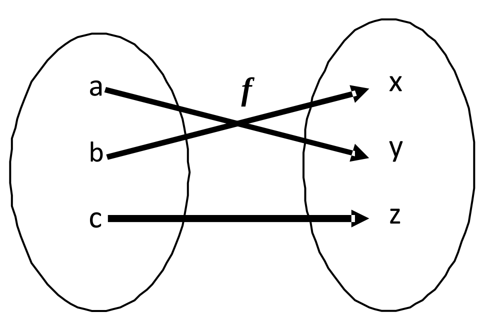
Here f^{-1} has two meanings, as the inverse of an element of B and as the inverse of a subset of B.

3.11 INVERSE FUNCTION

Let f be a function of A into B. In general, $f^{-1}(b)$ could consist of more than one element or might even be empty set ∅. Now if f: A → B is a one-one function and an onto function, then for each $b \in B$ the inverse $f^{-1}(b)$ will consist of a single element in A. We therefore have a rule that assigns to each $b \in B$ a unique element $f^{1}(b)$ in A. Accordingly, f^{-1} is a function of B into A and we can write f^{1} : B → A

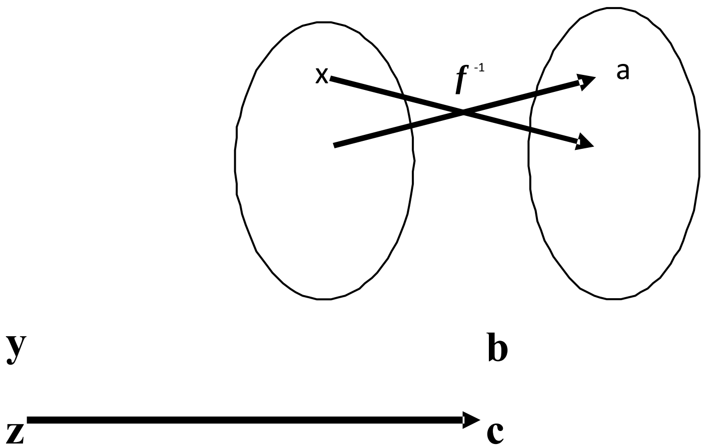
In this situation, when f: A ® B is one-one and onto, we call f^{-1} the inverse function of f.

Example 10.1: Let the function f: A → B be defined by the diagram

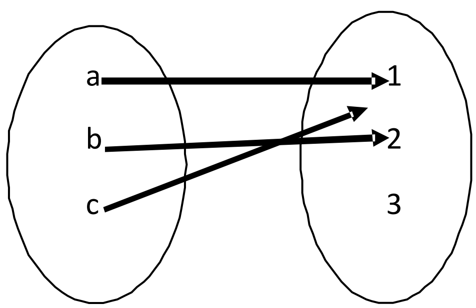


Notice that f is one-one and onto. Therefore f^{-1}, the inverse function exists.

We describe f^{-1}: B → A by the diagram



Example 6.1: Let the function f be defined by the diagram:

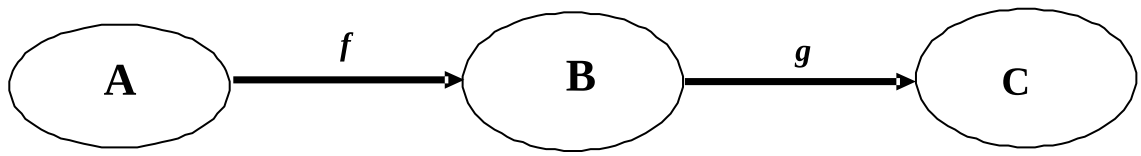


Then f is a constant function since 3 is assigned to every element in A.

Example 6.3: Let f: $\Re \rightarrow \Re$ be defined by the formula f(x) = 5. Then f is a constant Function since 5 is assigned to every element.

3.12 PRODUCT FUNCTION

Let f be a function of A and B and let g be a function of B, the co-domain of f, into C. We illustrate the function below.



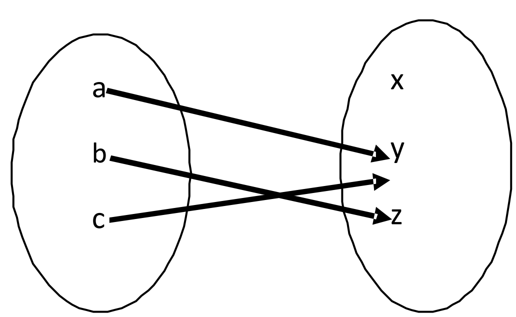
Let a $\in$ A; then its image f (x) is in B which is the domain of g. Accordingly, we can find the image of f (a) under the mapping of g, that is, we can find g (f(a)). Thus, we have a rule which assigns to each element a$\in$ A a corresponding element (f(a)) $\in$ C. In other words, we have a function of A into C. This new function is called the ***product function*** or ***composition function*** of f and g and it is denoted by (g ^{o} f) or (gf)

More briefly, if f: A → B and g: B → C then we define a function

Notice further, that if we send the arrows in the opposite direction in the first diagram of f we essentially have the diagram of f $^{-1}$.

Example 10.2: Let the function f: A → B be defined by the diagram

Since f(a) = y and f(c) = y, the function f is not one-one. Therefore, the inverse function f $^{-1}$ does not exist. As f $^{-1}$ (y) = {a, c}, we cannot assign both a and c to the element y $\in$ B.

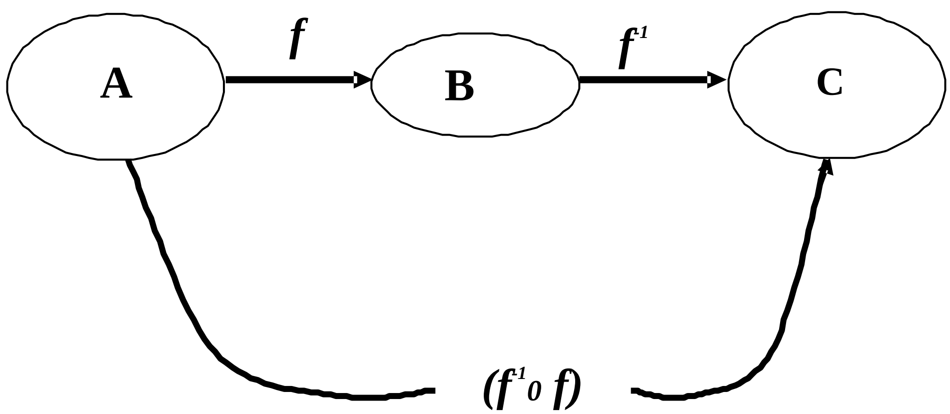


Example 10.3: Let f: $\Re \rightarrow \Re$, the real numbers, be defined by $f(x) = x^3$. Notice that f is one-one and onto.

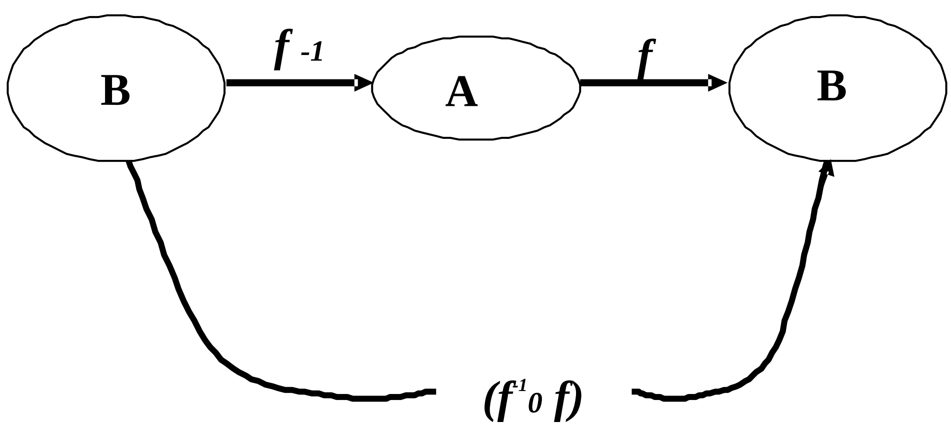
Hence $f^{-1}: \Re \rightarrow \Re$ exists. In fact, we have a formula which defines the inverse function, $f^{-1}(x) = \sqrt[3]{x}$.

3.12.1 Theorems on the inverse Function

Let a function f: A → B have an inverse function f^{-1}: B → A. Then we see by the diagram



That we can form the product ($f^{-1} \circ f$) which maps A into A, and we see by the diagram



That we can form the product function ($f \circ f^{-1}$) which maps B into B. We now state the basic theorems on the inverse function:

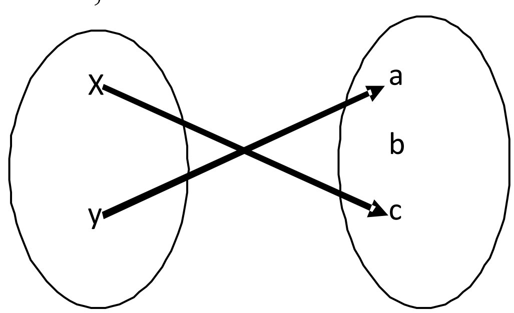
Theorem 4.2: Let the function $f: A \rightarrow B$ be one-one and onto; i.e. the inverse function $f^{-1}: B \rightarrow A$ exists. Then the product function

$$(f^{-1} \circ f): A \rightarrow A$$

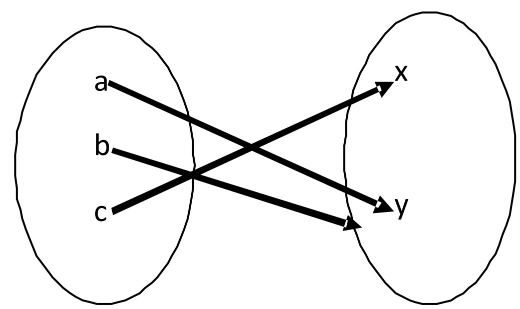
is the identity function on A, and the product function $(f^{\circ}\ f^{1}): B \rightarrow B$ is the identity function on B.

Theorem 4.3: Let $f: A \rightarrow B$ and $g: B \rightarrow A$. Then g is the inverse function of f, i.e. $g = f - 1$, if the product functions $(g^{\circ} f): A \rightarrow A$ is the identity function on A and $(f \circ g): B \rightarrow B$ is the identity function on B.

Both conditions are necessary in Theorem 4.3 as we shall see from the example below;



(a)



(b)

Now define a function $g: B \rightarrow A$ by the diagram (b) above.

We compute $(g^{\circ} f): A \rightarrow A$, $(g^{\circ} f)(x) = g(f(x)) = g(c) = x$ and $(g^{\circ} f)(y) = g(f(y))$ $= g(a) = y$

Therefore the product function $(g^{\circ} f)$ is the identity function on A. But g is not the inverse function of f because the product function $(f^{\circ}\ G)$ is not the identity function on B, f not being an auto function.

4.0 CONCLUSION

I believe that by now you fully grasp the idea of functions, mappings and transformations. This knowledge will be built upon in subsequent units.

5.0 SUMMARY

Recall that in this unit we have studied concepts such as mappings and functions. We have also examined the concepts of one-to-one and onto functions. This concept has allowed us to explain equality between two sets. We also established in the unit that the inverse of f: A – B usually denoted f^1, exit, if f is a one-to-one and onto function.

It is instructive to note that Inverse function is not studied in isolation but more importantly a useful and powerful tool in understanding calculus.

6.0 TUTOR – MARKED ASSIGNMENTS

1. Let the function f: R# → R# be defined by f (X) = {1 if x is rational] {-1 if x is irrational.

a. Express f in words

b. Suppose the ordered pairs (x + y, 1) and (3, x – y) are equal.

Find x and y.

2. Let M = {1, 2, 3, 4, 5} and let the function f: M → $\Re$ be defined by $f(x) = x^2 + 2x - 1$ Find the graph of f.

3. Prove: $A \times (B \cap C) = (A \times B) \cap (A \times C)$

4. Prove $A \subset B$ and $C \subset D$ implies $(A \times C) \subset (B \times D)$.

7.0 REFERENCES AND FURTHER READINGS

Seymour L.S. (1964). Outline Series: Theory and Problems of Set Theory and related topics, pp. 1 – 133.

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MODULE TWO

UNIT 1: PROBABILITY THEORY AND APPLICATIONS CONTENTS

1.0 Introduction

2.0 Objectives

3.0 Main Content

3.1 Definitions of Probability

3.2 Law of Probability

3.3 Computational Formula for Multiple Occurrence of an Event

3.4 Joint, Marginal, Conditional Probabilities, and the Bayes Theorem

3.4.1 Joint Probabilities

3.4.2 Marginal Probabilities

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7.0 References/Further Reading

1.0 INTRODUCTION

In this unit, we pay a special attention to the concept of probability, probability laws, computation of probabilities, and their applications to business decisions. At the end of this lecture, students will be expected to be able to make effective decisions under uncertainties. The basic elements of probability theory are the outcomes of the process or phenomenon under study. Each possible type of occurrence is referred to as **an event.** The collection of all the possible events is called the **sample space.** A **compound or joint event** is an event that has two or more characteristics. For example, the event of a student who is "an economics major and B or above average" is a joint or compound event since the student must be an economics major and have a B or above average. The event "black ace" is also a compound event since the card must be both black and ace in order to qualify as a black ace.

Probability is a concept that most people understand naturally, since such words as "chance," "likelihood," "possibility" and "proportion are used as part of everyday speech. For example, most of the following, which might be heard in any business situation, are in fact statements of probability. a) "There is a 30% chance that this job will not be finished in time". b) "There is every likelihood that the business will make a profit next year". c) "Nine times out of ten, he arrives late for his appointments". In statistical sense, probability simply puts a well defined structure around the concept of everyday probability, enabling a logical approach to problem solving to be followed.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

• explain the theory of probability
• define probability
• recognise the laws of probability
• calculate probabilities
• apply probabilities in making decisions involving uncertainties.

3.0 MAIN CONTENT

3.1 Definitions of Probability

There are basically two separate ways of calculating probability. 1. Calculation based on **theoretical probability**. This is the name given to probability that is calculated without an experiment that is, using only information that is known about the physical situation. 2. Calculation based **on empirical probability**. This is probability calculated using the results of an experiment that has been performed a number of times. Empirical probability is often referred to as *relative frequency* or *Subjective probability.*

Definition of Theoretical Probability Let E represent an event of an experiment that has an equally likely outcome set, U, then the theoretical probability of event E occurring when the experiment is written as Pr (E) and given by:

$$Pr(E) = \frac{\text{number of different ways that the event can occur}}{\text{number of different possible outcomes}}$$

$$= \frac{n(E)}{n(U)}$$

Where n (E) = the number of outcomes in event set

E n (U) = total possible number of outcomes in outcome set, U. If, for example, an ordinary six-sided die is to be rolled, the equally likely outcome set, U, is {1,2,3,4,5,6} and the event "even number" has event set {2,4,6}. It follows that the theoretical probability of obtaining an even number can be calculated as:

$$Pr(\text{even numbers}) = \frac{n(\text{even numbers})}{n(U)} = \frac{3}{6} = 0.50$$

Other Examples

A wholesaler stocks heavy (2B), medium (HB), fine (2H) and extra fine (3H) pencils which come in packs of 10. Currently in stock are 2 packs of 3H, 14 packs of 2H, 35 packs of HB and 8 packs of 2B. If a pack of pencil is chosen at random for inspection, what is the probability that they are:

(a) medium (b) heavy (c) not very fine (d) neither heavy nor medium?

Solutions

Since the pencil pack is chosen at random, each separate pack of pencils can be regarded as a single equally likely outcome. The total number of outcomes is the number of pencil packs, that is, 2+14+35+8 = 59.

Thus, n (U) = 59

$$\text{(a)} \quad Pr(\text{medium}) = \frac{n(\text{medium})}{n(U)} = \frac{35}{59} = 0.5932$$

$$\text{(b)} \quad Pr(\text{heavy}) = \frac{n(\text{heavy})}{n(U)} = \frac{8}{59} = 0.1356$$

(c) Pr (not very fine). Note that the number of pencil packs that are not very fine is 14+35+8 = 57

$$\text{Therefore, } Pr(\text{not very fine}) = \frac{n(\text{not very fine})}{n(U)} = \frac{57}{59} = 0.966$$

(d) “Neither heavy nor medium” is equivalent to “fine” or “very fine” in the problem. There is 2+14 = 16 of these pencil packs.

Thus, Pr (neither heavy nor medium)

$$= \frac{n(\text{neither heavy nor medium})}{n(U)} = \frac{16}{59} = 0.2781$$

Definition of Empirical (Relative Frequency) Probability

If E is some event of an experiment that has been performed a number of times, yielding a frequency distribution of events or outcomes, then the empirical probability of event E occurring when the experiment is performed one more time is given by:

$$Pr(E) = \frac{\text{number of times the event occurred}}{\text{number of times the experiment was performed}} = \frac{f(E)}{\sum f}$$

Where f(E) = the frequency of event E

Σf = total frequency of the experiment.

Put differently, the empirical probability of an event E occurring is simply the proportion of times that event E actually occurred when the experiment was performed. For example, if, out of 60 orders received so far this financial year, 12 were not completely satisfied, the proportion, 12/60 = 0.2 is the empirical probability that the next order received will not be completely satisfied.

Other Examples

A number of families of a particular type were measured by the number of children they have, given the following frequency distribution:

Number of children:	0	1	2	3	4	5 or more
Number of families:	12	28	22	8	2	2

Use this information to calculate the (relative frequency) probability that another family of this type chosen at random will have:

(a) 2 children (b) 3 or more children (c) less than 2 children

Solutions Here, Σf = total number of families = 74

(a) $Pr\,(2\text{ children}) = \dfrac{f(2\text{ children})}{\sum f} = \dfrac{22}{74} = 0.297$

(b) f (3 or more children) = 8+2+2 = 12

Thus, Pr(3 or more children = 12/74 = 0.162

(c) $Pr(\text{less than 2 children}) = \dfrac{f(\text{ less than 2 children}}{\sum f} = \dfrac{12+28}{74} = 0.541$

3.2 Laws of Probability

There are four basic laws of probability.

1. Addition Law for mutually exclusive events

2. Addition Law for events that are not mutually exclusive

3. Multiplication Law for Independent events

4. Multiplication Law for Dependent events.

Addition Law for Mutually Exclusive Events

Two events are said to be mutually exclusive events if they cannot occur at the
same time. The addition law states that if events A and B are mutually exclusive events, then: Pr (A or B) = Pr (A) + Pr (B)

Examples

The purchasing department of a big company has analysed the number of orders
placed by each of the 5 departments in the company by type as follows:

Table 9.1: Departmental Orders

Type of Order		Department			Total
	Sales	Purchasing	Production		
Accounts Maintenance					
Consumables	10	12	8	4	38
4			1	1	15
Equipment	1	3	1	2	7
9			10	7	60
Special	0	0			
4					
Total	11	15			
17					

An error has been found in one of these orders. What is the probability that the incorrect order:

a) came from maintenance?

b) came from production?

c) came from maintenance or production?

d) came from neither maintenance nor production?

Solutions

a) since there are 7 maintenance orders out of the 60,

Pr (maintenance) = 7/60 = 0.117

b) Similarly, Pr (Production) = 17/60 = 0.283

c) Maintenance and production departments are two mutually exclusive events so that, Pr (maintenance or production) = Pr (maintenance) + Pr(production = 0.117 + 0.283 = 0.40

d) Pr (neither maintenance nor production)

= 1-Pr (maintenance or production) = 1-0.4 = 0.6

Addition Law for Events that are Not Mutually Exclusive Events

If events A and B are not mutually exclusive, that is, they can either occur together or occur separately, then according to the Law:

Pr (A or B or Both) = Pr (A) +Pr (B) – Pr (A).Pr (B)

Example

Consider the following contingency table for the salary range of 94 employees:

Table 9.2: Contingency Table for the Salary range of 94 Employees

Salary/month	Women	Total
Men	37	57
N10, 000 and above	22	37
20	78	
Below 10,000		
15		

Total	59	94
35		

What is the probability of selecting an employee who is a man or earns below N10, 000 per month?

Solution

The two events of being a man and earning below N10, 000 is not mutually exclusive. It follows that:

Pr(man or below N10,000) = Pr(man) + Pr(below N10,0000) - Pr(man) · Pr(below N10,0000)

35/94 + 37/94 – (35)/(94)(37)/(94)

=0.372+0.394 – (0.372)(0.394)

=0.766 – 0.147

= 0.619 or 61.9%

Multiplication Law for Independent Events

This law states that if A and B are independent events, then:

Pr (A and B) = Pr (A). Pr (B)

As an **example**, suppose, in any given week, the probability of an assembly line failing is 0.03 and the probability of a raw material shortage is 0.1.

If these two events are independent of each other, then the probability of an assembly line failing and a raw material shortage is given by:

Pr (Assembly line failing and Material shortage) = (0.03)(0.1) = 0.003

Multiplication Law for Dependent Events

This Law states that if A and B are dependent events, then:

Pr (A and B) = Pr(A).Pr(B/A)

Note that Pr (B/A) in interpreted as probability of B given that event A has occurred.

Example

A display of 15 T-shirts in a Sports shop contains three different sizes: small, medium and large. Of the 15 T-shirts:

3 are small

6 are medium

6 are large.

If two T-shirts are randomly selected from the T-shirts, what is the probability of selecting both a small T-shirt and a large T-shirt, the first not being replaced before the second is selected?

Solution

Since the first selected T-shirt is not replaced before the second T-shirt is selected, the two events are said to be dependent events. It follows that:

Pr (Small T-shirt and Large T-shirt)

= Pr(Small).Pr(Large/Small)

= (3/15)(4/14)

$= (0.2)(0.429)$

$= 0.086$

3.3 Computational Formula for Multiple Occurrence of an Event

The probability of an event, E occurring X times in n number of trials are given by the formula:

$$Pr\ (E^{nx}) = C_{n,x}p^{x}q_{(n-x)}$$

Where $C_{n,x} = \dfrac{n!}{X!(n-x)!}$

p = probability of success

q = probability of failure

p + q = 1

Example Assume there is a drug store with 10 antibiotic capsules of which 6 capsules are effective and 4 are defective. What is the probability of purchasing the effective capsules from the drug store?

Solution

From the given information: The probability of purchasing an effective capsule is:
P = 6/10 = 0.60

Since p + q = 1; q = 1 – 0.60 = 0.40; n = 10; x = 6

Pr (E106) = probability of purchasing the 6 effective capsules

= C10,6(0.6)6(0.4)4

= 10!/(6!(10 – 6)!)(0.047)(0.026)

= 10.9.8.7.6!(0.0012) 6! 4!

= 210(0.0012) = 0.252

Hence, the probability of purchasing the 6 effective capsules out of the 10 capsules is 25.2 percent

3.4 Joint, Marginal, Conditional Probabilities, and the Bayes Theorem

3.4.1 Joint Probabilities

A joint probability implies the probability of joint events. Joint probabilities can be conveniently analysed with the aid of joint probability tables.

The Joint Probability Table

A joint probability table is a contingency table in which all possible events for a variable are recorded in a row and those of other variables are recorded in a column, with the values listed in corresponding cells as in the following example.

Example: Consider a research activity with the following observations on the number of customers that visit XYZ supermarket per day. The observations (or events) are recorded in a joint probability table as follows:

Table 9.3: Joint Probability Table

Table 9.3: Joint Probability Table	Male (M)	Female (F)	Total
Age (Years)			
Below 30 (B)	60	70	130
30 and Above (A)	60	20	80

Total	120	90	210

We can observe four joint events from the above table:

Below 30 and Male (B∩M) = 60
Below 30 and Female (B∩F) = 70
30 and Above and Male (A∩M) = 60
30 and Above and Female (A∩F) = 20

Total events or sample space = 210

The joint probabilities associated with the above joint events are

$$\Pr(B \cap M) = \frac{60}{210}$$

Pr (B ∩ F) 70/120 = 0 3333

Pr (A ∩ M) 60/210 = 0.2857

Pr (A ∩ F) 20/210 = 0.0952

3.4.2 Marginal Probabilities

The Marginal Probability of an event is its simple probability of occurrence, given the sample space. In the present discussion, the results of adding the joint probabilities in rows and columns are known as marginal probabilities.

The marginal probability of each of the above events:

Male (M), Female (F), Below 30 (B), and Above 30 (A) are as follows:

Pr (M) = Pr(B∩M) + Pr(A∩M) = 0.2857 + 0.2857 = 0.57

Pr (F) = Pr(B∩F)+Pr(A∩F) = 0..3333 + 0.0952 = 0.43

Pr (B) = Pr(B∩M)+Pr(B∩F) = 0.2857+0.3333 = 0.62

Pr (A) = {r(A∩M)+Pr(A∩F) = 0.2857+0.0952 = 0.38

The joint and marginal probabilities above can be summarised in a contingency table as follows:

Table 9.4: Joint and Marginal Probability Table.

Age	Male (M)	Female (F)	Marginal Probability
Below 30 (B)	0.2857	0.3333	0.62
30 and Above (A)	0.2857	0.0952	0.38
Marginal Probability	0.57	0.43	1.00

3.4.3 Conditional Probability

Assuming two events, A and B, the probability of event A, given that event B has occurred is referred to as the conditional probability of event A.

In symbolic term:

$$Pr(A\ B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{Pr(A).Pr(B)}{Pr(B)} = Pr(A)$$

Where Pr (A/B) = conditional probability of event A

Pr (A∩B) = joint probability of events A and B

Pr (B) = marginal probability of event B

In general,

$$Pr(A/B) = \frac{\text{Joint Probability of events A and B}}{\text{Marginal Probability of event B}}$$

3.4.4 The Bayes Theorem

Bayes theorem is a formula which can be thought of as “reversing” conditional probability. That is, it finds a conditional probability, A/B given, among other things, its inverse, B/A. According to the theorem, given events A and B,

$$Pr(A/B) = \frac{Pr(A) \cdot Pr((B/A)}{Pr(B)}$$

As an example in the use of Bayes theorem, if the probability of meeting a business contract date is 0.8, the probability of good weather is 0.5 and the probability of meeting the date given good weather is 0.9, we can calculate the probability that there was good weather given that the contract date was met.

Let G = good weather, and m = contract date was met

Given that: Pr (m) = 0.8; Pr (G) = 0.5; Pr (m/G) = 0.9, we need to find Pr (G/m):

From the Bayes theorem:

$$Pr(G/m) = \frac{Pr(G) \cdot Pr(m/G)}{Pr(m)} = \frac{(0.5)(0.9)}{0.8}$$

= 0.5625 or 56.25%

3.5 Probability and Expected Values

The expected value of a set of values, with associated probabilities, is the arithmetic mean of the set of values. If some variable, X, has its values specified with associated probabilities, P, then:

Expected value of X = E (X) = Σ PX

Example

An ice-cream salesman divides his days into 'Sunny' 'Medium' or 'Cold'. He estimates that the probability of a sunny day is 0.2 and that 30% of his days are cold. He has also calculated that his average revenue on the three types of days is N220, N130, and N40 respectively. If his average total cost per day is N80, calculate his expected profit per day.

An ice-cream salesman divides his days into 'Sunny' 'Medium' or 'Cold'. He estimates that the probability of a sunny day is 0.2 and that 30% of his days are cold. He has also calculated that his average revenue on the three types of days is N220, N130, and N40 respectively. If his average total cost per day is N80, calculate his expected profit per day.

Solution

We first calculate the different values of profit that are possible since we are required to calculate expected profit per day, as well as their respective probabilities.

Given that Pr (sunny day) = 0.2; Pr (cold day) = 0.3

Since in theory, Pr (sunny day) + Pr (cold day) +Pr (medium day) = 1

It follows that:
Pr (medium day) = 1 - 0.2 - 0.3 = 0.5

The total costs are the same for any day (#80), so that the profits that the salesman makes on each day of the three types of day are;

Sunnyday:N(220-80)=N140
Medium day: N (130-80) = N50

Cold day: N(40-80)= -N40 (loss)

We can summarise the probability table as follows:

SELF ASSESSMENT EXERCISE 2

1. What do we mean by the terms "mutually exclusive events" and"independent events?"
2. State, with simple examples, the four laws of probability.

4.0 CONCLUSION

Probability is a concept that most people understand naturally, since such words as a chance, likelihood, possibilities and proportion are used as part of everyday speech. It is a term used in making decisions involving uncertainty. Though the concept is often viewed as very abstract and difficult to relate to real world activities, it remains the best tool for solving uncertainties problems.

To remove some of the abstract nature of probabilities, this unit has provided you with the simplest approach to understanding and calculating, as well as applying the probability concept. It defines probability in two basic forms:

(i) the theoretical definition; and

(ii) the empirical definition.

5.0 SUMMARY

The issues discussed in this unit can be summarised in the following way: there are basically two separate ways of calculating probability which are as stated below:

i. Theoretical probability: this is calculated without an experiment, that is, using only information that is known about the physical situation.

ii. Calculation based on **empirical probability**. This is probability calculated using the results of an experiment that has been performed a number of times. Empirical probability is often referred to as **Relative frequency** or **Subjective probability.**

There are four basic **laws of probability**:

1. addition law for mutually exclusive events
2. addition law for events that are not mutually exclusive
3. multiplication law for independent events
4. multiplication law for dependent events.

The probability of an event, E occurring an X time in n number of trials is given by the formula:

$$Pr(E_{nx}) = C_{n,x} p_x q_{(n-x)}$$

Where $C_{n,x} = \dfrac{n!}{X!(n-x)!}$

p = probability of success

q = probability of failure

p + q = 1

A **joint probability** implies the probability of joint events. Joint probabilities can be conveniently analysed with the aid of joint in which all possible events for a variable are recorded in a row and those of other variables are recorded in a column, with the values listed in corresponding cells.

The **Marginal Probability** of an event is its simple probability of occurrence, given the sample space.

Assuming two events, A and B, the probability of event A, given that event B has occurred is referred to as the **conditional probability** of event A. The expected value of a set of values, with associated probabilities, is the arithmetic mean of the set of values. If some variable, X, has its values specified with associated probabilities, P, then: Expected value of X = E (X) = Σ PX

6.0 TUTOR-MARKED ASSIGNMENT

A firm has tendered for two independent contracts. It estimates that it has probability 0.4 of obtaining contract A and probability 0.1 of obtaining contract B.
Find the probability that the firm:
(a) obtains both contracts
(b) obtains neither of the contracts
(c) obtains exactly one contract

7.0 REFERENCES/FURTHER READING

Francis, A. (1998). *Business Mathematics and Statistics, 5th edition.* Great Britain: Ashford Colour Pres

UNIT 2 DECISION ANALYSIS

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1.0 INTRODUCTION

Decision analysis is the modern approach to decision making both in economics and in business. It can be defined as the logical and quantitative analysis of all the factors influencing a decision. The analysis forces decision makers to assume some active roles in the decision-making process. By so doing, they rely more on rules that are consistent with their logic and personal behaviour than on the mechanical use of a set of formulas and tabulated probabilities. The primary aim of decision analysis is to increase the likelihood of good outcomes by making good and effective decisions. A good decision must be consistent with the information and preferences of the decision maker. It follows that decision analysis provides decision-making framework based on available information on the business environment, be it sample information, judgemental information, or a combination of both.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- identify and analyse decision problems
- analyse decisions that are made under conditions of certainty and
- uncertainties
- make expected monetary value decisions

3.0 MAIN CONTENT

3.1 Certainty and Uncertainty in Decision Analysis

Most decision-making situations involve the choice of one among several alternative actions. The alternative actions and their corresponding payoffs are usually known to the decision-maker in advance. A prospective investor choosing one investment from several alternative investment opportunities, a store owner determining how many of a certain type of commodity to stock, and a company executive making capital-budgeting decisions are some examples of a business decision maker selecting from a multitude of alternatives. The decision maker however, does not know which alternative will be best in each case, unless he/she also knows with certainty the values of the economic variables that affect profit. These economic variables are referred to, in decision analysis, as *states of nature* as they represent different events that may occur, over which the decision maker has no control.

The states of nature in decision problems are generally denoted by si (i = 1, 2, 3… k), where k is the number of or different states of nature in a given business and economic environment. It is assumed here that the states of nature are mutually exclusive, so that no two states can be in effect at the same time, and collectively exhaustive, so that all possible states are included within the decision analysis.

The alternatives available to the decision maker are denoted by ai (i = 1, 2, 3, …, n), where n is the number of available alternatives. It is also generally assumed that the alternatives constitute a mutually exclusive, collectively exhaustive set.

When the state of nature, si, whether known or unknown, has no influence on the outcomes of given alternatives, we say that the decision maker is operating under *certainty.* Otherwise, he/she is operating under *uncertainty.*

Decision making *under certainty* appears to be simpler than that under uncertainty. Under certainty, the decision maker simply appraises the outcome of each alternative and selects the one that best meets his/her objective. If the number of alternatives is very high however, even in the absence of uncertainty, the best alternative may be difficult to identify. Consider, for example, the problem of a delivery agent who must make 100 deliveries to different residences scattered over Lagos metropolis. There may literally be thousands of different alternative routes the agent could choose. However, if the agent had only 3 stops to make, he/she could easily find the least-cost route.

Decision making *under uncertainty* is always complicated. It is the probability theory and mathematical expectations that offer tools for establishing logical

procedures for selecting the best decision alternatives. Though statistics provides the structure for reaching the decision, the decision maker has to inject his/her intuition and knowledge of the problem into the decision-making framework to arrive at the decision that is both theoretically justifiable and intuitively appealing.
A good theoretical framework and commonsense approach are both essential ingredients for decision making under uncertainty. To understand these concepts, consider an investor wishing to invest N100, 000 in one of three possible investment alternatives, A, B, and C. Investment A is a Savings Plan with returns of 6 percent annual interest. Investment B is a government bond with 4.5 percent annual interest. Investments A and B involve no risks. Investment C consists of shares of mutual fund with a wide diversity of available holdings from the securities market. The annual return from an investment in C depends on the uncertain behaviour of the mutual fund under varying economic conditions.

The investors available actions (ai; I = 1, 2, 3, 4) are as follows

a1: do not invest

a2: select investment A the 6% bank savings plan.

a3: select investment B, the 4.5 % government bond.

a4: select investment C, the uncertain mutual fund Observe that actions a1 to a3 do not involve uncertainty as the outcomes associated with them do not depend on uncertain market conditions.

Observe also that action a 2 dominates actions a1 and a3. In addition, action a1 is clearly inferior to the risk-free positive growth investment alternatives a2 and a3 as it provides for no growth of the principal amount.

Action a4 is associated with an uncertain outcome that, depending on the state of the economy, may produce either a negative return or a positive return. Thus there exists no apparent dominance relationship between action a4 and action a2, the best among the actions involving no uncertainty.

Suppose the investor believes that if the market is down in the next year, an investment in the mutual fund would lose 10 percent returns; if the market stays the same, the investment would stay the same; and if the market is up, the investment would gain 20 percent returns. The investor has thus defined the states of nature for his/her investment decision-making problem as follows:

s1: the market is down.

s2: the market remains unchanged.

s3: the market is up.

A study of the market combined with economic expectations for the coming year may lead the investor to attach subjective probabilities of 0.25, 0.25, and 0.50, respectively, the states of nature, s1, s2, and s3. The major question is then, how can the investor use the foregoing information regarding investments A, B, and C, and the expected market behaviour serves as an aid in selecting the investment that best satisfies his/her objectives? This question will be considered in the sections that follow.

3.2 Analysis of the Decision Problem

In problems involving choices from many alternatives, one must identify all the actions that may be taken and all the states of nature which occurrence may influence decisions. The action to take none of the listed alternatives whose outcome is known with certainty may also be included in the list of actions. Associated with each action is a list of payoffs. If an action does not involve risk, the payoff will be the same no matter which state of nature occurs.

The payoffs associated with each possible outcome in a decision problem should be listed in a *payoff table*, defined as a listing, in tabular form, of the value payoffs associated with all possible actions under every state of nature in a decision problem. The payoff table is usually displayed in grid form, with the states of nature indicated in the columns and the actions in the rows. If the actions are labeled a1, a2… an, and the states of nature labeled s1, s2, …, sk, a payoff table for a decision problem appears as in table 14.1 below. Note that a payoff is entered in each of the nk cells of the payoff table, one for the payoff associated with each action under every possible state of nature.

Table 14.1: The Payoff Table

State of Nature

ACTION	**s1**	**s2**	**s3**	**……..**	**sk**
a1					
a2					
a3 . .					
. An					

Example

The managing director of a large manufacturing company is considering three potential locations as sites at which to build a subsidiary plant. To decide which location to select for the subsidiary plant, the managing director will determine the degree to which each location satisfies the company's objectives of minimising transportation costs, minimising the effect of local taxation, and having access to an ample pool of available semi-skilled workers. Construct a payoff table and payoff measures that effectively rank each potential location according to the degree to which each satisfies the company's objectives.

Solution

Let the three potential locations be sites A, B, and C. To determine a payoff measure to associate with each of the company's objectives under each alternative, the managing director subjectively assigns a rating on a 0 to 10 scale to measure the degree to which each location satisfies the company's objectives. For each objective, a 0 rating indicates complete dissatisfaction, while a 10 rating indicates complete dissatisfaction. The results are presented in table 14.2 below:

Table 14.2: Ratings for three alternative plant sites for a Manufacturing Company

	ALTERNATIVE		
COMPANY OBJECTIVE	Site A	Site B	Site C
Transportation Costs	6	4	10
Taxation Costs	6	9	5
Workforce Pool	7	6	4

To combine the components of payoff, the managing director asks himself, what are the relative measures of importance of the three company objectives I have considered as components of payoff? Suppose the managing director decides that minimising transportation costs is most important and twice as important as either the minimisation of local taxation or the size of workforce available. He/she thus assigns a weight of 2 to the transportation costs and weights of 1 each to taxation costs and workforce. This will give rise to the following payoff measures:

Payoff (Site A) = 6(2) + 6(1) + 7(1) = 25

Payoff (Site B) = 4(2) + 9(1) + 6(1) = 23

Payoff (Site C) = 10(2) + 5(1) + 4(1) = 29

3.3 Expected Monetary Value Decisions

A decision-making procedure, which employs both the payoff table and prior probabilities associated with the states of nature to arrive at a decision, is referred to as the *Expected Monetary Value* decision procedure. Note that by *prior probability,* we mean probabilities representing the chances of occurrence of the identifiable states of nature in a decision problem prior to gathering any sample information. The *expected monetary value decision* refers to the selection of available action based on either the expected opportunity loss or the expected profit of the action.

Decision makers are generally interested in the *optimal monetary value decisions*. The optimal expected monetary value decision involves the selection of the action associated with the minimum *expected opportunity loss* or the action associated with the maximum *expected profit*, depending on the objective of the decision maker.

The concept of expected monetary value applies mathematical expectation, where opportunity loss or profit is the random variable and the prior probabilities represent the probability distribution associated with the random variable.

The ***expected opportunity loss*** is computed by:

$E(L_i) = \Sigma_{all\ j} L_{ij}P(s_j), (i = 1, 2, \ldots, n)$

Where L_{ij} is the opportunity loss for selecting action a_i given that the state of nature, s_j, occurs and $P(s_j)$ is the prior probability assigned to the state of nature, s_j.

The ***expected profit for each action is*** computed in a similar way:

$E(\pi_i) = \Sigma_{all\ j}\ \pi_{ij}P(s_j)$

Where π_{ij} represents profits for selecting action a_i

Example

By recording the daily demand for a perishable commodity over a period of time, a retailer was able to construct the following probability distribution for the daily demand levels:

Table 14.3: Probability Distribution for the Daily Demand

sj	P(sj)
1	0.5
2	0.3
3	0.2
4 or more	0.0

The opportunity loss table for this demand-inventory situation is as follows:

Table 14.4: The Opportunity Loss Table

State of Nature, Demand

Action, Inventory	s1(1)	s2(2)	s3(3)
a1 (1)	0	3	6
a2 (2)	2	0	3
a3 (3)	4	2	0

We are required to find the inventory level that minimises the expected opportunity loss.

Solution

Given the prior probabilities in the first table, the expected opportunity loss is computed as follows:

E (Li) = Σj=13LijP (sj), for each inventory level, I = 1, 2, 3.

The expected opportunity losses at each inventory level become:

E (L1) = 0(0.5) + 3(0.3) + 6(0.2) = N2.10

E (L2) = 2(0.5) + 0(0.3) + 3(0.2) = N1.60

E (L3) = 4(0.5) + 2(0.3) + 0(0.2) = N2.60

It follows that in order to minimise the expected opportunity loss, the retailer should stock 2 units of the perishable commodity. This is the optimal decision.

3.4 Decision Making Involving Sample Information

Prior probabilities are acquired either by subjective selection or by computation from historical data. No current information describing the probability of occurrence of the states of nature was assumed to be available.

In many cases, observational information or other evidences are available to the decision maker either for purchase or at the cost of experimentation. For example, a retailer whose business depends on the weather may consult a meteorologist before making decisions, or an investor may hire a market consultant before investing. Market surveys carried out before the release of a new product represent another area in which the decision maker may seek additional information. In each of these examples, the decision maker attempts to acquire information relative to the occurrence of the states of nature from a source other than that from which the prior probabilities were computed.

When such information is available, *Baye s Law* can be employed to revise the prior probabilities to reflect the new information. These revised probabilities are referred to as *posterior probabilities.*

By definition, the *posterior probability* represented symbolically by *P (sk/x)* is the probability of occurrence of the state of nature sk, given the sample information, x. The probabilities, *P(x/si)* are the conditional probabilities of observing the observational information, x, under the states of nature, si, and the probabilities *P (si)* are the prior probabilities.

The expected monetary value decisions are formulated in the same way as before, except that the posterior probabilities are used instead of prior probabilities. If the objective is to minimise the expected opportunity loss, the quantity is computed for each action . The expected opportunity loss in this case is computed by:

$E (Li) = \Sigma all\ I\ LijP(si/x)\ I = 1, 2, 3, \ldots, n$

Example

It is known that an assembly machine operates at a 5 percent or 10 percent defective rate. When running at a 10 percent defective rate, the machine is said to be out of control. It is then shut down and readjusted.

From past experience, the machine is known to run at 5 percent defective rate 90 percent of the time. A sample of size n = 20 has been selected from the output of the machine, and y = 2 defectives have been observed. Based on both the prior and sample information, what is the probability that the assembly machine is in control (running at 5 percent defective rate)?

Solution

The states of nature in this example relates to the assembly machine defective rates. Thus the states of nature include:

s1 = 0.05, and s2 = 0.10 with the assumed prior probabilities of occurrence of 0.90 and 0.10. We are required to use these prior probabilities, in line with the observed sample information, to find the posterior probability associated with the state of nature, s1.

In this problem, the "experimental information, x" is the observation of y = 2 defectives from a sample of n = 20 items selected from the output of the assembly machine. We need to find the probability that the experimental information, x, could arise under each state of nature, si. This can be done by referring to the binomial probability distribution table found in the appendix.

Under the state of nature s1 = 0.05, we obtain:

$P(x/0.05) = P(n = 20,\ y = 2/0.05) = 0.925 - 0.736 = 0.189$ (from the binomial distribution table) Under the state of nature, s2 = 0.10,

we obtain: $P(x/0.10) = P(n = 20, y = 2/0.10)$ = 0.677 – 0.392 = 0.285 (from the binomial distribution table). We now employ the Baye's Law to find the posterior probability that the machine is in control (s1) based on both the prior and experimental information. To make the work easy, we use the Columnar's approach to the use of Baye's Law as illustrated below:

Table 14.5: Columnar Approach to Use of Baye's Law

(1) State of nature S1	(2) Prior (Ps1)	(3) Experimental Information P(x/si)	(4) Product $P(s_i)P(x/s_i)$	(5) Posterior $P(s_i/x)$

S1 0.05	0.90	0.189	0.1701	0.86
S2 0.10	0.10	0.285	0.0285	0.14
	1.00		0.1986	1.00

Looking at column (4), we observe the product of the entries in columns (2) and (3). These values measure the *joint probabilities*. The sum of the entries in column (4) is the term in the denominator of the formula for Baye's Law and measures the *marginal probability* of observing the experimental information, x. The posterior probabilities, column (5), are obtained by taking each entry in column (4) and dividing by the sum of the entries in column (4).

