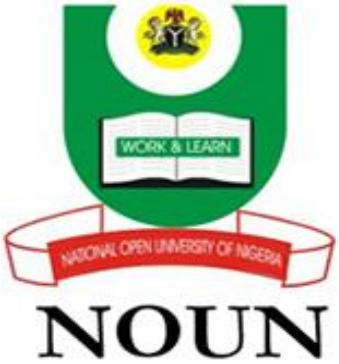

NATIONAL OPEN UNIVERSITY OF NIGERIA

INTRODUCTION TO ECONOMETRICS II

ECO 356

FACULTY OF SOCIAL SCIENCES

COURSE GUIDE

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Main Introduction

ECO 306 is a logical extension of the first-semester course on regression analysis. As such, it introduces the concept of thesimultaneous equation and their estimation. Essentially, this course examines the possible solutions to problems arising from the breakdown of the ordinary least squares assumptions and sampling theories. To this end, it covers topics likemulticollinearity, heteroscedasticity, autocorrelation and Econometrics Modeling: Specification and Diagnostic Testing. It also examines the use of regression analyses, correlation, variance and dummy variables.For this reason, experiential case studies that apply the techniques to real-life data are stressed and discussed throughout the course, and students are required to get acquaintedwith their several models and theories that deal with the measurement of economic relationships.
The course would be a very useful material to you in your academic pursuit and could helpto broaden your understanding further in this case. Once this understanding and application areestablished, you are then able to have a broadened knowledge of econometrics while distinguishing it from mathematical economics.

This course is therefore developed in a manner to guide you further on what econometrics entails, what course materials in line with a course learning structure you will be using. The learning structure suggests some general guidelines for a time frame required of you on each unit to achieve the course aims and objectives effectively. Further work in this course would expose you to introductory levels of topics like; vector autoregressions, unit roots, cointegration, time-series analysis and errors in variables.

Course Outline

ECO 306 is made up of **five modules** with **seventeen units** spread across **twelve lectures weeks.** The modules cover areas such as theconcept of thesimultaneous equation and their estimation, ordinary least squares assumptions, multicollinearity, heteroscedasticity, autocorrelation and econometrics modeling: Specification and Diagnostic Testing, use of dummy variables and time-lags as independent variables.

Aims

The aimof this course is to give you thorough understanding and an appreciative importance of econometrics being concerned with more than measurement in economics. But more importantly, how econometrics as a method of causal inference isapplied to economics. That is, this method of causal inference is a statistical inference combined with the logic of causal order; which is to infer or learn something about the real world by analysing a sample of data.

Specifically, the aims of the course are to:

- Equip you with the application of statistical methods to the measurement and critical assessment of assumed economic relationships using data.
- Provide an improvedintroductory understanding of how the economy works, at either the microeconomic or macroeconomic level.

Course Objectives

To achieve the aims mentioned above alsoto the overall stated course objectives. Each unit, in the beginning, has its specific objectives. You should read them before you start working through the unit. You may want to refer to them during your study of the unit to check on your progress and should always take a look back at the objectives after completion. In this way, you can be certain you have done what was necessary to you by the unit. The course objectives are set below for you to achieve the aims of the course. On successful conclusion of the course, you should be able to:

- Know the basic principles of econometric analysis
- Express relationships between economic variables using mathematical concepts and theories
- Understand both the fundamental techniques and wide array of applications involving linear regression estimation
- Analyse the strengths and weaknesses of the basic regression model.
- Outline the assumptions of the normal linear regression model and discuss the significance of these assumptions
- Explain the method of ordinary least squares
- Test hypotheses of model parameters and joint hypotheses concerning more than one variable
- Discuss the consequences of multicollinearity, the procedures for identifying multicollinearity, and the techniques for dealing with it
- Explain what ismeant by heteroscedasticity, and the consequences for ordinary least square (OLS) estimators and prediction based on those estimators
- Assess the methods used to identify heteroscedasticity, including data plots and more formal tests, and the various techniques to deal with heteroscedasticity, including model transformations and estimation by weighted least squares
- Explain autocorrelation, and discuss the consequences of autocorrelated disturbances for the properties of OLS estimator and prediction based on those estimators
- Outline and discuss the methods used to identify autocorrelated disturbances, and what can be done about it, including estimation by generalised least squares
- Discuss the consequences of disturbance terms not being normally distributed, tests for non-normal disturbances, and methods to deal with non-normal disturbances, including the use of dummy variables
- Discuss the consequences of specifying equations incorrectly
- Discuss the tests used to identify correct model specification and statistical criteria for choosing between models