Even though we found that 10 percent of the items in the sample is defective (that is, 2 out of the 20 items is defective), the posterior probability that the machine is running at the 10 percent defective rate (running out of control) is only 0.14, which is a little greater than the prior probability that the machine is out of control (0.10).

It follows that the probability that the machine is not running out of control is 0.86.

SELF ASSESSMENT EXERCISE

Give the justification for using an expected monetary value objective in decision problems.

4.0 CONCLUSION

This unit has uncovered the fact that decision analysis is the modern approach to decision making both in economics and in business. It can be defined as the logical and quantitative analysis of all the factors influencing a decision. Decision analysis forces decision makers to assume some active roles in the decision-making process. By so doing, they rely more on rules that are consistent with their logic and personal behaviour than on the mechanical use of a set of formulas and tabulated probabilities.

The primary aim of decision analysis is to increase the likelihood of good outcomes by making good and effective decisions. A good decision must be consistent with the information and preferences of the decision maker. It follows that decision analysis provides decision-making framework based on available information on the business environment, be it sample information, judgemental information, or a combination of both.

5.0 SUMMARY

The unit considered the following important elements of decision analysis:

1. Decision Problems and Analysis

In problems involving choices from many alternatives, one must identify all the actions that may be taken and all the states of nature whose occurrence may influence decisions. The action to take none of the listed alternatives whose outcome is known with certainty may also be included in the list of actions. Associated with each action is a list of payoffs. If an action does not involve risk, the payoff will be the same no matter which state of nature occurs.

2. Expected Monetary Value Decisions

A decision-making procedure, which employs both the payoff table and prior probabilities associated with the states of nature to arrive at a decision, is referred to as the *Expected Monetary Value* decision procedure. Note that by *prior probability* we mean probabilities representing the chances of occurrence of the identifiable states of nature in a decision problem prior to gathering any sample information. The *expected monetary value decision* refers to the selection of available action based on either the expected opportunity loss or the expected profit of the action.

3. Decision Making Involving Sample Information

In many decisions, observational information or other evidence are available to the decision maker either for purchase or at the cost of experimentation. For example, a retailer whose business depends on the weather may consult a meteorologist before making decisions, or an investor may hire a market consultant before investing. Market surveys carried out before the release of a new product represent another area in which the decision maker may seek additional information. In each of these examples, the decision maker attempts to acquire information relative to the occurrence of the states of nature from a source other than that from which the prior probabilities were computed. When such information is available, *Baye s Law* can be employed to revise the prior probabilities to reflect the new information. We refer to such probabilities as *posterior probabilities.*

6.0 TUTOR-MARKED ASSIGNMENT

A special commission appointed by the governor of Enugu state is currently studying the issue of site location for a nuclear power-generating plant. The alternatives being currently considered are (a1) to locate the plant at the mouth of Ever-Valley River, (a2) to locate the plant inland, and (a3) not to build the plant at all. Because of the heat generated in cooling operations, a1 may prove damaging to a nearby fishing industry vital to the economy of Enugu state. However, there may be insufficient water flow available at site a2 for adequate cooling. If alternative a3 is chosen, sufficient energy resources may not be available to meet future needs.

(a) If you were the economic adviser to the commission, what would you recommend to be used as a measure of payoff for evaluating the three alternatives?

(b) Suppose the commission has decided that the important attributes of value in the decision model are

(i) the ability of each alternative to meet future commercial power needs,

(ii) the ability of each alternative to meet future residential power needs,

(iii) the effects on the state's fishing industry, and

(iv) other environmental effects. What recommendation would you give to the commission regarding the incorporation of these four attributes into a model to derive some reasonable measure of payoff associated with each alternative?

7.0 REFERENCES/FURTHER READING

Francis, A. (1998). *Business Mathematics and Statistics, 5th edition* (Great Britain: Ashford Colour Press).

UNIT 3: TYPES OF DECISION SITUATIONS

1.0 Introduction
2.0 Objectives
3.0 Main Content
  3.1 Elements of Decision Situation
  3.2 Types of Decision Situations
    3.2.1 Decision Making Under Condition of Certainty
    3.2.2 Decision Making Under Conditions of Uncertainty
    3.2.3 Decision Making Under Conditions of Risk
    3.2.4 Decision Under Conflict
4.0 Conclusion
5.0 Summary
6.0 Tutor Marked Assignment
7.0 References

1.0 INTRODUCTION

Recall that in the previous unit we presented five decision criteria – Maximax, Maximin, Laplace's, Minimax Regret, and Hurwicz criterion. We also stated that the criteria are used for analysing decision situations under uncertainty. In this unit, we shall delve fully into considering these situations and learn how we can use different techniques in analysing problems in certain decision situations i.e Certainty, Uncertainty, Risk, and Conflict situations.

2.0 OBJECTIVES

After studying this unit, you should be able to

(1) Identify the four conditions under which decisions can be made

(2) Describe each decision situation

(3) Identify the techniques for making decision under each decision situation

(4) Solve problems under each of the decision situation

3.0 MAIN CONTENT

3.1 ELEMENTS OF DECISION SITUATION

Dixon – Ogbechi (2001) presents the following elements of Decision Situation:

1 The Decision Maker: The person or group of persons making the decision.

2 Value System: This is the particular preference structure of the decision maker.

3 Environmental Factors: These are also called states of nature. They can be

i.	Political	v	Cultural factors
ii.	Legal	vi.	Technological factors
iii.	Economic factors	viii	Natural Disasters
iv.	Social factors		

4 Alternative: There are various decision options available to the decision maker.

5 Choice: The decision made.

6 Evaluation Criteria: These are the techniques used to evaluate the situation at hand.

3.2 TYPES OF DECISION SITUATIONS

According to Gupta and Hira (2012), there are four types of environments under which decisions can be made. These differ according to degree of certainty. The degree of certainty may vary from complete certainty to complete uncertainty. The region that lies between corresponds to decision making under risk.

3.2.1 DECISION MAKING UNDER CONDITION OF CERTAINTY

In this environment, only one state of nature exits for each alternative. Under this decision situation, the decision maker has complete and accurate information about future outcomes. In other words, the decision maker knows with certainty the consequence of every alternative course of action. It is easy to analyse the situation and make good decisions. Since the decision maker has perfect knowledge about the future outcomes, he simply chooses the alternative with the optimum payoff. The approach to analysing such decision problem is deterministic. Decision techniques used here include simple arithmetic for simple problem, and for complex decision problems, methods used include cost-volume analysis when information about them is precisely known, linear programming, transportation and assignment models, deterministic inventory models, deterministic quelling models and network model. We shall discuss these models later.

3.2.2 DECISION MAKING UNDER CONDITIONS OF UNCERTAINTY

Here, more than one state of nature exists, but the decision maker lacks sufficient knowledge to allow him assign probabilities to the various state of nature. However, the decision maker knows the states of nature that may possibly occur but does not have information which will enable him to determine which of these states will actually occur. Techniques that can be used to analyse problem under this condition include the Maximax criterion, equally likely or Laplace's criterion, and Hurwicz criterion or Criterion of Realism. These techniques have earlier been discussed. We shall consider a more difficult problem for further illustration.

EXAMPLE 1: Word Problem

A farmer is considering his activity in the next farming season. He has a choice of three crops to select from for the next planting season – Groundnuts, Maize, and Wheat. Whatever is his choice of crop; there are four weather conditions that could prevail: heaving rain, moderate rain, light rain, and no rain. In the event that the farmer plants Ground nuts and there is heavy rain, he expects to earn a proceed of ₦650,000 at the end of the farming season, if there is moderate rain N1,000,000, high rain – ₦450,000 and if there is no rain – (-N1,000)If the farmer plants Maize, the following will be his proceeds after the harvest considering the weather condition: heavy rain – N1,200,000, moderate rain – N1,500,000, Light rain – ₦600,000 and no rain N2000. And if the farmer decides to plant wheat, he expects to make the following: heavy rain – ₦1,150,000, moderate rain – ₦1,300,000, Light rain- ₦800,000 and No rain – ₦200 -000.

The farmer has contact you, an expert in OR to help him decide on what to do.

Question: Construct a payoff matrix for the above situation, analyse completely and advise the farmer on the course of action to adopt. Assume $\alpha = 0.6$.

Solution

First, construct a contingency matrix from the above problem.

Contingency Matrix 1a

Alterative Crops	**Weather conditions**			
	Heavy Rain (S_1) N	**Moderate Rain (S_2) N**	**Light Rain (S_3) N**	**No Rain (S_4) N**
Groundnut(d_1)	750,000	1,000.000	450,000	-1,000

Maize (d_2)	1,200,000	1,500,000	600,000	2000
Wheat (d_3)	1,150,000	1,300,000	800,000	-200,000

Fig. 1a: Pay- off Table

Contingency Matrix 1b

Alterative Crops	**Weather conditions**				**Max col**	**Min Col**
	S_1 (₦'000)	**S_2 $S_3$$S_4$ (₦'000)**	**(₦'000)**	**(₦'000)**		
d_1	750	1,000	450	-1	1,000	-1
d_2	1,200	1,500	600	2	1,500	2
d_3	1,150	1,300	800	-200	1,300	-200

Fig. 1b: Pay- off Table

Regret Matrix 1

Alterative Crops	**Weather conditions**				**Max col**	**Min Col**
	S_1 (₦'000)	**S_2S_3 (₦'000)**	**(₦'000)**	**(S_4) (₦'000)**		
d_1	1200 – 750 450	1500-1000 500	800-450 350	2-(1) 3	500	
d_2	1200 – 1200 0	1500-1500 0	800-600 200	2-2 0	200	200
d_3	1200-1150 50	1500-1300 200	800-800 0	2-(-200) 202	202	
Col max	1200	1500	800	2		

Fig. 2: Regret Matrix1

1. **Maximax Criterion**

Alt.	**Max Col.**
d_1	1,000
d_2	**<u>1,500</u>**

d_3	1,300

Recommendation: Using the maximax criterion, the farmer should select alternative d_2 and plant maize worth ₦1,500,000.

2. **Maximin criterion**

Alt.	Min. Col.
d_1	-1
d_2	**2**
d_3	-200

Recommendation: Using the maximum criterion, the farmer should select alternative d_2 and plant maize worth ₦2,000.

3. **Minimax Regret Criterion**

Choice of crops	Weather conditions				Max Col	Min Col
	S_1 S_2 S_3 (S_4)					
d_1	450	500	350	3	500	
d_2	0	0	200	0	200	200
d_3	50	200	0	202	202	

Fig. 3: Pay- off Table

Recommendation: Using the Mini Max Regret Criterion, the decision maker should select alternative d_2 and plant maize to minimize loss worth ₦200,000

4. **Laplace Criterion**

$$d1 = \frac{750 + 1000 + 450 - 1}{4} = 549.75$$

$$d2 = \frac{1200 + 1500 + 600 + 2}{4} = \underline{\mathbf{825.50}}$$

$$d3 = \frac{1150 + 1300 + 800 - 200}{4} = 762.50$$

Recommendation: Using the Equally Likely or Savage Criterion, the farmer should select alternative d_2 to plant maize worth ₦825,500.

(5) Hurwicz Criterion

$\alpha = 0.6$, $1 - \alpha = 0.4$

$CRi = (\text{max in row}) + (1-\alpha)(\text{min in row})$

$CR_1 = 0.6 (1000) + (0.4) (-1) = 600 + (-0.4) = 599.6$

$CR_2 = 0.6 (1500) + (0.4) (2) = 900 + 0.8 = \underline{\mathbf{900.8}}$

$CR_3 = 0.6 (1300) + (0.4) (-200) = 780 + (-80) = 700$

Recommendation: Using the Hurwicz criterion the farmer should select alternative d2 and cultivate maize worth ₦900,800.00.

3.2.3 DECISION MAKING UNDER CONDITIONS OF RISK

Under the risk situation, the decision maker has sufficient information to allow him assign probabilities to the various states of nature. In other words, although the decision maker does not know with certainty the exact state of nature that will occur, he knows the probability of occurrence of each state of nature. Here also,

more than one state of nature exists. Most Business decisions are made under conditions of risk. The probabilities assigned to each state of nature are obtained from past records or simply from the subjective judgement of the decision maker.

A number of decision criteria are available to the decision maker. These include.

(i) Expected monetary value criterion (EMV)

(ii) Expected Opportunity Loss Criterion (EOL)

(iii) Expected Value of Perfect Information (EVPI)

(Gupta and Hira, 2012)

We shall consider only the first two (EMV and EOL) criteria in details in this course.

i. EXPECTED MONETARY VALUE (EMV) CRITERION

To apply the concept of expected value as a decision making criterion, the decision maker must first estimate the probability of occurrence of each state of nature. Once the estimations have been made, the expected value of each decision alternative can be computed. The expected monetary value is computed by multiplying each outcome (of a decision) by the corresponding probability of its occurrence and then summing the products. The expected value of a random variable is written symbolically as E(x), is computed as follows:

$$E(x) = \sum_{i=1}^{n} x_i P(x_i)$$

(Taylor III, 2007)

EXAMPLE 3.2

A businessman has constructed the payoff matrix below. Using the EMV criterion, analyse the situation and advise the businessman on the kind of property to invest on.

Contingency Matrix 2

Decision to invest	**State of Nature**		
	Good Economic Conditions (N)	Poor Economic Condition (N)	Turbulent Economic Condition (N)
Apartment building (d_1)	50,000	30,000	15,000
Office building (d_2)	100,000	40,000	10,000
Warehouse (d_3)	30,000	10,000	-20,000
Probabilities	0.5	0.3	0.2

Fig. 4: Pay- off Table. Adapted from Taylor, B.W. III (2007) Introduction to Management Science, New Jersey: Pearson Education Inc.

SOLUTION

EVd_1 = 50,000 (0.5) + 30,000 (0.3) + 15,000 (0.2)
= 25,000 + 9,000 + 3,000
= ₦37,000
EVd_2 = 100,000 (0.5) + 40,000 (0.3) + 10,000 (0.2)
=₦50,000 +12,000 + 2000
=₦64,000

EVd_3 = 30,000 (0.5) + 10,000 (0.3) + (-20,000(0.2)
= 15,000 + 3000 - 4000
= ₦14,000

Recommendation: Using the EMV criterion, the businessman should select alternative d_2 and invest in office building worth ₦64,000.

Under this method, the best decision is the one with the greatest expected value. From the above EXAMPLE, the alternative with the greatest expected value is EVd_1 which has a monetary value of ₦37,000. This does not mean that N37,000 will result if the investor purchases apartment buildings, rather, it is assumed that one of the payoffs values will result in ₦25,000 or ₦9,000 or ₦ 3,000. The expected value therefore implies that if this decision situation occurs a large number of times, an average payoff of N37,000 would result, Alternatively, if the payoffs were in terms of costs, the best decision would be the one with the lowest expected value.

ii. EXPECTED OPPORTUNITY LESS (EOL)

The expected opportunity Loss criterion is a regret criterion. It is used mostly in minimization problems. The minimization problem involves the decision maker either trying to minimize loss or minimize costs. It is similar the Minimax Regret Criterion earlier discussed. The difference however, is that is has probabilities attached to each state of nature or occurrence.

The difference in computation between the EMV and EOL methods is that, unlike the EMV methods, a regret matrix has to be constructed from the original matrix before the EOL can be determined.

EXAMPLE3.3

We shall determine the best alternative EOL using contingency matrix 2 above

First, we construct a regret matrix from contingency matrix 2 above. Remember how the Regret matrix table is constructed? Ok. Let us do that again here.

Quick Reminder

To construct a regret matrix, determine the highest value in each state of nature and subtract every payoff in the same state of nature from it. Your will observe that

most of the payoff will become negative values and zero.

Decision to invest	Regret Matrix 2 State of Nature		
	(N)	(N)	(N)
Apartment building (d_1)	(100,000 - 50,000) 50,000	(40,000 - 30,000) 10,000	(15,000 – 15,000) 0
Office building (d_2)	(100,000 -100,000) 0	(40,000 – 40,000) 0	(15,000 -10,000) 5,000
Warehouse (d_3)	(100,000 – 30,000) 70,000	(40,000 – 40,000) 30,000	(15,000+20,000) 35,00

Probabilities	0.5	0.3	0.2

Fig. 3.3: Regret Matrix 2

$EOLd_1$ = 50,000 (0.5) + 10,000 (0.3) + 0(2)
= 25,000 + 3,000+0
= ₦28,000

$EOLd_2$ = 0.(0.5) + 0(0.3) + 5,000 (0.2)
= 0 + 0 + 1,000
= ₦1,000

$EOLd_3$ = 70,000 (0.5) + 30,000 (0.3) + 35,000 (0.2)
= 35,000 + 9,000 + 7,000
= ₦51,000

Recommendation: Using the EOL criterion, the decision maker should select alternative d_2 and invest in office building worth ₦1,000.

The Optimum investment option is the one which minimizes expected opportunity losses, the action calls for investment in office building at which point the minimum expected loss will be ₦1,000.

You will notice that the decision rule under this criterion is the same with that of the Minimax Regret criterion. This is because both methods have the same objectives that is, the minimization of loss. They are both pessimistic in nature. However, loss minimization is not the only form minimization problem. Minimisation problems could also be in the form of minimisation of cost of production or investment. In analyzing a problem involving the cost of production you do not have to construct a regret matrix because the pay-off in the table already represents cost.

NOTE: It should be pointed out that EMV and EOL decision criteria are completely consistent and yield the same optimal decision alternative.

iii EXPECTED VALUE OF PERFECT INFORMATION

Taylor III (2007) is of the view that it is often possible to purchase additional information regarding future events and thus make better decisions. For instance, a farmer could hire a weather forecaster to analyse the weather conditions more accurately to determine which weather condition will prevail during the next farming season. However, it would not be wise for the farmer to pay more for this information than he stands to gain in extra yield from having this information. That is, the information has some maximum yield value that represents the limit of what the decision maker would be willing to spend. This value of information can be computed as an expected value – hence its name, expected value of perfect information (EVPI).

The expected value of perfect information therefore is the maximum amount a decision maker would pay for additional information. In the view of Adebayo et al (2007), the value of perfect information is the amount by which the profit will be increased with additional information. It is the difference between expected value of optimum quantity under risk and the expected value under certainty. Using the EOL criterion, the value of expected loss will be the value of the perfect information.

Expected value of perfect information can be computed as follows

$EVPI = EVwPI – EMV_{max}$

Where

EVPI = Expected value of perfect information

EVwPI = Expected value with perfect information

EMV_{max} = Maximum expected monetary valueor Expected value without perfect information

(Or minimum EOL for a minimization problem)

EXAMPLE3.4

Using the data on payoff matrix 3 above,

Decision to invest	**State of Nature**		
	Good (N)	**Poor** (N)	**Turbulent** (N)
Apartment building (d_1)	50,000	30,000	15,000
Office building (d_2)	100,000	40,000	10,000
Warehouse (d_3)	30,000	10,000	-20,000
Probabilities	0.5	0.3	0.2

Fig. 3.3: Pay-off Tale

EVwPI = $\sum P_j$ x best out on each state of nature (S_j).

The expected value with perfect information can be obtained by multiplying the best outcome in each state of nature by the corresponding probabilities and summing the results.

We can obtain the EVwPI from the table above as follows.

EVwP1 = 100,000 x 0.5 + 40,000 x 0.3 + 15,000 x 0.2

= 50,000 + 12,000 + 3,000

= N**65,000**

Recall that our optimum strategy as calculated earlier was ₦64,000.

EVP1 = EVwP1 – EMV_{max}

= ₦65000 - 64,000

= **₦1,000**

The expected value of perfect information (EVPI) is N1000. This implies that the maximum amount the investor can pay for extra information is N1000. Because it is difficult to obtain perfect information, and most times unobtainable, the decision maker would be willing to pay some amount less than N1000 depending on how accurate the decision maker believes the information is. Notice that the expected value of perfect information (N1000) equals our expected opportunity loss (EOL) of N1000 as calculated earlier.

Taylor III (2007) provides a justification for this. According to him, this will always be the case, and logically so, because regret reflects the difference between the best decision under a state of nature and the decision actually made. This is the same thing determined by the expected value of perfect information.

3.2.4 DECISION UNDER CONFLICT

Decision taken under conflict is a competitive decision situation. This environment occurs when two or more people are engaged in a competition in which the action taken by one person is dependent on the action taken by others in the competition.
In a typical competitive situation the player in the competition evolve strategies to outwit one another. This could by way intense advertising and other promotional efforts, location of business, new product development, market research, recruitment of experienced executives and so on. An appropriate techniques to use in solving problems involving conflicts is the Game Theory (Adebayo et al 2007).

Practice Exercise

(1) Identify and discuss the situations under which decision are made.

(2) An investor is confronted with a decision problem as represented in the matrix below. Analyse the problem using the EMV and EOL criteria and advise the decision maker on the best strategy to adopt.

State of Nature	**Alternatives**			
	Expand	**Construct**	**Subcontract**	**Prob.**
High (₦)	50,000	70,000	30,000	0.5
Moderate (₦)	25,000	30,000	15,000	03
Low (₦)	25,000	-40,000	-1,000	0.15
Nil (₦)	-45,000	-80,000	-10,000	0.05

Hint: Note that the positions of the states of nature and the alternative strategies have changed.

4.0 CONCLUSION

Business Organisations are confronted with different situations under which they make decisions. There are different ways to approach a situation; the technique for analysing a particular decision problem depends upon the prevailing situation under which problem presents itself. It is important for decision makers to always identify the situations they are faced with and fashion out the best technique for analysing the situation in order to arrive at the best possible alternative course of action to adopt.

5.0 SUMMARY

In this unit, we have discussed the different situations under which a decision maker is faced with decision problems. These decision situations include Certainty, Uncertainty, Risk and Conflict situation. Decision situations could also be referred to as decision environments. We have also identified and discussed various techniques used in solving problems under these situations. The deterministic approach to decision analysis which includes simple arithmetic techniques for simple problems and cost-volume analysis, linear programming, transportation model, assignment models quenching modes etc. for complex problems could be used to solve problems under situation of certainty. Techniques that can be used to solve problem under uncertainty include: Maximax criterion, Minimax criterion Minimax Regret criterion, Equally-Likely or Laplace criterion, and Hurwicz criterion. Decisions under Risk Situations can be analysed using the Expected Monetary Value (EMV) or Expected Opportunity Loss (EOL) criterion. Finally, Game theory can be used to analyse decision under conflict with is a situation that involve competition.

6.0 TUTOR MARKED ASSIGNMENT

(1) Who is a decision maker?

(2) Identify and explain the environmental factors or states of Nature that affect a decision situation.

(3) List and explain four situations under which decisions can be made.

(4) Identify the techniques that can be used to analyse decision problems under the following situations

- (i) Certainty
- (ii) Uncertainty
- (iii) Risk
- (iv) Conflict

(5) Consider the contingency matrix below

Alternatives	States of Nature		
	S1 (N)	**S_2**(N)	**S_3**(N)
A_1	100,000	75,000	1000

A_2	625,000	12,000	920
A_3	11,900	750	-73

Analyse the situation completely and advise the decision maker on the optimal strategy to adopt under each criterion.

(6) Using the table in question (5) above, assume that strategy

S_1 = Strong Economic Growth, with a probability of 0.45

S_2= Weak Economic Growth, with a probability of 0.3

S_3 = Economic Recession, with a probability of 0.25

Assuming also, that the decision maker has the opportunity of purchasing extra information which will help him take perfect decisions,

A_1 = Build new manufacturing plant

A_2 = Increase present plant size

A_3= Employ more professionals to run present plant

(i) What is the optimal investment strategy using the EMV technique?

(ii) Using the EOL techniques to analyse the situation and advise the investor on the course of action to adopt.

(iii) What is the maximum amount that the investor can pay for additional information?

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UNIT 4: DECISION TREES

1.0 Introduction
2.0 Objectives
3.0 Main Content
  3.1 Definition
  3.2 Benefits of using decision tree
  3.3 Disadvantage of the decision tree
  3.4 Components of the decision tree
  3.5 Structure of a decision tree
  3.6 How to analyse a decision tree
  3.7 The Secretary Problem
    3.7.1 Advantages of The Secretary Problem Over the General decision tree
    3.7.2 Analysis of the Secretary Problem
4.0 Conclusion
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1.0 INTRODUCTION

So far, we have been discussing the techniques used for decision analysis. We have demonstrated how to solve decision problems by presenting them in a tabular form. However, if decision problems can be presented on a table, we can also represent the problem graphically in what is known as a decision tree. Also the decision problems discussed so far dealt with only single stage decision problem.

That is, the payoffs, alternatives, state of nature and the associated probabilities were not subject to change. We now consider situations that involve multiple stages. They are characterized by a sequence of decisions with each decision influencing the next. Such problems, called sequential decision problems, are analysed best with the help of decisions trees.

2.0 OBJECTIVES

After studying this unit, you should be able to

1. Describe a decision tree
2. Describe what Decision nodes and outcome nodes are
3. Represent problems in a decision trees and perform the fold back and tracing forward analysis
4. Calculate the outcome values using the backward pass
5. Identify the optimal decision strategy

3.0 MAIN CONTENT

3.1 DEFINITION

A decision tree is a graphical representation of the decision process indicating decision alternatives, states of nature, probabilities attached to the states of nature and conditional benefits and losses (Gupta & Hira 2012). A decision tree is a pictorial method of showing a sequence of inter-related decisions and outcomes. All the possible choices are shown on the tree as branches and the possible outcomes as subsidiary branches. In summary, a decision tree shows: the decision points, the outcomes (usually dependent on probabilities and the outcomes values) (Lucey, 2001).

The decision tree is the simplest decision making model in the face of an uncertain future. In such a model, a plan of action must account for all contingencies (Chance outcome) that can arise. A decision tree represents the uncertainty of choice graphically. This makes it easy to visualize the contingency plans which are called strategies (Denardo, 2002).

3.2 BENEFITS OF USING DECISION TREE

Dixon-Ogbechi (2001) presents the following advantages of using the decision tree

- They assist in the clarification of complex decisions making situations that involve risk.
- Decision trees help in the quantification of situations.
- Better basis for rational decision making are provided by decision trees.
- They simplify the decision making process.

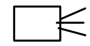
3.3 DISADVANTAGE OF THE DECISION TREE

- The disadvantage of the decision tree is that it becomes time consuming, cumbersome and difficult to use/draw when decision options/states of nature are many.

3.4 COMPONENTS OF THE DECISION TREE

It is important to note the following components of the structure of a decision problem

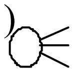
- **The choice or Decision Node**: Basically, decision trees begin with choice or decision nodes. The decision nodes are depicted by square (□). It is a point in the decision tree were decisions would have to be made. Decision nodes



are immediately by alternative courses of action in what can be referred to as the decision fork. The decision fork is depicted by a square with arrows or lines emanating from the right side of the square ( ).The number of lines emanating from the box depend on the number of alternatives available.

- **Change Node**: The chance node can also be referred to as state of nature node or event node. Each node describes a situation in which an element of uncertainty is resolved. Each way in this uncertainty can be resolved is represented by an arc that leads rightward from its chance node, either to another node or to an end-point. The probability on each such arc is a conditional probability, the condition being that one is at the chance node to its left. These conditional probabilities sum to 1 (0ne), as they do in probability tree (Denardo, 2002).

The state of nature or chance nodes are depicted by circles ( ), it implies that at this point, the decision maker will have to compute the expected monetary value (EMV) of each state of nature. Again the chance event node is depicted this ( )



3.5 STRUCTURE OF A DECISION TREE

The structure and the typical components of a decision tree are shown in the diagram below.

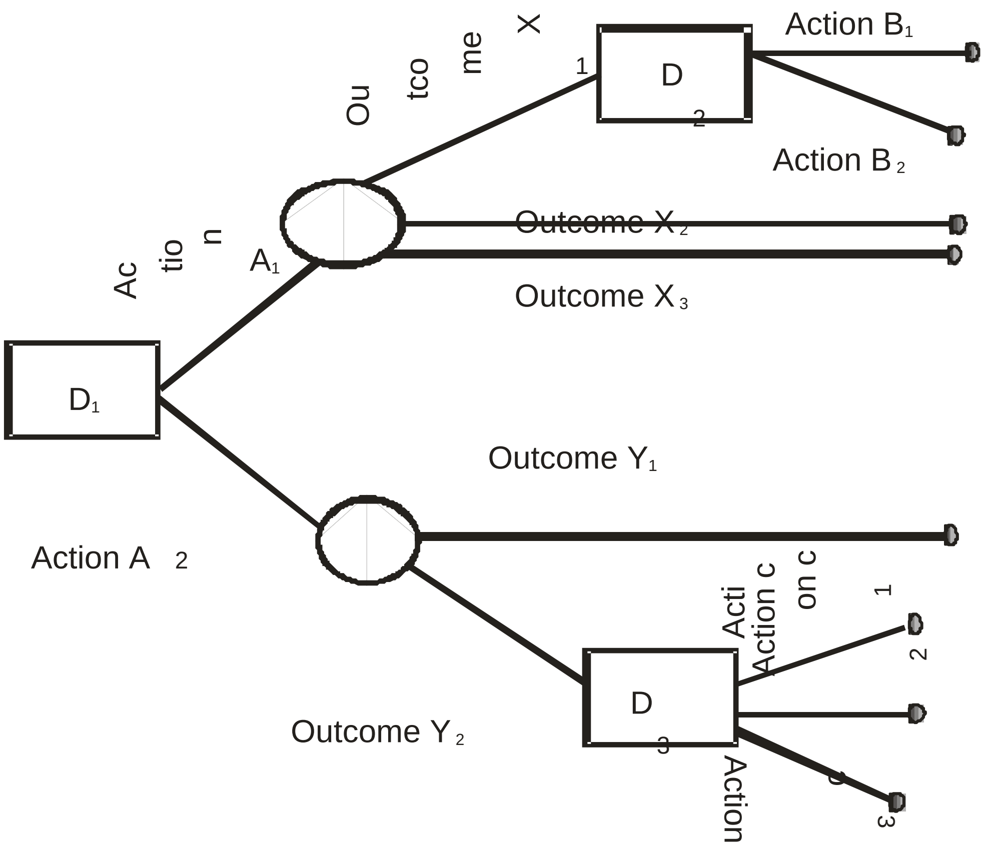


Fig.13: Adapted from Lucey, T (2001), Quantitative Techniques, 5th London: Continuum

The above is a typical construction of a decision tree. The decision tree begins with a decision node D_1 signifying that the decision maker is first of all presented with a decision to make. Immediately after the decision node, there are two courses of Action A_1 and A_2.If the decision maker chooses A1, there are three possible outcomes – X_1 X_2, X_3. And if chooses A_2, there will be two possible outcomes Y_1 and Y_2 and so on.

3.6 HOW TO ANALYSE A DECISION TREE

The decision tree is a graphical representation of a decision problem. It is multi-
state in nature. As a result, a sequence of decisions are made repeatedly over a period of time and such decisions depend on previous decisions and may lead to a set of probabilistic outcomes. The decision tree analysis process is a form of probabilistic dynamic programming (Dixon-Ogbechi, 2001).

Analysing a decision tree involves two states

i. **Backward Pass**: This involves the following steps

- starting from the right hand side of the decision tree, identify the nearest terminal. If it is a chance event, calculate the EMV (Expected Monetary Value). And it is a decision node, select the alternative that satisfies your objective.
- Repeat the same operation in each of the terminals until you get to the end of the left hand side of the decision tree.

ii. **Forward Pass**: The forward pass analysis involves the following operation.

- Start from the beginning of the tree at the right hand side, at each point, select the alternative with the largest value in the case of a minimization problem or profit payoff, and the least payoff in the case of a minimization problem or cost payoff.
- Trace forward the optimal contingency strategy by drawing another tree only with the desired strategy.

These steps are illustrate below

EXAMPLE 4.1

Contingency Matrix 1

States of Nature	Alternatives		Probability
	Stock Rice (A_1)	Stock Maize (A_2)	
High demand (S_1) (N)	8,000	12,000	0.6
Low demand (S_2) (N)	4,000	-3,000	0.4

Fig. 4.1: Pay-off Matrix

Question: Represent the above payoff matrix on a decision tree and find the optimum contingency strategy.

We can represent the above problem on a decision tree thus:

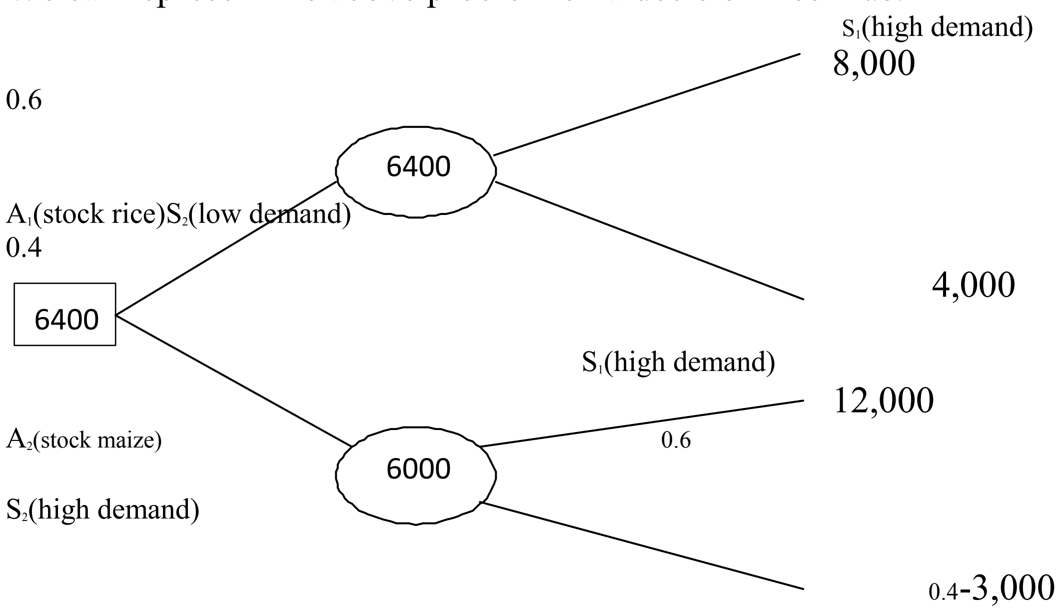


Fig. 4.2: A Decision Tree.

Next, we compute the EMY for alternatives A_1 and A_2.

EMV_{A1} = 8,000 x 0.6 + 4,000 x 0.4 = 6400
= 4800 x 1,600

EMV_{A2}= 12,000 x 0.6 + (-3,000) x 0.4
= 7,200 – 1,200 = N6,000

EMV_{A1} gives the highest payoff

We can now draw our optimal contingency strategy thus:

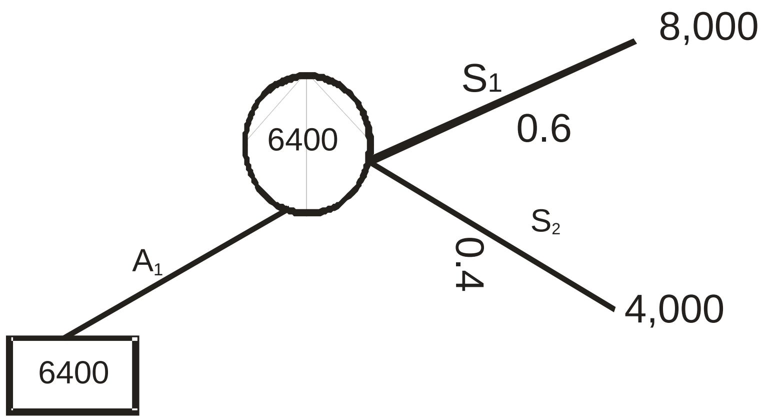


Fig. 4.3: Optimal Contingency Strategy

The above decision tree problem is in its simplest form. They also could be word problem to be represented on a decision tree diagram unlike the above problem that has already been put in tabular form. Let us try one of such problems.

EXAMPLE 4.2

A client has contracted NOUNCIL, a real estate firm to help him sell three properties A,B,C that he owns in Banana Island. The client has agreed to pay NOUNCIL 5% commission on each sale. The agent has specified the following conditions: NOUNCIL must sell property A first, and this he must do within 60days. If and when A is sold, NOUNCIL receives 5% commission on the sale, NOUNCIL can then decide to back out on further sale or go ahead and try to sell the remaining two property B and C within 60 days. If they do not succeed in selling the property within 60days, the contract is terminated at this stage. The following table summarises the prices, selling Costs (incurred by NOUNCIL whenever a sale is made) and the probabilities of making sales

Property	Prices of property	Selling Cost	Probability
A	12,000	400	0.7
B	25,000	225	0.6
C	50,000	450	0.5

Fig. 4.3: Pay-off Matrix

(i) Draw an appropriate decision tree representing the problem for NOUNCIL.

(ii) What is NOUNCIL's best strategy under the EMV approach?

(Question Adapted from Gupta and Hira (2012))

SOLUTION

Hint: Note that the probabilities provided in the table are probabilities of sale. Therefore, to get the probability of no sale, we subtract the prob. Of sale from 1

Prob. of no Sales = 1 – prob. of sales

NOUNCIL gets 5% Commission if they sell the properties and satisfy the specified conditions.

The amount they will receive as commission on sale of property A,B, and C are as follows

Commission on A = 5/100 x 12,000 = ₦6000

Commission on B = 5/100 x 25,000 = ₦1250

Commission on C = 5/100 x 50,000 = N2500

The commission calculated above are conditional profits to NOUNCIL. To obtain the actual profit accrued to NOUNCIL from the sale of the properties, we subtract the selling cost given in the table above from the commission.

NOUNCIL’S Actual profit

A = ₦600 – ₦400 = ₦200

B = ₦1250 - ₦225 = ₦1025

C = ₦2500 – ₦450 = ₦2050

We now construct our decision tree

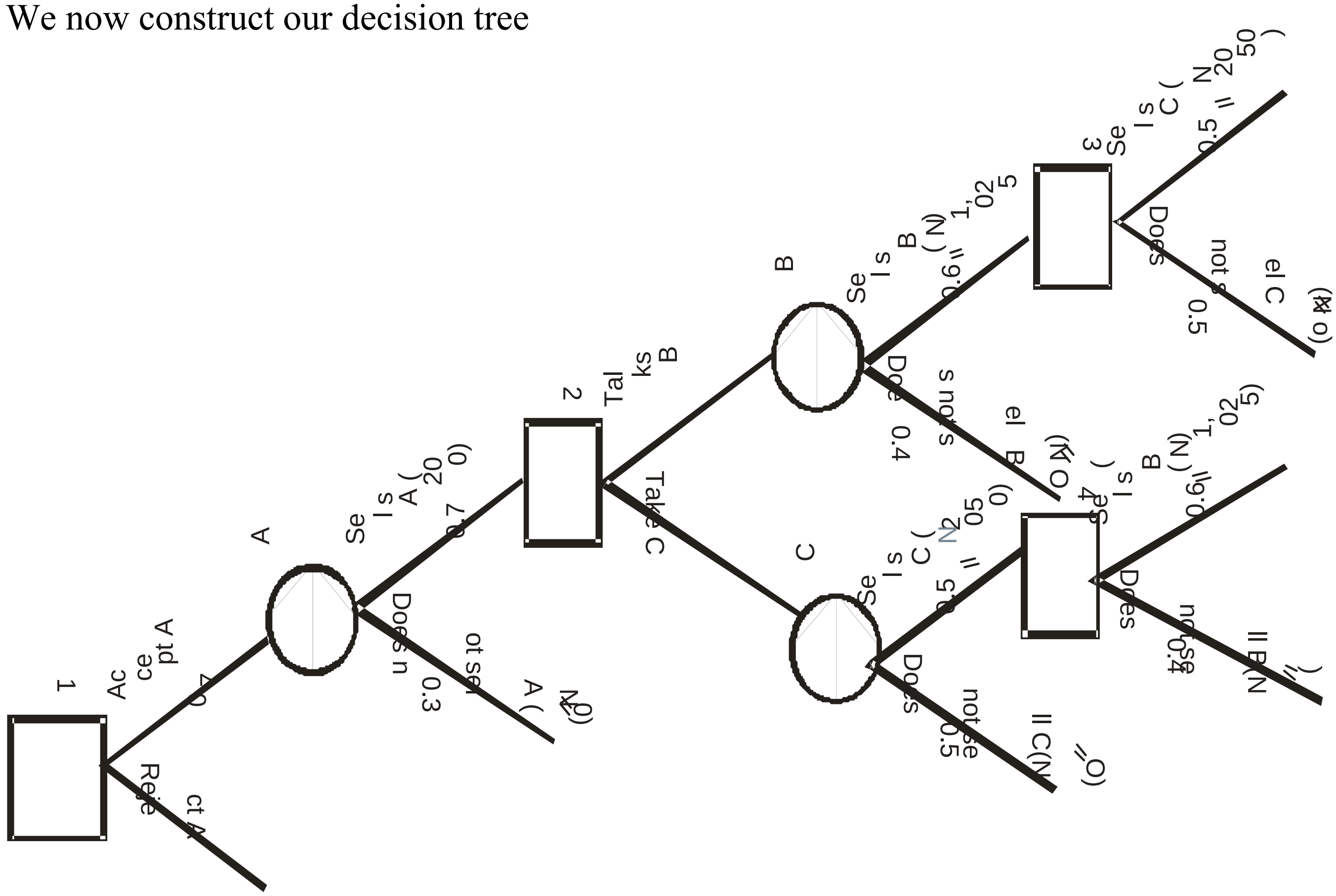


Fig. 4.4: A Decision Tree

BACKWARD PASS ANALYSIS

EMV of Node 3 = ₦ (0.5 2050 + 0.5 x0) = ₦1025

EMV of Node 4 = ₦ (0.6 x 1,025 + 0.4 x 0) =₦615

EMV of Node B = [0.6 (1025 + 1025) + 0.4 x 0] = 1230

Note: 0.6 (EMV of node 3 + profit from sales of B at node B)

EMV of Node C = [0.5 (2050 + 615) + 0.5 x 0] = ₦1332.50

Note: same as EMV of B above

EMV of Node 2 =₦1332.50 (Higher EMV at B and C)

EMV of Node A = ₦[0.7 (200 + 1,332.50) + 0.3 x 0] = ₦1072.75

EMV of Node 1 =₦1072.75

Optimal contingency strategy

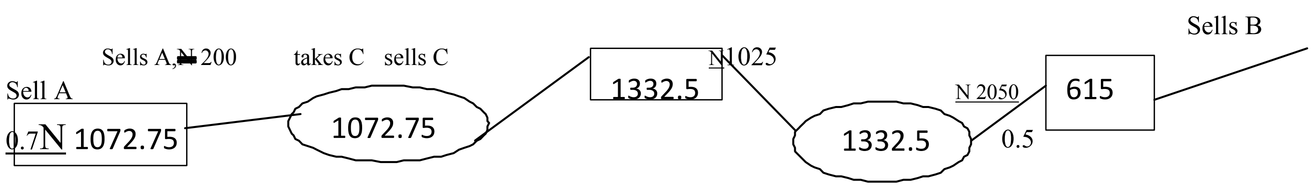


Fig. 4.4b: Optimal Contingency Strategy

The optimal contingency strategy path is revealed above. Thus the optimum strategy for NOUNCIL is to sell A, if they sell A, then try sell C and if they sell C, then try sell B to get an optimum expected amount of ₦1072.75.

Let us try another example as adapted from Dixon – Ogbechi (2001).