Working through the Course

This course highlights on critical thinking and the application of both logical and quantitative skills.It also stresses on the application of econometric methods to economic theory and practical problems. Therefore, to complete this course, you are required to read the study units, referenced books and other materials on the course.
Each unit contains self-assessment exercises called Student Assessment Exercises (SAE). At some points in the course, you will be required to submit assignments for assessment purposes. At the end of the course, there is a final examination. This

course should take about twelve weeks to complete, and some components of the course areoutlined under the course material subsection.

Course Material

The major component of the course, what you have to do and how you should allocate your time to each unit to complete the course successfully and on time are as follow:

1. Course guide
2. Study unit
3. Textbook
4. Assignment file
5. Presentation schedule

Study Unit

In this course, there are five modules that subdivided into 17 units which should be studied thoroughly.

Module 1: **Sampling Theory, Variance, and Correlation**
Unit 1: Random Variables and Sampling Theory
Unit 2: Covariance and Variance
Unit 3: Correlation

Module 2: **Regression Models, Hypotheses Testing, and Dummy Variables**
Unit 4: Simple Regression Analyses
Unit 5: Properties of the Regression Coefficients and Hypothesis Testing
Unit 6: Multiple Regression Analysisand Multicollinearity
Unit 7: Transformations of Variables
Unit 8: Dummy Variables
Unit 9: Specification of regression variables: A preliminary skirmish

Module 3: **Heteroscedasticity/Heteroskedasticity**
Unit 10: Heteroscedasticity and Its Implications
Unit 11: Solution to Heteroscedasticity Problem
Unit 12: Other Tests

Module 4: **Autocorrelation, Error, and Econometric Modelling**
Unit 13: Stochastic Regression and measurement errors
Unit 14: Autocorrelation
Unit 15: Econometric Modellingand Models Using Time Series Data

Module 5: **Simultaneous Equation, Binary Choice, and Maximum Likelihood Estimation**
Unit 16: Simultaneous Equations
Unit 17: Binary Choice and Maximum Likelihood Estimation.

The general aim of module 1 (units 1-3) is to provide you with a thorough understanding of the basic statistical tools needed for regression analyses in the subsequent modules. The Random variables and sampling theory, covariance,

variance, and correlationaredemystified for proper understanding. By the end of this module, you would have been able to understand the basics of regression analysis.

Module 2 (units 4-9) explains single-equation regression models. It shows how a hypothetical linear relationship between two variables can be quantified using appropriate data. The principle of least squares regression analysis explained, and expressions for the coefficients are derived.Multicollinearityand multiple regression analysis looked at in units 6. Transformations of Variablesdiscussed in unit 7 while dummy variables as well preliminary skirmish of the specification of regression variables are the topics in units 8 and 9. An exploration of what happens when there is a violation of one of the classical assumptions; equal variances (homoscedastic) is carried out in module 3. It demonstrates how properties of estimators of the regression coefficients depend on the properties of the disturbance term in the regression model. Also, in this module, we shall look at some of the problems that arise when violations of the Gauss–Markov conditions; the assumptions relating to the disturbance term, are not satisfied. Basic understanding of heteroscedasticity (unequal-variances) will gain thoroughexplanation.

The module 4 (unit 13-15) covers an understanding of the basics of econometric modelling. It goes further to give some details on stochastic regression and measurement errors, autocorrelation, econometric modelling and models using time series data. More detail description of an introduction to Consequences of Measurement Errors.Intercorrelation among the Explanatory Variables and Measurement Errors in the Dependent Variable are brought to the students' knowledge here. Also, possible causes of Autocorrelation and Detection of First-Order Autocorrelation using the Durbin–Watson Test are presented in units 14and 15 of the same module 4. While module 5 with units 16 and 17provide you with a thorough understanding of the basic rudiments of Simultaneous Equation, Binary Choice, and Maximum Likelihood Estimation.

Respectively, study unit will take at least two hours which include anintroduction, objective, main content, examples, In-Text Questions (ITQ) and their solutions, self-assessment exercise, conclusion, summary, and reference. Additional areas border on the Tutor-Marked Assessment (TMA) questions. Some of the ITQ and self-assessment exercise will require you free-associating and solve with some of your colleagues. You are advised to do so to grasp and get familiar with how significant econometrics is in being concerned with measurement and also as a method of causal inference application to economics.
There are also econometrics materials, textbooks under the reference and other (on-line and off-line) resources for further studies. These are intended to give you extra facts whenever you allow yourself of such prospect. You are required to study the materials; practise the ITQ, self-assessment exercise and TMA questions for better and thorough understanding of the course. In doing these, the identified learning objectives of the course would have been attained.

For further reading in this course, the following reference texts and materials are suggested:

Textbooks and References

Robert D. Coleman, 2006, The Aims and Methodology of Econometrics Harvard Business School, USA

Gujarati, Damodar N., 1988, Basic Econometrics, Second Edition. New York: McGraw-Hill

Dougherty C., 2014, Elements of Econometrics; anUndergraduate study in Economics, Management, Finance and the Social Sciences, London School of Economics and Political Science, Oxford Revised Edition.

Hill, R. Carter, William E. Griffiths and George G. Judge, 2001, Undergraduate Econometrics, second edition. New York: John Wiley & Sons

Maddala, G.S., 1992, Introduction to Econometrics, second edition. New York: Macmillan Publishing Company.

Assignment Folder

The assignments given in this course are for you to attempt all of them by following the timetable recommended regarding when to do them and submission of same for grading by your lecturer. The marks you obtain for these assignments will count toward the final mark you obtain for this course. Further information on assignments will be found in the Assignment File itself and later in this Course Guide in the section on Assessment.

There are fiveassignments in this course:
Assignment 1 - All TMAs' question in Units 1 – 3 (Module 1)
Assignment 2 - All TMAs' question in Units 4 – 8 (Module 2)
Assignment 3 - All TMAs' question in Units 9 – 11 (Module 3)
Assignment 4 - All TMAs' question in Units 12 – 14 (Module 4)
Assignment 5 - All TMAs' question in Units 15 – 17 (Module 5)

Presentation Plan

The presentation plan included in your course materials gives you the important dates in the year for the completion of tutor-marking assignments and tutorial attendance. Remember, you are required to submit all your assignments by thedue date. You should guide against dropping behind in your assignments submission.

Assessment

Two types of assessments are available in this course; Tutor-Marked Assignment and a written examination at the end of the course.

For the assignments, you are expected to apply lessons learnt during the course. The assignments must be submitted to your lecturer for proper valuation in agreement with the deadlines stated in the Presentation Schedule and the Assignments File. The assignment works you are to submit to your lecturer for evaluation would count for 30% of your total course grade.

At the end of the course, you will need to sit for a final written examination of three hours duration. This examination will also count for 70% of your total course grade.

Tutor-Marked Assignments (TMAs)

There are six tutor-marked assignments in this course. You will submit all the assignments. You are encouraged to attempt all the questions carefully. The TMAs constitute 30% of the total marks.

Assignment questions for the units contained in this course are in the Assignment File. You will be able to complete your assignments from the information and materials contained in your textbooks and study units. However, it is desirable that you demonstrate that you have readand solved a lot of problems relating to each topic in a module. You could use other reference materials to have a broader viewpoint of each subject in this course.

When you have completed each assignment, send it together with a TMA form to your lecturer. Make sure that each assignment reaches your lecturer on or before the due dates given in the Presentation File. If for any reason, you cannot complete your assignment on time, contact your lecturer before the assignment is due, so as to discuss the possibility of an extension. Extensions will not be granted after the due date unless there are exceptional circumstances.

Concluding Examination and Grading

Final examination on the course will be for three hours duration and has a value of 70% of the total course grade. The examination will consist of questions which reflect the types of self-assessment practice exercises and tutor-marked problems you have previously encountered. There is an evaluation of all areas of the course.
You are advised to use the time between finishing the last unit and sitting for the examination to revise the entire course materials. You might find it useful to review your In-Text Questions (ITQ) and self-assessment exercises, tutor-marked assignments and comments on them before the examination. The final examination covers the entire course outline.