EXAMPLE 4.3

The management of the school of Basic Remedial and Vocational Studies of NOUN is contemplating investing in two diploma programmes- Diploma in Business and Diploma in Law. Dip Bus will cost ₦9millionwith 0.6 chance of success. Dip Law will cost ₦4.5million, but has only 0.35 chance of success. In the event of success, management has to decide whether or not to advertise the product heavily or lightly. Heavy advertisement will cost N3,600,000 but has 0.65 probability of full acceptance as against partial acceptance by the market while light advertising will cost N1,200,000 and has a probability of 0.45 of full acceptance. Full acceptance in Dip Bus Programme would be worth N36million while that of DipLaw programme would be worth 27 million. Partial market acceptance will worth ₦21 million and ₦24 million respectively.

Advise the management of the school of Remedial and Vocational Studies on the diploma programme and level of advertising to embark on.

SOLUTION

We begin by drawing the decision tree.

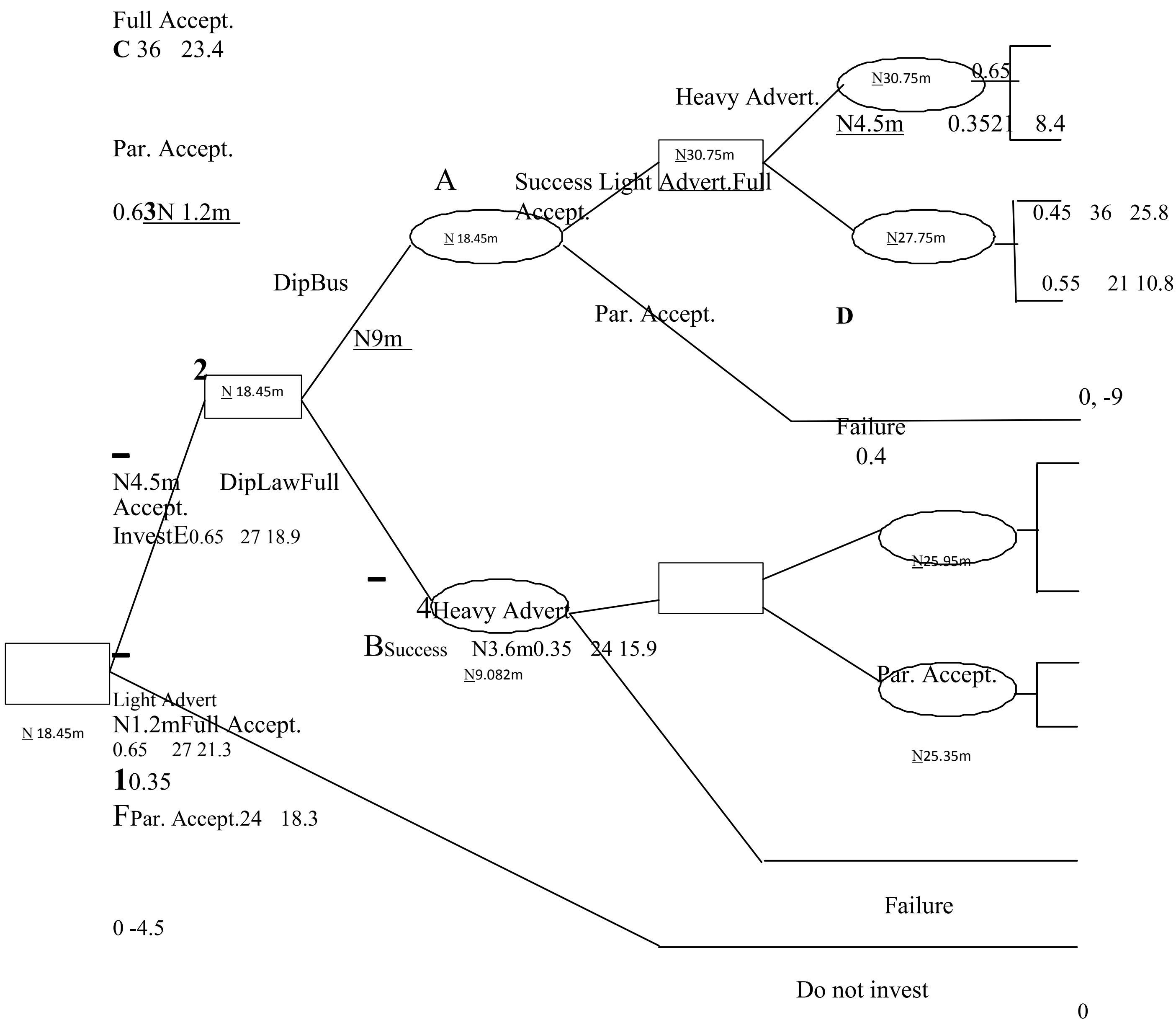


Fig. 4.5: A Decision Tree

BACKWARD PASS ANALYSIS

EMV of Node C = N36,000,000 x 0.65 + 21,000,000 x N0.36

= N23,400,000 + 7,350,000

= N30750,000

EMV of Node D = ₦36,000,000 x 0.45 + ₦21,000,000 x 0.55
= ₦16,200,000 + ₦11,500,000
= 27,750,000

EMV of Node E = ₦27,000,000 x 0.65+ ₦24,000,000 x 0.35
= ₦17,550,000 + ₦8,400,000
= ₦25,950,000

EMV of Node F = ₦27,000 x 0.45 + 24,000,000 x 0.55
= ₦12,150,000 + 13,200,000
= ₦25,350,000

EMV of Node A = ₦30,750,000 x 0.6 + ₦0 + 0.4
= ₦18,450,000 + ₦0
= ₦18,450,000

EMV of Node B = ₦25,950,000 x 0.35 + ₦0 x 0.65
= ₦9082500 + 0
= ₦9,082,500

Note: Decision made at each of the decision nodes were arrived at by
comparing the values in each chance event nodes and selecting the highest value.

Optimal Contingency Strategy

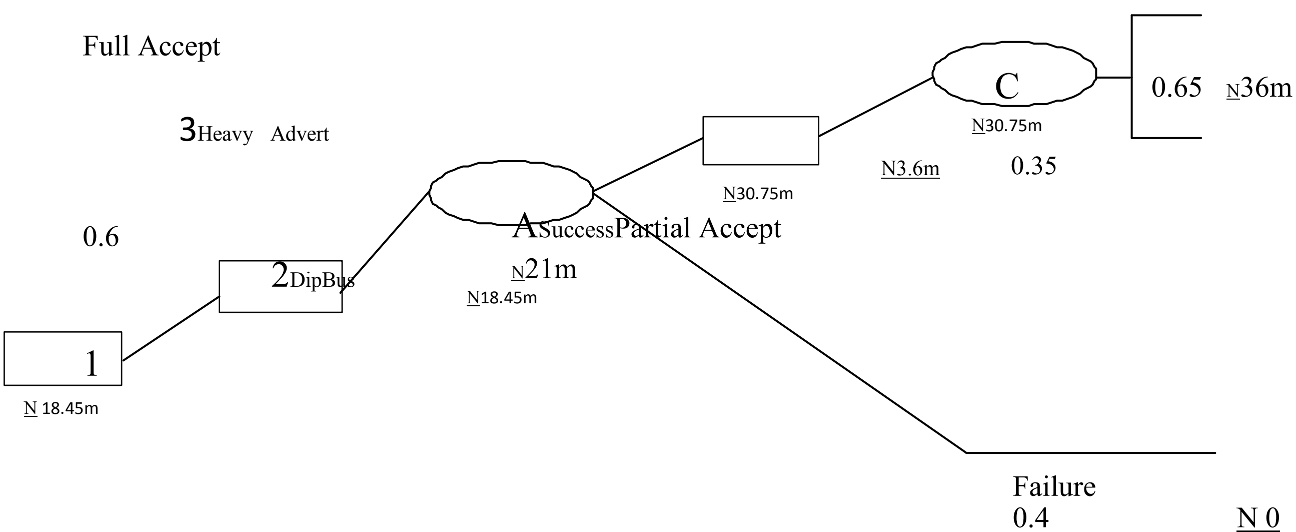


Fig. 4.6: The Optimum Contingency Strategy

Recommendation: Using the EMV method, the management of school of Remedial and vocational studies should invest in DipBus and embark on heavy advertisement to get on optimum expected amount of ₦18,450,000.

3.7 THE SECRETARY PROBLEM

The secretary problem was developed to analyse decision problems that are complex and repetitive in nature. This type of decision tree is a modification upon general decision tree in that it collapses the branches of the general tree and once an option is jettisoned, it cannot be recalled.

3.7.1 Advantages of the Secretary Problem Over the General Decision Tree

In addition to the advantages of the general decision try the secretary problem has the following added advantages

(1) It is easy to draw and analyse.

(2) It saves time.

3.7.2 Analysis of the Secretary Problem

The analysis of a secretary decision tree problem is similar to that of the general decision tree. The only difference is that since the multi stage decision problem could be cumbersome to formulate when the branches become too many, the secretary problem collapses the different states of nature into one. This will be demonstrated in the example below.

EXAMPLE

The management of Bureau for public Enterprises (BPE) has invited bids for the Distribution arm of the power holding company of Nigeria (PHCN) PLC. Three categories of bids are expected – high, fair, and low.

High bid is worth N100m, a fair bid is worth N60m and a low bid is worth N30M.
The probabilities of the first prospective bidder are 0.5; 0.3; 0.2 for high; fair; and low respectively. Those of the second prospective bidder are 0.5; 0.2; and 0.3 respectively, while those of the third bidder are 0.4; 0.2; 0.4 respectively.

Question

(i) Formulate the problem as:

(a) A general decision tree

(b) A secretary problem

(ii) Analyse the situation completely

(iii) What is the optimal contingency strategy?

(Dixon – Ogechi, 2001)

Solution

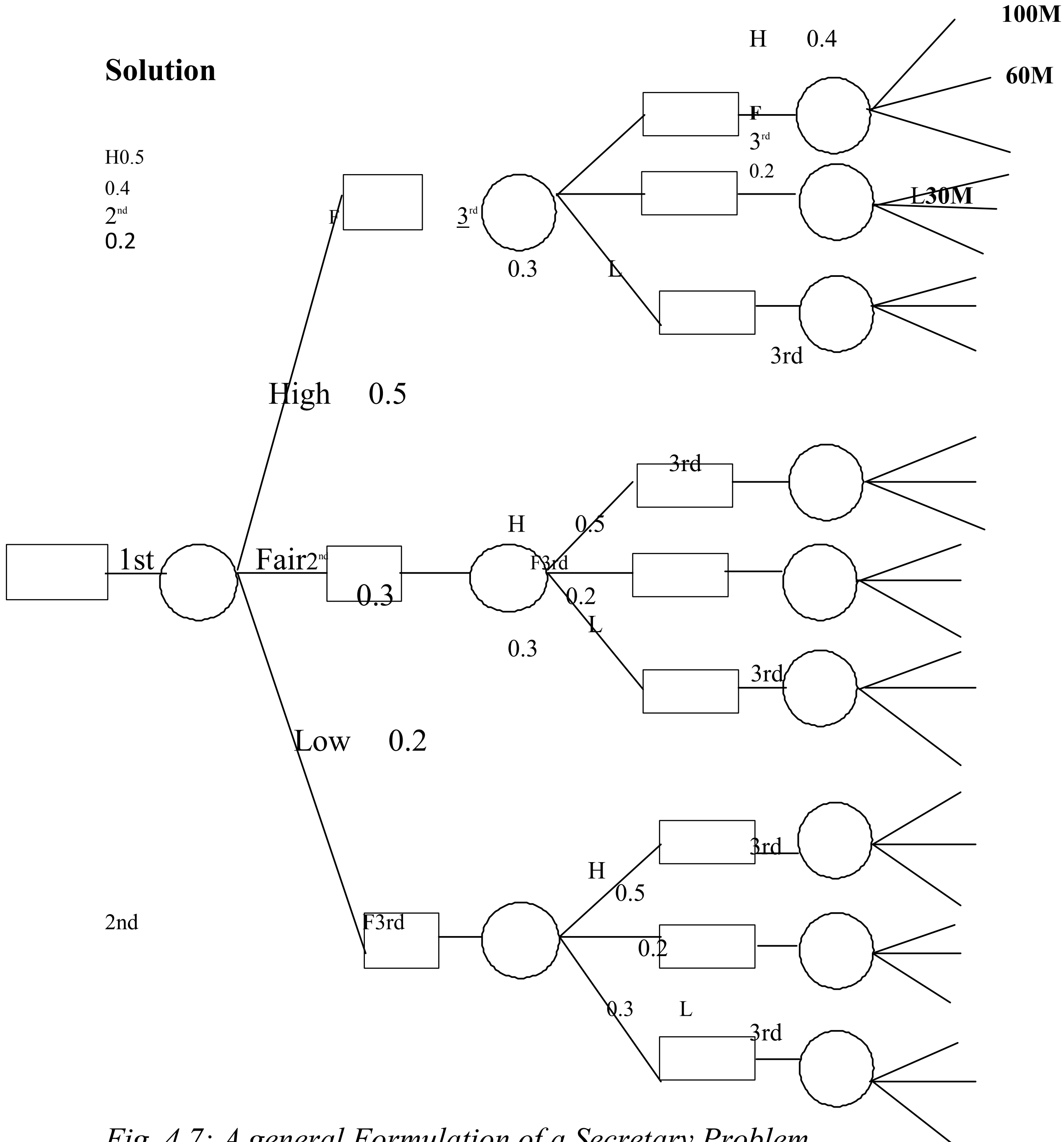


Fig. 4.7: A general Formulation of a Secretary Problem

The bids and their corresponding probabilities, and worth are to be repeated throughout the 3rd chance event fork like has been done in the first. You can try that in your note and see how it would look like

You can see how cumbersome the general decision tree formulation of the above problem is. It is very time consuming to formulate because it has too many branches. As a result, the secretary formulation was developed to help analyse decision problems of this nature without going through the process as indicated above.

Now let us see the secretary formulation of the same problem.

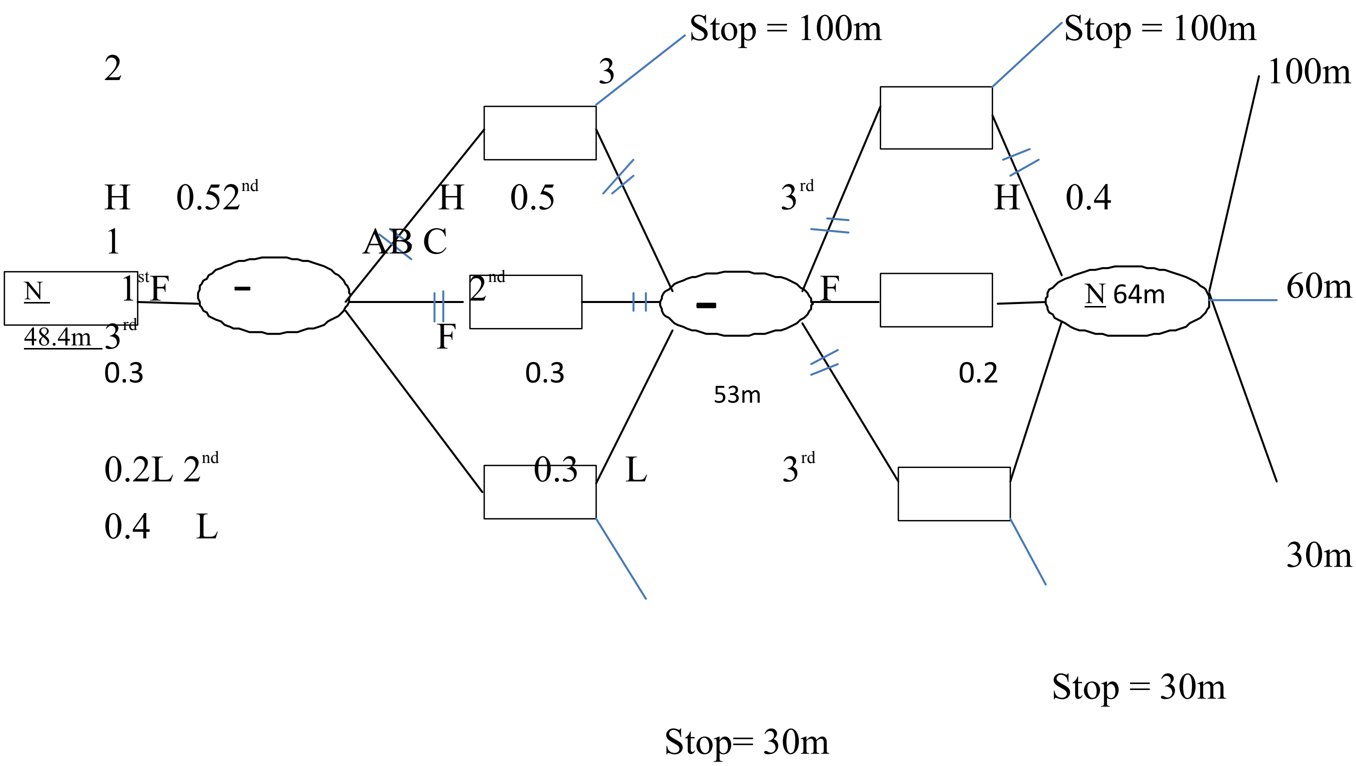


Fig. 4.8: Formulation of a Secretary Problem

$EMV_C = 100 \times 0.4 + 60 \times 0.2 + 30 \times 0.4$

$40 + 12 + 12$

$= N64m$

$EmV_B = 64 \times 0.5 + 60 \times 0.2 + 30 \times 0.3$

$= 32 + 12 + 9$

$= N53M$

$EMV_B = 64 \times 0.5 + 60 \times 0.2 + 30 \times 0.3$
$= 32 + 12 + 9$
$= ₦53M$

$EMV_A = 53 \times 0.5 + 53 \times 0.3 + 30 \times 0.2$
$= 26.5 + 15.9 + 6$
$= ₦48.4M$

Optimal contingency Strategy

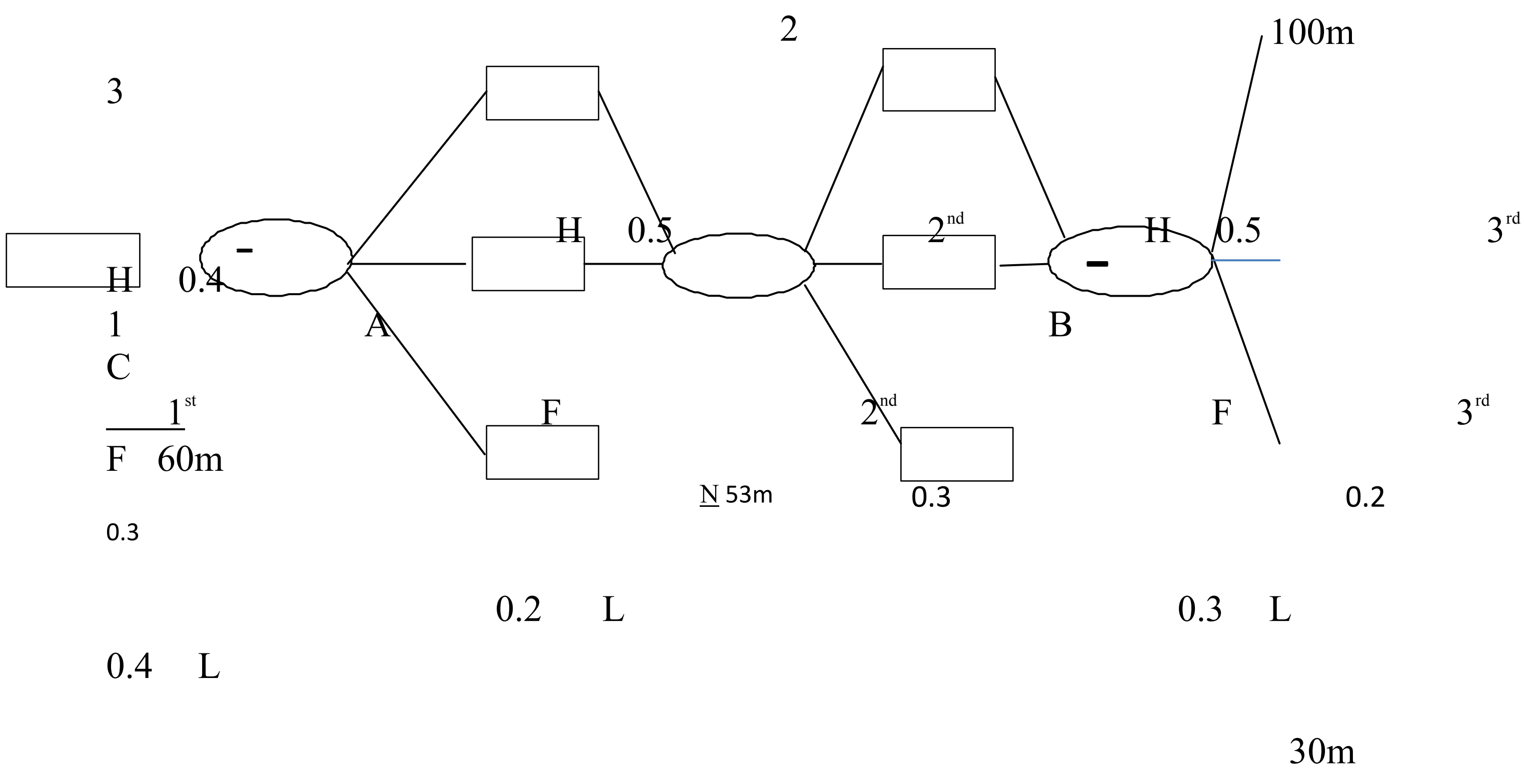


Fig. 4.9: Optimal Contingency Strategy

We can see that the major difference between the secretary formulation and the general decision tree formulation is that at decision nodes, instead of the tree of proceed to different chance event nodes and develop different branches, the branches are collapsed into one from the three decision nodes to form one change event node.

4.0 CONCLUSION

Decision trees provide a graphical method of presenting decision problems. The problems are represented in a form of a tree diagram with the probabilities and payoffs properly labelled for easier understanding, interpretation, and analysis. Once a decision problem can be represented in tabular form, it can also be presented in form of a decision tree.

However, the general decision tree could become complex and cumbersome to understand and analysed if the nature of the problem is also complex and involves a large number of options. The secretary formulation method of the general decision tree was developed as an improvement upon the general formulation to be used for analysing complex and cumbersome decision problems. Generally, the decision tree provides a simple and straight forward way of analysing decision problems.

5.0 SUMMARY

Now let us cast our minds back to what we have learnt so far in this unit. We learnt that the decision tree is mostly used for analysing a multi-stage decision problem. That is, when there is a sequence of decisions to be made with each decision having influence on the next. A decision tree is a pictorial method of showing a sequence of inter-related decisions and outcomes. It is a graphical representation that outlines the different states of nature, alternatives courses of actions with their corresponding probabilities. The branches of a decision tree are made up of the decision nodes at which point a decision is to be made, and the chance node at which point the EMV is to be computed.

The decision tree assist the clarification of a complex decision problem, it helps in the quantification of a decision situation, and to simplify the decision making process. On the other hand, the decision tree could become time consuming, cumbersome and difficult to use or draw when the options and states of nature are too many.

In order to tackle this problem of the decision tree becoming too cumbersome, the secretary formulation of the decision tree was developed. The secretary formulation of the decision tree collapses the different states of nature into one in situation where the states of nature are repetitive in nature.

Whichever formulation of the tree diagram, be it the general or the secretary formulation, the decision tree has two methods of analysis – the backward pass and the forward pass.

6.0 TUTOR MARKED ASSIGNMENT

1 What do you understand by the term decision tree?

2 Identify the two formulations of the decision tree and give the difference between them.

3 Outline the advantages and disadvantage of a decision tree.

4 Write short notes on the following:

   i Decision Node

   ii Chance Even Node

5 Identify and discuss the two method of analysis of a decision tree

6 Consider the matrix below

Decision to purchase	States of Nature	
	Good Economic condition	Poor Economic Condition
Apartment Building	₦50,000	₦30,000
Office Building	100,000	- 40,000
Warehouse	30,000	10,000
Probability	**0.60**	**0.40**

Adapted from: Taylor III (2007, P. 490)

Formulate the above decision problem as a decision tree, obtain the optimal contingency strategy, and advice the decision maker on the strategy to adopt.

7 Ajaokuta steel company is confronted with the choice of selecting between three operational options: (i) Produce commercially (ii) Build pilot plant, and (iii) Stop operating the steel plant. The management has estimated the following probabilities- if the pilot plant is built, it has a 0.8 chance of high yield and 0.2 chance of low yield. If the pilot plant does show a high yield, management estimates a probability of 0.75 that the commercial plant will have a high yield. If the pilot plant shows a low yield, there is only a 0.1 chance that the commercial plant will show a high yield. Finally, management's best assessment of the yield on a commercial size plant without building a pilot plant first has a 0.6 probability of high yield. A pilot plant will cost N3,000,000. The profits earned under high and low yields conditions are ₦12,000,000 and – ₦1,200,000 respectively. Find the optimum decision for the company. (Cupta & Hira 2012: 777)

Hint: The above problem is relatively simple. Start with a decision node ▭ Draw three branches from the decision representing the three alternative decisions. From the branch representing commercial production, draw a chance event node and put the corresponding probabilities and profits and stop.

From the branch representing Pilot plant, draw a chance event fork with two branches representing high and low yields with their probabilities. From each of the chance event branches, construct another decision fork representing commercial production on one branch and stop at the other.

Finally, construct a chance event fork in each of the commercial production branches with the probabilities well represented, then put the profits for both high and low yields.

Form the third branch emanating from the first decision fork, label it stop operation. Do not continue construction because it has no yields.

7.0 REFERENCES

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MODULE THREE

UNIT 1: MATHEMATICAL PROGRAMMING (LINEAR PROGRAMMING)

1.0 Introduction
2.0 Objectives
3.0 Main Content
  3.1 Requirements for Linear Programming Problems
  3.2 Assumptions in Linear Programming
  3.3 Application of Linear Programming
  3.4 Areas of Application of Linear Programming
  3.5 Formulation of Linear Programming Problems
  3.6 Advantages Linear Programming Methods
  3.7 Limitation of Linear programming Models
  3.8 Graphical Methods of Linear Programming Solution
4.0 Conclusion
5.0 Summary
6.0 Tutor Marked Assignment
7.0 References

1.0 INTRODUCTION

Linear programming deals with the optimization (maximization or minimization) of a function of variables known as objective function, subject to a set of linear equations and/or inequalities known as constraints. The objective function may be profit, cost, production capacity or any other measure of effectiveness, which is to be obtained in the best possible or optimal manner. The constraints may be imposed by different resources such as market demand, production process and equipment, storage capacity, raw material availability, etc. By linearity is meant a mathematical expression in which the expressions among the variables are linear e.g., the expression $a_1x_1 + a_2x_2 + a_3x_3 + ... + a_nx_n$ is linear. Higher powers of the variables or their products do not appear in the expressions for the objective function as well as the constraints (they donot have expressions like x_1^3, $x_2^{3/2}$, x_1x_2, $a_1x_1 + a_2 \log x_2$, etc.). The variables obey the properties of proportionality (e.g., if a product requires 3 hours of machining time, 5 units of it will require 15 hours) and additivity (e.g., amount of a resource required for a certain number of products is equal to the sum of the resource required for each).

It was in 1947 that George Dantzig and his associates found out a technique for solving military planning problems while they were working on a project for U.S. Air Force. This technique consisted of representing the various activities of an organization as a linear programming (L.P.) model and arriving at the optimal programme by minimizing a linear objective function. Afterwards, Dantzig suggested this approach for solving business and industrial problems. He also developed the most powerful mathematical tool known as “simplex method” to solve linear programming problems.

2.0 OBJECTIVES

At the end of this study unit, you should be able to

1. Explain the requirements for Linear Programming
2. Highlight the assumptions of Linear Programming
3. Identify the Areas of application of Linear Programming

4. Formulate a Linear Programming problem
5. Solve various problems using Linear Programming

3.0 MAIN CONTENT

3.1 REQUIREMENTS FOR A LINEAR PROGRAMMING PROBLEM

All organizations, big or small, have at their disposal, men, machines, money and materials, the supply of which may be limited. If the supply of these resources were unlimited, the need for management tools like linear programming would not arise at all. Supply of resources being limited, the management must find the best allocation of its resources in order to maximize the profit or minimize the loss or utilize the production capacity to the maximum extent. However this involves a number of problems which can be overcome by quantitative methods, particularly the linear programming.

Generally speaking, linear programming can be used for optimization problems if the following conditions are satisfied:

1. There must be a well-defined objective function (profit, cost or quantities produced) which is to be either maximized or minimized and which can be expressed as a linear function of decision variables.

2. There must be constraints on the amount or extent of attainment of the objective and these constraints must be capable of being expressed as linear equations or inequalities in terms of variables.

3. There must be alternative courses of action. For example, a given product may be processed by two different machines and problem may be as to how much of the product to allocate to which machine.

4. Another necessary requirement is that decision variables should be interrelated and nonnegative. The non-negativity condition shows that linear programming deals with real life situations for which negative quantities are generally illogical.

5. As stated earlier, the resources must be in limited supply. For example, if a firm starts producing greater number of a particular product, it must make smaller number of other products as the total production capacity is limited.

3.2 ASSUMPTIONS IN LINEAR PROGRAMMING MODELS

A linear programming model is based on the following assumptions:

1. Proportionality: A basic assumption of linear programming is that proportionality exists in the objective function and the constraints. This assumption implies that if a product yields a profit of #10, the profit earned from the sale of 12 such products will be # (10 x 12) = #120. This may not always be true because of quantity discounts. Further, even if the sale price is constant, the manufacturing cost may vary with the number of units produced and so may vary the profit per unit. Likewise, it is assumed that if one product requires processing time of 5 hours, then ten such products will require processing time of 5 x 10 = 50 hours. This may also not be true as the processing time per unit often decreases with increase in number of units produced. The real world situations may not be strictly linear. However, assumed linearity represents their close approximations and provides very useful answers.

2. Additivity: It means that if we use t_1 hours on machine A to make product 1 and t_2 hours to make product 2, the total time required to make products 1 and 2 on machine A is $t_1 + t_2$ hours. This, however, is true only if the change-over time from product 1 to product 2 is negligible. Some processes may not behave in this way.

For example, when several liquids of different chemical compositions are mixed, the resulting volume may not be equal to the sum of the volumes of the individual liquids.

3. Continuity: Another assumption underlying the linear programming model is that the decision variables are continuous i.e., they are permitted to take any non-negative values that satisfy the constraints. However, there are problems wherein variables are restricted to have integral values only. Though such problems, strictly speaking, are not linear programming problems, they are frequently solved by linear programming techniques and the values are then rounded off to nearest integers to satisfy the constraints. This approximation, however, is valid only if the variables have large optimal values. Further, it must be ascertained whether the solution represented by the rounded values is a feasible solution and also whether the solution is the best integer solution.

4. Certainty: Another assumption underlying a linear programming model is that the various parameters, namely, the objective function coefficients, R.H.S. coefficients of the constraints and resource values in the constraints are certainly and precisely known and that their values do not change with time. Thus the profit or cost per unit of the product, labour and materials required per unit, availability of labour and materials, market demand of the product produced, etc. are assumed to be known with- certainty. The linear programming problem is, therefore, assumed to be deterministic in nature.

5. Finite Choices: A linear programming model also assumes that a finite (limited) number of choices (alternatives) are available to the decision-maker and that the decision variables are interrelated and non-negative. The non-negativity condition

shows that linear programming deals with real-life situations as it is not possible to produce/use negative quantities.

Mathematically these non-negativity conditions do not differ from other constraints. However, since while solving the problems they are handled differently from the other constraints, they are termed as non-negativity restrictions and the term constraints is used to represent constraints other than non-negativity restrictions and this terminology has been followed throughout the book.

3.3 APPLICATIONS OF LINEAR PROGRAMMING METHOD

Though, in the world we live, most of the events are non-linear, yet there are many instances of linear events that occur in day-to-day life. Therefore, an understanding of linear programming and its application in solving problems is utmost essential for today's managers.

Linear programming techniques are widely used to solve a number of business, industrial, military, economic, marketing, distribution and advertising problems. Three primary reasons for its wide use are:

1. A large number of problems from different fields can be represented or at least approximated to linear programming problems.

2. Powerful and efficient techniques for solving L.P. problems are available.

3. L.P. models can handle data variation (sensitivity analysis) easily.

However, solution procedures are generally iterative and even medium size problems require manipulation of large amount of data. But with the development of digital computers, this disadvantage has been completely overcome as these computers can handle even large L.P. problems in comparatively very little time at a low cost.

3.4 AREAS OF APPLICATION OF LINEAR PROGRAMMING

Linear programming is one of the most widely applied techniques of operations research in business, industry and numerous other fields. A few areas of its application are given below.

1. INDUSTRIAL APPLICATIONS

(a) Product mix problems: An industrial concern has available a certain production capacity (men, machines, money, materials, market, etc.) on various manufacturing processes to manufacture various products. Typically, differents products will have different selling prices, will require different amounts of production capacity at the several processes and will, therefore, have different unit profits; there may also be stipulations (conditions) on maximum and/or minimum product levels. The problem is to determine the product mix that will maximize the total profit.

(b) Blending problems: These problems are likely to arise when a product can be made from a variety of available raw materials of various compositions and prices. The manufacturing process involves blending (mixing) some of these materials in varying quantities to make a product of the desired specifications.

For instance, different grades of gasoline are required for aviation purposes. Prices and specifications such as octane ratings, tetra ethyl lead concentrations, maximum vapour pressure etc. of input ingredients are given and the problem is to decide the proportions of these ingredients to make the desired grades of gasoline so that (i) maximum output is obtained and (ii) storage capacity restrictions are satisfied.

Many similar situations such as preparation of different kinds of whisky, chemicals, fertilisers and alloys, etc. have been handled by this technique of linear programming.

(c) Production scheduling problems: They involve the determination of optimum production schedule to meet fluctuating demand. The objective is to meet demand, keep inventory and employment at reasonable minimum levels, while minimizing the total cost Production and inventory.

(d) Trim loss problems: They are applicable to paper, sheet metal and glass manufacturing industries where items of standard sizes have to be cut to smaller sizes as per customer requirements with the objective of minimizing the waste produced.

(e) Assembly-line balancing: It relates to a category of problems wherein the final product has a number of different components assembled together. These components are to be assembled in a specific sequence or set of sequences. Each assembly operator is to be assigned the task / combination of tasks so that his task time is less than or equal to the cycle time.

(f) Make-or-buy (sub-contracting) problems: They arise in an organisation in the face of production capacity limitations and sudden spurt in demand of its products. The manufacturer, not being sure of the demand pattern, is usually reluctant to add additional capacity and has to make a decision regarding the products to be manufactured with his own resources and the products to be sub-contracted so that the total cost is minimized.

2. MANAGEMENT APPLICATIONS

(a) Media selection problems: They involve the selection of advertising mix among different advertising media such as T.V., radio, magazines and newspapers that will maximize public exposure to company's product. The constraints may be on the total advertising budget, maximum expenditure in each media, maximum number of insertions in each media and the like.

(b) Portfolio selection problems: They are frequently encountered by banks, financial companies, insurance companies, investment services, etc. A given amount is to be allocated among several investment alternatives such as bonds, saving certificates, common stock, mutual fund, real estate, etc. to maximize the expected return or minimize the expected risk.

(c) Profit planning problems: They involve planning profits on fiscal year basis to maximize profit margin from investment in plant facilities, machinery, inventory and cash on hand.

(d) Transportation problems: They involve transportation of products from, say, n sources situated at different locations to, say, m different destinations. Supply position at the sources, demand at destinations, freight charges and storage costs, etc. are known and the problem is to design the optimum transportation plan that minimizes the total transportation cost (or distance or time).

(e) Assignment problems: They are concerned with allocation of facilities (men or machines) to jobs. Time required by each facility to perform each job is given and the problem is to find the optimum allocation (one job to one facility) so that the total time to perform the jobs is minimized.

(f) Man-power scheduling problems: They are faced by big hospitals, restaurants and companies operating in a number of shifts. The problem is to allocate optimum man-power in each shift so that the overtime cost is minimized.

3. MISCELLANEOUS APPLICATIONS

(a) Diet problems: They form another important category to which linear programming has been applied. Nutrient contents such as vitamins, proteins, fats, carbohydrates, starch, etc. in each of a number of food stuffs is known. Also the minimum daily requirement of each nutrient in the diet as well as the cost of each type of food stuff is given and the problem is to determine the minimum cost diet that satisfies the minimum daily requirement of nutrients.

(b) Agriculture problems: These problems are concerned with the allocation of input resources such as acreage of land, water, labour, fertilisers and capital to various crops so as to maximize net revenue.

(c) Flight scheduling problems: They are devoted to the determination of the most economical patterns and timings of flights that result in the most efficient use of aircrafts and crew.

(d) Environment protection: They involve analysis of different alternatives for efficient waste disposal, paper recycling and energy policies.

(e) Facilities location: These problems are concerned with the determination of best location of public parks, libraries and recreation areas, hospital ambulance depots, telephone exchanges, nuclear power plants, etc.

Oil refineries have used linear programming with considerable success. Similar trends are developing in chemical industries, iron and steel industries, aluminium industry, food processing industry, wood products manufacture and many others. Other areas where linear programming has been applied include quality control inspection, determination of optimal bombing patterns, searching of submarines, design of war weapons, vendor quotation analysis, structural design, scheduling military tanker fleet, fabrication scheduling, steel production scheduling, balancing of assembly lines and computations of maximum flows in networks.

In fact linear programming may be used for any general situation where a linear objective function has to be optimised subject to constraints expressed as linear equations/inequalities.

3.5 FORMULATION OF LINEAR PROGRAMMING PROBLEMS

First, the given problem must be presented in linear programming form. This requires defining the variables of the problem, establishing inter-relationships between them and formulating the objective function and constraints. A model, which approximates as closely as possible to the given problem, is then to be developed. If some constraints happen to be nonlinear, they are approximated to appropriate linear functions to fit the linear programming format. In case it is not possible, other techniques may be used to formulate and then solve the model.

EXAMPLE 9.1 (Production Allocation Problem)

A firm produces three products. These products are processed on three different machines. The time required to manufacture one unit of each of the three products and the daily capacity of the three machines are given in the table below.

TABLE 9.1

Machine	Time per unit (minutes)			Machine capacity (minutes/day)
	Product 1	Product 2	Product 3	
M_1	2	3	2	440
M_2	4	-	3	470
M_3	2	5	-	430

It is required to determine the daily number of units to be manufactured for each product. The profit per unit for product 1, 2 and 3 is #4, #3 and #6 respectively. It is assumed that all the amounts produced are consumed in the market. Formulate the mathematical (L.P) model that will maximize the daily profit.

Formulation of Linear Programming Model

Step 1:

From the study of the situation find the key-decision to be made. It this connection, looking for variables helps considerably. In the given situation key decision is to decide the extent of products 1, 2 and 3, as the extents are permitted to vary.

Step 2:

Assume symbols for variable quantities noticed in step 1. Let the extents. (mounts) of products, 1, 2 and 3 manufactured daily be x_1, x_2 and x_3 units respectively.

Step 3:

Express the feasible alternatives mathematically in terms of variables. Feasible alternatives are those which are physically, economically and financially possible.
In the given situation feasible alternatives are sets of values of x_1, x_2 and x_3,

where $x_1, x_2, x= \geq 0$,

since negative production has no meaning and is not feasible.

Step 4:

Mention the objective quantitatively and express it as a linear function of variables.
In the present situation, objective is to maximize the profit.

i.e., maximize $Z = 4x_1 + 3x_2 + 6x_3$.

Step 5:

Put into words the influencing factors or constraints. These occur generally because of constraints on availability (resources) or requirements (demands). Express these constraints also as linear equations/inequalities in terms of variables.

Here, constraints are on the machine capacities and can be mathematically expressed as

$2x_1 + 3x_2 + 2x_3 \leq 440$,

$4x_1 + 0x_2 +3x_3 \leq 470$,

$2x_1 + 5x_2 + 0x_3 \leq 430$.

EXAMPLE 9.2 (Diet Problem)

A person wants to decide the constituents of a diet which will fulfil his daily requirements of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from four different types of foods. The yields per unit of these foods are given in table 2.2.

TABLE 9.2

Food type	Yield per unit			Cost per unit (#)
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
Minimum requirement	800	200	700	

Formulate linear programming model for the problem.

Formulation of L.P Model

Let x_1, x_2, x_3 and x4 denote the number of units of food of type 1, 2, 3 and 4 respectively.

Objective is to minimize the cost i.e.,

Minimize Z = #($45x_1 + 40x_2 + 85x_3 + 65x_4$).

Constraints are on the fulfilment of the daily requirements of the various constituents.

i.e., for protein, $3x_1 + 4x_2 + 8x_3 + 6x_4 \geq 800$,

for fats, $2x_1 + 2x_2 + 7x_3 + 5x_4 \geq 200$,

and for carbohydrates, $6x_1 + 4x_2 + 7x_3 + 4x_4 \geq 700$,

EXAMPLE 9.3 (Blending Problem)

A firm produces an alloy having the following specifications:

(i) specific gravity ≤ 0.98,

(ii) chromium $\geq 8\%$,

(iii) melting point $\geq 450°C$.

Raw materials A, B and C having the properties shown in the table can be used to make the alloy.

Table 9.3

Property	Properties of raw material		
	A	B	C
Specific gravity	0.92	0.97	1.04
Chromium	7%	13%	16%
Melting point	$440^{o}C$	$490^{o}C$	$480^{o}C$

Costs of the various raw materials per ton are: #90 for A, #280 for B and #40 for C. Formulate the L.P model to find the proportions in which A, B and C be used to obtain an alloy of desired properties while the cost of raw materials is minimum.

Formulation of Linear Programming Model

Let the percentage contents of raw materials A, B and C to be used for making the alloy be x_1, x_2 and x_3 respectively.

Objective is to minimize the cost

i.e., minimize $Z = 90x_1 + 280x_2 + 40x_3$.

Constraints are imposed by the specifications required for the alloy.

They are

$0.92x_1 + 0.97x_2 + 1.04x_3 \leq 0.98$,

$7x_1 + 13x_2 + 16x_3 \geq 8$,

$440x_1 + 490x_2 + 480x_3 \geq 450$,

and $x_1 + x_2 + x_3 = 100$,

as x_1, x_2 and x_3 are the percentage contents of materials A, B and C in making the alloy.

Also x_1, x_2, x_3, each ≥ 0.

EXAMPLE 9.4 (Advertising Media Selection Problem)

An advertising company wishes to plan its advertising strategy in three different media television, radio and magazines. The purpose of advertising is to reach as large a number of potential customers as possible. Following data have been obtained from market survey:

TABLE 9.4

	Television	Radio	Magazine I	Magazine II
Cost of an advertising unit	# 30,000	# 20,000	# 15,000	# 10,000
No. of potential customers reached per unit	200,000	600,000	150,000	100,000
No. of female customers reached per unit	150,000	400,000	70,000	50,000

The company wants to spend not more than #450,000 on advertising. Following are the further requirements that must be met:

at least I million exposures take place among female customers,

advertising on magazines be limited to #150,000,

at least 3 advertising units be bought on magazine I and 2 units on magazine II,

the number of advertising units on television and radio should each be between 5 and 10.

Formulation of Linear Programming Model

Let x_1, x_2, x_3 and x4 denote the number of advertising units to be bought on television, radio, magazine I and magazine II respectively.