Marking Scheme

Table 1 presents the total marks (100%) allocation.

Table 1: Mark Allotment

Assessment	**Marks**
Assignment (Best three assignment out of the five marked)	30%
Final Examination	70%
Total	**100%**

Overview

Table 2 shows the units, number of weeks and assignments to be taken by you to complete the course successfully;Introduction to Econometrics (ECO 306).

Table 2: Assignment Schedule

***** *Comprise of a single module (Module 3) not broken into theunit.*

Unit	Unit Title	Week's Activity	Assessment (end of unit)
	Course Guide		
❖	**Sampling Theory, Variance, and Correlation**		
1	Random variables and sampling theory	Week 1	
2	Covariance and Variance	Week 2	
3	Correlation	Week 3	Assignment 1
❖	**Regression Models, Hypotheses Testing, and Dummy Variables**		
4	Simple Regression Analyses	Week 4	
5	Properties of the regression coefficients and hypothesis testing	Week 5	
6	Multiple regression analysis and Multicollinearity	Week 6	
7	Transformations of Variables	Week 7	
8	Dummy Variables	Week 8	
9	Specification of regression variables: A preliminary skirmish	Week 9	Assignment 2
❖	**Heteroscedasticity/Heteroskedasticity**		
*10	Heteroscedasticity and its Effects	Week 10	
*11	Solution to Heteroscedasticity Problem	Week 11	
*12	Other Tests	Week12	Assignment 3
❖	**Autocorrelation, Error and Econometric Modelling**		
13	Stochastic Regression and measurement errors	Week 13	
14	Autocorrelation	Week 14	
15	Econometric Modellingand Models Using Time Series Data	Week 15	Assignment 4
❖	**Simultaneous Equation, Binary Choice, and Maximum Likelihood Estimation**		
16	Simultaneous Equation	Week 16	
17	Binary Choice and Maximum Likelihood Estimation	Week 17	Assignment 5
	Total	**17 Weeks**	
			Examination

Making the Most of this Course

An advantage of the distance learning is that the study units replace the university lecturer. You can read and work through specially designed study materials at your tempo and at a time and place that goes well with you.

Consider doing it yourself insolving and providing solutions to econometric problems in the lecture instead of listening and copying solution being provided by a lecturer. In the same way, that a lecturer might set you some practice exercisesand ITQ to do, the study units tell you when to solve problems and read your books or other material, and when to embark on adiscussion with your colleagues. Just as a lecturer might give you an in-class exercise, your study units provide exercises for you to do at appropriate points.

Each of the study units follows a common format. The first item is an introduction to the subject matter of the unit and how a particular unit is integrated with the other units and the course as a whole. Nextis a set of learning objectives. These objectives let you know what you should be able to do by the time you have completed the unit.
You should use these objectives to guide your study. When you have finished the unit, you must go back and check whether you have achieved the objectives. If you make a habit of doing this, you will significantly improve your chances of passing the course and getting the best grade.

The main body of the unit guides you through the required understanding from other sources. This will usually be either from your textbooks or a reading section. Some units require you to undertake apractical overview of real life econometric events. You will find when you need to embark on discussion and guided through the tasks you must do.
The purpose of the practical overview of real life econometric events is in twofold. First, it will enhance your understanding of the material in the unit. Second, it will give you practical experience and skills to evaluate economic arguments, and understand the roles of econometric in guiding current economic problems, measurements, analysis, solutions and debates outside your studies. In any event, most of the critical thinking skills you will develop during studying are applicable in normal working practice, so it is important that you encounter them during your studies.

Self-assessments are available throughout the units, and answers are at the ends of the units. Working through these tests will help you to achieve the objectives of the unit and prepare you for the assignments and the examination. You should do each self-assessment exercises as you come to it in the study unit. Also, ensure to master some major econometricstheorems and models while studying the material.