The objective is to maximize the total number of potential customers reached.
i.e., maximize $Z = 10\,(2x_1 + 6x_2 + 1.5x_3 + x_4)$.
Constraints are
on the advertising budget: $30{,}000x_1+20{,}000x_2+15{,}000x_3+10{,}000x_4 \leq 450{,}000$
or $30x_1+20x_2+15x_3+10x_4 \leq 450$,
on number of female customers reached by
the advertising campaign: $150{,}000x_1+400{,}000x_2+70{,}000x_3+50{,}000x_4= \geq 100{,}000$
or $15x_1+40x_2+7x_3+5x_4 \geq 100$
on expenses on magazine
advertising: $15{,}000x_3+10{,}000x_4 \leq 150{,}000$ or $15x_3+10x_4 \leq 150$
on no. of units on magazines: $x_3 \geq 3$,
$x_4 \geq 2$,

on no. of units on television: : $5 \leq x_1 \leq 10$ or $x_1 \geq 5$, $x_1 \leq 10$

on no. of units on radio: $5 \leq x_2 \leq 10$ or $x_2 \geq 5$, $x_2 \leq 10$

where x_1, x_2, x_3, x_4, each ≥ 0.

EXAMPLE 9.5 (Inspection Problem)

A company has two grades of inspectors, I and II to undertake quality control inspection. At least 1,500 pieces must be inspected in an 8-hour day. Grade I inspector can check 20 pieces in an hour with an accuracy of 96%. Grade II inspector checks 14 pieces an hour with an accuracy of 92%.

Wages of grade I inspector are #5 per hour while those of grade II inspector are #4 per hour. Any error made by an inspector costs #3 to the company. If there are, in all, 10 grade I inspectors and 15 grade II inspectors in the company find the optimal assignment of inspectors that minimizes the daily inspection cost.

Formulation of L.P Model

Let x_1 and x_2 denote the number of grade I and grade II inspectors that may be assigned the job of quality control inspection.

The objective is to minimize the daily cost of inspection. Now the company has to incur two types of costs: wages paid to the inspectors and the cost of their inspection errors. The cost of grade I inspector/hour is

$(5 + 3 \times 0.04 \times 20)$ = #7.40.

Similarly, cost of grade II inspector/hour is

$(4 + 3 \times 0.08 \times 14)$ = #7.36.

.:The objective function is

minimize $Z = 8(7.40x_1 + 7.36x_2) = 59.20x_1 + 58.88x_2$.

Constraints are on the number of grade I inspectors : $x_1 \leq 10$,

on the number of grade II inspectors : $x_2 \leq 15$,

on the number of pieces to be inspected daily:

$20 \text{ X } 8x_1 + 14 \text{ X } 8x_2 \geq 1,500$

or $160x_1 + 112x_2 \geq 1,500$,

where $x_1, x_2 \geq 0$.

EXAMPLE 9.6 (Product Mix Problem)

A chemical company produces two products, X and Y. Each unit of product X requires 3 hours on operation I and 4 hours on operation IL while each unit of product Y requires 4 hours on operation I and 5 hours on operation II. Total available time for operations I and 11 is 20 hours and 26 hours respectively. The production of each unit of product Y also results in two units of a by-product Z at no extra cost.

Product X sells at profit of #I0/unit, while Y sells at profit of #20/unit. By-product Z brings a unit profit of #6 if sold; in case it cannot be sold, the destruction cost is # 4/unit. Forecasts indicate that not more than 5 units of Z can be sold. Formulate the L.P. model to determine the quantities of X and Y to be produced, keeping Z in mind, so that the profit earned is maximum.

Formulation of L.P Model

Let the number of units of products X, Y and Z produced be x_1, x_2, x_Z, where

x_Z = number of units of Z produced

= number of units of Z sold + number of units of Z destroyed

= $x_3 + x_4$(say).

Objective is to maximize the profit. Objective function (profit function) for products X and Y is linear because their profits (#10/unit and #20/unit) are constants irrespective of the number of units produced. A graph between the total profit and quantity produced will be a straight line. However, a similar graph for product Z is non-linear since it has slope +6 for first part, while a slope of -4 for the second. However, it is piece-wise linear, since it is linear in the regions (0 to 5) and (5 to 2Y). Thus splitting x into two parts, viz, the number of units of Z sold (x_3) and number of units of Z destroyed (x_4) makes the objective function for product Z also linear.

Thus the objective function is

maximize $Z = l0x_1 + 20x_2 + 6x_3 - 4x_4$.

Constraints are

on the time available on operation I: $3x_1 + 4x_2 \leq 20$,

on the time available on operation II: $4x_1 + 5x_2 \leq 26$,

on the number of units of product Z sold: $x_3 \leq 5$,

on the number of units of product Z produced: $2Y = Z$

or $2x_2 = x_3 + x_4$ or $-2x_2 + x_3 + x_4 = 0$,

where x_1, x_2, x_3, x_4, each ≥ 0.

EXAMPLE 9.7 (Product Mix Problem)

A firm manufactures three products A, B and C. Time to manufacture product A is twice that for B and thrice that for C and if the entire labour is engaged in making product A, 1,600 units of this product can be produced. These products are to be produced in the ratio 3: 4: 5. There is demand for at least 300, 250 and 200 units of products A, B and C and the profit earned per unit is #90, #40 and #30 respectively.

Formulate the problem as a linear programming problem.

TABLE 9.5

Raw material	Requirement per unit of product (kg)			Total availability kg
	A	B	C	
P Q	6 4	5 7	2 3	5,000 6,000

Formulation of L.P. Model

Let x_1, x_2 and x_3 denote the number of units of products A, B and C to be manufactured.

Objective is to maximize the profit.

i.e., maximize $Z = 90x_1 + 40x_2 + 30x_3$.

Constraints can be formulated as follows:

For raw material P, $6x_1 + 5x_2 + 2x_3 \leq 5{,}000$,

and for raw material Q, $4x + 7x_2 + 3x_3 \leq 6{,}000$.

Product B requires 1/2 and product C requires 1/3rd the time required for product A.

Let t hours be the time to produce A. Then $t/_2$ and $t/_3$ are the times in hours to

produce B and C and since 1,600 units of A will need time 1,600t hours, we get the constraint,

$t\,x_1 + t/2\,x_2 + t/3 x_3 \leq 1{,}600t$ or $x_1 + x_2/2 + x_3/3 \leq 1{,}600$.

Market demand requires.

$x_1 \geq 300$,

$x_2 \geq 250$,

and $x_3 \geq 200$.

Finally, since products A, B and C are to be produced in the ratio 3: 4: 5, x_1: x_2: x_3:: 3: 4: 5

or $x_1/3 = x_2/4$,

and $x_2/4 = x_3/5$.

Thus there are two additional constraints

$4x_1 - 3x_2 = 0$,

$5x_2 - 4x_3 = 0$,

where $x_1, x_2, x_3 \geq 0$.

EXAMPLE 9.8 (Trim Loss Problem)

A paper mill produces rolls of paper used in making cash registers. Each roll of paper is 100m in length and can be used in widths of 3, 4, 6 and 1 (km. The company production process results in rolls that are 24 cm in width. Thus the company must cut its 24cm roll to the desired widths. It has six basic cutting alternatives as follows:

Cutting alternatives	Width of rolls (cm)				Waste (cm)
	3	4	6	10	
1	4	3	-	-	
2				-	-
	-	3	2	-	
3	1	1	1	1	1
4	-	-	2	1	2

5	-	4	1	-	2
6	3	2	1	-	1

The minimum demand for the four rolls is as follows:

Roll width (cm)	Demand
2	2,000
4	3,600
6	1,600
10	500

The paper mill wishes to minimize the waste resulting from trimming to size. Formulate the L.P model.

Formulation of L.P. Model

Key decision is to determine how the paper rolls be cut to the required widths so that trim losses (wastage) are minimum.

Let x, (j = 1, 2, ..., 6) represent the number of times each cutting alternative is to be used.

These alternatives result/do not result in certain trim loss.

Objective is to minimize the trim losses.

i.e., minimize $Z = x_3+2x_4+2x_5+x_6$.

Constraints are on the market demand for each type of roll width:

For roll width of 3cm, $4x_1 + x_3 + 3x_6 \geq 2{,}000$,

for roll width of 4 cm, $3x_1 + 3x_2 + x_3 + 4x_5 + 2x_6 \geq 3{,}600$,

for roll width of 6cm, $2x_2 + x_3 + 2x_4 + x_5 + x_6 \geq 1{,}600$,

and for roll width of 10cm, $x_3 + x_4 \geq 500$.

Since the variables represent the number of times each alternative is to be used, they cannot have negative values.

∴ $x_1, x_2, x_3, x_4, x_5, x_6$, each ≥ 0.

EXAMPLE 9.9 (Production Planning Problem)

A factory manufactures a product each unit of which consists of 5 units of part A and 4 units of part B. The two parts A and B require different raw materials of which 120 units and 240 units respectively are available. These parts can be manufactured by three different methods. Raw material requirements per production run and the number of units for each part produced are given below.

TABLE 9.6

Method	Input per run (units)		Output per run (units)	
	Raw material 1	Raw material 2	Part A	Part B
1	7	5	6	4
2	4	8	7	5
3	2	9	7	3

Formulate the L.P model to determine the number of production runs for each method so as to maximize the total number of complete units of the final product.

Formulation of Linear Programming Model

Let x_1, x_2, x_3 represent the number of production runs for method 1, 2 and 3 respectively.

The objective is to maximize the total number of units of the final product. Now, the total number of units of part A produced by different methods is $6x_1 + 5x_2 + 7x_3$ and for part B is $4x_1 + 8x_2 + 3x_1$. Since each unit of the final product requires 5 units of part A and 4 units of part B, it is evident that the maximum number of units of the final product cannot exceed the smaller value of

$$\frac{6x_1 + 5x_2 + 7x_3}{5} \text{ and } \frac{4x_1 + 8x_2 + 3x_3}{4}$$

Thus the objective is to maximize

$$Z = \text{Minimum of } \left\{ \frac{6x_1 + 5x_2 + 7x_3}{5}, \frac{4x_1 + 8x_2 + 3x_3}{4} \right\}$$

Constraints are on the availability of raw materials. They are, for raw material 1, $7x_1 + 4x_2 + 2x_3 \leq 120$,
and raw material 2, $5x_1 + 7x_2 + 9x_3 \leq 240$.
The above formulation violates the linear programming properties since the objective function is non-linear. (Linear relationship between two or more variables is the one in which the variables are directly and precisely proportional).

However, the above model can be easily reduced to the generally acceptable linear programming format.

$$\text{Let } y = \left\{ \frac{6x_1 + 5x_2 + 7x_3}{5}, \frac{4x_1 + 8x_2 + 3x_3}{4} \right\}$$

It follows that $\frac{6x_1 + 5x_2 + 7x_3}{5} \geq y$ and $\frac{4x_1 + 8x_2 + 3x_3}{4} \geq y$

i.e., $6x_1 + 5x_2 + 7x_3 - 5y \geq 0$, and $4x_1 + 8x_2 + 3x_3 - 4y \geq 0$.
Thus the mathematical model for the problem is
Maximize $Z = y$,
subject to constraints $7x_1 + 4x_2 + 2x_3 \leq 120$,

$5x_1 + 7x_2 + 9x_3 \leq 240$,

$6x_1 + 5x_2 + 7x_3 - 5y \geq 0,$
$4x_1 + 8x_2 + 3x_3 - 4y \geq 0,$
where$x_1, x_2, x_3, y \geq 0.$

EXAMPLE 9.10 (Fluid Blending Problem)

An oil company produces two grades of gasoline P and Q which it sells at #30 and #40 per litre. The company can buy four different crude oils with the following constituents and Costs:

TABLE 2.7

Crude oil	Constituents			Price/litre (#)
	A	B	C	
1	0.75	0.15	0.10	20.00
2	0.20	0.30	0.50	22.50
3	0.70	0.10	0.20	25.00
4	0.40	0.10	0.50	27.50

Gasoline P must have at least 55 per cent of constituent A and not more than 40 per cent of C. Gasoline Q must not have more than 25 per cent of C. Determine how the crudes should be used to maximize the profit.

Formulation of Mathematical Model

Key decision to be made is how much of each crude oil be used in making each of the two grades of gasoline. Let these quantities in litres be represented by x_{IJ}, where i = crude oil 1, 2, 3, 4 and j = gasoline of grades P and Q respectively. Thus

x_{1p} = amount in litres of crude oil 1 used in gasoline of grade P

x_{2p} = amount in litres of crude oil 2 used in gasoline of grade P

… …

… …

x_{1q} = amount in litres of crude oil 1 used in gasoline of grade Q

X_{2q} = amount in litres of crude oil 2 used in gasoline of grade Q

… …

… …

Objective is to maximize the net profit.

i.e., maximize $Z = \# [30(x_{ip} + x_{2p} + x_{3p}, + x_{4p}) + 40(_{xiq} + x_{2q} + x_{3q} + x_{4q})$

$$-20(x_{1p} + X_{ia}) - 22.50\ (x_{2p} + x_{2q}) - 25\ (x_{3p}\ x_{3q}) - 27.50(x_{4p} - x_{4q})$$

or maximize $Z = \#[10x_{1p} + 7.50x_{2p} + 5x_{3p}, + 2.50x_{4p} + 20_{1q} + 17.50x_{2q} + 15x_{3q})$

Constraints are on the quantities of constituents A and C to be allowed in the two grades of gasoline.

i.e., $0.75x_{1p} + 0.20x_{2p} + 0.70x_{3p} + 0.40x_{4p} \geq 0.55\ (x_{1p}, + x_{2p} + x_{3p} + x_{4p})$,

$$0.10x_{1p} + 0.50x_{2p} + 0.20x_{3p} + 0.50x_{4p} \leq 0.40\ (_{x1} + x_{2p} + x_{3p} + x_{4p}),$$

and $0.10x_{1q} + 0.50x_{2q} + 0.20x_{3q} + 0.50x_{4q} \leq 0.25\ (x_{1q} + x_{2q} + x_{3q} + x_{4q})$,

where $x_{1p}, x_{2p}, x_{3p}, x_{4p}, x_{1q}, x_{2q}, x_{3q}, x_{4q}$, each ≥ 0.

EXAMPLE 9.11 (Production Planning Problem)

A company manufacturing air coolers has, at present, firm orders for the next 6 months. The company can schedule its production over the next 6 months to meet orders on either regular or overtime basis. The order size and production costs over the next six months are as follows:

Month :	1	2	3	4	5	6

Orders :	640	660	700	750	550	650
Cost/unit (#) for regular production :	40	42	41	45	39	40
Cost/unit (#) for overtime production :	52	50	53	50	45	43

With 100 air coolers in stock at present, the company wishes to have at least 150 air coolers in stock at the end of 6 months. The regular and overtime production in each month is not to exceed 600 and 400 units respectively. The inventory carrying cost for air coolers is #12 per unit per month. Formulate the L.R model to minimize the total cost.

Formulation of L.P. Model

Key decision is to determine the number of units of air coolers to he produced on regular as well as overtime basis together with the number of units of ending inventory in each of the six months.

Let x_{ij} be the number of units produced in month j (j = 1, 2, ..., 6), on a regular or overtime basis (i = 1, 2). Further let y_j represent the number of units of ending inventory in month j (j= 1, 2, ..., 6).

Objective is to minimize the total cost (of production and inventory carrying).

i.e., minimize $Z = (40_{x11} + 42_{x12} + 41_{x13} + 45_{x14} + 39_{x15} + 40_{x16})$

$+ (52_{x21} + 50_{x22} + 53_{x23} + 50_{x24} + 45_{x25} + 43_{x26})$

$+12(y_1 + y_2 + y_3 + y_4 + y_5 + y_6)$

Constraints are

for the first month, $100 + x_{11} + x_{21} - 640 = y_1$, for the second month, $y_1 + x_{12} + x_{22} - 660 = y_2$,

for the third month, $y_2 + x_{13} + x_{23} - 700 = y_3$

for the fourth month, $y_3 + x_{14} + x_{24} - 750 = y_4$

for the fifth month, $y_4 + x_{15} + x_{25} - 550 = y_5$

and for the sixth month, $y_5 + x_{16} + x_{26} - 650 = y_6$

Also, the ending inventory constraint is

$$Y_6 \geq 150$$

Further, since regular and overtime production each month is not to exceed 600 and 400 units respectively,

$x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}$, each ≤ 600,

and $x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}$, each ≤ 400.

Also $x_{ij} \geq 0$ (i=1, 2; j=1, 2,..., 6), $y_j \geq 0$.

EXAMPLE 9.12 (Transportation Problem)

A dairy firm has two milk plants with daily milk production of 6 million litres and 9 million litres respectively. Each day the firm must fulfil the needs of its three distribution centres which have milk requirement of 7, 5 and 3 million litres respectively. Cost of shipping one million litres of milk from each plant to each distribution centre is given, in hundreds of naira below. Formulate the L.P model to minimize the transportation cost.

Distribution Centres

	1	2	3	Supply
	2	3	11	6
	1	9	6	9
Plants	7	5	3	
			Demand	

Formulation of L.P Model

Key decision is to determine the quantity of milk to be transported from either plant to each distribution centre.

Let x_1, x_2 be the quantity of milk (in million litres) transported from plant I to distribution centre no. 1 and 2 respectively. The resulting table representing transportation of milk is shown below.

Distribution Centres

1	2	3
x_1	x_2	$6-x_1-x_2$
$7-x_1$	$5-x_2$	$9-(7-x_1)$ $(5-x_2)$
7	5	3

Objective is to minimize the transportation cost.

i.e., minimize $Z = 2x_1 + 3x_2 + 11(6 - x_1 - x_2) + (7 - x_1) + 9(5 - x_2)$

$+ 6[9 - (7 - x_1) - (5 - x_2)] = 100 - 4x_1 - 11x_2.$

Constraints are

$6 - x_1 - x_2 \geq 0$ or $x_1 + x_2 \leq 6$,

$7 - x_1 \geq 0$ or $x_1 \leq 7$,

$5 - x_2 \geq 0$ or $x_2 \leq 5$,

and $9 - (7 - x_1) - (5 - x_2) \geq 0$ or $x_1 + x_2 \geq 3$,

where $x_1, x_2 \geq 0$.

EXAMPLE 9.13 (Product Mix Problem)

A plant manufactures washing machines and dryers. The major manufacturing departments are the stamping department, motor and transmission deptt. and assembly deptt. The first two departments produce parts for both the products while the assembly lines are different for the two products. The monthly deptt.capacities are

Stamping deptt.	: 1,000 washers or 1,000 dryers
Motor and transmission deptt.	: 1,600 washers or 7,000 dryers
Washer assembly line	: 9,000 washers only
Dryer assembly line	: 5,000 dryers only.

Profits per piece of washers and dryers are #270 and #300 respectively. Formulate the L.P model.

Formulation of Linear Programming Model

Let x_1 and x_2 represent the number of washing machines and dryers to be manufactured each month.

The objective is to maximize the total profit each month.

i.e. maximize $Z = 270x_1 + 300x_2$.

Constraints are on the monthly capacities of the various departments.

For the stamping deptt., $\underline{x_1} + \_\_ \; x_2 \leq 1$, 1,000 1,000

For the motor and transmission deptt.,

$$\frac{x_1}{1,600} + \frac{x_2}{7,000} \leq 1$$

for the washer assembly deptt., $x_1 \leq 9,000$

and for the dryer assembly deptt., $x_2 \leq 5,000$

where $x_1 \geq 0$, $x_2 \geq 0$.

EXAMPLE 9.14 (Product Mix Problem)

A certain farming organization operates three farms of comparable productivity. The output of each farm is limited both by the usable acreage and by the amount of water available for irrigation. Following are the data for the upcoming season:

Farm	Usable acreage	Water available in acre feet
1	400	1,500
2	600	2,000
3	300	900

The organization is considering three crops for planting which differ primarily in their expected profit per acre and in their consumption of water. Furthermore, the total acreage that can be devoted to each of the crops is limited by the amount of appropriate harvesting equipment available.

Crop	Minimum acreage	Water consumption in acre feet per acre	Expected profit per acre
A	400	5	# 400
B	300	4	# 300

C	300	3	# 100

However, any combination of the crops may be grown at any of the farms. The organization wishes to know how much of each crop should be planted at the respective farms in order to maximize expected profit. Formulate this as a linear programming problem.

Formulation of Linear Programming Model

The key decision is to determine the number of acres of each farm to be allotted to each crop.

Let x, (i = farm 1, 2, 3; j = crop A, B, C) represent the number of acres of the ith farm to be allotted to the jth crop.

The objective is to maximize the total profit.

i.e., maximize $Z = \# \left[400 \sum x_{1A} + 300 \sum x_{1B} + 100 \sum x_{1C}. \right]$

Constraints are formulated as follows:

For availability of water in acre feet,

$5x_{1A} + 4x_{1B} + 3x_{1c} \leq 1{,}500,$

$5x_{2A} + 4x2_{B} + 3x_{2C} \leq 2{,}000,$

$5x_{3A} + 4x_{3B} + 3x_{3C} \leq 900.$

For availability of usable acreage in each farm,

$x_{1A} + x_{1B} + x_{1C} \leq 400,$

$x_{2A} + x_{2B} + x_{3C} \leq 600,$

$x_{3A} + x_{3B} + x_{3C} \leq 300.$

For availability of acreage for each crop,

$x_{1A} + x_{2A} + x_{3A} \geq 400,$

$x_{1B} + x_{2B} + x_{3B} \geq 300,$

$x_{1C} + x_{2C} + x_{3C} \geq 300.$

To ensure that the percentage of usable acreage is same in each farm,

$$\frac{x_{1A} + x_{1B} + x_{1C}}{400} \times 100 = \frac{x_{2A} + x_{2B} + x_{2C}}{600} \times 100 = \frac{x_{3A} + x_{3B} + x_{3C}}{300} \times 100$$

or $\qquad 3(x_{1A} + x_{1B} + x_{1C}) = 2(x_{2A} + x_{2B} + x_{2C}),$

and $\qquad (x_{2A} + x_{2B} + x_{2C}) = 2(x_{3A} + x_{3B} + x_{3C}).$

where $x_{1A}, x_{1B}, x_{IC}, x_{2A}, x_{2B}, x_{2C}, x_{3A}, x_{3B}, x_{3C}$, each ≥ 0.

The above relations, therefore, constitute the L.P. model.

EXAMPLE 9.15 (Product Mix Problem)

Consider the following problem faced by a production planner in a soft drink plant. He has two bottling machines A and B. A is designed for 8-ounce bottles and B for 16-ounce bottles. However; each can be used on both types of Bottles with some lass of efficiency. The following data are available:

Machine	8-ounce bottles	16-ounce bottles
A	100/minute	40/minute
B	60/minute	75/minute

The machines can be run 8-hour per day, 5 days a week. Profit on 8-ounce bottle is 15 paise and on 16-ounce bottle is 25 paise. Weekly production of the drink cannot exceed 300,000 ounces and the market can absorb 25,000 eight-ounce bottles and 7,000 sixteen-ounce bottles per week. The planner wishes to maximize his profit subject, of course, to all the production and marketing constraints. Formulate this as L.P problem.

Formulation of Linear Programming Model

Key decision is to determine the number of 8-ounce bottles and 16-ounce bottles to be produced on either of machines A and B per week. Let xA1, xB1 be the number of 8-ounce bottles and xA2, xB2 be the number of 16-ounce bottles to be produced per week on machines A and B respectively.

Objective is to maximize .the weekly profit.

i.e., maximize Z = #[0.15 (x_{A1} +x_{B1}) + 0.25(x_{A2} +x_{B2})].

Constraints can be formulated as follows:

Since an 8-ounce bottle takes 1/100minute and a 16-ounce bottle takes 1/40 minute on

machine A and the machine can be run for 8 hours a day and 5 days a week, the time constraint on machine A can be written as

$$\frac{x_{A1}}{100} + \frac{x_{A2}}{40} \leq 5 \text{ X } 8 \text{ X } 60$$

$$\leq 2{,}400$$

Similarly, time constraint on machine B can be written as

$$\frac{x_{B1}}{60} + \frac{x_{B2}}{75} \leq 2{,}400.$$

Since the total weekly production cannot exceed 300,000 ounces,

$8(x_{A1}+X_{B1}) + 16(x_{A2} + x_{B2}) \leq 300{,}000.$

The constraints on market demand yield

$x_{A1} + x_{B1} \geq 25{,}000,$

$x_{A2} + x_{B2} \geq 7{,}000,$

where x_{A1}, x_{B1}, x_{A2}, x_{B2}, each ≥ 0.

3.6 ADVANTAGES OF LINEAR PROGRAMMING METHODS

Following are the main advantages of linear programming methods:

1. It helps in attaining the optimum use of productive factors. Linear programming indicates how a manager can utilize his productive factors most effectively by a better selection and distribution of these elements. For example, more efficient use of manpower and machines can be obtained by the use of linear programming.

2. It improves the quality of decisions. The individual who makes use of linear programming methods becomes more objective than subjective. The individual having a clear picture of the relationships within the basic equations, inequalities or constraints can have a better idea about the problem and its solution.

3. It also helps in providing better tools for adjustments to meet changing conditions. It can go a long way in improving the knowledge and skill of future executives.

4. Most business problems involve constraints like raw materials availability, market demand, etc. which must be taken into consideration. Just because we can produce so many units of products does not mean that they can be sold. Linear programming can handle such situations also since it allows modification of its mathematical solutions.

5. It highlights the bottlenecks in the production processes. When bottlenecks occur, some machines cannot meet demand while others. remain idle, at least part of the time. Highlighting of bottlenecks is one of the most significant advantages of linear programming.

3.7 LIMITATIONS OF'LINEAR PROGRAMMING MODEL

This model, though having a wide field, has the following limitations:

1. For large problems having many limitations and constraints, the computational difficulties are enormous, even when assistance of large digital computers is available. The approximations required to reduce such problems to meaningful sizes may yield the final results far different from the exact ones.

2. Another limitation of linear programming is that it may yield fractional valued answers for the decision variables, whereas it may happen that only integer values of the variables are logical.

For instance, in finding how many lathes and milling machines to be produced, only integer values of the decision variables, say x_1 and x_2 are meaningful. Except when the variables have large values, rounding the solution values to the nearest integers will not yield an optimal solution. Such situations justify the use of special techniques like integer programming.

3. It is applicable to only static situations since it does not take into account the effect of time. The O.R. team must define the objective function and constraints which can change due to internal as well as external factors.

4. It assumes that the values of the coefficients of decision variables in the objective function as well as in all the constraints are known with certainty. Since in most of the business situations, the decision variable coefficients are known only probabilistically, it cannot be applied to such situations.

5. In some situations it is not possible to express both the objective function and constraints in linear form. For example, in production planning we often have non-linear constraints on production capacities like setup and takedown times which are often independent of the quantities produced. The misapplication of linear programming under non-linear conditions usually results in an incorrect solution.

6. Linear programming deals with problems that have a single objective. Real life problems may involve multiple and even conflicting objectives. One has to apply goal programming under such situations.

When comparison is made between the advantages and disadvantages/limitations of linear programming, its advantages clearly outweigh its limitations. It must be clearly understood that linear programming techniques, like other mathematical tools only help the manager to take better decisions; they are in no way a substitute for the manager.

3.8 GRAPHICAL METHOD OF SOLUTION

Once a problem is formulated as mathematical model, the next step is to solve the problem to get the optimal solution. A linear programming problem with only two variables presents a simple case, for which the solution can be derived using a graphical or geometrical method. Though, in actual practice such small problems are rarely encountered, the graphical method provides a pictorial representation of the solution process and a great deal of insight into the basic concepts used in solving large L.P. problems. This method consists of the following steps:

1. Represent the given problem in mathematical form i.e., formulate the mathematical model for the given problem.

2. Draw the x_1 and x_2-axes. The non-negativity restrictions $x_1 \geq 0$ and $x_2 \geq 0$ imply that the values of the variables x_1 and x_2 can lie only in the first quadrant. This eliminates a number of infeasible alternatives that lie in 2nd, 3rd and 4th quadrants.

3. Plot each of the constraint on the graph. The constraints, whether equations or inequalities are plotted as equations. For each constraint, assign any arbitrary value to one variable and get the value of the other variable. Similarly, assign another

arbitrary value to the other variable and find the value of the first variable. Plot these two points and connect them by a straight line. Thus each constraint is plotted as line in the first quadrant.

4. 1dentify the feasible region (or solution space) that satisfies all the constraints simultaneously. For type constraint, the area on or above the constraint line i.e., away from the origin and for type constraint, the area on or below the constraint line i.e., towards origin will be considered. The area common to all the constraints is called feasible region and is shown shaded. Any point on or within the shaded region represents a feasible solution to the given problem. Though a number of infeasible points are eliminated, the feasible region still contains a large number of feasible points

5. Use iso-profit (cost) function line approach. For this plot the objective function by assuming Z = 0. This will be a line passing through the origin. As the value of Z is increased from zero, the line starts moving to the right, parallel to itself. Draw lines parallel to this line till the line is farthest distant from the origin (for a maximization problem). For a minimization problem, the line be nearest to the origin. The point of the feasible region through which this line passes will be optimal point;It is possible that this line may coincide with one of the edges of the feasible region. In that case, every point on that edge will give the same maximum/minimum value of the objective function and will be the optimal point.

Alternatively use extreme point enumeration approach. For this, find the co-ordinates each extreme point (or corner point or vertex) of the feasible region. Find the value of the objective function at each extreme point. The point at which objective function is maximum/minimum optimal point and its co-ordinates give the optimal solution.

4.0 CONCLUSION

Linear programming involves with the optimization (maximization or minimization) of a function of variables known as objective function, subject to a set of linear equations and/or inequalities known as constraints. The objective function may be profit, cost, production capacity or any other measure of effectiveness, which is to be obtained in the best possible or optimal manner. The constraints may be imposed by different resources such as market demand, production process and equipment, storage capacity, raw material availability and so on.

5.0 SUMMARY

All organizations, big or small, have at their disposal, men, machines, money and materials, the supply of which may be limited. If the supply of these resources were unlimited, the need for management tools like linear programming would not arise at all. Supply of resources being limited, the management must find the best allocation of its resources in order to maximize the profit or minimize the loss or utilize the production capacity to the maximum extent. However this involves a number of problems which can be overcome by quantitative methods, particularly the linear programming. Generally speaking, linear programming can be used for optimization problems if the following conditions are satisfied- there must be a well-defined objective function; there must be constraints on the amount or extent of attainment of the objective and these constraints must be capable of being expressed as linear equations or inequalities in terms of variables; there must be alternative courses of action; decision variables should be interrelated and nonnegative; and the resources must be in limited supply. Linear Programming has the following assumptions- Proportionality, Additivity, Continuity, Certainty, and Finite Choices. LP solution methods can be applied in solving industrial problems, management related problems, and a host of other problem areas.

6.0 TUTOR MARKED ASSIGNMENT

1. Briefly discuss what linear programming involves.
2. Identify and discuss five assumptions of linear programming.
3. List and explain three areas where linear programming can be applied.
4. Highlight four limitations of linear programming.
5. Give five advantages of the linear programming method.
6. A manufacturer has two milk plants with daily milk production of 9 million litres and 11 million litres respectively. Each day the firm must fulfil the needs of its three distribution centres which have milk requirement of 9, 6 and 4 million litres respectively. Cost of shipping one million litres of milk from each plant to each distribution centre is given, in hundreds of naira below. Formulate the L.P model to minimize the transportation cost.

Distribution Centres

	1	2	3	
Supply				
	3	4	12	9
	2	10	7	11
Plants	9	6	4	
				Demand

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UNIT 2 OPERATIONS RESEARCH (OR)

1.0 Introduction
2.0 Objectives
3.0 Main Content
    3.1 Development of Operations Research
    3.2 Definition of Operations Research
    3.3 Characteristics of OR
    3.4 Scientific Method in Operations Research
        3.4.1 The Judgement Phase
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1.0 INTRODUCTION

We mentioned in Unit 1, module 1, that the subject Business Decision Analysis takes its root from the discipline Operations Research or Operational Research (OR). This unit is devoted to giving us background knowledge of OR. It is however, not going to be by any way exhaustive as substantial literature been developed about quantitative approaches to decision making. The root of this literature are centuries old, but much of it emerged only during the past half century in tandem with the digital computer (Denardo, 2002). The above assertion relates only to the development of the digital computer for use in solving OR problems. The proper roots of OR can be traced to the early 1800s. But it was in 1885 when Ferderick Taylor emphasized the application scientific analysis to methods of production, that it really began (Gupta & Hira 2012).

This unit provides only an overview of OR with emphasis on the definition of OR, characteristics, Scope, application, objectives, and phases of OR.

2.0 OBJECTIVES

At the end of this study unit, you should be able to:

1 Briefly trace the development of OR.

2 Define OR.

3 Outline the characteristics of OR.

4 Give reasons why operations research is necessary in industries.

5 Discuss the scope of OR.

6 List and explain the areas of application of OR.

7 Outline the objectives of OR.

3.0 MAIN CONTENT

3.1 DEVELOPMENT OF OPERATIONS RESEARCH

Gupta and Hira (2012) traced the development of Operations Research (OR) thus:

i THE PERIOD BEFORE WORLD WAR II

The roots of OR are as old as science and society. Though the roots of OR extend to even early 1800s, it was in 1885 when Ferderick, W. Taylor emphasized the application of scientific analysis to methods of production, that the real start took place. Taylor conducted experiments in connection with a simple shovel. His aim was to find that weight load of Ore moved by shovel would result in the maximum amount of ore move with minimum fatigue. After many experiments with varying weights, he obtained the optimum weight load, which though much lighter than that commonly used, provided maximum movement of ore during a day. For a "first-class man" the proper load turned out to be 20 pounds. Since the density of Ore differs greatly, a shovel was designed for each density of Ore so as to assume the proper weight when the shovel was correctly filled. Productivity rose substantially after this change.

Henry L. Gantt, also of the scientific management era, developed job sequencing and scheduling methods by mapping out each job from machine to machine, in order to minimize delay. Now, with the Gantt procedure, it is possible to plan machine loading months in advance and still quote delivery dates accurately.

Another notable contributor is A.K. Erlang a Danish Mathematician who published his work on the problem of congestion of telephone traffic. During that period, operators were unable to handle the calls the moment they were made, resulting in delayed calls. A few years after its appearance, his work was accepted by the British Post Office as the basis calculating circuit facilities.

Other early contributors include F.W. Harris, who published his work in the area of inventory control in 1915, H.C. Levinson an American Astronomer who applied scientific analysis to the problems of merchandizing.

However, the first industrial Revolution was the main contributing factor towards the development of OR. Before this period, most of the industries were small scale, employing only a handful of men. The advent of machine tools – the replacement of man by machine as a source of power and improved means of transportation and communication resulted in fast flourishing industries. If became increasingly difficult for a single man to perform all the managerial functions (Planning, sales, purchasing production, etc). Consequently, a division of management functions took place. Managers of production marketing, finance, personal, research and development etc. began to appear. For example, production department was sub-divided into sections like maintenance, quality control, procurement, production planning etc.

ii WORLD WAR II

During War II, the military management in England called on a team of scientists to study the strategic and tactical problems of air and land defence. This team was under the leadership of Professor P. M. S. Blackett of University of Manchester and a former Naval Officer. "Blackett's circus", as the group was called, included three Physiologists, two Mathematical Physicists, one Astrophysicist, one Army officer, one Surveyor, one general physicist and two Mathematicians. The objective of this team was to find out the most effective allocation of limited military resources to the various military operations and to activities within each operation. The application included effective use of newly invented radar, allocation of British Air Force Planes to missions and the determination best patters for searching submarines. This group of scientist formed the first OR team. The name Operations Research (or Operational Research) was coined in 1940 because the team was carrying out research on military operations. The encouraging results of the team's efforts lead to the formation of more of such teams in the British Armed services and the use of such scientific teams soon spread to the western allies – United States, Canada, and France. Although the science of Operations Research originated in England, the United States soon took the lead. In the United States, OR terms helped in developing strategies for mining operations, inventing new flight patterns, and planning of sea mines.

iii **POST WORLD WAR II**

Immediately after the war, the success of military teams attracted the attention of industrial mangers who were seeking solutions to their problems. Industrial operations research in U.K and USA developed along different lines, and in UKthe critical economic efficiency and creation of new markets. Nationalisation of new key industries further increased the potential field for OR. Consequently OR soon spread from military to government, industrial, social and economic planning.
In the USA, the situation was different impressed by is dramatic success in UK, defence operations research in USA was increased. Most of the war-experienced OR workers remained in the military services. Industrial executives did not call for much help because they were returning to peace and many of them believed that it was merely a new applicati0on of an old technique. Operations research has been known by a variety of names in that country such as Operational Analysis, Operations Evaluation, Systems Analysis, Systems Evaluation, Systems Research, Decision Analysis, Quantitative Analysis, Decision Science, and Management Science.
In 1950, OR was introduced as a subject for academic study in American Universities. They were generally schools of Engineering, Public Administration, Business Management, Applied Mathematics, Economics, Computer Sciences, etc.

Since this subject has been gaining ever increasing importance for the student in Mathematics, statistics, commerce, Economics, Management and Engineering, to increase the impact of operations research, the Operations Research Society of America (ORSA) was formed in 1950. In 1953 the Institute of Management Science (IMS) was established. Other countries followed suit and in 1959, International Federation of OR Societies was established, and in many countries, International Journals OR began to appear.

Today, the impact of Operations Research can be felt in many areas. This is shown by the ever increasing member of educational institutions offering it at degree level. The fast increase in the number of management consulting firms speak of the popularity of OR. Lately, OR activities have spread to diverse fields such as hospitals, libraries, city planning, transportations systems, crime investigation etc.

3.2 DEFINITION OF OPERATIONS RESEARCH

Many definitions of OR have been suggested by writers and experts in the field of operations Research. We shall consider a few of them.

1 Operations Research is the applications of scientific methods by inter disciplinary teams to problems involving the control of organized (Man-Machine) Systems so as to provide solutions which best serve the purpose of the organization as a whole (Ackoff & Sasieni, 1991).

2 Operations Research is applied decision theory. It uses any scientific, Mathematical or Logical means to attempt to cope with the problems that confront the executive when he tries to achieve a thorough going rationality in dealing with his decision problems (Miller and Starr, 1973).

3 Operations research is a scientific approach to problem solving for executive management (Wagner, 1973).

4 Operations Research is the art of giving bad answers to problems, to which, otherwise, worse answers are given (Saaty, 1959).

5 OR, in the most general sense, can be characterized as the application of scientific methods, tools, and techniques to problems involving the operations of systems so as to provide those in control of the operations with optimum solutions to the problems. (Churchman, Ackoff, and Arnoff, 1957).

It could be noticed that most of the above definitions are not satisfactory. This is because of the following reasons.

i They have been suggested at different times in the development of operations research and hence emphasis only one other aspect.

ii The interdisciplinary approach which is an important characteristic of operations research is not included in most of the definitions.

iii It is not easy to define operations research precisely as it is not a science representing any well-defined social, biological or physical phenomenon.

3.3 CHARACTERISTICS OF OR

Ihemeje (2002) presents four vital characteristics of OR.

1 The OR approach is to develop a scientific model of the system under investigation with which to compare the probable outcomes of alternative management decision or strategies.

2 OR is essentially an aid to decision making. The result of an operation study should have a direct effect on managerial action, management decision based on the finding of an OR model are likely to be more scientific and better informed.

3 It is based on the scientific method. It involves the use of carefully constructed models based on some measurable variables. It is, in essence, a

quantitative and logical approach rather than a qualitative one. The dominant techniques of OR are mathematical and statistical.

4 OR model will be constructed for a particular "problem area". This means that the model has "boundaries" and only considers a small part of a large organization or system. This may result in sub-optimisation of solution to a problem. An OR project is often a team effort involving people drawn from many different backgrounds including accountants, economists, engineers as well as OR experts themselves.

Other characteristics of OR are:

5 It is system (Executive) Oriented
6 It makes use of interdisciplinary teams
7 Application of scientific method
8 Uncovering of new problems
9 Improvement in quality of decision
10 Use of computer
11 Quantitative solution
12 Human factors

(Gupta & Hira, 2012)

3.4 SCIENTIFIC METHOD IN OPERATIONS RESEARCH

Of these three phases, the research phase is the longest. The other two phases are equally important as they provide the basis of the research phase. We now consider each phase briefly as presented by Gupta & Hira (2012).

3.4.1THE JUDGEMENT PHASE

The judgement phases of the scientific method of OR consists of the following:

A **Determination of the Operation:** An operation is the combination of different actions dealing with resources (e.g men and machines) which form a structure from which an action with regards to broader objectives is maintained. For example an act of assembling an engine is an operation.

B **Determination of Objectives and Values Associated with the operation** :
In the judgement phase, due care must be given to define correctly the frame of references of operations. Efforts should be made to find the type of situation, e.g manufacturing, engineering, tactical, strategic etc.

C **Determination of Effectiveness Measures**: The measure of effectiveness implies the measure of success of a model in representing a problem and providing a solution. It is the connecting link between the objectives and the analysis required for corrective action.

D **Formulation of the Problem Relative to the Objective**: Operation analysis must determine the type of problem, its origin, and causes. The following are some types of problems:

i Remedial type with its origin in actual or threatened accidents e.g. air crashes, job performance hazards.

ii Optimization type e.g. performing a job more efficiently.

iii Transference type consisting of applying the new advances, improvements and inventions in one field to other fields.

iv Prediction typee.g. forecasting and problems associated with future developments and inventions

3.4.2 THE RESEARCH PHASE

The research phase of OR includes the following:

a. Observation and Data Collection: This enhances better understanding of the problem.

b. Formulation of Relevant Hypothesis: Tentative explanations, when formulated as propositions are called hypothesis. The formulation of a good hypothesis depends upon the sound knowledge of the subject–matter. A hypothesis must provide an answer to the problem in question.

c. Analysis of Available Information and Verification of Hypothesis: Quantitative as well as qualitative methods may be used to analyse available data.

d. Prediction and Generalisation of Results and Consideration of Alternative Methods: Once a model has been verified, a theory is developed from the model to obtain a complete description of the problem. This is done by studying the effect of changes in the parameters of the model. The theory so developed may be used to extrapolate into the future.

3.4.3 THE ACTION PHASE

The action phase consists of making recommendations for remedial action to those who first posed the problem and who control the operations directly. These recommendations consists of a clear statement of the assumptions made, scope and limitations of the information presented about the situation, alternative courses of action, effects of each alternative as well as the preferred course of action.

A primary function of OR group is to provide an administrator with better understanding of the implications of the decisions he makes. The scientific method supplements his ideas and experiences and helps him to attain his goals fully.

3.5 NECESSITY OF OPERATIONS RESEARCH IN INDUSTRY

Having studied the scientific methods of operations research, we now focus on why OR is important or necessary in industries. OR came into existence in connection with war operations, to decide the strategy by which enemies could be harmed to the maximum possible extent with the help of the available equipment.
War situations required reliable decision making. But the need for reliable decision techniques is also needed by industries for the following reasons.

- **Complexity:** Today, industrial undertakings have become large and complex. This is because the scope of operations has increased. Many factors interact with each other in a complex way. There is therefore a great uncertainty about the outcome of interaction of factors like technological, environmental, competitive etc. For instance, a factory production schedule will take the following factors into account:

i Customer demand.
ii Raw material requirement.
iii Equipment Capacity and possibility of equipment failure.
iv Restrictions on manufacturing processes.

It could be seen that, it is not easy to prepare a schedule which is both economical and realistic. This needs mathematical models, which in addition to optimization, help to analyse the complex situation. With such models, complex problems can be split into simpler parts, each part can be analysed separately and then the results can be synthesized to give insights into the problem.

- **Scattered Responsibility and Authority:** In a big industry, responsibility and authority of decision-making is scattered throughout the organization and thus the organization, if it is not conscious, may be following inconsistent goals. Mathematical quantification of OR overcomes this difficulty to a great extent.

- **Uncertainty:** There is a lot of uncertainty about economic growth. This makes each decision costlier and time consuming. OR is essential from the point of view of reliability.

- **Knowledge Explosion:** Knowledge is increasing at a very fast pace. Majority of industries are not up-to-date with the latest knowledge and are therefore, at a disadvantage. OR teams collect the latest information for analysis purpose which is quit useful for the industries.