The following is a practical strategy for working through the course. If students encounter any difficulties they may consult the lecturer concerned. Remember that part of the lecturers' responsibility is to help the students when necessary. When in need of help, do not hesitate to contact the lecturer though the available channels, for the required assist.

1. Read this Course Guide thoroughly.
2. Organize a study schedule. Refer to the `Course overview' for more details. Note the time you are expected to spend on each unit and how the assignments relate to the units. Important information, e.g. details of your tutorials, and the date of the first day of the semester is available from study centre. You need to gather together all this information in one place, such as your dairy or a wall calendar. Whatever method you choose to use, you should decide on and write in your dates for working breach unit.
3. Once you have created your study schedule, do everything you can to stick to it. The major reason students fail is that they get behind with their course work. If you get into difficulties with your schedule, please let your tutor know before it is too late for help.
4. Turn to Unit 1 and read the introduction and the objectives for the unit.

5. Assemble the study materials. Information about what you need for a unit is available in the `Overview' at the beginning of each unit. You will also need both the study unit you are working on and one of your textbooks on your desk at the same time.
6. Work through the unit. The content of the unit itself has been arranged to provide a sequence for you to follow. As you work through the unit, you will be instructed to read sections from your textbooks or other articles. Use the unit to guide your reading.
7. Up-to-date course information will be deliveredcontinuously to you at the study centre.
8. Work before the relevant due date (about four weeks before due dates) get the Assignment File for the next required assignment. Keep in mind that you will learn a lot by doing the assignments carefully. They have been designed to help you meet the objectives of the course and, therefore, will help you pass the exam. Submit all assignments no later than the due date.
9. Review the objectives for each study unit to confirm that you have achieved them. If you feel unsure about any of the objectives, review the study material or consult your lecturer.
10. When you are confident that you have achieved a unit's objectives, you can then start on the next unit. Proceed unit by unit through the course and try to pace your study so that you keep yourself on schedule.
11. When you have submitted an assignment to your tutor for marking, do not wait for its return `before starting on the next units. Keep to your schedule. When returning the assignment, pay particular attention to your lecturer's comments, both on the tutor-marked assignment form and also written on the assignment. Consult your lecturer as soon as possible if you have any questions or problems.
12. After completing the last unit, review the course and prepare yourself for the final examination. Check that you have achieved the unit objectives (listed at the beginning of each unit) and the course objectives (listed in this Course Guide).

Tutors and Tutorials

There are some hours of tutorials (2-hours sessions) provided in support of this course. You should get notifications of dates, times, and location for these tutorials. Together with the name and phone number of your lecturer, as soon as thetutorial group allocated are made.

Your lecturer will mark and comment on your assignments, keep a close watch on your progress and on any difficulties you might encounter, and provide assistance to you during the course. You must mail your tutor-marked assignments to your lecturer well before the due date (at least two working days are required). They will be marked by your lecturer and returned to you as soon as possible.

Do not hesitate to contact your lecturer by telephone, e-mail, or discussion board if you need help. The following might be circumstances in which you would find help necessary. Contact your lecturer if.

• You do not understand any part of the study units or the assigned readings
• You have difficulty with the self-assessment exercises

• You have a question or problem with an assignment, with your lecturer's comments on an assignment or with the grading of an assignment.

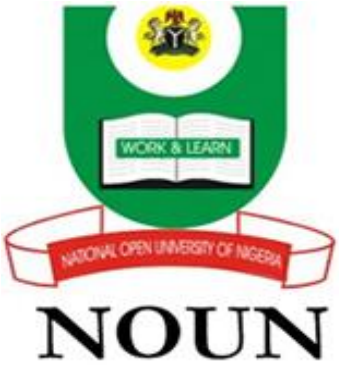
You should try your best to attend the tutorials. Such avenues are the only chance to have face to face contact with your lecturer and to ask questions which are given instant answers instantly. You can raise any problem encountered in the course of your study. To gain the maximum benefit from course tutorials, prepare a question list before attending them. You will learn a lot from participating in discussions actively.

Summary

The course, Introduction to Econometrics II (ECO 306) presents you with general background and applications of the concept of Random Variables, Sampling Theory and how to be able to identify functions and problems associated with estimation. This course also examines ordinary least squares assumptions and sampling theories. Topics like, multicollinearity, heteroscedasticity, autocorrelation and Econometrics modeling had illustrative examples used for further explanations. For this reason, use of regression analyses, correlation, variance and dummy variables with experiential case studies that apply the techniques to real-life data are stressed and discussed throughout the course.

This course is therefore developed in a manner to guide you further on what econometrics entails, what course materials in line with a course learning structure you will be using. The learning structure suggested some general guidelines for a time frame required of you on each unit to achieve the course aims and objectives.

Conclusively, you would have developed critical thinking skills with the material necessary for anefficient introductory understanding of econometrics. Nevertheless, to achieve a lot more from the course, please try to solve econometrics problems independently, do presentation and interpretation of findings in any assignment given both in your academic programme and other spheres of life. Further work in this course would expose you to introductory levels of topics like; vector autoregressions, unit roots, cointegration, and time-series analysis.
We wish you the very best in your schoolwork.



NATIONAL OPEN UNIVERSITY OF NIGERIA

Course Code: **ECO 356**

Course Title: **Introduction to Econometrics II**

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INTRODUCTION TO ECONOMETRICS II

INTRODUCTION TO ECONOMETRICS II

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MODULE 1 SAMPLING THEORY, VARIANCE, AND CORRELATION

The general aim of this module is to provide the student with a thorough understanding of the basic statistical tools that will be needed for regression analyses in the subsequent module. As well as random variables and sampling theory, Covariance, variance, and correlation are to bedemystified for proper understanding. By the end of this module,students would have been able to understand the basic parts of regression analysis.

The units to be studied in this module are;

Unit 1: Random variables and sampling theory

Unit 2: Covariance and Variance

Unit 3: Correlation

UNIT 1: RANDOM VARIABLES AND SAMPLING THEORY

CONTENTS

1.1.1.0 INTRODUCTION

Random variable or stochastic variable is a variable whose possible values are numerical products of a chance occurrence. As a function, a random variable is required to be quantifiable, which rules out certain uncontrolled circumstances where the quantity which the random variable returns is considerably sensitive to small changes in the outcome. It is common that these outcomes depend on some physical variables that are not well understood. For example, when you toss a coin, which outcome will be observed is not certain. The domain of a random variable is the set of possible outcomes. In the case of the coin, there are only two possible outcomes, namely heads or tails. The domain of the random variable leads us into the concept of sampling theory which is concerned with the theory involved in the selection of a subset of individuals from within a statistical population estimates the characteristics of the whole population.

1.1.2.0 OBJECTIVE

The main objective of this unit is to provide a broad understanding of the topic, Random Variables, and Sampling Theory, which is preparatory to the more widely used simple and multiple regression analyses.

1.1.3.0 MAIN CONTENTS

1.1.3.1 Random Variables and Sampling Theory

A variable X is said to be a random variable if for every real number a there exist a probability $P(X \leq a)$ that X takes on a value less than or equal to a. That is, a Random variable is a variable whose value cannot be predicted exactly. It can assume any value. Random variables could be discrete or continuous. A discreterandom variable is one that has a specific set of possible values or a finite set of values. An example is a total score when two dice are thrown. A continuous variable, e.g. the temperature in a particular room, is a variable that can assume any value in thecertain range. It can take any form of the continuing range of values.

The set of all possible values of a random variable is known as a**population** where thesample or a random variable can be drawn for inferential analysis.

1.1.3.2 Expected values of discrete random variable

The expected value of a discrete random variable is the weighted average of all its possible values, taking the probability of each outcome as its weight. It can be calculated by multiplying each possible value of the random variable by its probability and adding. In mathematical terms, if X denotes the random variable, its expected value is denoted by $E(X)$.

Let us suppose that X can take $n$particular values ofx_1, x_2,...,$x_n$and that the probability of $x_i$isp_i.

Then,

$$E(X) = x_i p_i + ... + x_n p_n \quad = \sum_{i=1}^{n} x_i p_i \qquad ...[1.01]$$

Table 1.0 shows an example of expected value of variable X with two dice.