3.6 SCOPE OF OPERATIONS RESEARCH

We now turn our attention towards learning about the areas that OR covers. OR as a discipline is very broad and is relevant in the following areas.

- **In Industry:** OR is relevant in the field or industrial management where there is a chain of problems or decisions starting from the purchase of raw materials to the dispatch of finished goods. The management is interested in having an overall view of the method of optimizing profits. In order to take scientific decisions, an OR team will have to consider various alternative methods of producing the goods and the returns in each case. OR should point out the possible changes in overall structure like installation of a new machine, introduction of more automation etc.

Also, OR has been successfully applied in production, blending, product mix, inventory control, demand forecast, sales and purchases, transportation, repair and maintenance, scheduling and sequencing, planning, product control, etc.

- **In Defence**: OR has wide application in defence operations. In modern warfare, the defence operations are carried out by a number of different agencies, namely – Air force, Army, and Navy. The activities perfumed by each of these agencies can further be divided into sub-activities viz: operations, intelligence, administration, training, etc. There is thus a need to coordinate the various activities involved in order to arrive at an optimum strategy and to achieve consistent goals.

- **In Management:** Operations Research is a problem-solving and decision-making science. It is a tool kit for scientific and programmable rules providing the management a qualitative basis for decision making regarding the operations under its control. Some of the area of management where OR techniques have been successfully applied are as follows:

A Allocation and Distribution

i Optimal allocation of limited resources such as men, machines, materials, time, and money.

ii Location and size of warehouses, distribution centres retail depots etc

iii Distribution policy

B Production and Facility Planning

i Selection, location and design of production plants, distribution centre and retail outlets.

ii Project scheduling and allocation of resources.

iii Preparation of forecast for the various inventory items and computing economic order quantities and reorder levels.

iv Determination of number and size of the items to be produced.

v Maintenances policy and preventive maintenance.

vi Scheduling and sequencing of production runs by proper allocation of machines.

C Procurement

i What, how, and when to purchase at minimum purchase cost.

ii Bidding and replacement policies.

iii Transportation planning and vendor analysis.

D Marketing

i Product selection, timing, and competitive actions.

ii Selection of advertisement media.

iii Demand forecast and stock levels.

iv Customer preference for size, colour and packaging of various products.

v Best time to launch a product.

E Finance

i Capital requirement, cash-flow analysis.

ii Credit policy, credit risks etc.

iii Profit plan for the organisation.

iv Determination of optimum replacement policies.

v Financial planning, dividend policies, investment and portfolio management, auditing etc.

F Personnel

i Selection of personnel, determination of retirement age and skills.

ii Recruitment policies and assignment of jobs.

iii Wages/salaries administration.

G Research and Development

i Determination of areas for research and development.

ii Reliability and control of development projects.

iii Selection of projects and preparation of budgets.

3.7 SCOPE OF OPERATIONS RESEARCH IN FINANCIAL MANAGEMENT

The scope of OR in financial management covers the following areas

i Cash Management: Linear programming techniques are helpful in determining the allocation of funds to each section. Linear programming techniques have also been applied to identify sections having excess funds; these funds may be diverted to the sections that need them.

ii Inventory Control: Inventory control techniques of OR can help management to develop better inventory policies and bring down the investment in inventories. These techniques help to achieve optimum balance between inventory carrying costs and shortage cost. They help to determine which items to hold, how much to hold, when to order, and how much to order.

iii Simulation Technique: Simulation considers various factors that affect and present and projected cost of borrowing money from commercial banks, and tax rates etc. and provides an optimum combination of finance (debt, equity, retained earnings) for the desired amount of capital. Simulation replaces subjective estimates, judgement and hunches of the management by providing reliable information.

iv Capital Budgeting

It involves evaluation of various investment proposals (viz, market introduction of new products and replacement of equipment with a new one). Often, decisions have been made by considering internal rate of return or net present values. Also the EMV method as discussed early can be used to evaluate investment proposals/project.

4.0 CONCLUSION

Operations Research as we know it today, as developed during War II, when the military management in England called on a team of scientists to study the strategic and tactical problems of air and land defence. Ever since that period, the impact of Operations Research can be felt in many areas. This is shown by the ever increasing member of educational institutions offering it at degree level. The fast increase in the number of management consulting firms speak of the popularity of OR. Lately, OR activities have spread to diverse fields such as hospitals, libraries, city planning, transportations systems, crime investigation etc. In business, Operations Research has been used as a problem-solving and decision-making science. It is a tool kit for scientific and programmable rules providing the management a qualitative basis for decision making regarding the operations under its control. OR techniques have been successfully applied in areas of allocation and Distribution, Production and Facility Planning, Procurement, Marketing, Finance, Personnel and, Research and Development.

5.0 SUMMARY

This has provided us with background information on the area of Operations Research. As stated in the opening unit of this study material, the discipline Business Decision Analysis or Analysis for Business Decisions takes its root from operations research. The History and Development of operations research which is as old as science and society. Though the roots of or extend to even early 1800s, it was in 1885 when Frederick, w. Taylor emphasized the application of scientific analysis to methods of production, that the real start took place. Taylor conducted experiments in connection with a simple shovel. However, Operations Research as we know it today can be traced to the period during War II, when the military management in England called on a team of scientists to study the strategic and tactical problems of air and land defence. This team was under the leadership of Professor P. M. S. Blackett of University of Manchester and a former Naval Officer. Immediately after the war, the success of military teams attracted the attention of industrial mangers who were seeking solutions to their problems. Industrial operations research in U.K and USA developed along different lines, and in UK the critical economic efficiency and creation of new markets. The modern day Operations Research is defined as the applications of scientific methods by inter disciplinary teams to problems involving the control of organized (Man-Machine) Systems so as to provide solutions which best serve the purpose of the organization as a whole (Ackoff & Sasieni, 1991). The Characteristics of OR include the fact that its approach is to develop a scientific model of the system under investigation with which to compare the probable outcomes of alternative management decision or strategies.

6.0 TUTOR MARKED ASSIGNMENT

1. Trace the history and development of operations to the founding fathers of the field of management.
2. Give two definitions of operations research with identified authors.
3. Identify the four main characteristics of operations research.
4. Identify and briefly discuss the phases involved in the Scientific Method in Operations research.
5. Give reason why reliable decision techniques are needed by industries.
6. Identify and discuss the areas where operations research is relevant.
7 Highlight the key areas where operations research is important in Financial Management.

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UNIT 3: MODELLING IN OPERATIONS RESEARCH

1.0 Introduction
2.0 Objectives
3.0 Main Content
   3.1 Definition
   3.2 Classification of Models
   3.3 Characteristics of Good models
   3.4 Advantages of Models
   3.5 Limitations of Models
   3.6 model Construction
   3.7 Approximation (Simplification) of Models
   3.8 Types of mathematical Models
4.0 Conclusion
5.0 Summary
6.0 References
7.0 Tutor Marked Assignment

1.0 Introduction

The construction and use of models is at the core of operations research. Operations research is concerned with scientifically deciding how to best design and operate man-machine systems, usually under conditions requiring the allocation of scarce resources. Modelling is a scientific activity that aims to make a particular part or feature of the world easier to understand, define, quantify, visualize, or simulate. Models are typically used when it is either impossible or impractical to create experimental conditions in which scientists can directly measure outcomes. Direct measurement of outcomes under controlled conditions will always be more reliable than modelled estimates of outcomes.

This unit introduces us to the subject of modelling and exposes us to characterisation, classification, uses, and construction of scientific models.

2.0 OBJECTIVES

At the end of this unit, you should be able to

1. Define a Model
2. Describe modelling
3. Give a classification of models
4. Outline the advantages and disadvantages of models
5. Explain the limitations of model
6. Describe how models are constructed

3.0 Main Content

3.1 Definition

Scientific modelling is an activity the aim of which is to make a particular part or feature of the world easier to understand, define, quantify, visualize, or simulate. It requires selecting and identifying relevant aspects of a situation in the real world and then using different types of models for different aims, such as conceptual models to better understand, operational models to operationalize, mathematical models to quantify, and graphical models to visualize the subject (http://en.wikipedia.org/wiki/Scientific modelling)

Adebayo et al (2010) define Modelling as a process whereby a complex life problem situation is converted into simple representation of the problem situation. They further described a model as a simplified representation of complex reality.

Thus, the basic objective of any model is to use simple inexpensive objects to represent complex and uncertain situations. Models are developed in such a way that they concentrate on exploring the key aspects or properties of the real object and ignore the other objects considered as being insignificant. Models are useful not only in science and technology but also in business decision making by focusing on the key aspects of the business decisions(Adebayo et al, 2010).

A model as used in Operations Research is defined as an idealised representation of real life situation. It represents one of the few aspects of reality. Diverse items such as maps, multiple activity charts, an autobiography, PERT network, break-even equations, balance sheets, etc, are all models because they each one of them represent a few aspects of the real life situation. A map for instance represents the physical boundaries but ignores the heights and the various places above sea levels (Gupta and Hira, 2012). According to Reeb and Leavengood (1998), Models can be defined as representations of real systems. They can be iconic (made to look like the real system), abstract, or somewhere in between.

3.2 Classification of Models

The following are the various schemes by which models can be classified:

i. By degree of abstraction
ii. By function
iii. By structure
iv. By nature of the environment
v. By the extent of generality
vi. By the time horizon

Let us now briefly discuss the above classifications of models as presented by Gupta and Hira (2012)

i. By Degree of Abstraction.

Mathematical models such as Linear Programming formulation of the blending problem, or transportation problem are among the most abstract types of models since they require not only mathematical knowledge, but also great concentration to the real idea of the real-life situation they represent.

Language models such as languages used in cricket or hockey match commentaries are also abstract models.

Concrete models such as models of the earth, dam, building, or plane are the least abstract models since they instantaneously suggest the shape or characteristics of the modelled entity.

ii. By Function

The types of models involved here include

Descriptive models which explain the various operations in non-mathematical language and try to define the functional relationships and interactions between various operations. They simply describe some aspects of the system on the basis of observation, survey, questionnaire, etc. but do not predict its behaviour. Organisational charts, pie charts, and layout plan describe the features of their respective systems.

Predictive models explain or predict the behaviour of the system. Exponential smoothing forecast model, for instance, predict the future demand

Normative or prescriptive models develop decision rules or criteria for optimal solutions. They are applicable to repetitive problems, the solution process of which can be programmed without managerial involvement. Liner programming is also a prescriptive or normative model as it prescribes what the managers must follow.

iii. By Structure

- **Iconic or physical models**

In iconic or physical models, properties of real systems are represented by the properties themselves. Iconic models look like the real objects but could be scaled downward or upward, or could employ change in materials of real object. Thus, iconic models resemble the system they represent but differ in size, they are images. They thus could be full replicas or scaled models like architectural building, model plane, model train, car, etc.

- **Analogue or Schematic Models**

Analogue models can represent dynamic situations and are used more often than iconic models since they are analogous to the characteristics of the system being studied. They use a set of properties which the system under study possesses. They are physical models but unlike iconic models, they may or may not look like the reality of interest. They explain specific few characteristics of an idea and ignore other details of the object. Examples of analogue models are flow diagrams , maps, circuit diagrams, organisational chart etc.

- **Symbolic or mathematical models**

Symbolic models employ a set of mathematical symbols (letters, numbers etc.) to represent the decision variables of the system under study. These variables are related together by mathematical equations/in-equations which describe the properties of the system. A solution from the model is, then, obtained by applying well developed mathematical techniques. The relationship between velocity, acceleration, and distance is an example of a mathematical model. Similarly, cost-volume-profit relationship is a mathematical model used in investment analysis.

iv. By Nature of Environment

- **Deterministic models**

In deterministic models, variables are completely defined and the outcomes are certain. Certainty is the state of nature assumed in these models. They represent completely closed systems and the parameters of the systems have a single value that does not change with time. For any given set of input variables, the same output variables always result. E.O.Q model is deterministic because the effect of changes in batch size on total cost is known. Similarly, linear programming, transportation, and assignment models are deterministic models.

- **Probabilistic Models**

These are the products of the environment of risk and uncertainty. The input and/or output variables take the form of probability distributions. They are semi-closed models and represent the likelihood of occurrence of an event. Thus, they represent to an extent the complexity of the real world and uncertainty prevailing in it. As a example, the exponential smoothing method for forecasting demand a probabilistic model.

v. By Extent of Generality

- **General Models:** Linear programming model is known as a general model since it can be used for a number of functions e.g. product mix, production scheduling, and marketing of an organisation.

- **Specific Models:** Sales response curve or equation as a function of advertising is applicable to the marketing function alone.

vi. By the Time Horizon

a. Static Models: These are one time decision models. They represent the system at specified time and do not take into account the changes over time.
In this model, cause and effect occur almost simultaneously and the lag between the two is zero. They are easier to formulate, manipulate and solve.
EOQ is a static model.

b. Dynamic Models: These are models for situations for which time often plays an important role. They are used for optimisation of multi-stage decision problems which require a series of decisions with the outcome of each depending upon the results of the previous decisions in the series. Dynamic programming is a dynamic model

3.3 Characteristics of Good Models

The following are characteristics of good models as presented by Gupta and Hira (2012)

1. The number of simplifying assumptions should be as few as possible.
2. The number of relevant variables should be as few as possible. This means the model should be simple yet close to reality.
3. It should assimilate the system environmental changes without change in its framework.
4. It should be adaptable to parametric type of treatment.
5. It should be easy and economical to construct.

3.4 Advantages of a Model

1 It provides a logical and systematic approach to the problem.
2. It indicates the scope as well as limitation of the problem.
3. It helps in finding avenues for new research and improvement in a system.
4. It makes the overall structure of the problem more comprehensible and helps in dealing with the problem in its entirety.
5. It permits experimentation in analysis of a complex system without directly interfering in the working and environment of the system

3.5 Limitations of a Model

1. Models are more idealised representations of reality and should not be regarded as absolute in any case.
2. The reality of a model for a particular situation can be ascertained only by conducting experiments on it.

3.6 Constructing a Model

Formulating a problem requires an analysis of the system under study. This analysis shows the various phases of the system and the way it can be controlled.

Problem formulation is the first stage in constructing a model. The next step involves the definition of the measure of effectiveness that is, constructing a model in which the effectiveness of the system is expressed as a function of the variables defining the system. The general Operations Research form is

$$E = f(x_i, y_i),$$

Where E = effectiveness of the system,

x_i = controllable variables,

y_i = uncontrollable variables but do affect E.

Deriving a solution from such a model consists of determining those values of control variables xi, for which the measure of effectiveness is measure of effectiveness is optimised. Optimised includes both maximisation (in case of profit, production capacity, etc.) and minimisation (in case of losses, cost of production, etc.).

The following steps are involved in the construction of a model

1. Selecting components of the system
2. Pertinence of components
3. Combining the components
4. Substituting symbols

3.7 Types of Mathematical Models

The following are the types of mathematical models available:

1. Mathematical techniques
2. Statistical techniques
3. Inventory models
4. Allocation models
5. Sequencing models
6. Project scheduling by PERT and CPM
7. Routing models
8. Competitive models
9. Queuing models
10. Simulation techniques.

4.0 Conclusion

We have seen that models and model construction are very critical in the practice of operations research because they provide the process whereby a complex life problem situation is converted into simple representation of the problem situation. They further described a model as a simplified representation of complex reality. The basic objective of any model is to use simple inexpensive objects to represent complex and uncertain situations. Models are developed in such a way that they concentrate on exploring the key aspects or properties of the real object and ignore the other objects considered as being insignificant. Models are useful not only in science and technology but also in business decision making by focusing on the key aspects of the business decisions. As a result, no meaningful progress can be done in the field of operations research without representing a problem in the form of a model.

5.0 Summary

This unit introduced us to the concept of models. We have learnt about the importance of models to operations research. The unit opened with a consideration of various definitions of models. Among the definitions is that by Adebayo et al (2010) who defined modelling as a process whereby a complex life problem situation is converted into simple representation of the problem situation. A model as used in Operations Research is defined as an idealised representation of real life situation. It represents one of the few aspects of reality. Next, we gave the following categorisation of models- by degree of abstraction, by structure, by nature of the environment, by the extent of generality, by the time horizon. Further, we considered the advantages of a model, limitations of a model, characteristics of good model, constructing a model, and types of mathematical models.

The above topics have helped us in developing introductory knowledge of models, how they are constructed, their uses and characteristics. In subsequent chapters, we shall consider in details, some models and how they are applied in solving practical problems in operations research.

6.0 Tutor Marked Assignment

1. Differentiate between model and modelling.
2. List the different classifications of models we have.
3. List and explain the classification of models by structure.
4. Outline five characteristics of a good model.
5. List the advantages of a model.
6. Formulate and clearly describe a simplified version of and OR model.
7. List the types of mathematical models you know.

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UNIT 4 SIMULATION

1.0 Introduction

2.0 Objectives

3.0 Main Content

- 3.1 Definition
- 3.2 Advantages of Simulation Technique
- 3.3 Application of Simulation
- 3.4 Limitations of Simulation Technique
- 3.5 Monte Carlo Simulation

4.0 Conclusion

5.0 Summary

6.0 References

7.0 Tutor Marked Assignment

1.0 Introduction

Simulation is primarily concerned with experimentally predicting the behaviour of a real system for the purpose of designing the system or modifying behaviour (Budnick et al., 1988). The main reason for a researcher to resort to simulation is twofold. First of all, simulation is probably the most flexible tool imaginable. Take queuing as an example. While it is very difficult to incorporate reneging, jumping queues and other types of customer behaviour in the usual analytical models this presents no problem for simulation. A systemmay have to run for a very long time to reach a steady state. As a result, a modeller may be more interested in transient states, which are easily available in a simulation.

The second reason is that simulation is very cheap. Building a model that simulates the opening of a new restaurant will most certainly be a lot less expensive than trying it out. Even if costs are no subject, the time frame can be compressed in a simulation. For instance, if we were to observe the demand structure of a product, a long time would be required, so that results would probably be available when the product has become technologically obsolete anyway (Eiselt and Sandblom, 2012). This unit exposes us to the subject of simulation, and its various components.

2.0 OBJECTIVES

At the end of this unit, you should be able to

1. Define Simulation
2. Identify when to use simulation
3. Outline the advantages of simulation technique
4. Identify the areas of application of simulation
5. Describe the limitations of simulation
6. Explain the Monte Carlo Simulation

3.0 Main Content

3.1 Definition

According Budnick et al (1988), Simulation is primarily concerned with experimentally predicting the behaviour of a real system for the purpose of designing the system or modifying behaviour. In other words, simulation is a tool that builds a model of a real operation that is to be investigated, and then feeds the system with externally generated data. We generally distinguish between deterministic and stochastic simulation. The difference is that the data that are fed into the system are either deterministic or stochastic. This chapter will deal only with stochastic simulation, which is sometimes also referred to as Monte Carlo simulation in reference to the Monte Carlo Casinos and the (hopefully) random outcome of their games of chance.

According to Gupta and Hira (2012), simulation is an imitation of reality. “They further stated that simulation is the representation of reality through the use of models or other device which will react in the same manner as reality under given set of conditions. Simulation has also been defined the use of a system model that has the designed characteristics of reality in order to produce the essence of actual operation. According to Donald G. Malcom, a simulated model may be defined as one which depicts the working of a large scale system of men, machine, materials, and information operating over a period of time in a simulated environment of the actual real world condition.

A good example of simulation is a children amusement or a cyclical park where children enjoy themselves in a simulated environment like Amusement Parks, Disney Land, Planetarium shows where boats, train rides, etc. are done to simulate actual experience.

In simulation, operational information of the behaviour of a system which aides in decision making is obtained unlike that which exist in analytical modelling technique where optimal solution attempt is made to obtain descriptive information through experimentation. Generally, a simulation model is the totality of many simple models, and model interrelationship among system variables and components. A model can thus, be decomposed into many simple but related models. Models can be used for predicting the behaviour of a system under varying conditions. It focuses mainly on detailed physical or financial operations of a system. Model development through the use of computers for simulation has resulted in techniques for identifying possible optimal solution for a decision problem by evaluating various suggested alternatives and then suggesting results (Adebayo et al, 2010).

3.2 Advantages of Simulation Technique

When the simulation technique is compared with the mathematical programming and slandered probability analysis, offers a number of advantages over these techniques. Some of the advantages are:

1. Simulation offers solution by allowing experimentation with models of a system without interfering with the real system. Simulation is therefore a bypass for complex mathematical analysis.
2. Through simulation, management can foresee the difficulties and bottlenecks which may come up due to the introduction of new machines, equipment or process. It therefore eliminates the need for costly trial and error method of trying out the new concept on real methods and equipment.
3. Simulation is relatively free from mathematics, and thus, can be easily understood by the operating personnel and non-technical managers. This helps in getting the proposed plan accepted and implemented.
4. Simulation models are comparatively flexible and can be modified to accommodate the changing environment of the real situation.
5. Simulation technique is easier to use than mathematical models and its considered quite superior to mathematical analysis.
6. Computer simulation can compress the performance of a system over several years and involving large calculation into few minutes of computer running time.
7. Simulation has the advantage of being used in training the operating and managerial staff in the operation of complex plans.

3.3 Application of Simulation

Simulation is quite versatile and commonly applied technique for solving decision problems. It has been applied successfully to a wide range of problems of science and technology as given below:

1. In the field of basic sciences, it has been used to evaluate the area under a curve, to estimate the value of [illegible], in matrix inversion and study of particle diffusion.
2. In industrial problems including shop floor management, design of computer systems, design of queuing systems, inventory control, communication networks, chemical processes, nuclear reactors, and scheduling of production processes.
3. In business and economic problems, including customer behaviour, price determination, economic forecasting, portfolio selection, and capital budgeting.
4. In social problems, including population growth, effect of environment on health and group behaviour.
5. In biomedical systems, including fluid balance, distribution of electrolyte in human body, and brain activities.
6. In the design of weapon systems, war strategies and tactics.
7. In the study of projects involving risky investments.

3.4 Limitations of Simulation Technique

Despite the many advantages of simulation, it might suffer from some deficiencies in in large and complex problems. Some of these limitations are given as follows:

i. Simulation does not produce optimum results when the model deals with uncertainties, the results of simulation only reliable approximations subject to statistical errors.

ii. Quantification of variables is difficult in a number of situations; it is not possible to quantify all the variables that affect the behaviour of the system.

iii. In very large and complex problems, the large number of variables and the interrelationship between them make the problem very unwieldy and hard to program.

iv. Simulation is by no means, a cheap method of analysis.

v. Simulation has too much tendency to rely on simulation models. This results in application of the technique to some simple problems which can more appropriately be handled by other techniques of mathematical programming.

3.5 Monte Carlo Simulation

The Monte Carlo method of simulation was developed by two mathematicians Jon Von Neumann and Stainslaw Ulam, during World War II, to study how far neurone would travel through different materials. The technique provides an approximate but quite workable solution to the problem. With the remarkable success of this technique on the neutron problem, it soon became popular and found many applications in business and industry, and at present, forms a very important tool of operation researcher's tool kit.

The technique employs random number and is used to solve problems that involve probability and where physical experimentation is impracticable, and formulation of mathematical model is impossible. It is a method of simulation by sampling technique. The following are steps involved in carrying out Monte Carlo simulation.

1. Select the measure of effectiveness (objective function) of the problem. It is either to be minimised or maximised.
2. Identify the variables that affect the measure of effectiveness significantly. For example, a number of service facilities in a queuing problem or demand, lead time and safety stock in inventory problem.
3. Determine the cumulative probability distribution of each variable selected in step 2. Plot these distributions with the values of the variables along the x-axis and cumulative probability values along the y-axis.
4. Get a set of random numbers.
5. Consider each random number as a decimal value of the cumulative probability distribution. Enter the cumulative distribution along the y-axis. Project this point horizontally till it meets the distribution curve. Then project the point of distribution down on the x-axis.
6. Record the value (or values if several variables are being simulated) generated in step 5. Substitute the formula chosen for measure of effectiveness and find its simulated value.
7. Repeat steps 5 and 6 until sample is large enough to the satisfaction of the decision maker.

Let us consider a simple example as presented by Gupta and Hira (2012).

EXAMPLE

Customers arrive at a service facility to get required service. The interval and service times are constant and are 1.8minutes and minutes respectively. Simulate the system for 14minutes. Determine the average waiting time of a customer and the idle time of the service facility.

Solution

The arrival times of customers at the service station within 14 minutes will be:

Customer :	1	2	3	4	5	6	7	8
Arrival time : *(minutes)*	0	1.8	3.6	5.4	7.2	9.0	10.8	12.6

The time at which the service station begins and ends within time period of 14 minutes is shown below. Waiting time of customers and idle time of service facility are also calculated

Customer	*Service Begins*	*Service ends*	*Waiting time of customer*	*Idle time of service facility*
1	0	4	0	0
2	4	8	4-1.8 = 2.2	0
3	8	12	8-3.6 = 4.4	0
4	12	16	12-5.4 = 6.6	0

The waiting time of the first four customers is calculated above. For the remaining, it is calculated below.

Customer :	5	6	7	8
Waiting time (min) :	14 – 7.2 = 6.8	5.0	3.2	1.4

Therefore, average waiting time of a customer

$$= \frac{0 + 2.2 + 4.4 + 6.6 + 6.8 + 5 + 3.2 + 1.4}{8} = \frac{29.6}{8} = 3.7 \text{ minutes}$$

Idle time of facility = nil.

4.0 Conclusion

Simulation is a very important tool in OR. Most times, it is seen as the last resort when all other efforts have failed, and simulation is considered as the last resort.

This is because simulating a real life system could be quite expensive and time consuming. In simulation, operational information of the behaviour of a system which aides in decision making is obtained unlike that which exist in analytical modelling technique where optimal solution attempt is made to obtain descriptive information through experimentation. Generally, a simulation model is the totality of many simple models, and model interrelationship among system variables and components. A model can thus, be decomposed into many simple but related models. Models can be used for predicting the behaviour of a system under varying conditions. However, simulation has its own weaknesses as it does not produce optimum results when the model deals with uncertainties, the results of simulation only reliable approximations subject to statistical errors. Quantification of variables is difficult in a number of situations; it is not possible to quantify all the variables that affect the behaviour of the system. In very large and complex problems, the large number of variables and the interrelationship between them make the problem very unwieldy and hard to program.

5.0 Summary

This unit provides for us an overview of simulation. It takes us through various conceptualisations on the definition of simulation. Simulation has been defined as the representation of reality through the use of models or other device which will react in the same manner as reality under given set of conditions. A good example of simulation is a children amusement or a cyclical park where children enjoy themselves in a simulated environment like Amusement Parks, Disney Land, Planetarium shows where boats, train rides, etc. are done to simulate actual experience. It is quite versatile and commonly applied technique for solving

decision problems such as basic sciences, in industrial problems including shop floor management, in business and economic problems etc.

However, simulation does not produce optimum results when the model deals with uncertainties, the results of simulation only reliable approximations subject to statistical errors. Quantification of variables is difficult in a number of situations; it is not possible to quantify all the variables that affect the behaviour of the system.

Finally, we discussed the concept of Monte Carlo Simulation which was developed by two mathematicians Jon Von Neumann and Stainslaw Ulam, during World War II, to study how far neurone would travel through different materials. The technique provides an approximate but quite workable solution to the problem. With the remarkable success of this technique on the neutron problem, it soon became popular and found many applications in business and industry, and at present, forms a very important tool of operation researcher's tool kit.

6.0 Tutor Marked Assignment

1. What do you understand by the term Simulation?
2. Explain six (6) advantages if Simulation.
3. Identify and explain five (5) areas of application of Simulation.
4. Give five limitations of Simulation.
5. What is Monte Carlo Simulation?
6. Describe the steps involved in Monte Carlo Simulation.

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UNIT 5: SYSTEMS ANALYSIS

1.0 Introduction

2.0 Objectives

3.0 Main content

- 3.1 Definition
- 3.2 The Systems Theory
- 3.3 Elements of a System
- 3.4 Types of Systems
- 3.5 Forms of Systems
  - 3.5.1 Conceptual System
  - 3.5.2 Mechanical System
  - 3.5.3 Social System
  - 3.5.4 Deterministic System
  - 3.5.5 Probabilistic System
- 3.6 The Concept of Entropy in a System

4.0 Conclusion

5.0 Summary

6.0 Tutor Marked Assignment

7.0 References

1.0 Introduction

The word system has a long history which can be traced back to Plato (Philebus), Aristotle (Politics) and Euclid (Elements). It had meant "total", "crowd" or "union" in even more ancient times, as it derives from the verb sunìstemi, uniting, putting together.

"System" means "something to look at". You must have a very high visual gradient to have systematization. In philosophy, before Descartes, there was no "system". Plato had no "system". Aristotle had no "system"(McLuhan. 1967)

In the 19th century the first to develop the concept of a "system" in the natural sciences was the French physicist Nicolas Léonard Sadi Carnot who studied thermodynamics. In 1824 he studied the system which he called the working substance, i.e. typically a body of water vapour, in steam engines, in regards to the system's ability to do work when heat is applied to it. The working substance could be put in contact with either a boiler, a cold reservoir (a stream of cold water), or a piston (to which the working body could do work by pushing on it). In 1850, the German physicist Rudolf Clausius generalized this picture to include the concept of the surroundings and began to use the term "working body" when referring to the system.

One of the pioneers of the general systems theory was the biologist Ludwig von Bertalanffy. In 1945 he introduced models, principles, and laws that apply to generalized systems or their subclasses, irrespective of their particular kind, the nature of their component elements, and the relation or 'forces' between them.

Significant development to the concept of a system was done by Norbert Wiener and Ross Ashby who pioneered the use of mathematics to study systems. (*Cybernetics,* 1948 *)(Chapman& Hall, 1956)*

In business, System Analysis and Design refers to the process of examining a business situation with the intent of improving it through better procedures and methods. System analysis and design relates to shaping organizations, improving performance and achieving objectives for profitability and growth. The emphasis is on systems in action, the relationships among subsystems and their contribution to meeting a common goal.

2.0 Objectives

After studying this unit, you should be able to

1. Define a system
2. Identify and describe the types of systems
3. Highlight the different forms of systems we have
4. Describe how a system is analysed
5. Discuss the concept of entropy

3.0 Main content

3.1 Definition

The term system is derived from the Greek word *systema*, which means an organized relationship among functioning units or components. A system exists because it is designed to achieve one or more objectives. We come into daily contact with the transportation system, the telephone system, the accounting system, the production system, and, for over two decades, the computer system. Similarly, we talk of the business system and of the organization as a system consisting of interrelated departments (subsystems) such as production, sales, personnel, and an information system. None of these subsystems is of much use as a single, independent unit. When they are properly coordinated, however, the firm can function effectively and profitably.

There are more than a hundred definitions of the word system, but most seem to have a common thread that suggests that a system is an orderly grouping of interdependent components linked together according to a plan to achieve a specific objective. The word component may refer to physical parts (engines, wings of aircraft, car), managerial steps (planning, organizing and controlling), or a system in a multi-level structure. The component may be simple or complex, basic or advanced. They may be single computer with a keyboard, memory, and printer or a series of intelligent terminals linked to a mainframe. In either case, each component is part of the total system and has to do its share of work for the system to achieve the intended goal. This orientation requires an orderly grouping of the components for the design of a successful system.

The study of systems concepts, then, has three basic implications:

1. A system must be designed to achieve a predetermined objective.
2. Interrelationships and interdependence must exist among the components.
3. The objectives of the organization as a whole have a higher priority than the objectives of its subsystems. For example, computerizing personnel applications must conform to the organization's policy on privacy, confidentiality and security, as well as making selected data (e.g. payroll) available to the accounting division on request.(Jawahar, 2006)

A system can also be defined is a collection of elements or components or units that are organized for a common purpose. The word sometimes describes the organization or plan itself (and is similar in meaning to method, as in "I have my own little system") and sometimes describes the parts in the system (as in "computer system").

According to the International Council of Systems Engineers (INCOSE), a system can be broadly defined as an integrated set of elements that accomplish a defined objective. People from different engineering disciplines have different perspectives of what a "system" is. For example, software engineers often refer to an integrated set of computer programs as a "system." Electrical engineers might refer to complex integrated circuits or an integrated set of electrical units as a "system." As can be seen, "system" depends on one’s perspective, and the “integrated set of elements that accomplish a defined objective” is an appropriate definition.

3.2 The Systems Theory

The general systems theory states that a system is composed of inputs, a process, outputs, and control. A general graphic representation of such a system is shown below.

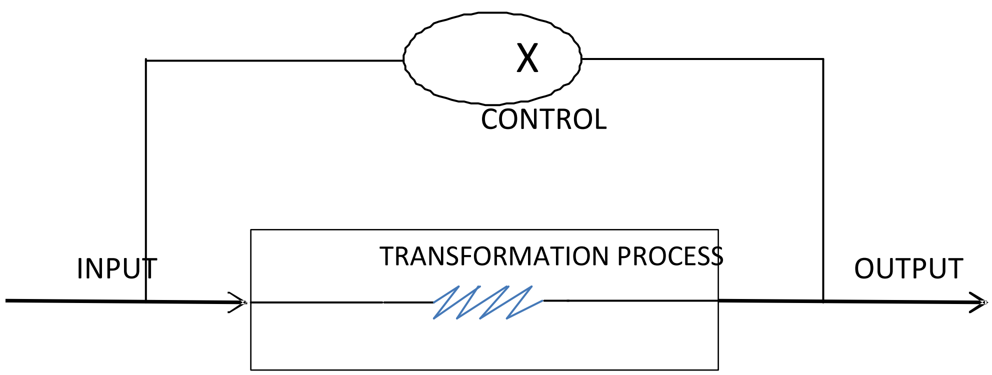

Fig. 8.1: An Operational System
Adapted from Ihemeje, (2002), Fundamentals of Business Decision Analysis, Lagos- Sibon Books Limited.

The input usually consists of people, material or objectives. The process consists of plant, equipment and personnel. While the output usually consists of finished goods, semi-finished goods, policies, new products, ideas, etc.

The purpose of a system is to transform inputs into outputs. The system theory is relevant in the areas of systems design, systems operation and system control. The systems approach helps in resolving organisational problems by looking at the organisation as a whole, integrating its numerous complex operations, environment, technologies, human and material resources. The need to look at the organisation in totality is premised on the fact that the objective if the different units of the organisation when pursued in isolation conflict with one another. For instance, the operation of a manufacturing department favours long and uninterrupted production runs with a view to minimising unit cost of production, including set-up costs. However, this will result in large inventories, and leading to high inventory costs. The finance department seeks to minimise costs as well as capital tied down in inventories. Thus, there is a desire for rapid inventory turnover resulting in lower inventory levels. The marketing department seeks favourable customer service and as a result, will not support any policy that encourages stock outs or back ordering. Back ordering is a method of producing later to satisfy a previously unfulfilled order. Consequently, marketing favours the maintenance of high inventory levels in a wide variety of easily accessible locations which in effect means some type of capital investment in warehouse or sales outlets. Finally, personnel department aims at stabilizing labour, minimizing the cost of firing and hiring as well as employee discontentment. Hence, it is desirable from the point of view of personnel to maintain high inventory level of producing even during periods of fall in demand.

Therefore, pursuing the interest of a section of the organization can result in solution which will not be optional from the point of view of the total organization, yet 'optimal' from the point of view of the section concerned. Such a situation is called sub-optimization. Adoption of the systems approach will eliminate this. The systems approach will produce an optimal solution, which attempts to resolve the conflicting objectives of the various sub-units of the organization. The adoption of the systems approach in OR methodology therefore puts its shoulder above other focus mainly on the solution of functional areas-based management problems only, i.e. those that adopt the piecemeal approach.

3.3 Elements of a System

Following are considered as the elements of a system in terms of Information systems:

- Input
- Output
- Processor
- Control
- Feedback
- Boundary and interface
- Environment
- INPUT: Input involves capturing and assembling elements that enter the system to be processed. The inputs are said to be fed to the systems in order to get the output. For example, input of a 'computer system' is input unit consisting of various input devices like keyboard, mouse, joystick etc.
- OUTPUT: The element that exists in the system due to the processing of the inputs is known as output. A major objective of a system is to produce output that has value to its user. The output of the system maybe in the form of cash, information, knowledge, reports, documents etc. The system is defined as output is required from it. It is the anticipatory recognition of output that helps in defining the input of the system. For example, output of a 'computer system' is output unit consisting of various output devices like screen and printer etc.
- PROCESSOR(S): The processor is the element of a system that involves the actual transformation of input into output. It is the operational component of a system. For example, processor of a 'computer system' is central

processing unit that further consists of arithmetic and logic unit (ALU), control unit and memory unit etc.

- CONTROL: The control element guides the system. It is the decision-making sub-system that controls the pattern of activities governing input, processing and output. It also keeps the system within the boundary set. For example, control in a 'computer system' is maintained by the control unit that controls and coordinates various units by means of passing different signals through wires.
- FEEDBACK: Control in a dynamic system is achieved by feedback. Feedback measures output against a standard in some form of cybernetic procedure that includes communication and control. The feedback may generally be of three types viz., positive, negative and informational. The positive feedback motivates the persons in the system. The negative indicates need of an action, while the information. The feedback is a reactive form of control. Outputs from the process of the system are fed back to the control mechanism. The control mechanism then adjusts the control signals to the process on the basis of the data it receives. Feed forward is a protective form of control. For example, in a 'computer system' when logical decisions are taken, the logic unit concludes by comparing the calculated results and the required results.
- BOUNDARY AND INTERFACE: A system should be defined by its boundaries-the limits that identify its components, processes and interrelationships when it interfaces with another system. For example, in a 'computer system' there is boundary for number of bits, the memory size etc. That is responsible for different levels of accuracy on different machines

(like 16-bit,32-bit etc.). The interface in a 'computer system' may be CUI (Character User Interface) or GUI (Graphical User Interface).

- ENVIRONMENT: The environment is the 'supersystem' within which an organisation operates. It excludes input, processes and outputs. It is the source of external elements that impinge on the system. For example, if the results calculated/the output generated by the 'computer system' are to be used for decision-making purposes in the factory, in a business concern, in an organisation, in a school, in a college or in a government office then the system is same but its environment is different.

3.4 Types of Systems

Systems are classified in different ways:

1. Physical or abstract systems.
2. Open or closed systems.
3. 'Man-made' information systems.
4. Formal information systems.
5. Informal information systems.
6. Computer-based information systems.
7. Real-time system.

Physical systems are tangible entities that may be static or dynamic in operation.
An open system has many interfaces with its environment. i.e. system that interacts freely with its environment, taking input and returning output. It permits interaction across its boundary; it receives inputs from and delivers outputs to the outside. A closed system does not interact with the environment; changes in the environment and adaptability are not issues for closed system.

3.5 Forms of systems

A system can be conceptual, mechanical or social. A system can also be deterministic or probabilistic. A system can be closed or open.

Conceptual system

A system is conceptual when it contains abstracts that are linked to communicate ideas. An example of a conceptual system is a language system as in English language, which contains words, and how they are linked to communicate ideas. The elements of a conceptual system are words.

Mechanical system

A system is mechanical when it consists of many parts working together to do a work. An example of a social system is a typewriter or a computer, which consists of many parts working together to type words and symbols. The elements of the mechanical system are objects.

Social system

A system is social when it comprises policies, institutions and people. An example of a social system is a football team comprising 11 players, or an educational system consisting of policies, schools and teachers. The elements of a social system are subjects or people.

Deterministic system

A system is deterministic when it operates according to a predetermined set of rules. Its future behaviour can therefore be predicted exactly if it's present state and operating characteristics are accurately known. Example s of deterministic systems are computer programmes and a planet in orbit. Business systems are not deterministic owing to the fact that they interfere with a number of in determinant factors, such as customer and supplier behaviour, national and international situations, and climatic and political conditions.

Probabilistic system

A system is probabilistic when the system is controlled by chance events and so its future behaviour is a matter of probability rather than certainty. This is true of all social systems, particularly business enterprises. Information systems are deterministic enterprises in the sense that a pre-known type and content of information emerges as a result of the input of a given set of data. This assumes that the information system operates according to pre-decided and formulated rules – which it generally would do. In a broader sense, information systems can be regarded as probabilistic because the wide variability in the nature of their input introduces many indeterminate and of their future behaviour i.e. output is not absolutely certain.

Closed system

A system is closed when it does not interface with its environment i.e. it has no input or output. This concept is more relevant to scientific systems that to social systems. The nearest we can get to a closed social system would be a completely self-contained community that provides all its own food, materials and power, and does not trade, communicate or come into contact with other communities.

Open system

A system is open when it has many interfaces with its environment, and so needs to be capable of adopting their behaviour in order to continue to exist in changing environments. An information system falls into this category since it needs to adapt to the changing demands for information. Similarly, a business system must be capable of reorganizing itself to meet the conditions of its environment, as detected from its input; it will more rapidly tend towards a state of disorganization. When functioning properly, an open system reaches a state of dynamic equilibrium. This is a steady state in which the system readily adapts to environmental factors by re-organizing itself according to the internal forces of its sub-systems. With a manufacturing company, for instance, the steady state can be thought of as the purchasing of materials and productive means, and the manufacturing and selling of products. An environmental factor could be an increase in the selling prices of its products (Ihemeje, 2002)

3.6 The Concept of Entropy in a System

The term entropy is used as a measure of disorganisation. Thus, we can regard open systems as tending to increase their entropy unless they receive negative entropy in the form of information from their environment. In the above example, if increased cost of cost of materials were ignored, the product will become unprofitable and as a result, the organisation may become insolvent, that is, a state of disorganisation.

Systems analysis is an activity, process, or study of critically examining the ways performing frequently occurring tasks that depend on the movement processing of information by a number of people within an organisation. System analysis may be carried out to either install a new system or overhaul an already existing one. This implies that a system is analysed for three main purposes- system design, system operation, and system control (Ihemeje, 2002)

4.0 Conclusion

In our everyday life, the word system is widely used. It has become fashionable to attach the word system to add a contemporary flair when referring to things or processes. People speak of exercise system, investment system, delivery system, information system, education system, computer system etc. System may be referred to any set of components, which function in interrelated manner for a common cause or objective.

A system exists because it is designed to achieve one or more objectives. We come into daily contact with the transportation system, the telephone system, the accounting system, the production system, and, for over two decades, the computer system. Similarly, we talk of the business system and of the organization as a system consisting of interrelated departments (subsystems) such as production, sales, personnel, and an information system. None of these subsystems is of much use as a single, independent unit. When they are properly coordinated, however, the firm can function effectively and profitably. There are more than a hundred definitions of the word system, but most seem to have a common thread that suggests that a system is an orderly grouping of interdependent components linked together according to a plan to achieve a specific objective. The word component may refer to physical parts (engines, wings of aircraft, car), managerial steps (planning, organizing and controlling), or a system in a multi-level structure. The component may be simple or complex, basic or advanced. They may be single computer with a keyboard, memory, and printer or a series of intelligent terminals linked to a mainframe. In either case, each component is part of the total system and has to do its share of work for the system to achieve the intended goal. This orientation requires an orderly grouping of the components for the design of a successful system.

5.0 Summary

This unit discusses the concept of systems analysis. The origin of system analysis has been traced to the Greek word *systema*, which means an organized relationship among functioning units or components. A system exists because it is designed to achieve one or more objectives. It can be defined is a collection of elements or components or units that are organized for a common purpose. The general systems theory states that a system is composed of inputs, a process, outputs, and control. The input usually consists of people, material or objectives. The process consists of plant, equipment and personnel. While the output usually consists of finished goods, semi-finished goods, policies, new products, ideas, etc. A system consists of the following element: input, output, processor, control, feedback, boundary and interface, and environment. Depending on the usage, a system has the following are types of systems: Physical or abstract systems, Open or closed systems, Man-made information systems, Formal information systems, Informal information systems, Computer-based information systems and Real-time system.
A system can be conceptual, mechanical or social. A system can also exist in the following forms- it can be deterministic or probabilistic, closed or open, mechanical, social, and conceptual.