Table 1.0:*Expected value of variable X with two dice*

X	P	$X.P$
2	$\frac{1}{36}$	$\frac{2}{36}$
3	$\frac{2}{36}$	$\frac{6}{36}$
4	$\frac{3}{36}$	$\frac{12}{36}$
5	$\frac{4}{36}$	$\frac{20}{36}$
6	$\frac{5}{36}$	$\frac{30}{36}$
7	$\frac{6}{36}$	$\frac{42}{36}$
8	$\frac{5}{36}$	$\frac{40}{36}$
9	$\frac{4}{36}$	$\frac{36}{36}$
10	$\frac{3}{36}$	$\frac{30}{36}$
11	$\frac{2}{36}$	$\frac{22}{36}$
12	$\frac{1}{36}$	$\frac{12}{36}$
$E(X) = \sum_{i=1}^{n} x_i p_i$		$\frac{252}{36} = 7$

In the case of the two dice, the values x_i … $x_n$were the numbers 2 … 12: x_1 = 2, x_2= 3... x_{11}=12, and p_1= $\frac{1}{36}$, p_2= $\frac{2}{36}$... p_{11}= $\frac{1}{36}$. As shown in table 1.0, the expected value is 7. Also, the expected value of a random variable is described as population mean. In the case of the random variable *X*, the population mean is given asμ_x.

1.1.3.3 Expected value rules

There are three main rules of expected values that are equally valid for both discrete and continuous random variables. These are;

Rule 1: The expected value of the sum of several variables is equal to the sum of their respective expected values. For example, if you have three random variables *X*, *Y*, and *Z*,

$$E(X + Y + Z) = E(X) + E(Y) + E(Z) \qquad \ldots[1.02]$$

Rule 2: If you multiply a random variable by a constant, you multiply its expected value by the same constant. If X is a random variable and b is a constant,

$$E(bX) = bE(X) \quad \ldots[1.03]$$

Rule 3: The expected value of a constant is that constant. For example, if b is a constant.

$$E(b) = b \quad \ldots[1.04]$$

Putting the three rules together; suppose we wish to calculate $E(Y)$, where we have

$$Y = b_1 + b_2X \quad \ldots[1.05]$$

and$b_1$and b_2 are constants.

Then,

$$E(Y) = E(b_1 + b_2X) \quad \ldots[1.06]$$

$$= E(b_1) + E(b_2X) \text{ (using rule 1)} \quad \ldots[1.07$$

$$= b_1 + b_2E(X) \quad (using\ rule\ 2\ \&3) \ldots[1.08]$$

1.1.3.4 Sampling theory

The goals of a sample survey and an experiment are very different. The role of randomisation also differs. In both cases, without randomisation, there can be no inference. Without randomisation, the analyst can only describe the observations and cannot generalise the results. In the sample survey, randomisation is used to reduce bias and to allow the results of the sample to be generalised to the population from which the sample was drawn. In an experiment, randomisation is used to balance the effects of confounding variables. The objective of asample survey is often to estimate a population mean and variance.

1.1.3.4.1 Some terminology

i. **Element:** An element is an object on which a measurement is made,which could be a voter in an area, a product as it comes off the assembly line or a plant in a field that has either flowered or not.

ii. **Population:** A population is a collection of elements about which we wish to make an inference. The population must be clearly defined before the sample is taken.

iii. **Sampling units:** These are some overlapping collections of elements from the population that covers the entire population. The sampling units partition the population of interest. The sampling units could be households or individual voters.

iv. **Frame:** Is a list of sampling units.

v. **Sample:** This is a collection of sampling units drawn from a frame or frames. Data are obtained from the sample and are used to describe characteristics of the population. Two advantages of sampling are that the cost is lower and data collection is faster than measuring the entire population.

vi. **Census:** The enumeration of the total element of the population.

Example 1: Suppose we are interested in what voters in a particular area think about the drilling of oil in the national wildlife preserves. The elements are the registered voters in the area. The population is the collection of registered voters. The sampling units will likely be households in which there may be several registered voters. The frame is a list of households in the area.