It has been quite an exciting journey through the world of systems analysis.

6.0 Tutor Marked Assignment

1 What do you understand term system?

2 With the aid of a well labelled diagram, describe how a system works.

3 What understand by the concept of entropy of a system?

4 List and explain the elements of a system.

5 Differentiate between an open and a closed system.

6 Identify the elements that make up the process component of a system.

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MODULE FOUR

UNIT 1: SEQUENCING

1.0 Introduction

2.0 Objectives

3.0 Main Content

3.1 The Problem (Definition)

3.2 Assumptions Made in Sequencing Problems

3.3 Nature of Scheduling

3.4 Loading Jobs in Work Centres

3.4.1 Gantt Charts

3.4.2 Assignment Method

3.5 Priority Rules for Job Sequencing

3.6 Applicability

3.7 Types of Sequencing Problems

3.7.1 Sequencing Jobs in two Machines

4.0 Conclusion

5.0 Summary

6.0 Tutor Marked Assignment

7.0 References

1.0 INTRODUCTION

Sequencing problems involves the determination of an optimal order or sequence of performing a series jobs by number of facilities (that are arranged in specific order) so as to optimize the total time or cost. Sequencing problems can be classified into two groups:

The first group involves n different jobs to be performed, and these jobs require processing on some or all of m different types of machines. The order in which these machines are to be used for processing each job (for example, each job is to be processed first on machine A, then B, and thereafter on C i.e., in the order ABC) is given. Also, the expected actual processing time of each job on each machine is known. We can also determine the effectiveness for any given sequence of jobs on each of the machines and we wish to select from the $(n!)^m$ theoretically feasible alternatives, the one which is both technologically feasible and optimises the effectiveness measure.

The second group of problems deal with the shops having a number of machines and a list of tasks to be performed. Each time a task is completed by a machine, the next task to be started on it has to be decided. Thus, the list of tasks will change as fresh orders are received.

Unfortunately, types of problems are intrinsically difficult. While solutions are possible for some simple cases of the first type, only some empirical rules have been developed for the second type till now (Gupta and Hira, 2012).

OBJECTIVES

After completing this chapter, you should be able to:

1. Explain what scheduling involves and the nature of scheduling.
2. Understand the use of Gantt charts and assignment method for loading jobs in work centres.
3. Discuss what sequencing involves and the use of priority rules.
4. Solve simple problems on scheduling and sequencing.

3.1 DEFINITION

Scheduling refers to establishing the timing of the use of equipment, facilities and human activities in an organization, that is, it deals with the timing of operations. Scheduling occurs in every organization, regardless of the nature of its operation. For example, manufacturing organizations, hospitals, colleges, airlines e.t.c. schedule their activities to achieve greater efficiency. Effective Scheduling helps companies to use assets more efficiently, which leads to cost savings and increase in productivity. The flexibility in operation provides faster delivery and therefore, better customer service. In general, the objectives of scheduling are to achieve trade-offs among conflicting goals, which include efficient utilization of staff, equipment and facilities and minimization of customer waiting tune, inventories and process times (Adebayo et al, 2006).

Job sequencing refers to the order in which jobs should be processed at each workstation. Sequencing decisions determine both the order in which jobs are processed at various work centres and the order in which jobs are processed at individual workstations within the work centres. For example, suppose that 20 computers are to be repaired. In what order should they be repaired? Should it be done on the basis of urgency or first come first served? Job sequencing methods provide such detailed information. Typically, a number of jobs will be waiting for processing. Priority rules are the methods used for dispatching jobs to work centres (Adebayo et al, 2006)

A general sequencing problem may be defined as follows:

Let there be '*n*' jobs (J_1, J_2, J_3 ....J_n) which are to be processed on '*m*' machines (*A*, *B*, *C*,.........), where the order of processing on machines *i.e.* for example, *ABC* means first on machine A, second on machine *B* and third on machine *C* or *CBA* means first on machine *C*, second on machine *B*and third on machine A etc. and the processing time of jobs on machines (actual or expected) is known to us, then our job is to find the optimal sequence of processing jobs that minimizes the total processing time or cost. Hence our job is to find that sequence out of $(n!)m$ sequences, which minimizes the total elapsed time (*i.e.*. time taken to process all the jobs). The usual notations used in this problem are:

Ai = Time taken by i th job on machine A where i = I, 2,3…n. Similarly we can interpret for machine B and C *i.e.* Bi and Ci etc.

T = Total elapsed time which includes the idle time of machines if any and set up time and transfer time.

3.2 ASSUMPTIONS MADE IN SEQUENCING PROBLEMS

Principal assumptions made for convenience in solving the sequencing problems are as follows:

1 The processing times A_i and B_i etc. are exactly known to us and they are independent of order of processing the job on the machine. That is whether job is done first on the machine, last on the machine, the time taken to process the job will not vary it remains constant.

2 The time taken by the job from one machine to other after processing on the previous machine is negligible. (Or we assume that the processing time given also includes the transfer time and setup time).

3 Only one operation can be carried out on a machine at a particular time.

4 Each job once started on the machine, we should not stop the processing in the middle. It is to be processed completely before loading the next job.

5 The job starts on the machine as soon as the job and the machine both become idle (vacant). This is written as job is next to the machine and the

machine is next to the job. (This is exactly the meaning of transfer time is negligible).

6 No machine may process more than one job simultaneously. (This means to say that the job once started on a machine, it should be done until completion of the processing on that machine).

7 The cost of keeping the semi-finished job in inventory when next machine on which the job is to be processed is busy is assumed to be same for all jobs or it is assumed that it is too small and is negligible. That is in process inventory cost is negligible.

8 While processing, no job is given priority *i.e.* the order of completion of jobs has no significance. The processing times are independent of sequence of jobs.

9 There is only one machine of each type.

3.3 Nature of Scheduling

Scheduling technique depends on the volume of system output, the nature of operations and the overall complexity of jobs. Flow shop systems require approaches substantially different from those required by job shops. The complexity of operations varies under these two situations.

1. **Flow Shop**

Flow shop is a high-volume system, which is characterized by a continuous flow of jobs to produce standardized products. Also, flow shop uses standardized equipment (i.e. special purposed machines) and activities that provide mass production. The goal is to obtain a smooth rate of flow of goods or customer through the system in order to get high utilization of labour and equipment. Examples are refineries, production of detergents etc.

2. **Job Shop**

This is a low volume system, which periodically shift from one job to another. The production is according to customer's specifications and orders or jobs usually in small lots. General-purpose machines characterize Job shop. For example, in designer shop, a customer can place order for different design.

Job-shop processing gives rise to two basic issues for schedulers: how to distribute the workload among work centre and what job processing sequence to use.

3.4 Loading Jobs in Work Centres

Loading refers to the assignment of jobs to work centres. The operation managers are confronted with the decision of assigning jobs to work centres to minimize costs, idle time or completion time.

The two main methods that can be used to assign jobs to work centres or to allocate resources are:

1. Gantt chart
2. Assignment method of linear programming

3.4.1 Gantt Charts

Gantt charts are bar charts that show the relationship of activities over some time periods. Gantt charts are named after Henry Gantt, the pioneer who used charts for industrial scheduling in the early 1900s. A typical Gantt chart presents time scale horizontally, and resources to be scheduled are listed vertically, the use and idle times of resources are reflected in the chart.

The two most commonly used Gantt charts are the schedule chart and the load chart.

3.4.2 Assignment Method

Assignment Model (AM) is concerned specifically with the problem of job allocation in a multiple facility production configuration. That is, it is useful in situations that call for assigning tasks or jobs to resources. Typical examples include assigning jobs to machines or workers, territories to sales people e.t.c. One important characteristic of assignment problems is that only one job (or worker) is assigned to one machine (or project). The idea is to obtain an optimum matching of tasks and resources. A chapter in this book has treated the assignment method.

3.5 PRIORITY RULES FOR JOB SEQUENCING

Priority rules provide means for selecting the order in which jobs should be done (processed). In using these rules, it is assumed that job set up cost and time are independent of processing sequence. The main objective of priority rules is to minimize completion time, number of jobs in the system, and job lateness, while maximizing facility utilization. The most popular priority rules are:

I. First Come, First Serve (FCFS): Job is worked or processed in the order of arrivals at the work centre.

2. Shortest Processing Time (SPT): Here, jobs are processed based on the length of processing time. The job with the least processing time is done first.

3. Earliest Due Date (EDD): This rule sequences jobs according to their due dates, that is, the job with the earliest due date is processed first.

4. Longest Processing Time (LPT): The job with the longest processing time is started first.

5. Critical Ratio: Jobs are processed according to smallest ratio of time remaining until due date to processing time remaining.

The effectiveness of the priority rules is frequently measured in the light of one or more performance measures namely; average number of jobs, job flow time, job lateness, make span, facility utilisation etc.

3.6 Applicability

The sequencing problem is very much common in Job workshops and Batch production shops. There will be number of jobs which are to be processed on a series of machine in a specified order depending on the physical changes required on the job. We can find the same situation in computer centre where number of problems waiting for a solution. We can also see the same situation when number of critical patients waiting for treatment in a clinic and in Xerox centres, where number of jobs is in queue, which are to be processed on the Xerox machines. Like this we may find number of situations in real world.

3.7 Types of Sequencing Problems

There are various types of sequencing problems arise in real world. All sequencing problems cannot be solved. Though mathematicians and Operations Research scholars are working hard on the problem satisfactory method of solving problem is available for few cases only. The problems, which can be solved, are:

(a) 'n' jobs are to be processed on two machines say machine A and machine B in the order AB. This means that the job is to be processed first on machine A and then on machine B.

(b) 'n' jobs are to be processed on three machines A,B and C in the order ABC *i.e.* first on machine A, second on machine B and third on machine C.

(c) 'n' jobs are to be processed on 'm' machines in the given order.

d) Two jobs are to be processed on 'm' machines in the given order.

(Murthy, 2007)

- **Single Machine Scheduling Models**

The models in this section deal with the simplest of scheduling problems: there is only a single machine on which tasks are to be processed. Before investigating thesolutions that result from the use of the three criteria presented in the introduction

- **'N' Jobs and Two Machines**

If the problem given has two machines and two or three jobs, then it can be solved by using the Ganttchart. But if the numbers of jobs are more, then this method becomes less practical. (For understanding about the Gantt chart, the students are advised to refer to a book on Production and Operations Management (chapter on Scheduling). Gantt chart consists of X-axis on which the time is noted and Y-axis on which jobs or machines are shown. For each machine a horizontal bar is drawn. On these bars the processing of jobs in given sequence is marked. Let us take a small example and see how Gantt chart can be used to solve the same.

EXAMPLE 13.1

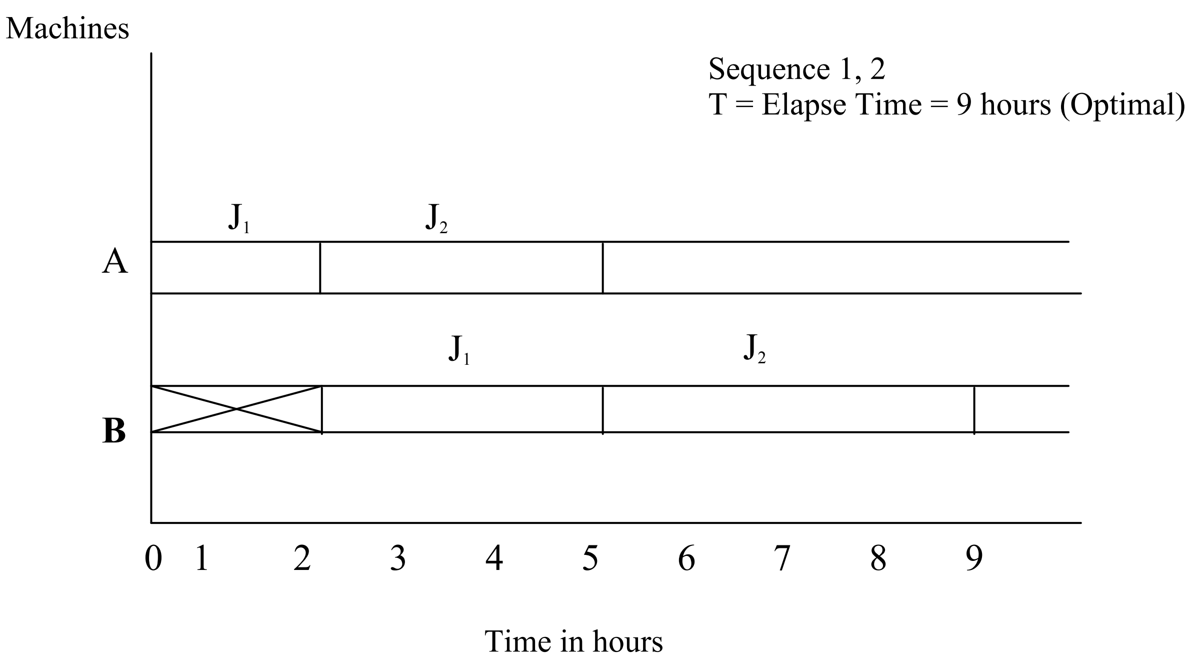
There are two jobs job 1 and job 2. They are to be processed on two machines, machine A and Machine B in the order AB. Job 1 takes 2 hours on machine A and 3 hours on machine B. Job 2 takes 3 hours on machine A and 4 hours on machine B. Find the optimal sequence which minimizes the total elapsed time by using Gantt chart.

Solution

Jobs.	*Machines (Time in hours)*	
	A	*B*
1	*2*	*3*
2	*3*	*4*

(a) Total elapsed time for sequence 1,2*i.e.* first job 1 is processed on machine A and then onsecond machine and so on.

Draw X - axis and Y- axis, represent the time on X - axis and two machines by two bars on Yaxis. Then mark the times on the bars to show processing of each job on that machine.



Sequence 1, 2
T = Elapse Time = 9 hours (Optimal sequence)

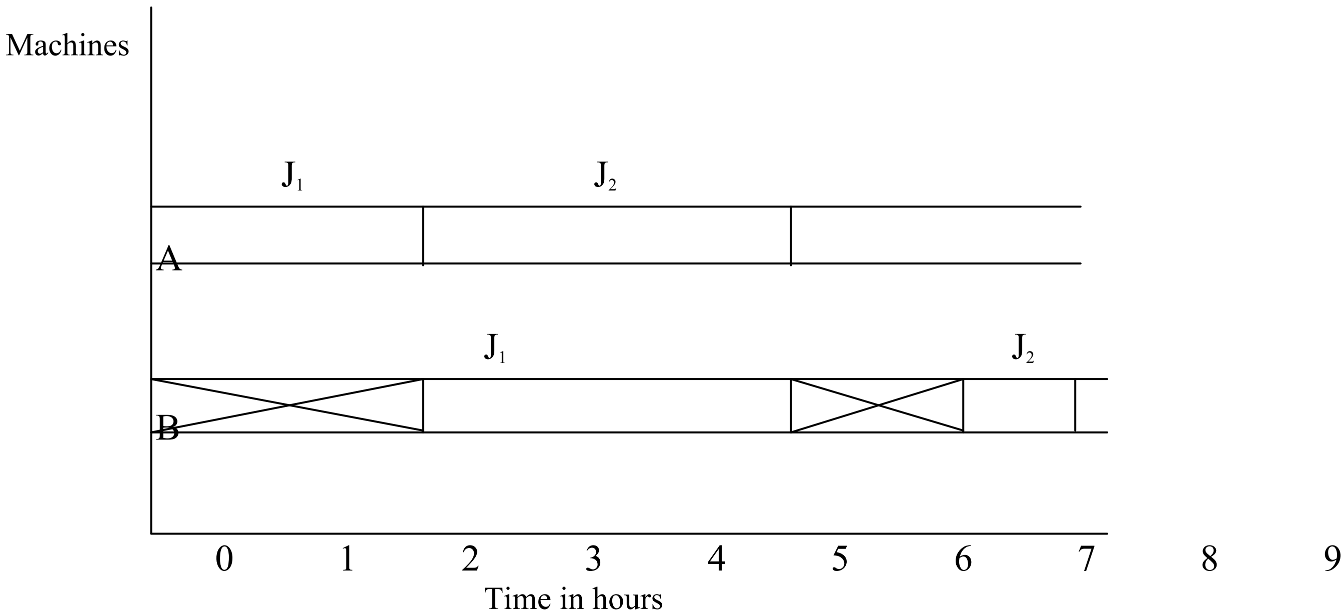


Fig. 13.1: *Gantt chart.*

Source: Murthy, R. P. (2007), Operations Research, 2nd ed., New Delhi: New Age International (P) LimitedPublisheN

Both the sequences shows the elapsed time = 9 hour

The drawback of this method is for all the sequences, we have to write the Gantt chart and find the total elapsed times and then identify the optimal solution. This is laborious and time consuming. If we have more jobs and more machines, then it is tedious work to draw the chart for all sequences.

Hence we have to go for analytical methods to find the optimal solution without drawing charts.

1 Analytical Method

A method has been developed by **Johnson and Bellman** for simple problems to determine a sequence of jobs, which minimizes the total elapsed time. The method:

1. 'n' jobs are to be processed on two machines A and B in the order AB (*i.e.* each job is to be processed first on A and then on B) and passing is not allowed. That is whichever job is processed first on machine A is to be first processed on machine B also, whichever job is processed second on machine A is to be processed second on machine B also and so on. That means each job will first go to machine A get processed and then go to machine B and get processed. *This rule is known as no passing rule***.**

2. Johnson and Bellman method concentrates on minimizing the idle time of machines. Johnson and Bellman have proved that optimal sequence of 'n' jobs which are to be processed on two machines A and B in the order AB necessarily involves the same ordering of jobs on each machine. This result also holds for three machines but does not necessarily hold for more than three machines. Thus total elapsed time is minimum when the sequence of jobs is same for both the machines.

3. Let the number of jobs be 1,2,3,…………n

The processing time of jobs on machine A be$A1$, $A2$, $A3$ …………. An

The processing time of jobs on machine B be$B1$, $B2$, $B3$ …………..Bn

Jobs	*Machine Time in Hours*		
	Machine A	*Machine B*	*Order of Processing is AB*
1	A_1	B_1	
2	A_2	B_2	
3	A_3	B_3	
I	A_I	B_I	
S	A_S	B_S	

T	A_T	B_T	
N	A_N	B_N	

4. Johnson and Bellman algorithm for optimal sequence states *that identify the smallest element in the given matrix. If the smallest element falls under column 1 i.e under machine I then do that job first.* As the job after processing on machine 1 goes to machine2, it reduces the idle time or waiting time of machine 2. *If the smallest element falls under column 2 i.e under machine 2 then do that job last.* This reduces the idle time of machine1. *i.e.* if r the job is having smallest element in first column, then do the r^{th} job first. If *s* the job has the smallest element, which falls under second column, then do the *s* the job last.

Hence the basis for Johnson and Bellman method is to keep the idle time of machines as low as possible. Continue the above process until all the jobs are over.

1	*2*	*3……….*		*n-1*	*n*
r					*s*

If there are '*n*' jobs, first write '*n*' number of rectangles as shown. Whenever the smallest elements falls in column 1 then enter the job number in first rectangle. If it falls in second column, then write the job number in the last rectangle. Once the job number is entered, the second rectangle will become first rectangle and last but one rectangle will be the last rectangle.

5. Now calculate the total elapsed time as discussed. Write the table as shown. Let us assume that the first job starts at Zero th time. Then add the processing time of job (first in the optimal sequence) and write in out column under machine 1. This is the time when the first job in the optimal sequence leaves

machine 1 and enters the machine 2. Now add processing time of job on machine 2. This is the time by which the processing of the job on two machines over. Next consider the job, which is in second place in optimal sequence. This job enters the machine 1 as soon the machine becomes vacant, i.e first job leaves to second machine. Hence enter the time in-out column for first job under machine 1 as the starting time of job two on machine 1. Continue until all the jobs are over. Be careful to see that whether the machines are vacant before loading. Total elapsed time may be worked out by drawing Gantt chart for the optimal sequence.

6. Points to remember:

(*a*) If there is tie i.e we have smallest element of same value in both columns, then:
(*i*) Minimum of all the processing times is A_r which is equal to B_s *i.e.* Min (A_i, B_i) = A_r =
B_s then do the *r* th job first and *s* th job last.
(*ii*) If Min (A_i, B_i) = A_r and also $A_r = A_k$(say). Here tie occurs between the two jobs having same minimum element in the same column *i.e.* first column we can do either *r*th job or *k* th job first. There will be two solutions. When the ties occur due to element in the same column, then the problem will have alternate solution. If more number of jobs have the same minimum element in the same column, then the problem
will have many alternative solutions. If we start writing all the solutions, it is a tedious
job. Hence it is enough that the students can mention that the problem has alternate solutions. The same is true with B_i s also. If more number of jobs have same minimum
element in second column, the problem will have alternate solutions.

Example 1.2

There are five jobs, which are to be processed on two machines *A* and *B* in the order *AB*. The processing times in hours for the jobs are given below. Find the optimal sequence and total elapsed time. *(Students has to remember in sequencing* *problems if optimal sequence is asked, it is the duty ofthe student to find the total elapsed time also).*

Jobs:	1	2	3	4	5
Machine A (Time in hN)	2	6	4	8	10
Machine B (Time in Hrs)	3	1	5	9	7

The smallest element is 1 it falls under machine *B* hence do this job last i.e in 5 the position. Cancel job 2 from the matrix. The next smallest element is 2, it falls under machine A hence do this job first, i.e in the first position. Cancel the job two from matrix. Then the next smallest element is 3 and it falls under machine *B*. Hence do this job in fourth position. Cancel the job one from the matrix. Proceed like this until all jobs are over the smallest element is 1 it falls under machine *B* hence do this job last i.e in 5 th position. Cancel job 2 from the matrix. The next smallest element is 2, it falls under machine A hence do this job first, i.e in the first position. Cancel the job two from matrix. Then the next smallest element is 3 and it falls under machine *B*. Hence do this job in fourth position. Cancel the job one from the matrix. Proceed like this until all jobs are over.

1	3	4	5	2

Total elapsed time:

OPTIMAL	*MACHINE –A*		*MACHINE - B*		*MACHINE IDLE JOB IDLE*			*REMARKS*
SEQUENCE	*IN*	*OUT*	*IN*	*OUT*	*A*	*B*		
1	0	2	2	5		2		As the Machine B Finishes Work at 5 Th hour will be Idle for 1 Hour. -do- 3 hr. -do- 1 hr. 1 hr as job finished early 1 hr idle
3	2	6	6	11		1		
4	6	14	14	23		3		
5	14	24	24	24		1		
2	24	30	31	32	1	2		

Total elapsed time = 32 hours (This includes idle time of job and idle time of machines).

The procedure: Let Job 1 is loaded on machine *A* first at zero th time. It takes two hours to process on the machine. Job 1 leaves the machine *A* at two hours and enters the machine 2 at 2-nd hour. Up to the time i.e first two hours, the machine *B* is idle. Then the job 1 is processed on machine *B* for 3 hours and it will be unloaded. As soon as the machine *A* becomes idle, *i.e.* at 2nd hour then next job 3 is loaded on machine *A*. It takes 4 hours and the job leaves the machine at 6 the hour and enters the machine *B* and is processed for 6 hours and the job is completed by 11 th hour. (Remember if the job is completed early and the Machine *B* is still busy, then the job has to wait and the time is entered in job idle column. In case the machine B completes the previous job earlier, and the machine *A* is still processing the next job, the machine has to wait for the job. This will be shown as machine idle time for machine *B*.). Job 4 enters the machine *A* at 6 th hour and processed for 8 hours and leaves the machine at 14 th hour. As the machine *B* has finished the job 3 by 11 the hour, the machine has to wait for the next job (job 4) up to 14 th hour. Hence 3 hours is the idle time for the machine B. In this manner we have to calculate the total elapsed time until all the jobs are over.

Example 13.3

There are seven jobs, each of which has to be processed on machine A and then on Machine B(order of machining is AB). Processing time is given in hour. Find the optimal sequence in which the jobs are to be processed so as to minimize the total time elapsed.

JOB:	1	2	3	4	5	6	7
MACHINE: A (TIME IN HOURS).	3	12	15	6	10	11	9
MACHINE: B (TIME IN HOURS).	8	10	10	6	12	1	3

Solution

By Johnson and Bellman method the optimal sequence is:

1	4	5	3	2	7	6

Optimal Sequence	Machine:A		Machine:B		Machine idle time		Job idle time	Remarks.
	In	out	In	out	A	B		
1	0	3	3	11		3	-	
4	3	9	11	17			2	Job finished early
5	9	19	19	31		2		Machine A take more time
3	19	34	34	44		3		Machine A takes more time.
2	34	46	46	56		2		- do-
7	46	55	56	59			1	Job finished early.
6	55	66	66	67	1	7		Machine A takes more time. Last is finished on machine A at 66 th hour.
	Total		**Elapsed**		**Time = 67 houN**			

Example 13.4

Assuming eight jobs are waiting to be processed. The processing time and due dates for the jobs are given below: Determine the sequence processing according to (a) FCFS (b) SPT (c) EDD and (d) LPT in the light of the following criteria:

(i) Average flow time,

(ii) Average number of jobs in the system,

(iii) Average job lateness,

(iv) Utilization of the workers

JOB	PROCESSING TIME	DUE DATE (DAYS)
A	4	9
B	10	18
C	6	6
D	12	19
E	7	17
F	14	20
G	9	24
H	18	28

Solution:

(a) **To determine the sequence processing according to FCFS**

The FCFS sequence is simply A-B-C-D-E-F-G-H- as shown below

Job	Processing Time	Flow time	Job due date	Job lateness (0 of negative)
A	4	4	9	0
B	10	14	18	0
C	6	20	6	14
D	12	32	19	13
E	7	39	17	22
F	14	53	20	33
G	9	62	24	38
H	18	80	28	52
	80	304		172

The first come, first served rule results is the following measures of effectiveness:

1. Average flow time = Sum of total flow time

Number of jobs

$= \frac{304\text{days}}{8} = 38\text{jobs}$

2. Average number of jobs in the system = $\frac{\text{Sum of total flow time}}{\text{Total processing time}}$

$= \frac{304\text{days}}{80} = 3.8\text{jobs}$

3. Average job lateness = $\frac{\text{Total late days}}{\text{Number of days}} = \frac{172}{8} \times 21.5 = 22\text{days}$

4. Utilization = $\frac{\text{Total processing time}}{\text{Sum of total flow time}} = \frac{80}{304} = 0.2631579$

0.2631579 x 100% = 26.31579 = 26.32%

(b) To determine the sequence processing according to SPT

SPT processes jobs based on their processing times with the highest priority given to the job with shortest time as shown below:

Job	Processing Time	Flow time	Job due date	Job lateness (0 of negative)
A	4	4	9	0
B	6	10	6	4
C	7	17	17	0
D	9	26	24	2
E	10	36	18	18
F	12	48	19	29
G	14	62	20	42
H	18	80	28	52
	80	283		147

The measure of effectiveness are:

1. Average flow time = $\frac{\text{Sum of total flow time}}{\text{Number of jobs}} = \frac{283}{8}$

$= 35.375\text{days} = 35.38$ days

2. Average number of jobs in the system = $\frac{\text{Sum of total flow time}}{\text{Total processing time}}$

$= \frac{283\text{days}}{80} = 3.54\text{jobs}$

3. Average job lateness = $\frac{\text{Total late days}}{\text{Number of days}} = \frac{147}{8}$

$= 18.375\text{days}$

$=18.38\text{days}$

4. Utilization = $\frac{\text{Total processing time}}{\text{Sum of total flow time}} = \frac{80}{283}$

$0.2826855 \times 100\%$

$= 28.27\%$

(c) To determine the sequence processing according to EDD

Using EDD, you are processing based on their due dates as shown below:

Job	Processing Time	Flow time	Job due date	Job lateness (0 of negative)
A	6	6	6	0
B	4	10	9	1
C	7	17	17	0
D	9	27	18	9
E	10	39	19	20
F	12	53	20	33
G	14	62	24	38
H	18	80	28	52
	80	294		153

The measure of effectiveness are:

1. Average flow time = $\frac{294}{8} = \underline{36.75\text{days}}$

2. Average number of jobs in the system = $\frac{294}{80}$

3.675 = 3.68days

3. Average job lateness = $\frac{153}{8}$ = 19.125

= 19.13days

=18.38days

4. Utilization = $\frac{80}{294}$ = 0.272108843

0.282108843 x 100

= 27.21%

(d) **To Determine the Sequence Processing According to LPT**

LPT selects the longer, bigger jobs first as presented below:

Job	Processing Time	Flow time	Job due date	Job lateness (0 of negative)
A	18	18	28	0
B	14	32	20	12
C	12	44	19	25
D	10	54	18	36
E	9	63	24	39
F	7	70	17	53
G	6	76	6	70
H	4	80	9	71
	80	437		306

The measure of effectiveness are:

1. Average flow time = $\frac{437}{8}$ = 54.625days

= 54.63days

2. Average number of jobs in the system = $\frac{437}{80}$ = 5.4625

5.46days

3. Average job lateness = $\frac{306}{8}$ = 38.25days

4. Utilization = $\frac{80}{437}$ = 0.183066361

0.183066361 x 100%

= 18.31%

The summary of the rules are shown in the table below:

	Average flow time (days)	**Average number of jobs in the system**	**Average job lateness job**	**Utilization%**
FCFS	38	3.8	21.5	26.32
SPT	35.38	3.54	18.38	28.27
EDD	36.75	3.68	19.13	27.21
LPT	54.63	5.46	38.25	18.31

As it can be seen from the table, SPT rule is the best of the four measures and is also the most superior in utilization of the system. On the other hand, LPT is the least effective measure of the three,

3.7.1 SEQUENCING JOBS IN TWO MACHINES

Johnson's rule is used to sequence two or more jobs in two different machines or work centres in the same order. Managers use Johnson rule method to minimize total timer for sequencing jobs through two facilities. In the process, machine total idle time is minimised. The rule does not use job priorities.

Johnson's rule involves the following procedures

1) List the jobs and their respective time requirement on a machine.

2) Choose the job with the shortest time. if the shortest time falls with the first machine, schedule that job first; if the time is at the second machine, schedule the job last. Select arbitrary any job if tie activity time occur.

3) Eliminate the scheduled job and its time

4) Repeat steps 2 and 3 to the remaining jobs, working toward the centre of the sequence until all the jobs are properly scheduled.

Example 13.5

Eight jobs have the following information

Job	Work Centre 1 Time (Hours)	Work Centre 2 Time (Hours)
A	9	6
B	7	10
C	12	8
D	14	14
E	11	16
F	5	5
G	15	13
H	16	4

Determine the sequence that will minimize the total completion time for these jobs.

Solution: (a) Steps Iteration 1

1st 2nd 3rd 4th 5th 6t7h 7th 8th

(b) The remaining job and their time are

Job	Work Centre 1 Time (Hours)	Work Centre 2 Time (Hours)

A	9	6
B	7	10
C	12	8
D	14	14
E	11	16
G	15	13

1st	2nd	3rd	4th	5th	6t7h	7th	8th

(d) Liberation 3:

1st	2nd	3rd	4th	5th	6t7h	7th	8th

(d) The remaining jobs and their times are

Job	**Work Centre 1 Time (Hours)**	**Work Centre 2 Time (Hours)**
D	14	14
G	15	13

There is a tie (at 14 hours) for the shortest remaining time. We can place job D in the first work centre or second work centre. Suppose it is placed in work centre 1.

(f) Liberation 4

1st	2nd	3rd	4th	5th	6t7h	7th	8th

(g) The sequential times are:

Work centre 1	5	7	11	14	15	12	9	16

Work Centre 2	5	10	16	14	13	8	6	4

Determination of throughput time and idle time at the work centres

Thus the eight jobs will be completed in 93 hours The work centre 2 will be wait for (5) hours for its first job, and also wait for two (2), one (1) and nine (9) hours after finishing jobs F,B and A respectively.

METHOD 2

We can also solve this problem using the tabulation method shown below

Job sequence	**1 Centre 1 Duration**	**11 Centre 1 in**	**111 Centre 1 out**	**IV Centre 2 Duration**	**V Centre 2**	**VI Centre 2 Out**	**VII Idle Time**
F	5	0	5	5	5	10	5
B	7	5	12	10	12	22	2
E	11	12	23	16	23	39	1
D	14	23	37	14	39	53	0
G	15	37	52	13	53	66	0
C	12	52	64	8	66	74	0
A	9	64	73	6	74	80	0
H	16	73	89	4	89	93	9
							17

- Columns I and IV are the durations for the jobs as given in the question
- In column 11 the starting point for F is 0; 5 + 0 5 for job B; 7 + 5 = 12 for job 11 ± 12=23 for job D etc,
- In column UT we obtain the cumulative time for I i.e first value is 5, next is+ 7 = 12 etc we can also obtain it by adding columns I and II
- In column V, we realize that the job at centre 2 cannot start until the job centre 1 ends. Thus the first value is 5 representing the duration of job F The

next value is the maximum of the sum of IV and V in centre 2 and the out time for the next job in centre I i.e max (5+5,12) 12. The value of 2 obtained next is max 10+12, 23) while the value of 39 obtained is max (16+23, 37) other values are similarly obtained using the same technique.

- Column VI is the sum of IV and V i.e 5 + 5 = 10, 10 + 12 = 22 etc.
- In column VII the first value is the duration of job F in centre I This represents the period that centre 2 has to wait before starting its first job. The next value of 2 is obtained by subtracting the time out for F from the time in for B i.e 12 - 10 = 2. This represents the time centre 2 will wait before starting job 13. Similarly 23 -22 = I is the time centre 2 will wait before starting job F. All other values are obtained in a similar manner.

Total idle time = 5 + 2 + 1 ±9 = 17

Total time for completion of all the job is 93

Example 13.6

You arc given the operation times in Hours for 6 jobs in two machines as follow:

Job	**Machine 1 Time (Hours)**	**Machines 2 Time (Hours)**
P	20	20
Q	16	12
R	33	36
S	8	28
T	25	33
U	48	60

(a) Determine the sequence that will minimize idle times on the two machines

(b) The time machine I will complete its jobs

(c) The total completion time for all the jobs

(d) The total idle time

Solution

Using the steps outlined earlier for optimum sequencing of jobs, we obtained

1st	2nd	3rd	4th	5th	6t7h
S	T	R	U	E	Q

We then use tabular method to solve the remaining questions

Job sequence	1 Machine 1 Duration	II Machine 1 in	III Machine I Out	IV Machine 2 Duration	V Machine 2 In	VI Machines 2 Out	VII Idle Time
S	8	0	8	28	8	36	8
T	25	8	33	33	36	69	0
R	33	33	66	36	69	105	0
U	48	66	114	60	114	174	9
P	20	114	134	20	174	194	0
Q	16	134	150	12	194	206	0

(a) Machine 1 will complete his job in150 hours

(b) Total completion time is 206 hours

(c) Total idle time is 17 hours

Note that machine 2 will wait 8 hours for its first job and also wait 9 hours after completing job R.

In general, idle time can occur either at the beginning of job or at the end of sequence of jobs. In manufacturing organizations, idle times can be used to do other jobs like maintenance, dismantling or setting up of other equipment.

4.0 CONCLUSION

Sequencing problems involves the determination of an optimal order or sequence of performing a series jobs by number of facilities (that are arranged in specific order) so as to optimize the total time or cost. Sequencing problems can be classified into two groups. The first group involves n different jobs to be performed, and these jobs require processing on some or all of m different types of machines. The order in which these machines are to be used for processing each job (for example, each job is to be processed first on machine A, then B, and thereafter on C i.e., in the order ABC) is given. Also, the expected actual processing time of each job on each machine is known. We can also determine the effectiveness for any given sequence of jobs on each of the machines and we wish to select from the $(n!)^m$ theoretically feasible alternatives, the one which is both technologically feasible and optimises the effectiveness measure.

5.0 SUMMARY

Scheduling, which occurs in every organisation, refers to establishing the timing of the use of equipment, facilities and human activities in an organization and so it deals with the timing of operations. Scheduling technique depends on the volume of system output, the nature of operations and the overall complexity of jobs. The complexity of operation varies under two situations, namely, Flow Shop system and Job Shop system. Flow Shop is a high volume system while Job Shop is a low volume system. Lading refers to assignment of jobs to work centres. The two main methods that can be used to assign jobs to work centres are used of Gant chart and Assignment Method. Job sequencing refers to the order in which jobs should be processed at each work station. Priority rules enables us to select the order in which job should be done. The main objective of priority rules is to minimize completion time, number of jobs in the system, and job lateness, while maximizing facility utilization. In FCFS, which means First Come First Served, job is processed in the order of arrivals at work centres. In Short Processing Time (SPT) jobs are processed based on the length of the processing time with the job with the least processing time being done first. In Earliest Due Date (EDD) the job with the earliest due data is processed first. In Longest Processing Time (LPT) the job with longest processing time are started first. Johnson's rule is used to sequence two more jobs in two different work centres in the same order.

6.0 TUTOR MARKED ASSIGNMENT

1. Explain the following concepts (a) Scheduling (b) Flow shop (c) Job shop(d) Sequencing
2. Describe two main methods used to assign jobs to work centres
3. Define the following (a) Average flow time (b) Average number of jobs in the system (c) Utilization
4. State the priority rules for sequencing
5. State the procedures for using Johnson's rules in sequencing N jobs in two machines
6. Information concerning six jobs that are to be process at a work centre is given below.

Job	Processing time (days)	Due date (days)
A	11	17
B	7	13
C	4	12
D	3	14
E	5	2

F	14	11

Determine the sequence processing according to

i. Average flow time
ii. Average number of jobs in the system
iii. Average job lateness
iv. Utilization of the work centre

7. The following seven jobs are waiting to be processed at a machine centre

Job	Due date	Processing time
A	13	8
B	20	14
C	10	10
D	23	16
E	21	11
F	24	18
G	28	13

In what sequence would the job be ranked according to the following decision rules (1) EDD, (2) SPT, (3) LPT, (4) FCFS)

8 Given the following processing time about six jobs in two machine follows

Job	Machine 1	Machine 2
A	13	8
B	10	7
C	8	10
D	6	11
E	5	9
F	7	6

Determine the sequence that will minimize the total completion time for these jobs

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UNIT 2: GAMES THEORY

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1.0 INTRODUCTION

The theory of games (or game theory or competitive strategies) is a mathematical theory that deals with the general features of competitive situations. This theory is helpful when two or more individuals or organisations with conflicting objectives try to make decisions. In such a situation, a decision made by on person affects the decision made by one or more of the remaining decision makers, and the final outcome depend depends upon the decision of all the parties. (Gupta and Hira, 2012)

According to Adebayo et al (2006), Game theory is a branch of mathematical analysis used for decision making in conflict situations. it is very useful for selecting an optimal strategy or sequence of decision in the face of an intelligent opponent who has his own strategy. Since more than one person is usually involved in playing of games, games theory can be described as the theory of multiplayer decision problem. The Competitive strategy is a system for describing games and using mathematical techniques to convert practical problems into games that need to be solved. Game theory can be described as a distinct and interdisciplinary approach to the study of human behaviour and such disciplines include mathematics, economics, psychology and other social and behavioural sciences. If properly understood it is a good law for studying decision- making in conflict situations and it also provides mathematical techniques for selecting optimum strategy and most rational solution by a player in the face of an opponent who already has his own strategy.

Adebayo et al (2006), attributed the development of game theory to John von Neumann, the great mathematician, in the last decade of 1940, whose first important theory, written in partnership with the great economist Oskar Morgan stein is titled “the theory of games and economic behaviour”. Oskar Morgan stein brought ideas from neo-classical economics into games theory. The key word between neo-economics and game theory is rationality, with emphasis being placed on the absolute rationality of men in making economic choice. It specifically advocates that human beings are rational in economic choices with each person aiming at maximising each or her rewards (profit, income or other subjective benefits) in the circumstances he faces. John von Neumann studied how players in Poker games maximize their rewards and found that they did it by bluffing and by being unpredictable. He was able to discover anew, unique I and unequivocal answer to the question of how players can maximise their payoffs in the game without any market forces, properties right, prices or other economic indicators in the picture. His discovery led to a very major extension to the concept of absolute rationality in neoclassical economics. However the discovery only applied to zero sum games. Other games theorists have since expanded the scope of the research on games theory.

The theory of games is based on the minimax principle put forward by J. Von Neumann which implies that each competitor will act as to minimise his maximum loss (or maximise his minimum gain) or achieve best of the worst. So far, only simple competitive problems have been analysed by this mathematical theory. The theory does not describe how a game should be played; it describes only the procedure and principles by which players should be selected (Gupta and Hira, 2012).

2.0 OBJECTIVES

By the end of this chapter, you will be able to:

- Define the concept of a game
- State the assumptions of games theory
- Describe the two-person zero-sum games
- Explain the concept of saddle point solution in a game
- Find pure and mixed strategies in games
- Use the simplex method to find the optimal strategies and value of a game

3.0 MAIN CONTENT

3.1 DECISIONMAKING

Making decision is an integral and continuous aspect of human life. For child or adult, man or woman, government official or business executive, worker or supervisor, participation in the process of decision- making is a common feature of everyday life. What does this process of decision making involve? What is a decision? How can we analyze and systematize the solving of certain types of decision problems? Answers of all such question are the subject matter of decision theory. Decision-making involves listing the various alternatives and evaluating them economically and select best among them. Two important stages in decision-making is: (*i*) making the decision and (*ii*) Implementation of the decision.

Analytical approach to decision making classifies decisions according to the amount and nature of the available information, which is to be fed as input data for a particular decision problems. Since future implementations are integral part of decision-making, available information is classified according to the degree of certainty or uncertainty expected in a particular future situation. With this criterion in mind, three types of decisions can be identified. First one is that these decisions are made when *future can be predicted with certainty*. In this case the decision maker assumes that there is only one possible future in conjunction with a particular course of action. The second one is that decision making under *conditions of risk*. In this case, the future can bring more than one state of affairs in conjunction with a specific course of action. The third one is decision making under *uncertainty*. In this case a particular course of action may face different possible futures, but the probability of such occurrence cannot be estimated objectively.

The Game theory models differ from *decision-making under certainty (DMUC) and decision making under risk (DMUR)* models in two respects. First the opponent the decision maker in a game theory model is an active and rational opponent in DMUC and DMUR models the opponent is the passive state of nature. Second point of importance is decision criterion in game model is the *maximin* or the *minimax* criterion. In DMUC and DMUR model s the criterion is the maximization or minimization of some measure of effectiveness such as profit or cost.

3.2 DESCRIPTION OF A GAME

In our day-to-day life we see many games like Chess, Poker, Football, Base ball etc. All the segames are pleasure-giving games, which have the character of a competition and are played according to well- structured rules and regulations and end in a *victory* of one or the other team or group or a player. But we refer to the word game in this unit the competition between two business organizations, which hasmoreearningcompetitivesituations.Inthischaptergameisdescribedas:

A competitive situation is called a game if it has the following characteristics (Assumption made to define a game):

1. There is finite number of competitors called *Players*. This is to say that the game is played by two or more number of business houses. The game may be for creating new market or to increase the market share or to increase the competitiveness of the product.

2. A list of finite or infinite number of possible *courses of action is available* to each player.

The list need not be the same for each player. Such a game is said to be in normal form. To explain this we can consider two business houses A and B. Suppose the player A has three strategies, as strategy I is to offer a car for the customer who is selected through advertising campaign. Strategy II may be a house for the winning customer, and strategy III may a cash prize of N10,00,000 for the winning customer. This means to say that the competitor A has three strategies or courses of action. Similarly, the player B may have two strategies, for example strategy I is A pleasure trip to America for 10 days and strategy II may be offer to spend with a cricket star for two days. In this game A has three courses of action and B has two courses of actions. The game can be represented by mans of a matrix as shown below:

		B	
		I	II
A	II		
	III		

3. A play is played when each player chooses one of his courses of actions. The choices are made simultaneously, so that no player knows his opponent's choice until he has decided his own course of action. But in real world, a player makes the choices after the opponent has announced his course of action.

Every play *i.e.* combination of courses of action is associated with an outcome, known as *pay off*- (generally money or some other quantitative measure for the satisfaction) which determines a set of gains, one to each player. Here also considered to be negative gain. Thus after each playoff the game, one player pays to other an amount determined by the courses of action chosen. For example consider the following matrix:

		B		
		I	II	III
A	I	2	4	–3
	II	–1	2	2

In the given matrix, we have two players. Among these the player who is named on the left side matrix is known as winner, *i.e*. here A is the winner and the matrix given is the matrix of the winner. The player named above is known as the loser. The loser's matrix is the negative version of the given matrix. In the above matrix, which is the matrix of A, a winner, we can describe as follows. If A selects first strategy, and *B* selects the second strategy, the outcome is +4 *i.e. A* will get4 units of money and *B* loses 4 units of money. *i.e*. *B* has to give 4 units of money to *A*. Suppose *A* selects second strategy and *B* selects first strategy A's outcome is–1, *i.e.* *A* loses one unit of money and he has to give that to *B*, it means *B* wins one unit of money.

4. All players act rationally and intelligently.

5. Each player is interested in maximizing his gains or minimizing his losses. The winner, *i.e*. the player on the left side of the matrix always tries to maximize his gains and is known as Maximin player. He is interested in maximizing his minimum gains. Similarly, the player B, who is at the top of the matrix, a loser always tries to minimize his losses and is known as Minimax player-*i.e*. who tries to minimize his maximum losses.

6. Each player makes individual decisions without direct communication between the players.

By principle we assume that the player play a strategy individually, without knowing opponent's strategy. But in real world situations, the players play strategy after knowing the opponent's choice to maximin or minimax his returns.

7. It is assumed that each player knows complete relevant information.

Game theory models can be classified in a number of ways, depending on such factors as the: (*i*) Number of players, (*ii*) Algebraic sum of gains and losses (*iii*) Number of strategies of each player, which decides the size of matrix.

Number of players: If number of players is two it is known as Two-person game. If the number of players is is '*n*' (where$n \geq 3$) it is known as *n*-person game. In real world two person games are more popular. If the number of players is '*n*', it has to be reduced to two person game by two constant collations, and then we have to solve the game, this is because, the method of solving *n*- person games are not yet fully developed.

Algebraic Sum of Gains and Losses*:* A game in which the gains of one player are the losses of other player or the algebraic sum of gains of both players is equal to zero, the game is known as Zero sum game (ZSG). In a zero sum game the algebraic sum of the gains of all players after play is bound to be zero. i.e. If gi as the pay of to a player in an-person game, then the game will be a zero sum game if sum of all gi is equal to zero.

In game theory, the resulting gains can easily be represented in the form of a matrix called pay–off matrix or gain matrix as discussed in 3 above. A pay- off matrix is a table, which shows how payments should be made at end of a play or the game. Zero sum game is also known as constant sum game. Conversely, if the sum of gains and losses does not equal to zero, the game is a non zero-sum game. A game where two persons are playing the game and the sum of gains and losses is equal to zero, the game is known as Two-Person Zero-Sum Game (TPZSG). A good example of two- person game is the game of chess. A good example of n-person game is the situation when several companies are engaged in an intensive advertising campaign to capture a larger share of the market (Murthy, 2007)

3.1 SOME IMPORTANT DEFINITIONS IN GAMES THEORY

Adebayo et al (2010) provide the following important definitions in game theory.

- **Player**: A player is an active participant in a game. The games can have two persons(Two-person game) or more than two persons (Multi person or n-person game)
- **Moves**: A move could be a decision by player or the result of a chance event.
- **Game**: A game is a sequence of moves that are defined by a set of rules that governs the players' moves. The sequence of moves may be simultaneous.
- **Decision maker**: A decision-maker is a person or group of people in a committee who makes the final choice among the alternatives. A decision-maker is then a player in the game.
- **Objective:** An objective is what a decision-maker aims at accomplishing by means of his decision. The decision-maker may end up with more than one objective.
- **Behaviour**: This could be any sequence of states in a system. The behaviours of a system are overt while state trajectories are covert.
- **Decision**: The forceful imposition of a constraint on a set of initially possible alternatives.
- **Conflict**: A condition in which two or more parties claim possession of something they cannot all have simultaneously. It could also be described as a state in which two or more decision-makers who have different objectives,

act in the same system or share the same resources. Examples are value conflicts, territorial conflict, conflicts of interests etc.

- **Strategy**: it is the predetermined rule by which a player decides his course of action from a list of courses of action during the game. To decide a particular strategy, the player needs to know the other's strategy.
- **Perfect information**. A game is said to have perfect information if at every move in the game all players know the move that have already been made. This includes any random outcomes.
- **Payoffs**. This is the numerical return received by a player at the end of a game and this return is associated with each combination of action taken by the player. We talk of "expected payoff' if its move has a random outcome.
- **Zero-sum Game**. A game is said to be zero sum if the sum of player's payoff is zero. The zero value is obtained by treating losses as negatives and adding up the wins and the losses in the game. Common examples are baseball and poker games.

3.2 ASSUMPTIONS MADE IN GAMES THEORY

The following are assumptions made in games theory.

- Each player (Decision-maker) has available to him two or more clearly specified choices or sequence of choices (plays).
- A game usually leads to a well-defined end-state that terminates the game. The end state could be a win, a loss or a draw.
- Simultaneous decisions by players are assumed in all games.
- A specified payoff for each player is associated with an end state (eg sum of payoffs for zero sum-games is zero in every end-state).

- Repetition is assumed. A series of repetitive decisions or plays results in a
- game
  Each decision-maker (player) has perfect knowledge of the game and of his opposition i.e. he knows the rules of the game in details and also the payoffs of all other players. The cost of collecting or knowing this information is not
- considered in game theory.
  All decision-makers are rational and will therefore always select among alternatives, the alternative that gives him the greater payoff.

The last two assumptions are obviously not always practicable in real life situation. These assumptions have revealed that game theory is a general theory of rational behaviour involving two or more decision makers who have a limit number of courses of action of plays, each leading to a well-defined outcome or ending with games and losses that can be expressed as payoffs associated with each courses of action and for each decision maker. The players have perfect knowledge of the opponent's moves and are rational in taking decision that optimises their individual gain.

The various conflicts can be represented by a mathx of payoffs. Game theory also proposes several solutions to the game. Two of the proposed solutions are:

1. **Minimax or pure Strategy**: In a minimax strategy each player selects a strategy that minimises the maximum loss his opponent can impose upon him.
2. **Mixed Strategy**: A mixed strategy which involves probability choices.

Lot of experiments have been performed on games with results showing conditions for (i) Cooperation (ii) Defection and (iii) Persistence of conflict,

3.3 DESCRIPTION AND TYPES OF GAMES

Games can be described in terms of the number of players and the type of sum obtained for each set of strategies employed. To this end we have the following types of games:

- Two-person zero-sum games. Here two players are involved and the sum of the
  Pay-offs for every set of strategies by the two players is zero
- Two-person non zero-sum games. Here two players are involved and there is one strategy set for which the sum of the payoffs is not equal to zero.
- Non- Constant sum games. The values of payoffs for this game vary.
- Multi-person non- Constant-Sum games. Many players are involved in the game and the payoffs for the players vary.
- Cooperative Games. In this game there is cooperation between some of the players and there are rules guiding the cooperation within the players Politics can be modelled as a cooperative game with some players forming alliance with prospective successful political parties while others defect from parties that they feel can fail in an election.
- Combinatorial games which makes use of combinatorial analysis
- Stochastic Games which is probabilistic in nature
- Two-person Zero-sum Stochastic Games
- Stochastic multi-generation game

Some other games are given interesting names to emphasise the issues being portrayed.

Examples are:

- Matching Penny Game

- Prisoners Dilemma
- Ultimatum
- Angel Problem
- Tragedy of the Commons
- Majority Rule

(Adebayo et al, 2006)

3.5.1 TWO-PERSON ZERO-SUM GAME

This game involves two players in which losses are treated as negatives and wins as positives and the sum of the wins and losses for each set of strategies in the game is zero. Whatever player one wins player two loses and vice versa. Each player seeks to select a strategy that will maximise his payoffs although he does not know what his intelligence opponent will do. A two-person zero-sum game with one move for each player is called a rectangular game.

Formally, a two-person zero-sum game can be represented as a triple (A, B, y) where A [al, a2.. . .a_{mj} and B [b 1, b2 bn] and are payoff functions, e_{ij} such that y [ai bj = eij. This game can be represented as an m x n matrix of payoffs from

player 2 to player 1 as follows:

$$\begin{bmatrix} [\gamma [a_1, b_1] & \gamma [aj, b_2] & \ldots\ldots\ldots & \gamma [a_i, b_n] \\ \gamma (a_m, b_1] & y [a_m b_2] & \ldots\ldots\ldots & \gamma [a_m, b_n] \end{bmatrix}$$

The two-person zero-sum games can also be represented as follows:

Suppose the choices or alternatives that are available for player 1 can be represented as l,2,3...m. While the options for player two can be represented as

1,2,3.,.n. If player 1 selects alternative i and player 2 selects alternative j then the payoff can be written as a. The table of payoffs is as follows:

		Alternatives for player 1				
		1	2	3	…	n
	1	$a_{11}a_{12}a_{13}$...				a_{1n}
Alternative for player 2	2	a_{21}	$a_{22}a_{23}$...			a_{2n}
	3	$a_{31}a_{32}a_{33}$...				a_{3n}
	.					
	.					
	m	a_{m1}	a_{m2}	a_{m3}	…	a_{mn}

A saddle point solution is obtained if the maximum of the minimum of rows equals the minimum of the maximum of columns i.e maximin = minimax

i.e max(min a9) = min(max a9)

Example 2.1

Investigate if a saddle point solution exists in this matrix

$$\begin{bmatrix} 2 & 1 & -4 \\ -3 & 6 & 2 \end{bmatrix}$$

Solution

$$\begin{array}{lcccc} & & & & \text{min} \\ & \left[\begin{matrix} 2 & 1 & 1 \\ -3 & 6 & 2 \end{matrix}\right. & & & \left.\begin{matrix} -4 \\ -3 \end{matrix}\right] \\ \text{Max} & 2 \quad 6 \quad 3 & & & \end{array}$$

$\max_i (\min_{ij}) = \max \; (-4, -3) = -3$

$\min_j (\max_j a_{ij}) = \min (2,6,3) = 2$
$\max_i (\min_j a_{ij}) \neq \min_j (\max_i a_{ij})$
So a saddle point solution does not exist.

Example 14.2

We shall consider a game called the "matching penny" game which is usually played by children. In this game two players agree that one will be even and the other odd. Each one then shows a penny. The pennies are shown simultaneously and each child shows a head or tail. If both show the same side "even" wins the penny from odd and if they show different sides odd wins from even. Draw the matrix of payoffs

Solution

The pay-off table is as follows:

		Odd	(Player 2)
		Head	Tail
	Head	(1,-1)	(-1, 1)
Even	Tail	(-1, 1)	(1,-1)
(Player 1)			

The sum in each cell is zero, hence it is a zero sum game. Now A (H, T), B (H, T) and y (H,H)= y(T,T) 1 while y (H,T)=(T,H)=-1, In matrix form, if row is for even and column is for odd we have the matrix of payoffs given to player I by players 2 as

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

SOLUTION OF TWO-PERSON ZERO-SUM GAMES

Every two-person zero-sum game has a solution given by the value of the game together with the optimal strategies employed by each of the two players in the game. The strategies employed in a two person zero sum game could be

i. Pure Strategies

ii. Dominating Strategies
iii. Mixed Strategies

3.5.2PURE STRATEGIES

In pure strategy, the maximin criterion enables one to obtain a saddle point solution. The maximin criterion states that for a two person zero sum game it is rational for each player to choose the strategy that maximises the minimum pay-off to be received by each of them. The pair of strategies and the payoffs such that each of the players maximises the minimum payoffs is the solution to the game.

So with his strategy player 1 (row player) can guarantee that the payoff is at least v, the lower value of the game where

$$\underline{v} = \sup_{a_i} \inf_{\underline{b_j}} \gamma(a_i,b_j)$$
$$= \max_i (\min_j a_{ij})$$

While player 2 (column player) can guarantee that player l's payoff is no more than v, the upper value of the game

$$\bar{v} = \inf_{b_j} \sup_{a_i} \gamma (a_i, b_j)$$
$$= \min_j (\max_i a_{ij})$$

For the maximin criterion which states that a saddle point solution exists in pure strategies we have

$$\bar{v} = \sup_{a_i} \inf_{b_j} \gamma(a1 , b,) = v = \bar{\inf}_{b_j} \sup_{a_i} \gamma (a_i, b_j)$$

$$\max_i (\min_j a_{ij}) = \min_j (\max_i a_{ij})$$

$$\begin{array}{cc} \begin{bmatrix} 1 & 1 & -1 & 1 & 1 \\ & -1 & 1 & -1 & -1 \\ 2 & 2 & 1 & -1 & -1 \\ 1 & 1 & 2 & 1 & 1 \end{bmatrix} & \begin{matrix} -1 \\ 1 \quad -1 \\ -1 \\ -1 \end{matrix} \end{array}$$

Solution

We find the row maximum and column minimum and then find the point where $\underline{V} = \underline{V}$ as follows:

Minimum of rows

$$\begin{array}{ccc} \begin{bmatrix} 1 & 1 & -1 & 1 & 1 \\ & & & & \\ & & & & \end{bmatrix} & -1 & -1 \end{array}$$

-1	1	-1	-1	1	-1	-1
2	2	1	-1	-1	-1	-1
1	1	2	1	1	-1	-1

maximum 2 2 2 1 1 -1
of columns
V = MAX (-1, -1, -1, -1) = -1. V = MIN (2, 2, 2, 1, 1, -1_ = -1
V = V So value is -1 and optimal strategy is (r_4, c_6)

Example 14.3

Find the solutions of this matrix game

$$\begin{bmatrix} -200 & -100 & -40 \\ 400 & 0 & 300 \\ 300 & -20 & 400 \end{bmatrix}$$

Solution

We check if max (min a_{ij}) min (max aij) in order to know whether it has a saddle point

i j i

solution. We first find the minimum of rows and miximum of columns as follows.

$$\begin{array}{lcccc} & \begin{bmatrix} -200 & -100 & -40 \\ 400 & 0 & 300 \\ 300 & -20 & 400 \end{bmatrix} & \begin{matrix} -200 \\ 0 \\ -20 \end{matrix} \\ \text{Max} & \begin{matrix} 400 & 0 & 400 \end{matrix} & \end{array}$$

So $\max_i (\min_j a_{ij})$ = max (-200, 0, -20) = 0

min(max a_{ij}) = mm (400, 0, 400). So a saddle point solution exists at (row2, column2)
i.e (r_2 c_2) The value of the game is 0.

Example 14.4

A modified version of a problem on game theory by Williams (1966) in Adedayo (2006) is hereby presented.

A man planning for the coming winter during summer time, has a home heating tank which has capacity for 200 gallons. Over the years, he observed that the heating oil consumption depends on the severity of the winter as follows

Mild winter: 100 gallons

Average winter: 150 gallons

Severe winter: 200 gallons

The price of oil also seems to fluctuate with severity of the winters as follows:

Mild winter: $1 per gallon

Average winter: $1.50 per gallon

Severe winter: $2 per gallon

He has to decide whether to stockpile 100 gallons, 150 gallons or 200 gallons at the present price of$ 1. If he stockpiles more than he needs, the unused will be wasted since he will be moving next summer. What is the best decision to take?

Solution:

You must recognise who the two players are. They are Nature and Man. Nature's strategies are three, based on severity: namely mild severity, average seventy and severe winter. Man's strategies are also those based on sizes of stockpile i.e. 100 gallons, 150 gallons, 200 gallons. The matrix of pay offs are obtained, using the value given and we get

Nature

$$
\begin{array}{ll}
 & \text{Mild winter} \quad \text{Average winter} \quad \text{Severe winter} \\
\begin{array}{r} \text{100 gal.} \\ \text{Max} \quad \text{150 gal.} \\ \text{200 gal} \end{array} &
\begin{bmatrix} -100 & -175 & -300 \\ -150 & -150 & -250 \\ -200 & -200 & -200 \end{bmatrix}
\end{array}
$$

The payoffs are negative since man is playing the row.

Note that for the 100 gallons stockpile in average winter, since 150 gallons are consumed. 50 extra gallons are needed at 1.50 per gallon 75. So total is 175. No extra is needed for 150 gallons and 200 gallons

(ii) For severe winter, 200 gallons are needed. So for 100 gallons stockpile, one needs I00 extra gallons at $2 per gallon = 200. So total = 300. Same argument goes for 150 stockpi1es: no extra is needed for 200 gallons stockpile.

$$\begin{array}{c}\begin{pmatrix} [-100 & -175 & -300] \\ [-150 & -150 & -250] \\ [1-200 & -200 & -200] \end{pmatrix}\end{array}\begin{array}{c} -300 \\ -250 \\ -200 \end{array}$$

Maximum of columns -100 -150 -200

v = max [-300, -250 -200] -200
v=min [-100, -150 -200] = -200
So the saddle point is at [a3, b3J

This implies that stockpiling 200 gallons is the optimal strategy.

3.5.3 DOMINATING STRATEGIES

In a pay-off matrix row dominance of i oven occurs if $a_i > a_j$, while column dominance of I over occurs if b_1 b_1. If dominance occurs, column j is not considered and we reduce the matrix by dominance until we are left with 1 x I matrix whose saddle point, solution can be easily found. We consider the matrix

$$\begin{pmatrix} 3 & 4 & 5 \\ 3 & 1 & 2 \\ 1 & 3 & 4 \end{pmatrix}\begin{array}{c} 3 \\ 3 \\ 4 \end{array}$$

Observation shows that every element in column 1 is less than or equal to that of column 4 and we may remove column 4 the dominating column. Similarly b_3 dominates b_2 and we remove the dominating column b_3. The game is reduced to

$$\begin{pmatrix} 3 & 4 \\ 3 & 1 \\ 1 & 3 \end{pmatrix}$$

In row dominance, we eliminate the dominated rows a, (where a.> a,) while in column dominance we eliminate the dominating column b_j (where$_i$≤b_j) since player 2 desired to concede the least payoff to the row player and thus minimise his losses.

This procedure is iterated using row dominance. Since a1 dominates a2 and also dominates a3 we remove the dominated rows a2 and a3. This is due to the fact that player 1, the row player, wishes to maximise his payoffs. We then have a 1 x 1

reduced game [3 4] which has a saddle point solution. Generally if a dominated strategy is reduced for a game, the solution of the reduced game is the solution of the original game.

3.5.4 MIXED STRATEGIES

Suppose the matrix of a game is given by

$$A = \begin{pmatrix} 2 & -1 & 3 \\ -1 & 3 & -2 \end{pmatrix}$$

Inspection shows that *i* column dominance cannot be used to obtain a saddle point solution. If no saddle point solution exists we randomise the strategies. Random choice of strategies is the main idea behind a mixed strategy. Generally a mixed strategy for player is defined as a pro6a6iffty distribution on the set of pure strategies. The minimax theorem put forward by von Neumann enables one to find the optimal strategies and value of a game that has no saddle point solution and he was able to show that every two-person zero-sum game has a solution in mixed if not in pure strategy.

3.5.5 OPTIMAL STRATEGIES IN 2 X 2 MATRIX GAME

Linear optimisation in linear programming enables one to calculate the value and optimal actions especially when the elements of A are more than 2. We now demonstrate how to solve the matching pennies matrix with a simple method applicable when A has two elements and B is finite. Here the value is given as maximin ($\theta\varphi$ [a_1,b_1] + (1-θ) φ (a_2 b_2), $\theta\varphi$,(a_1,b_2) + (1-θ) φ (a_2, b_2))

The matric is odd

$$\begin{array}{ccc} & \theta_1 & 1\theta_1 & \\ \theta 1 & \left[\begin{array}{cc} -1 & \\ & 1\text{-}\theta\text{-}1 \end{array}\right] & 1 \end{array}$$

We note here that the maximin criterion cannot hold since max (mm of row) max (-1, -1)-1 while min (max of columns) = min (1,1) 1 and no saddle point solution exists.

Let "even" choose randomised action (θ, 1 -θ) i.e θ = a (a_1) and (1-θ) = θ (a_2).
Using formula above, we have max mm (θ - 1 + θ, - θ + 1 - θ)
θ + -l (1 - θ) = -θ+ 1 (1 —θ) using principle of equalising expectations.
This gives 2θ -1, 1 -2θ
4θ =2. And θ =$^1/_2$

Similarly if optimal randomised action by player 2= θ_1,
then we get θ_1 + (1-θ_1)-1, θ,+1 -θ_1)

$\theta1(1) + -1(1-\theta_1) = \theta_1(-1) + (1-\theta_1)$. Simplify both sides of the equation to get $2\theta_1 - 1 = 1 - 2\theta_1\theta_1 = {}^1/_2$and so randomised action by player 1 is $({}^1/_2, {}^1/_2)$ and also $({}^1/_2, {}^1/_2)$ by player 1

The value can be obtained by substituting = $^1/_2$ into 2θ - 1 or I - 2θ or by substituting $\theta_1 = {}^1/_2$ into $2\theta_1$ - I or 1 $-2\theta_1$. If we do this we get a value of zero. So the solution is as follows:
Optimal strategies of $({}^1/_2, {}^1/_2)$ for player 1 and $({}^1/_2, {}^1/_2)$ for player 2 and the value of the game is 0.

It is Obvious that there is no optimal mixed strategy that is independent of the opponent.

Example 14.5

Two competing telecommunication companies MTN and Airtel both have objective of maintaining large share in the telecommunication industry. They wish to take a decision concerning investment in a new promotional campaign. Airtelwishes to consider the following options:

r_1: advertise on the Internet

r_2: advertise in all mass media

MTN wishes to consider these alternatives

c_1: advertise in newspapers only

c_2: run a big promo

If Airtel advertise on the Internet and MTN advertises in newspapers, MTN will increase its market share by 3% at the expense of V-Mobile. If MTN runs a big promo and Airtel advertises on the Internet, Airtel will lose 2% of the market share. If Airtel advertises in mass media only and MTh advertises in newspapers, Airtel will lose 4%. However, if Airtel advertises in mass media only and MTN

runs a big promo, Airtel will gain 5% of the market share.

a) Arrange this information on a payoff table

b) What is the best policy that each of the two companies should take?

Solution

a) The matrix of payoff is as follows

MTN
c_1 c_2
Airtelr_1

$$\begin{bmatrix} 3 & -2 \\ -4 & 5 \end{bmatrix}$$

We first cheek if a saddle point solution exists. We use the minimax criterion to do this. Now for the rows,
Minimax (3,5) = 3 while for the columns
Maximin = Max (-4, -2) = -2.

Since minimax is not equal to maximin, no saddle point solution exists. We then randomise and use the mixed strategy.
Let (θ, 1 - θ) be the mixed strategies adopted by Airtel while (8, 1-8) be the strategies adopted by MTN
Then for Airtel. θ (3)+-4(1 - θ)-2θ+5(1 -θ)
3θ - 4+4θ =-2θ +5 - 5θ
7θ - 4 -7θ +5.
Solving we obtain
$\theta = {}^9/_{14}$ and $1 - \theta {}^5/_{14}$
The randomised strategies by V-Mobile will be (${}^9/_4$)
For MTN, $3\theta -2(1-\theta)=-4\theta_1 i+5(1-\theta_1)$
$3\theta+2\theta_1 - 2=-4\theta+5-5\theta_1$
$5\theta_1-2= -9\theta_1+5$. Solving, we obtain $\theta_1={}^1/_2$and $1-\theta_1={}^1/_2$

The value of the game can be found by substituting 9/14 into 78-4 or – 79+5; or V2

into 5θ -2 or -9θ+5. When we do this we obtain the value ${}^1/_2$. So Airtel should advertise on the Internet${}^9/_{14}$ of the time and advertise on the mass media${}^5/_{14}$ of the time. On the other hand, MTN should advertise in the newspapers only 50% ($^1/_2$) of the time and run a big promo $^1/_2$ of the time. The expected gain of Airtel is $^1/_2$ of the market share.

3.5.6 EQUILIBRIUM PAIRS

In mixed strategies, a pair of optimal strategies a* and b* is in equilibrium if for any other a and b, E(a,b*) <E(a*,b*) <E(a*, b)

A pair of strategies (a*, b*) in a two person zero sum game is in equilibrium if and only if {(a*, b*), E(a*, b*)} is a solution to the game. Nash Theory states that any two person game (whether zero-sum or non-zero-sum) with a finite number of pure strategies has at least one equilibrium pair. No player can do better by changing strategies, given that the other players continue to follow the equilibrium strategy.

3.5.7 OPTIMAL STRATEGIES IN 2 X N MATRIX GAME

Suppose we have a matrix game of

$$\begin{bmatrix} 5 & 2 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

Now

$\max_i(\min_j a_{ij})= \max(2,3)=3$ while = min(max) 4.

The two players now have to look for ways of assuring themselves of the largest possible shares of the difference

$\max_i(\min_j a_{ij}) - \min_i(\max_j a_{ij}) \geq 0$

They will therefore need to select strategies randomly to confuse each other. When a player chooses any two or more strategies at random according to specific probabilities this device is known as a mixed strategy.

There are various method employed in solving 2x2, 2xn, mx2 and m x n game matrix and hence finding optimal strategies as we shall discuss in this and the next few sections. Suppose the matrix of game is m x n. If player one is allowed to select strategy I. with probability pi and player two strategy II with probability q.

then we can say player 1 uses strategy

$P=(P_1,P_2\ldots P_m)$

While player 2 selects strategy

$q=(q_1,q_2,\ldots q_n)$.

The expected payoffs for player 1 by player two can be explained in

$$E\sigma\ \sigma * \sum_{i=1} \sum_{j=1} pi\varphi\ (pi)q$$

In this game the row player has strategy q = (q1, q2.. .q). The max-mm reasoning is used to find the optimal strategies to be employed by both playeNWe demonstrate with a practical example:

Example 14.6

Let the matrix game be

$$\begin{bmatrix} 5 & 2 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

Solution

Inspection shows that this does not have a saddle point solution. The optimal strategy p" for the row player is the one that will give him the maximum pay-off.

Since p = (p p2). Let the expected value of the row be represented by E_1player. If player 2 plays column 1 is =

5p+3(1 .p) 2p+3p

If player 2 plays column 2 we have

$E_{2(p)} = 2P + 4\ (l-P) = -2P+4$

and if player 2 plays colunm 3 we have

$E_3(p)\ 4_{(p)}+5(1-p) = p+5$.So, $E_{1(p)} = 2p + 3$; $E_{2(p)} = 2p+4$ and $E_{3(p)} = p+5$

are the payoffs for player 1 against the three part strategies of player 2, we give

arbitrary values for p to check which of these strategies by player2 will yield the largest payoff for

player 1.

Let $p^3/_4$ ……. $E_1=-2x^3/_4+3=4^1/_2$

$E_{2(p)} = 2x¼+42^1/_2$ $E_{2(p)} =-^3/_4+5=4^1/_4$.

So the two largest are $E_{1(p)}$, $E_{3(p)}$ and we equate them to get

$$2p = 3 = p+5$$

so $\quad 3p= 2,\ p={}^{2}/_{3}$

$E_{j(p)} = (2 \text{ x } {}^{2}/_{3}) + 3\ 4^{1}/_{3}$

$E = -2_{(p)} -2 \text{ x } 2^{2}/_{3} + 4 = 2^{2}/_{3}$and $E_{3(p)} = {}^{2}/_{3} + 5 = 74^{1}/_{3}$

So$({}^{2}/_{3},\ {}^{1}/_{3})$ is optimal for player 1. To get the optimal strategy for player 2, we observe that it is advisable for player 2 to play column 2 in other to ensure that the payoff to row player is minimal. So the game is reduced to

$$\begin{bmatrix} 5 & 4 \\ 3 & 5 \end{bmatrix}$$

Let (q, l-q) be the strategy for player 2 in a required game.

So 5q + 4 (1-q) 3q +5(l-q)

5q+4-4q=3q+5-5q

q +45-2q

3q=l $\quad$ q=1/3

So it is optimal for player 2 to play mixed strategy with probability $q({}^{1}/_{3},O,{}^{2}/_{3})$.If we substitute $q = {}^{1}/_{3}$ into q+4 or 5-2q, we obtain $4^{1}/_{3}$ as before. This is the value of the game.

3.5.8 OPTIMAL STRATEGIES FOR M X 2 ZERO - SUM GAMES

The procedure here is to convert to 2 x m game by finding the transpose of the matrix of the payoffs and then multiplying 2 x m matrix by =1. The new game matrix is then solved using the procedure for 2 x m matrix described earlier

Example 14.7

Find the optimal strategies for the matrix game

$$X = \quad 1 \quad -2 \qquad \begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix}$$

Solution

$$X^T = \begin{bmatrix} -1 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$$

Next multiply each element by - ,to obtain the matrix game

$$\begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -2 \end{bmatrix}$$

We then solve this using the method described earlier to obtain randomised optimal strategies ($^3/_5$,$^2/_5$) for player I and ($^3/_5$,$^2/_5$,0) for player 2 with values of the games being V5. Graphical methods can also be employed to solve 2xn and mx2 games. Here the expected payoffs are plotted as the functions and the intersection of the lines gives the value of p(or q).

3.5.9 OPTIMAL STRATEGIES IN M X N TWO PERSONS ZERO- SUM GAME USING THE SIMPLEX METHOD

In this type of games the method usually used is the simplex method of linear programming. It involves converting the two persons zero-sum game into a Standard Maximum Problem (SMP). If any negative number exists in the payoff matrix we eliminate by adding a suitable constant to every entry to ensure that all the entries are positive. From the prime the dual of the matrix of payoffs is formed, and both are solved using the Simplex method. The optimal strategy for both row and column players are obtained by dividing each of the optimal value obtained by their sum. We now give details on how this method can be used, to solve the next example.

Example 14.8

Find the randomised optimal strategies for the matrix of payoffs

$$\begin{matrix} 1 & 0 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 1 \end{matrix}$$

Solution- This matrix does not have a saddle point and it cannot be solved by using the concept of dominating strategies. As a 3x3 matrix of payoff, we can use the simplex linear programming method to solve it. Since there are negative entries, we convert it to a matrix of positive entries by adding constant c 2, we then obtain

$$P_2 = \begin{pmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & 3 \end{pmatrix}$$

4.0 CONCLUSION

The theory of games (or game theory or competitive strategies) is a mathematical theory that deals with the general features of competitive situations. This theory is helpful when two or more individuals or organisations with conflicting objectives try to make decisions. In such a situation, a decision made by on person affects the decision made by one or more of the remaining decision makers, and the final outcome depend depends upon the decision of all the parties. The theory of games is based on the minimax principle put forward by J. Von Neumann which implies that each competitor will act as to minimise his maximum loss (or maximise his minimum gain) or achieve best of the worst. So far, only simple competitive problems have been analysed by this mathematical theory. The theory does not describe how a game should be played; it describes only the procedure and principles by which players should be selected

5.0 SUMMARY

Making decision is an integral and continuous aspect of human life. For child or adult, man or woman, government official or business executive, worker or supervisor, participation in the process of decision- making is a common feature of everyday life. A competitive situation is called a game if it has the following characteristics- there is finite number of competitors called Players. A list of finite or infinite number of possible courses of action is available to each player; a list of finite or infinite number of possible courses of action is available to each player; a play is played when each player chooses one of his courses of actions; all players act rationally and intelligently. Each player is interested in maximizing his gains or minimizing his losses; each player makes individual decisions without direct communication between the players; it is assumed that each player knows complete relevant information. A game in which the gains of one player are the losses of other player or the algebraic sum of gains of both players is equal to zero, the game is known as Zero sum game (ZSG). Next we defined some important elements in game theory like- player, moves, game, decision maker, objective, behaviour, decision, conflict, strategy, perfect information, payoffs, zero-sum, and game. Finally, we solved problems involving Two-Person Zero-Sum Game, Pure Strategies, Dominating Strategies, Mixed Strategies, Optimal Strategies in 2 X 2 Matrix Game, Equilibrium Pairs, Optimal Strategies in 2 X N Matrix Game, Optimal Strategies For M X 2 Zero - Sum Games.

6.0 TUTOR MARKED ASSIGNMENT

1. What do you understand by a game?
2. Write short notes on the following

- Player

- Moves
- Game
- Decision maker
- Objective
- Behaviour
- Decision
- Conflict
- Strategy
- Perfect information
- Pay-offs
- Zero-sum Game

3.0 Find the optimal strategies for the matrix game

$$X = \quad 3 \quad -2 \qquad \begin{bmatrix} -2 & 2 \\ -2 & 3 \end{bmatrix}$$

7.0 REFERENCES

Adebayo, O.A., Ojo, O., and Obamire, J. K. (2006). Operations Research in Decision Analysis, Lagos: Pumark Nigeria Limited.

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UNIT 3: INVENTORY CONTROL

1.0 Introduction
2.0 Objective
3.0 Main Content
   3.1 Definition of Inventory and Inventory Control
   3.2 Basic Concepts in Inventory Planning
   3.3 Necessity for Maintaining Inventory
   3.4 Causes of Poor Inventory Control Systems
   3.5 Classification of Inventories
   3.6 Costs Associated With Inventory
   3.7 Purpose of Maintaining Inventory or Objective of inventory cost Control
   3.8 Other Factors to be considered in inventory Control
   3.9 Inventory Control Problem
   3.10 The Classical EOQ Model (Demand Rate Uniform, Replenishment Rate Infinite)
4.0 Conclusion
5.0 Summary
6.0 Tutor Marked Assignment
7.0 References

1.0 INTRODUCTION

One of the basic functions of management is to employ capital efficiently so as to yield the maximum returns. This can be done in either of two ways or by both, *i.e.* (a) By maximizing the margin of profit; or (b) By maximizing the production with a given amount of capital, *i.e.* to increase the productivity of capital. This means that the management should try to make its capital work hard as possible. However, this is all too often neglected and much time and ingenuity are devoted to make only labour work harder. In the process, the capital turnover and hence the productivity of capital is often totally neglected.

Several new techniques have been developed and employed by modern management to remedy this deficiency. Among these, Materials Management has become one of the most effective. In Materials Management, Inventory Control play vital role in increasing the productivity of capital.

Inventory management or Inventory Control is one of the techniques of Materials Management which helps the management to improve the productivity of capital by reducing the material costs, preventing the large amounts of capital being locked up for long periods, and improving the capital - turnover ratio. The techniques of inventory control were evolved and developed during and after the Second World War and have helped the more industrially developed countries to make spectacular progress in improving their productivity.

The importance of materials management/inventory control arises from the fact that materials account for 60 to 65 percent of the sales value of a product, that is to say, from every naira of the sales revenue, 65 kobo are spent on materials. Hence, small change in material costs can result in large sums of money saved or lost. Inventory control should, therefore, be considered as a function of prime importance for our industrial economy.

Inventory control provides tools and techniques, most of which are very simple to reduce/control the materials cost substantially. A large portion of revenue (65 percent) is exposed to the techniques, correspondingly large savings result when they are applied than when attempts are made to saver on other items of expenditure like wages and salaries which are about 16 percent or overheads which may be 20 percent. By careful financial analysis, it is shown that a 5 percent reduction in material costs will result in increased profits equivalent to a 36 percent increase in sales (Murthy, 2007).

2.0 OBJECTIVES

At the this study unit, you should be able to

1. Define inventory control
2. Explain the basic concepts in inventory control
3. Identify the issues that necessitate maintaining inventory
4. Identify causes of poor inventory control systems
5. Discuss the various classifications of inventories
6. Highlight the costs associated with inventory
7. Identify objective of inventory cost control
8. Discuss the problems associated with inventory control
9. Solve problems associated with the classical EOQ model.

3.0 MAIN CONTENT

3.1 DEFINITION OF INVENTORY AND INVENTORY CONTROL

The word *inventory* means a physical stock of material or goods or commodities or other economic resources that are stored or reserved or kept in stock or in hand for smooth and efficient running of future affairs of an organization at the minimum cost of funds or capital blocked in the form of materials or goods (Inventories). The function of directing the movement of goods through the entire manufacturing cycle from the requisitioning of raw materials to the inventory of finished goods in an orderly manner to meet the objectives of maximum customer service with minimum investment and efficient (low cost) plant operation is termed as *inventory control.* (Murthy, 2007)

Gupta and Hira (2012) defined an inventory as consisting of usable but idle resources such as men, machines, materials, or money. When the resources involved are material, the inventory is called stock. An inventory problem is said to exist if either the resources are subject to control or if there is at least one such cost that decrease as inventory increases. The objective is to minimise total (actual or expected) cost. However, in situations where inventory affects demand, the objective may also be to minimise profit.

3.2 BASIC CONCEPTS IN INVENTORY PLANNING

For many organizations, inventories represent a major capital cost, in some cases the dominant cost, so that the management of this capital becomes of the utmost importance. When considering the inventories, we need to distinguish different classes of items that are kept in stock. In practice, it turns out that about 10% of the items that are kept in stock usually account for something in the order of 60% of the value of all inventories. Such items are therefore of prime concern to the company, and the stock of these items will need close attention. These most important items are usually referred to as “A items” in the ABC classification system developed by the General Electric Company in the 1950s. The items next in line are the B items, which are of intermediate importance. They typically represent 30% of the items, corresponding to about 30% of the total inventory value. Clearly, B items do require some attention, but obviously less than A items. Finally, the bottom 60% of the items are the C items. They usually represent maybe 10% of the monetary value of the total inventory. The control of C items in inventory planning is less crucial than that of the A and B items. The models in this chapter are mostly aimed at A items.

Due to the economic importance of the management of inventories, a considerable body of knowledge has developed as a specialty of operations research. We may mention just-in-time (JIT) systems that attempt to keep inventory levels in a production system at an absolute minimum, and put to work in Toyota's so-called kanban system. There are also material requirements planning (MRP) aimed at using the estimated demand for a final product in order to determine the need for materials and components that are part of a final product. Multi-echelon and supply-chain management systems also consider similar aspects of production-inventory control systems. Such topics are beyond the scope of this text, in which we can only cover some basic inventory models (Eiselt and Sandblom, 2012).

3.3 NECESSITY FOR MAINTAINING INVENTORY

Though inventory of materials is an idle resource (since materials lie idle and are not to be used immediately), almost every organisation. Without it, no business activity can be performed, whether it is service organisation like a hospital or a bank or it a manufacturing or trading organisation. Gupta and Hira (2012) present the following reasons for maintain inventories in organisations.

1. It helps in the smooth and efficient of an enterprise.
2. It helps in providing service to the customer at short notice.
3. In the absence of inventory, the enterprise may have to pay high prices due to piecemeal purchasing.
4. It reduces product cost since there is an added advantage of batching and long, uninterrupted production runs.

5. It acts as a buffer stock when raw materials are received late and shop rejection is too many.
6. Process and movement inventories (also called pipeline stock) are quite necessary in big enterprises where significant amount of time is required to tranship items from one location to another.
7. Bulk purchases will entail fewer orders and, therefore, less clerical cost.
8. An organisation may have to deal with several customers and vendors who may not be necessarily near it. Inventories therefore have to be built to meet the demand at least during the transit period.
9. It helps in maintaining economy by absorbing some of the fluctuations when the demand for an item fluctuates or is seasonal.

3.4 CAUSES OF POOR INVENTORY CONTROL SYSTEMS

a. Overbuying without regard to the forecast or proper estimate of demand to take advantages of favourable market.
b. Overproduction or production of goods much before the customer requires them
c. Overstocking may also result from the desire to provide better service to the custom.
d. Cancellation of orders and minimum quantity stipulations by the suppliers may also give rise to large inventories.

(Gupta and Hira, 2012)

3.5 CLASSIFICATION OF INVENTORIES

Inventories may be classified as those which play direct role during manufacture or which can be identified on the product and the second one are those which are required for manufacturing but not as a part of production or cannot be identified on the product. The first type is labelled as ***direct inventories*** and the second are labelled as ***indirect inventories***. Further classification of direct and indirect inventories is as follows:

A. Direct inventories

(*i*) *Raw material inventories or Production Inventories*: The inventory of raw materials is the materials used in the manufacture of product and can be identified on the product. In inventory control manager can concentrate on the

(*a*) Bulk purchase of materials to save the investment,

(*b*) To meet the changes in production rate,

(*c*) To plan for buffer stock or safety stock to serve against the delay in delivery of inventory against orders placed and also against seasonal fluctuations. Direct inventories include the following:

- **Production Inventories-** items such as raw materials, components and subassemblies used to produce the final products.
- **Work-in-progress Inventory-** items in semi-finished form or products at different stages of production.
- **Finished Goods Inventory**
- **Miscellaneous Inventory-** all other items such as scrap, obsolete and unsaleable products, stationary and other items used in office, factory and sales department, etc.

(*ii*) *Work-in -process inventories or in process inventories:* These inventories are of semi-finished type, which are accumulated between operations or facilities.As far as possible, holding of materials between operations to be minimized if not avoided. This is because; as we process the materials the economic value (added labour cost) and use values are added to the raw material, which is drawn from stores. Hence if we hold these semi-finished material for a long time the inventory carrying cost goes on increasing, which is not advisable in inventory control. These inventories serve the following purposes:

(*a*) Provide economical lot production,

(*b*) Cater to the variety of products,

(*c*) Replacement of wastages,

(*d*) To maintain uniform production even if sales varies.

(*iii*) *Finished goods inventories:* After finishing the production process and packing, the finished products are stocked in stock room. These are known as finished goods inventory. These are maintained to:

(*a*) To ensure the adequate supply to the customers,

(*b*) To allow stabilization of the production level and

(*c*) To help sales promotion programme.

(*iv*) *MRO Inventory or Spare parts inventories:* Maintenance, Repair, and Operation items such as spare parts and consumable stores that do not go into final products but are consumed during the production process. Any product sold to the customer, will be subjected to wear and tear due to usage and the customer has to replace the worn-out part. Hence the manufacturers always calculate the life of the various components of his product and try to supply the spare components to the market to help after sales service. The use of such spare parts inventory is:

(*a*) To provide after sales service to the customer,

(*b*) To utilize the product fully and economically by the customer.

(*iv*) ***Scrap or waste inventory or Miscellaneous Inventory:*** While processing the materials, we may come across certain wastages and certain bad components (scrap), which are of no use. These may be used by some other industries as raw material. These are to be collected and kept in a place away from main stores and are disposed periodically by auctioning.

B. Indirect Inventories

Inventories or materials like oils, grease, lubricants, cotton waste and such other materials are required during the production process. But we cannot identify them on the product. These are known as indirect inventories. In our discussion of inventories, in this chapter, we only discuss about the direct inventories.
Inventories may also be classified depending on their nature of use. They are:

(*i*) ***Fluctuation Inventories:*** These inventories are carried out to safeguard the fluctuation in demand, non-delivery of material in time due to extended lead-time.
These are sometimes called as Safety stock or reserves. In real world inventory situations, the material may not be received in time as expected due to trouble in transport system or some times, the demand for a certain material may increase unexpectedly. To safeguard such situations, safety stocks are maintained. The level of this stock will fluctuate depending on the demand and lead-time etc.

(*ii*) ***Anticipation inventory:*** When there is an indication that the demand for company's product is going to be increased in the coming season, a large stock of material is stored in anticipation. Some times in anticipation of raising prices, the material is stocked. Such inventories, which are stocked in anticipation of raising demand or raising rises, are known as anticipation inventories.

(*iii*) ***Lot size inventory or Cycle inventories:*** This situation happens in batch production system. In this system products are produced in economic batch quantities. It sometime happens that the materials are procured in quantities larger than the economic quantities to meet the fluctuation in demand. In such cases the excess materials are stocked, which are known as lot size or cycle inventories.

(*iv*) ***Transportation Inventories:*** When an item is ordered and purchased they are to be received from the supplier, who is at a far of distance. The materials are shipped or loaded to a transport vehicle and it will be in the vehicle until it is delivered to the receiver. Similarly, when a finished product is sent to the customer by a transport vehicle it cannot be used by the purchaser until he receives it. Such inventories, which are in transit, are known as Transportation inventories.

(*v*) ***Decoupling inventories:*** These inventories are stocked in the manufacturing plant as a precaution, in case the semi-finished from one machine does not come to the next machine, this stock is used to continue a production. Such items are known as decoupling inventories.

3.6 COSTS ASSOCIATED WITH INVENTORY

While maintaining the inventories, we will come across certain costs associated with inventory, which are known as *economic parameters*. Most important of them are discussed below:

A. **Inventory Carrying Charges, or Inventory Carrying Cost or Holding Cost or Storage Cost (C_1) or (i%)**

This cost arises due to holding of stock of material in stock. This cost includes the cost of maintaining the inventory and is proportional to the quantity of material held in stock and the time for which the material is maintained in stock. The components of inventory carrying cost are:

(i) Rent for the building in which the stock is maintained if it is a rented building. In case it is own building, depreciation cost of the building is taken into consideration. Sometimes for own buildings, the nominal rent is calculated depending on the local rate of rent and is taken into consideration.

(ii) It includes the cost of equipment if any and cost of racks and any special facilities used in the stores.

(iii) Interest on the money locked in the form of inventory or on the money invested in purchasing the inventory.

(iv) The cost of stationery used for maintaining the inventory.

(v) The wages of personnel working in the stores.

(vi) Cost of depreciation, insurance.

(vii) Cost of deterioration due to evaporation, spoilage of material etc.

(viii) Cost of obsolescence due to change in requirement of material or changed in process or change in design and item stored as a result of becomes old stock and become sales.

(ix) Cost of theft and pilferage *i.e.* indenting for the material in excess of requirement.

This is generally represented by C_1naira per unit quantity per unit of time for production model. That is manufacturing of items model. For purchase models it is represented by $i\%$ of average inventory cost. If we take practical situation into consideration, many a time we see that the inventory carrying cost (some of the components of the cost) cannot be taken proportional to the quantity of stock on hand. For example, take rent of the stores building. As and when the stock is consumed, it is very difficult to calculate proportion of rent in proportion to the stock in the stores as the rent will not vary day to day due to change in inventory level. Another logic is that the money invested in inventory may be invested in other business or may be deposited in the bank to earn interest. As the money is in the form of inventory, we cannot earn interest but losing the expected interest on the money. This cost of money invested, is generally compared to the interest rate i% and is taken as the inventory carrying cost. Hence the value of 'i' will be a fraction of a and will be $0 < i < 1$. In many instances, the bank rate of interest is somewhere between 16 to 20 % and other components like salary, insurance, depreciation etc. may work out to 3 to 5 %. Hence, the total of all components will be around 22 to 25 % and this is taken as the cost of inventory carrying cost and is expressed as i % of average inventory cost.

B. **Shortage cost or Stock - out - cost- (C_2)**

Sometimes it so happens that the material may not be available when needed or when the demand arises. In such cases the production has to be stopped until the procurement of the material, which may lead to miss the delivery dates or delayed production. When the organization could not meet the delivery promises, it has to pay penalty to the customer. If the situation of stock out will occur very often, then the customer may not come to the organization to place orders that is the organization is losing the customers In other words, the organization is losing the goodwill of the customers The cost of good will cannot be estimated. In some cases it will be very heavy to such extent that the organization has to forego its business. Here to avoid the stock out situation, if the organization stocks more material, inventory carrying cost increases and to take care of inventory cost, if the organization purchases just sufficient or less quantity, then the stock out position may arise. Hence the inventory manager must have sound knowledge of various factors that are related to inventory carrying cost and stock out cost and estimate the quantity of material to be purchased or else he must have effective strategies to face grave situations. The cost is generally represented as so many naira and is represented by C_2.

C. Set up cost or Ordering cost or Replenishment Cost (C_3)

For purchase models, the cost is termed as ordering cost or procurement cost and for manufacturing cost it is termed as set up cost and is represented by C_3.

(*i*) *Set up cost:* The term set up cost is used for production or manufacturing models. Whenever a job is to be produced, the machine is to set to produce the job. That is the tool is to be set and the material is to be fixed in the jobholder. This consumes some time. During this time the machine will be idle and the labour is working. The cost of idle machine and cost of labour charges are to be added to the cost of production. If we produce only one job in one set up, the entire set up cost is to be charged to one job only. In case we produce '*n*' number of jobs in one set up, the set up cost is shared by '*n*' jobs. In case of certain machines like N.C machines, or Jig boarding machine, the set up time may be 15 to 20 hours The idle cost of the machine and labour charges may work out to few thousands of naira. Once the machine set up is over, the entire production can be completed in few hours if we produce more number of products in one set up the set up cost is allocated to all the jobs equally. This reduces the production cost of the product. For example let us assume that the set up cost is N 1000/-. If we produce 10 jobs in one set up, each job is charged with N 100/- towards the set up cost. In case, if we produce 100 jobs, the set up cost per job will be N10/-. If we produce, 1000 jobs in one set up, the set up cost per job will be Re. 1/- only. This can be shown by means of a graph as shown in figure 15.1.

(***ii***) ***Ordering Cost or Replenishment Cost:*** The term Ordering cost or Replenishment cost is used in purchase models. Whenever any material is to be procured by an organization, it has to place an order with the supplier. The cost of stationary used for placing the order, the cost of salary of officials involved in preparing the order and the postal expenses and after placing the order enquiry charges all put together, is known as ordering cost. In Small Scale Units, this may be around N 25/- to N 30/- per order. In Larger Scale Industries, it will be around N 150 to N 200 /- per order. In Government organizations, it may work out to N 500/- and above per order. If the organization purchases more items per order, all the items share the ordering cost. Hence the materials manager must decide how much to purchase per order so as to keep the ordering cost per item at minimum.
One point we have to remember here, to reduce the ordering cost per item, if we purchase more items, the inventory carrying cost increases. To keep inventory carrying cost under control, if we purchase less quantity, the ordering cost increase. Hence one must be careful enough to decide how much to purchase? The nature of ordering cost can also be shown by a graph as shown in figure 8.1. If the ordering cost is C_3 per order (can be equally applied to set up cost) and the quantity ordered

/ produced is ‘*q*’ then the ordering cost or set up cost per unit will be $C3/q$ is inversely proportional to the quantity ordered, *i.e*. decreased with the increase in ‘*q*’ as shown in the graph below;

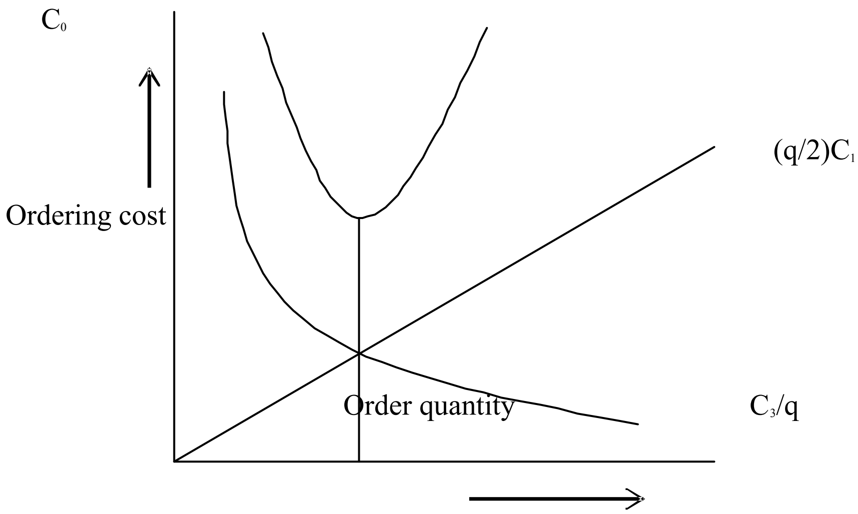

Fig. 15.1: Ordering Cost

Source : Murthy, P. R. (2007) Operations Research 2nd ed. New Delhi: New Age International Publishers

(***iii***) ***Procurement Cost :*** These costs are very much similar to the ordering cost / set up cost. This cost includes cost of inspection of materials, cost of returning the low quality materials, transportation cost from the source of material to the purchaser‘s site. This is proportional to the quantity of materials involved. This cost is generally represented by ‘*b*’ and is expressed as so many naira per unit of material. For convenience, it always taken as a part of ordering cost and many a time it is included in the ordering cost / set up cost.

D. Purchase price or direct production cost

This is the actual purchase price of the material or the direct production cost of the product. It is represented by '*p*'. *i.e*. the cost of material is N '*p*' per unit. This may be constant or variable. Say for example the cost of an item is N 10/- item if we purchase 1 to 10 units. In case we purchase more than 10 units, 10 percent discount is allowed. *i.e*. the cost of item will be N9/- per unit. The purchase manager can take advantage of discount allowed by purchasing more. But this will increase the inventory carrying charges. As we are purchasing more per order, ordering cost is reduced and because of discount, material cost is reduced. Materials manager has to take into consideration these cost – quantity relationship and decide how much to purchase to keep the inventory cost at low level.

3.7 PURPOSE OF MAINTAINING INVENTORY OR OBJECTIVE OF INVENTORY COST CONTROL

The purpose of maintaining the inventory or controlling the cost of inventory is to use the available capital optimally (efficiently) so that inventory cost per item of material will be as small as possible. For this the materials manager has to strike a balance between the interrelated inventory costs. In the process of balancing the interrelated costs *i.e*. Inventory carrying cost, ordering cost or set up cost, stock out cost and the actual material cost. Hence we can say that *the objective of controlling the inventories is to enable the materials manager to place and order at right time with the right source at right price to purchase right quantity.* The benefits derived from efficient inventory control are:

(i) It ensures adequate supply of goods to the customer or adequate of quantity of raw materials to the manufacturing department so that the situation of stock out may be reduced or avoided.

(ii) By proper inventory cost control, the available capital may be used efficiently or optimally, by avoiding the unnecessary expenditure on inventory.

(iii) In production models, while estimating the cost of the product the material cost is to be added. The manager has to decide whether he has to take the actual purchase price of the material or the current market price of the material. The current market price may be less than or greater than the purchase price of the material which has been purchased some period back. Proper inventory control reduces such risks.

(iv) It ensures smooth and efficient running of an organization and provides safety against late delivery times to the customer due to uncontrollable factors

(v) A careful materials manager may take advantage of price discounts and make bulk purchase at the same time he can keep the inventory cost at minimum.

(vi) It enables a manager to select a proper transportation mode to reduce the cost of transportation.

(vii) Avoids the chances of duplicate ordering.

(viii) It avoids losses due to deterioration and obsolescence etc.

(ix) Causes of surplus stock may be controlled or totally avoided.

(x) Proper inventory control will ensure the availability of the required material in required quantity at required time with the minimum inventory cost.

Though many managers consider inventory as an enemy as it locks up the available capital, but by proper inventory control they can enjoy the benefits of inventory control and then they can realize that the inventory is a real friend of a manager in utilizing the available capital efficiently.

3.8 OTHER FACTORS TO BE CONSIDERED IN INVENTORY CONTROL

There are many factors, which have influence on the inventory, which draws the attention of an inventory manager, they are:

(*i*) Demand

The demand for raw material or components for production or demand of goods to satisfy the needs of the customer, can be assessed from the past consumption/supply pattern of material or goods. We find that the demand may be deterministic in nature *i.e.*, we can specify that the demand for the item is so many units for example say '*q*' units per unit of time. Also the demand may be static, *i.e.* it means constant for each time period (uniform over equal period of times). Further, the demand may follow several patterns and so why it is uncontrolled variable, such as it may be uniformly distributed over period or instantaneous at the beginning of the period or it may be large in the beginning and less in the end etc. These patterns directly affect the total carrying cost of inventory.

(*ii*) Production of goods or Supply of goods to the inventory

The supply of inventory to the stock may deterministic or probabilistic (stochastic) in nature and many a times it is uncontrollable, because, the rate of production depends on the production, which is once again depends on so many factors which are uncontrollable / controllable factors Similarly supply of inventory depends on the type of supplier, mode of supply, mode of transformation etc. The properties of supply mode have its effect in the level of inventory maintained and inventory costs.

(*iii*) Lead time or Delivery Lags or Procurement time

Lead-time is the time between placing the order and receipt of material to the stock. In production models, it is the time between the decisions made to take up the order and starting of production. This time in purchase models depends on many uncontrollable factors like transport mode, transport route, agitations etc. It may vary from few days to few months depending on the nature of delay. The materials manager has to refer to the past records and approximately estimate the lead period and estimate the quantity of safety stock to be maintained. In production models, it may depend on the labour absenteeism, arrival of material to the stores, power supply, etc.

(*iv*) Type of goods

The inventory items may be discrete or continuous. Sometimes the discrete items are to be considered as continuous items for the sake of convenience.

(*v*) Time horizon

The time period for which the optimal policy is to be formulated or the inventory cost is to be optimized is generally termed as the Inventory planning period of Time horizon. This time is represented on X - axis while drawing graphs. This time may be finite or infinite.

(*vi*) Safety stock or Buffer stock

Whatever care taken by the materials manager, one cannot avoid the stock out situation due to many factors. To avoid the stock out position the manager sometimes maintains some extra stock, which is generally known as Buffer Stock, or Safety Stock. The level of this stock depends on the demand pattern and the lead-time. This should be judiciously calculated because, if we stock more the inventory carrying cost increases and there is chance of pilferage or theft. If we maintain less stock, we may have to face stock out position. The buffer stock or safety stock is generally the consumption at the maximum rate during the time interval equal to the difference between the maximum lead timesand the normal (average) lead time or say the maximum, demand during lead time minus the average demand during lead time. Depending on the characteristics above discussed terms, different types of inventory models may be formulated. These models may be deterministic models or probabilistic model depending on the demand pattern.

In any inventory model, we try to seek answers for the following questions:

(*a*) **When should the inventory be purchased for replenishment?** For example, the inventory should be replenished after a period't' or when the level of the inventory is qo.

(*b*) **How much quantity must be purchased or ordered or produced at the time of replenishment so as to minimize the inventory costs?** For example, the inventory must be purchased with the supplier who is supplying at a cost of Np/ per unit. In addition to the above depending on the data available, we can also decide from which source we have to purchase and what price we have to purchase? But in general time and quantity are the two variables, we can control separately or in combination.

3.9 INVENTORY CONTROL PROBLEM

The inventory control problem consists of the determination of three basic factors:

1. When to order (produce or purchase)?
2. How much to order?
3. How much safety stock to be kept?

When to order: This is related to lead time (also called delivery lag) of an item. Lead time may interval between the placement of an order for an item and its receipt in stock. It may be replenishment order on an outside or within the firm. There should be enough stock for each item so that customers' orders can be reasonably met from this stock until replenishment. This stock level known as *reorder level,* has to be determined for each item. It is determined by balancing the cost of maintaining these stocks and the disservice to the customer if his orders are not met.

How much to order: Each order has an associated ordering cost or cost of acquisition. To keep this cost low, the number of orders has to be as reduced as possible. To achieve limited number of orders, the order size has to be increased.
But large order size would imply high inventory cost. Thus, the problem of how much to order is solved by compromising between the acquisition cost and the inventory carrying cost.

How much should the safety stock be. This is important to avoid overstocking while ensuring that no stock out takes place.

The inventory control policy of an organisation depends upon the demand characteristics. The demand for an item may be dependent or independent. For instance, the demand for the different models of television sets manufactured by a company does not depend upon the demand for any other item, while the demand for its components will depend upon the demand for the television sets.

3.10 THE CLASSICAL EOQ MODEL (Demand Rate Uniform, Replenishment Rate Infinite)

According Gupta and Hira 2012, the EOQ model is one of the simplest inventory models we have. A store keeper has an order to supply goods to customers at a uniform rate R per unit. Hence, the demand is fixed and known. Not shortages are allowed, consequently, the cost of shortage $C_2$is infinity. The store keeper places an order with a manufacturer every *t* time units, where *t* is fixed; and the ordering cost per order is C_3. Replenishment time is negligible, that is, replenishment rate is infinite so that the replacement is instantaneous (lead time is zero). The holding cost is assumed to be proportional to the amount of inventory as well as the time inventory is held. Hence the time of holding inventory I for time T is C_1IT, where C_1, C_2 and C_3 are assumed to be constants. The store keeper's problem is therefore to the following

i. How frequently should he place the order?
ii. How many units should he order in each order placed?

This model is represented schematically below.

If orders are placed at intervals t, a quantity $q = Rt$ must be ordered in each order. Since the stock in small time dt is $Rt\,dt$ the stock in time period t will be

$$\int_0^t Rt.dt = \frac{1}{2}Rt^2 = \frac{1}{2}qt = \text{Area of inventory triangle OAP}.$$

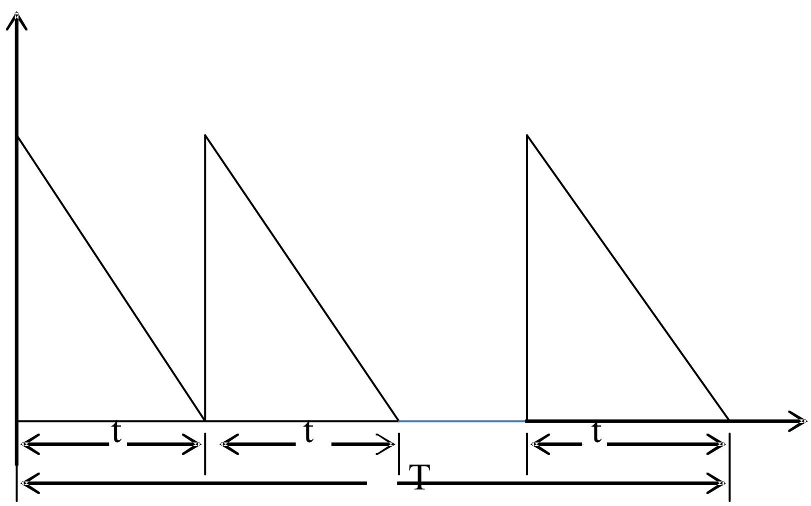

Fig. Inventory situation for EOQ model

$\therefore$ Cost of holding inventory during time $t = \frac{1}{2} C_1Rt^2$.

Order cost to place an order $= C_3$.

$\therefore$ Total cost during time $t = \frac{1}{2} C_1Rt^2 + C_3$.

$\therefore$ Average total cost per unit, $C(t) = \frac{1}{2} C_1 R t + \frac{C_3}{t}$ …………… *(1)*

C will be minimum if $\frac{dC(t)}{dt} = 0$ and $\frac{d^2C(t)}{dt^2}$ is positive.

Differentiating equation (1) w.r.t 't

$$\frac{d^2C(t)}{dt} = \frac{1}{2} C_1 R - \frac{C_3}{t^2} = 0, \text{ which gives } t = \sqrt{\frac{2C_3}{C_1 R}}.$$

Differentiating w.r.t. t

$\frac{d^2C(t)}{dt^2} = \frac{2C_3}{t^3}$, which is positive for value of t given by the above equation.

Thus $C(t)$ is minimum for optimal time interval,

$$t_o = \sqrt{\frac{2C_3}{C_1 R}} \quad \text{…………………………. } (2)$$

Optimum quantity q_0 to be ordered during each order,

$$q_0 = Rt_0 = \sqrt{\frac{2C_3 R}{C_1}} \quad \text{…………………… } (3)$$

This is known as the optimal lot size (or economic order quantity) formula by r. H. Wilson. It is also called Wilson's or square root formula or Harris lot size formula.

Any other order quantity will result in a higher cost.

The resulting minimum average cost per unit time,

$$C0(q) = \frac{1}{2} C_1 R \sqrt{\frac{2C_3}{C1R}} + C_3 \sqrt{\frac{C_1 R}{2C_3}}$$

$$= \frac{1}{\sqrt{2}} \sqrt{C_1 C_3 R} + \frac{1}{\sqrt{2}} \sqrt{C_1 C_3 R} = \sqrt{2C_1 C_3 R} \text{ …… } (4)$$

Also, the total minimum cost per unit time, including the cost of the item

$$= \sqrt{2C_1C_3R} + CR, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \textit{(5)}$$

Where C is cost/unit of the item
Equation *(1)* can be written in an alternative form by replacing *t* by *q*/R as

$$C(q) = \frac{1}{2}C_1 q + \frac{C_3 R}{q}$$

The average inventory is $\frac{q(t) + 0}{2} = \frac{q(t)}{2}$ and it is time dependent.

It may be realised that some of the assumptions made are not satisfied in actual practice. For instance, in real life, customer demand is usually not known exactly and replenishment time is usually not negligible.

Corollary 1. In the above model, if the order cost is $C_3 + bq$ instead of being fixed, where b is the cost of order per unit of item, we can prove that there no change in the optimum order quantity due to changed order cost.

Proof. The average cost per unit of time, $C(q) = \frac{1}{2}C_1 q + \frac{R}{q}(C_3 + bq)$.
From equation *(5),*

$\frac{dC(q)}{dq} = 0$ and $\frac{d^2 C(q)}{dq^2}$ is positive

That is, $\frac{1}{2}C_1 - \frac{RC_3}{q^2} = 0$ or $q = \sqrt{\frac{2RC_3}{C_1}}$,
and $\frac{d^2 C(q)}{dq^2} = \frac{2RC_3}{q^3}$, which is necessarily positive for above value of *q*.

$$q_0 = \sqrt{\frac{2C_3R}{C_1}}$$, which is the same as equation *(3)*

Hence, there is no change in the optimum order quantity as a result of the change in the cost of order.

Corollary 2. In the model in figure …… discussed above, the lead time has been assumed to be zero. However, most real life problems have positive lead time L from the order for the item was placed until it is actually delivered. The ordering policy of the above model therefore, must satisfy the reorder point.

If L is the lead time in days, and R is the inventory consumption rate in units per day, the total inventory requirements during the lead time = LR. Thus we should place an order q as soon as the stock level becomes LR. This is called reorder point p = LR.

In practice, this is equivalent to continuously observing the level of inventory until the reorder point is obtained. That is why economic lot size model is also called continuous review model.

If the buffer stock B is to maintained, reorder level will be

$$P = B + LR \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

Furthermore, if D days are required for reviewing the system,

$$p = B + LR + \frac{RD}{2} = B + R\left[L + \frac{D}{2}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

Assumptions in the EOQ Formula

The following assumptions have been made while deriving the EOQ formula:

1. Demand is known and uniform (constant)
2. Shortages are not permitted; as soon as the stock level becomes zero, it is instantaneously replenished.
3. Replenishment stock is instantaneous or replenishment rate is infinite.
4. Lead time is zero. The moment the order is placed, the quantity ordered is automatically received.
5. Inventory carrying cost and ordering cost per order remain constant over time. The former has a linear relationship with the quantity ordered and the latter with the number of order.
6. Cost of the item remains constant over time. There are no price- breaks or quantity discounts.
7. The item is purchased and replenished in lots or batches.

8. The inventory system relates to a single item.

Limitations of the EOQ Model

The EOQ formula has a number of limitations. It has been highly controversial since a number of objections have been raised regarding its validity. Some of these objections are:

1. In practice, the demand neither known with certainty nor it is uniform. If the fluctuations are mild, the formula can be applicable but for large fluctuations, it loses its validity. Dynamic EOQ models, instead, may have to be applied.
2. The ordering cost is difficult to measure. Also it may not be linearly related to the number of orders as assumed in the derivation of the model. The inventory carrying rate is still more difficult to measure and even to define precisely.
3. It is difficult to predict the demand. Present demand may be quite different from the past history. Hardly any prediction is possible for a new product to be introduced in the market.
4. The EOQ model assumes instantaneous replenishment of the entire quantity ordered. The practice, the total quantity may be supplied in parts. EOQ model is not applicable in such a situation.
5. Lead time may not be zero unless the supplier is next-door and has sufficient stock of the item, which is rarely so.
6. Price variations, quantity discounts and shortages may further invalidate the use of the EOQ formula.

However, the flatness of the total cost curve around the minimum is an answer to the many objections. Even if we deviate from EOQ within reasonable limits, there is no substantial change in cost. For example, if because of inaccuracies and errors, we have selected an order quantity 20%more (or less) than $q_0$the increase in total cost will be less than 20%.

EXAMPLE 15.1

A stock keeper has to supply 12000 units of a product per year to his customer. The demand is fixed and known and the shortage cost is assumed to be infinite. The inventory holding cost is N 0.20k per unit per month, and the ordering cost per order is N350. Determine

i. The optimum lot size q_0

ii. Optimum scheduling period t_0

iii. Minimum total variable yearly cost.

Solution

Supply rate R = $\frac{12,000}{12} = 1,000 \; unit/month,$

C1 = N 0.20K per unit per month, C3 = ₦350 per order.

i. $q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2X350X1000}{0.20}} = 1870 units/order$

ii. $t_0 = \sqrt{\frac{2C_3}{C_1R}} = \sqrt{\frac{2X350}{0.20X1000}} = 1.87 \; months = 8.1 \; weeks \; between \; orders$

iii. $C_0 = \sqrt{2C_1C_3R} = \sqrt{2X0.2X12X350X(1000X12)} = ₦4490 \; per \; year$

EXAMPL15.2

A particular item has a demand of 9000 unit/year. The cost of a single procurement is ₦100 and the holding cost per unit is ₦ 2.40k per year. The replacement is instantaneous and no shortages are allowed. Determine

i. The economic lot size,

ii. The number of orders per year,

iii. The time between orders

iv. The total cost per if the cost of one unit is ₦1

Solution

R = 9000 units/year

C3 = N100/procurement, C1 = N2.40/unit/year

i. $q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 X 100 X 9000}{2.40}} = 866\ units/procurement$

ii. $n_0 = \frac{1}{t_0} = \sqrt{\frac{2.40 X 9000}{2 X 100}} = \sqrt{108} = 10.4\ orders/year$

iii. $t_0 = \frac{1}{n_0} = \frac{1}{10.4} = 0.0962\ years =$
$1.15\ months\ between\ procurement$

iv. $C_0 = 900 X 1 + \sqrt{2C_1C_3R} = 9000 + \sqrt{2 X 2.40 X 100 X 9000}$

= 9000 + 2080 = ₦11080/year

EXAMPL 15.3

A stockist has to supply 400 units of a product every Monday to his customer. He gets the product at ₦ 50 per unit from the manufacturer. The cost of ordering and transportation from the manufacturer is ₦75 per order. The cost of carrying the inventory is 7.5% per year of the cost of the product. Find

i. The economic lot size
ii. The total optimal cost (including the capital cost)
iii. The total weekly profit if the item is sold for ₦ 55 per unit

Solution

R = 400 units/week

C3 = N75per order
C1 = 7.5% per year of the cost of the product

$= N\left(\frac{7.5}{100} X 50\right)$ per unit per year

$= \left(\frac{7.5}{100} X \frac{50}{2}\right)$ per unit per week

$= N\frac{3.75}{52}$ per unit per week

i. $q_0 = \sqrt{\frac{2C_3 R}{C_1}} = \sqrt{\frac{2 \times 75 \times 400 \times 52}{52}} = 912$ units per order

ii. $C_0 = 400 \times 50 + \sqrt{2C_1 C_3 R}$

$= 200000 + \sqrt{\frac{2 \times 3.75}{52}} \times 75 \times 400 = 200000 + 65.8$

$= N20065.80$ per week

iii. $Profit\ P = 55 \times 400 - C_0 = 22000.80 = N1934.20$ per week

4.0 CONCLUSION

Inventory management or Inventory Control is one of the techniques of Materials Management which helps the management to improve the productivity of capital by reducing the material costs, preventing the large amounts of capital being locked up for long periods, and improving the capital - turnover ratio. The techniques of inventory control were evolved and developed during and after the Second World War and have helped the more industrially developed countries to make spectacular progress in improving their productivity. Inventory control provides tools and techniques, most of which are very simple to reduce/control the materials cost substantially. A large portion of revenue (65 percent) is exposed to

the techniques, correspondingly large savings result when they are applied than when attempts are made to saver on other items of expenditure like wages and
salaries which are about 16 percent or overheads which may be 20 percent. By careful financial analysis, it is shown that a 5 percent reduction in material costs will result in increased profits equivalent to a 36 percent increase in sales.

5.0 SUMMARY

It has been an interesting journey through the subject of inventory control systems.
This unit has provided us with vital information about the inventory control model.
An inventory control model has been defined an inventory as consisting of usable but idle resources such as men, machines, materials, or money. When the resources involved are material, the inventory is called stock. Though inventory of materials is an idle resource (since materials lie idle and are not to be used immediately), almost every organisation. It helps in the smooth and efficient of an enterprise. It helps in providing service to the customer at short notice. In the absence of inventory, the enterprise may have to pay high prices due to piecemeal purchasing.
It reduces product cost since there is an added advantage of batching and long, uninterrupted production runs. It acts as a buffer stock when raw materials are received late and shop rejection is too many.

Overbuying without regard to the forecast or proper estimate of demand to take advantages of favourable market can may result in poor inventory control system.
Inventories can be classified into direct and indirect. Direct inventories include raw material inventories or production inventories, work-in -process inventories or in process inventories, finished goods inventories, MRO inventory or spare parts inventories. While indirect inventory type include, fluctuation inventories,

anticipation inventory, lot size inventory or cycle inventories, transportation inventories, and decoupling inventories. The purpose of maintaining the inventory or controlling the cost of inventory is to use the available capital optimally (efficiently) so that inventory cost per item of material will be as small as possible. The inventory control problem arises from, when to order, how much to order, how much should the safety stock be.

Finally, we discussed the EOQ model which is one of the simplest inventory models we have. A store keeper has an order to supply goods to customers at a uniform rate R per unit. Hence, the demand is fixed and known.

6.0 TUTOR MARKED ASSIGNMENT

1. What do you understand by the term inventory control?
2. Identify and discuss the different classifications of inventories.
3. Give six limitations of the EOQ model.
4. Outline the assumptions of the EOQ formula
5. List and explain three inventory control problems.
6. Clearly write out the EOQ formula and explain all its components.

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Unit 4: CASE ANALYSIS

1.0 Introduction
2.0 Objectives
3.0 Main Content
    3.1 What is a Case Study
    3.2 The Case Method As A Learning Tool
    3.3 How to do a Case Study
    3.4 Preparing a Case Study
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1.0 INTRODUCTION

This introduces us to case analysis. A case study is a description of an actual administrative situation involving a decision to be made or a problem to be solved.
It can be a real situation that actually happened just as described, or portions have been disguised for reasons of privacy. It is a learning toolin which students and Instructors participate in direct discussion of case studies, as opposed to the lecture

method, where the Instructor speaks and students listen and take notes. In the case method, students teach themselves, with the Instructor being an active guide, rather than just a talking head delivering content.

2.0 OBJECTIVES

After studying this unit, you should be able to

1. Discuss what case analysis involves
2. Analyse a case as a learning tool
3. Highlight the stages in preparing a case.
4. Analyse case data.
5. Outline key decision criteria in a case.

3.1 WHAT IS A CASE STUDY?

A case study is a description of an actual administrative situation involving a decision to be made or a problem to be solved. It can be a real situation that actually happened just as described, or portions have been disguised for reasons of privacy. Most case studies are written in such a way that the reader takes the place of the manager whose responsibility is to make decisions to help solve the problem. In almost all case studies, a decision must be made, although that decision might be to leave the situation as it is and do nothing.

3.2 THE CASE METHOD AS A LEARNING TOOL

The case method of analysis is a learning tool in which students and Instructors participate in direct discussion of case studies, as opposed to the lecture method, where the Instructor speaks and students listen and take notes. In the case method, students teach themselves, with the Instructor being an active guide, rather than just a talking head delivering content. The focus is on students learning through their joint, co-operative effort.

Assigned cases are first prepared by students, and this preparation forms the basis for class discussion under the direction of the Instructor. Students learn, often unconsciously, how to evaluate a problem, how to make decisions, and how to orally argue a point of view. Using this method, they also learn how to think in terms of the problems faced by an administrator. In courses that use the case method extensively, a significant part of the student's evaluation may rest with classroom participation in case discussions, with another substantial portion resting on written case analyses. For these reasons, using the case method tends to be very intensive for both students and Instructor.

Case studies are used extensively throughout most business programs at the university level, and The F.C. Manning School of Business Administration is no exception. As you will be using case studies in many of the courses over the next four years, it is important that you get off to a good start by learning the proper way to approach and complete them.

3.3 HOW TO DO A CASE STUDY

While there is no one definitive "Case Method" or approach, there are common steps that most approaches recommend be followed in tackling a case study. It is inevitable that different Instructors will tell you to do things differently; this is part of life and will also be part of working for others. This variety is beneficial since it will show you different ways of approaching decision making. What follows is intended to be a rather general approach, portions of which have been taken from an excellent book entitled, Learning with Cases, by Erskine, Leenders, & Mauffette-Leenders, published by the Richard Ivey School of Business, The University of Western Ontario, 1997.

Beforehand (usually a week before), you will get:

1. the case study,
2. (often) some guiding questions that will need to be answered, and
3. (sometimes) some reading assignments that have some relevance to the case subject.

Your work in completing the case can be divided up into three components:

1. what you do to prepare before the class discussion,
2. what takes place in the class discussion of the case, and
3. anything required after the class discussion has taken place.

For maximum effectiveness, it is essential that you do all three components. Here are the subcomponents, in order. We will discuss them in more detail shortly.

1. Before the class discussion:
1. Read the reading assignments (if any)
2. Use the Short Cycle Process to familiarize yourself with the case.
3. Use the Long Cycle Process to analyze the case
4. Usually there will be group meetings to discuss your ideas.
5. Write up the case (if required)
2. In the class discussion:
1. Someone will start the discussion, usually at the prompting of the Instructor.

2. Listen carefully and take notes. Pay close attention to assumptions. Insist that they are clearly stated.

3. Take part in the discussion. Your contribution is important, and is likely a part of your evaluation for the course.

3. After the class discussion:

1. Review ASAP after the class. Note what the key concept was and how the case fits into the course.

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3.4 PREPARING A CASE STUDY

It helps to have a system when sitting down to prepare a case study as the amount of information and issues to be resolved can initially seem quite overwhelming. The following is a good way to start.

Step 1: The Short Cycle Process

1. Quickly read the case. If it is a long case, at this stage you may want to read only the first few and last paragraphs. You should then be able to

2. Answer the following questions:

1. Who is the decision maker in this case, and what is their position and responsibilities?

2. What appears to be the issue (of concern, problem, challenge, or opportunity) and its significance for the organization?

3. Why has the issue arisen and why is the decision maker involved now?

4. When does the decision maker have to decide, resolve, act or dispose of the issue? What is the urgency to the situation?

3. Take a look at the Exhibits to see what numbers have been provided.

4. Review the case subtitles to see what areas are covered in more depth.

5. Review the case questions if they have been provided. This may give you some clues are what the main issues are to be resolved.

You should now be familiar with what the case study is about, and are ready to begin the process of analysing it. You are not done yet! Many students mistakenly believe that this is all the preparation needed for a class discussion of a case study.

If this was the extent of your preparation, your ability to contribute to the discussion would likely be limited to the first one quarter of the class time allotted.

You need to go further to prepare the case, using the next step. One of the primary reasons for doing the short cycle process is to give you an indication of how much work will need to be done to prepare the case study properly.

Step 2: The Long Cycle Process

At this point, the task consists of two parts:

1. A detailed reading of the case, and then
2. Analysing the case.

When you are doing the detailed reading of the case study, look for the following sections:

1. Opening paragraph: introduces the situation.
2. Background information: industry, organization, products, history, competition, financial information, and anything else of significance.
3. Specific (functional) area of interest: marketing, finance, operations, human resources, or integrated.
4. The specific problem or decision(s) to be made.
5. Alternatives open to the decision maker, which may or may not be stated in the case.

6. Conclusion: sets up the task, any constraints or limitations, and the urgency of the situation.

Most, but not all case studies will follow this format. The purpose here is to thoroughly understand the situation and the decisions that will need to be made. Take your time, make notes, and keep focussed on your objectives.

Analysing the case should take the following steps:

1. Defining the issue(s)
2. Analysing the case data
3. Generating alternatives
4. Selecting decision criteria
5. Analysing and evaluating alternatives
6. Selecting the preferred alternative
7. Developing an action/implementation plan

Defining the issue(s)/Problem Statement

The problem statement should be a clear, concise statement of exactly what needs to be addressed. This is not easy to write! The work that you did in the short cycle process answered the basic questions. Now it is time to decide what the main issues to be addressed are going to be in much more detail. Asking yourself the following questions may help:

1. What appears to be the problem(s) here?
2. How do I know that this is a problem? Note that by asking this question, you will be helping to differentiate the symptoms of the problem from the problem itself. Example: while declining sales or unhappy employees are a problem to most

companies, they are in fact, symptoms of underlying problems which need to addressed.

3. What are the immediate issues that need to be addressed? This helps to differentiate between issues that can be resolved within the context of the case, and those that are bigger issues that needed to addressed at a another time (preferably by someone else!).

4. Differentiate between importance and urgency for the issues identified. Some issues may appear to be urgent, but upon closer examination are relatively unimportant, while others may be far more important (relative to solving our problem) than urgent. You want to deal with important issues in order of urgency to keep focussed on your objective. Important issues are those that have a significant effect on:

1. Profitability,
2. strategic direction of the company,
3. source of competitive advantage,
4. morale of the company's employees, and/or
5. customer satisfaction.

The problem statement may be framed as a question, e.g. what should Joe do? or How can Mr Smith improve market share? Usually the problem statement has to be re-written several times during the analysis of a case, as you peel back the layers of symptoms or causation.

3.5 ANALYSING CASE DATA

In analysing the case data, you are trying to answer the following:

1. Why or how did these issues arise? You are trying to determine cause and effect for the problems identified. You cannot solve a problem that you cannot

determine the cause of! It may be helpful to think of the organization in question as consisting of the following components:

1. resources, such as materials, equipment, or supplies, and
2. people who transform these resources using
3. processes, which creates something of greater value.

Now, where are the problems being caused within this framework, and why?

2. Who is affected most by this issues? You are trying to identify who are the relevant stakeholders to the situation, and who will be affected by the decisions to be made.

3. What are the constraints and opportunities implicit to this situation? It is very rare that resources are not a constraint, and allocations must be made on the assumption that not enough will be available to please everyone.

4. What do the numbers tell you? You need to take a look at the numbers given in the case study and make a judgement as to their relevance to the problem identified. Not all numbers will be immediately useful or relevant, but you need to be careful not to overlook anything. When deciding to analyse numbers, keep in mind why you are doing it, and what you intend to do with the result. Use common sense and comparisons to industry standards when making judgements as to the meaning of your answers to avoid jumping to conclusions.

3.6 GENERATING ALTERNATIVES

This section deals with different ways in which the problem can be resolved. Typically, there are many (the joke is at least three), and being creative at this stage helps. Things to remember at this stage are:

1. Be realistic! While you might be able to find a dozen alternatives, keep in mind that they should be realistic and fit within the constraints of the situation.

2. The alternatives should be mutually exclusive, that is, they cannot happen at the same time.

3. Not making a decision pending further investigation is not an acceptable decision for any case study that you will analyse. A manager can always delay making a decision to gather more information, which is not managing at all! The whole point to this exercise is to learn how to make good decisions, and having imperfect information is normal for most business decisions, not the exception.

4. Doing nothing as in not changing your strategy can be a viable alternative, provided it is being recommended for the correct reasons, as will be discussed below.

5. Avoid the meat sandwich method of providing only two other clearly undesirable alternatives to make one reasonable alternative look better by comparison. This will be painfully obvious to the reader, and just shows laziness on your part in not being able to come up with more than one decent alternative.

6. Keep in mind that any alternative chosen will need to be implemented at some point, and if serious obstacles exist to successfully doing this, then you are the one who will look bad for suggesting it.

Once the alternatives have been identified, a method of evaluating them and selecting the most appropriate one needs to be used to arrive at a decision.

3.7 KEY DECISION CRITERIA

A very important concept to understand, they answer the question of how you are going to decide which alternative is the best one to choose. Other than choosing randomly, we will always employ some criteria in making any decision. Think about the last time that you make a purchase decision for an article of clothing.

Why did you choose the article that you did? The criteria that you may have used could have been:

1. fit
2. price
3. fashion
4. colour
5. approval of friend/family
6. availability

Note that any one of these criteria could appropriately finish the sentence, the brand/style that I choose to purchase must.... These criteria are also how you will define or determine that a successful purchase decision has been made. For a business situation, the key decision criteria are those things that are important to the organization making the decision, and they will be used to evaluate the suitability of each alternative recommended.

Key decision criteria should be:

1. Brief, preferably in point form, such as

a. improve (or at least maintain) profitability,
b. increase sales, market share, or return on investment,
c. maintain customer satisfaction, corporate image,
d. be consistent with the corporate mission or strategy,
e. within our present (or future) resources and capabilities,
f. within acceptable risk parameters,
g. ease or speed of implementation,
h. employee morale, safety, or turnover,
i. retain flexibility, and/or

j. minimize environmental impact.

2. Measurable, at least to the point of comparison, such as alternative A will improve profitability more that alternative B.
3. Be related to your problem statement, and alternatives. If you find that you are talking about something else, that is a sign of a missing alternative or key decision criteria, or a poorly formed problem statement.

Students tend to find the concept of key decision criteria very confusing, so you will probably find that you re-write them several times as you analyse the case. They are similar to constraints or limitations, but are used to evaluate alternatives.

3.8 EVALUATION OF ALTERNATIVES

If you have done the above properly, this should be straightforward. You measure the alternatives against each key decision criteria. Often you can set up a simple table with key decision criteria as columns and alternatives as rows, and write this section based on the table. Each alternative must be compared to each criteria and its suitability ranked in some way, such as met/not met, or in relation to the other alternatives, such as better than, or highest. This will be important to selecting an alternative. Another method that can be used is to list the advantages and disadvantages (pros/cons) of each alternative, and then discussing the short and long term implications of choosing each. Note that this implies that you have already predicted the most likely outcome of each of the alternatives. Some students find it helpful to consider three different levels of outcome, such as best, worst, and most likely, as another way of evaluating alternatives.

3.9 RECOMMENDATION

You must have one! Business people are decision-makers; this is your opportunity to practice making decisions. Give a justification for your decision (use the KDC's). Check to make sure that it is one (and only one) of your Alternatives and that it does resolve what you defined as the Problem.

4.0 CONCLUSION

The case method of analysis is a learning tool in which students and Instructors participate in direct discussion of case studies, as opposed to the lecture method, where the Instructor speaks and students listen and take notes. In the case method, students teach themselves, with the Instructor being an active guide, rather than just a talking head delivering content. The focus is on students learning through their joint, co-operative effort.

5.0 SUMMARY

This study unit has exposed us to the subject of case analysis in OR. We opened the unit with a description of what case analysis is all about. We defined a case asa description of an actual administrative or operational situation involving a decision to be made or a problem to be solved. It can be a real situation that actually happened just as described, or portions have been disguised for reasons of privacy. Most case studies are written in such a way that the reader takes the place of the manager whose responsibility is to make decisions to help solve the problem.The stages involved in case analysis include the following the short cycle process, and the long cycle process. Next, we considered the key decision criteria when doing a case analysis. These have to be brief, measurable, and be related to your problem statement. Finally we considered how to analyse alternatives.

6.0 TUTOR MARKED ASSIGNMENT

1. What do you understand by case analysis?

2 Discuss the key decision criteria used in analysing a case.

3. Identify and briefly discuss the stages involved in case analysis.

4. Present a brief analogy on how to do a case study.

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