1.1.3.4.2 Reasons for sampling

Information could be obtained by taking a complete enumeration of the whole population or aggregate. This is usually difficult as information on every element is rarely available. Therefore, it is better to employ sampling method to obtained information than complete enumeration for the following reasons:

i. **Reduce cost:** if data are secured from only a small fraction of the aggregate, expenditures are smaller than if a complete census is attempted. With large populations result accurate enough to be useful can be obtained from samples that represent only a small fraction of the population.

ii. **Greater speed:** for the same reason the data can be collected and summarized more quickly with a sample than with a complete count. This is a vital consideration when the information is urgently needed.

iii. **Greater Scope:** a complete census is impracticable; the choice lies between obtaining the information by sampling or not at all. Thus surveys that rely on sampling have more scope and flexibility regarding the kind of information that can be obtained.

iv. **Greater Accuracy:** here, personnel of higher quality can be employed and given intense training. This would allow for much more careful supervision of the field work. Processing and analysing of the results become feasible because the volume of work is now reduced. The sample would most likely produce amore accurate result than the complete enumeration.

1.1.3.4.3 Types of sampling technique

i. **Probability sampling technique:** Simple random sampling, systematic random sampling, stratified random sampling, cluster sampling, etc.

ii. **Non-probability sampling technique:** Snowball sampling, quota sampling technique, accidental or convenient sampling technique, etc.

Sample designs that utilise planned randomness are called**probability samples**while non-probability doesn't apply randomness as it is based on the subjective dictate of the researcher since all elements are not given equal chance of being selected. The most fundamental probability sample is the simple random sample. In a simple random sample, a sample of *n* sampling units is selected in such a way that each sample of size *n* has the same chance of being selected. In practice, other more sophisticated probability sampling methods are commonly used, but we would focus here on simple random sampling technique.

1.1.3.4.4 Simple Random Sampling technique

Suppose the observations$y_1, y_2, \ldots y_n$are to be sampled from a population with mean, standard deviation, and size *N* in such a way that every possible sample of size *n* has an equal chance of being selected. Then the sample $y_1, y_2, \ldots y_n$ was selected in a simple random sample. If the sample mean is denoted by $\bar{y}$ then we have;

$$E(\bar{y}) = \mu \qquad \ldots[1.09]$$

$$\mathbf{V}(\bar{\boldsymbol{y}}) = \frac{\boldsymbol{\sigma^2}}{\boldsymbol{n}}\left(\frac{\boldsymbol{N-n}}{\boldsymbol{N-1}}\right) \qquad \ldots[1.10]$$

The term $\left(\frac{N-n}{N-1}\right)$ in the above expression is known as the **finite population correction factor**. For the sample variance s^2, it can be shown that

$$\boldsymbol{E(\ s^2) = \left(\frac{N}{N-1}\right).\sigma^2} \qquad \ldots[1.11]$$

When using s^2 as an estimate ofσ^2, we must adjust with

$$\boldsymbol{\sigma^2 \backsimeq \left(\frac{N}{N-1}\right).E(\ s^2)} \qquad \ldots[1.12]$$

Consequently, an unbiased estimator of the variance of the sample mean is given by

$$\boldsymbol{\widehat{V}(\bar{y})} = \frac{\boldsymbol{s^2}}{\boldsymbol{n}}\left(\frac{\boldsymbol{N-n}}{\boldsymbol{N}}\right) \qquad \ldots[1.13]$$

As a rule of thumb, the correction factor $\left(\frac{N-n}{N-1}\right)$ can be ignored if it is greater than 0.9, or if the sample is less than 10% of the population.

Example 2; Consider the finite population with N = 4 elements$\{0,2,4,6\}$. For this population μ = 3 and σ^2= 5. Simple random samples without replacement of size n = 2 are selected from the population. All possible samples along with their summary statistics are listed in table 1.1.1.

Table 1.1.1 *Simple Random Sampling*

Samples	Probability	mean	Variance
(0,2)	$\frac{1}{6}$	1	2
(0,4)	$\frac{1}{6}$	2	8
(0,6)	$\frac{1}{6}$	3	18
(2,4)	$\frac{1}{6}$	3	2

(2,6)	$\frac{1}{6}$	4	8
(4,6)	$\frac{1}{6}$	5	2

We see in this example that;

$$V(\bar{y}) = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right) = \frac{5}{2}\left(\frac{4-2}{4-1}\right) = \frac{5}{2}\left(\frac{2}{3}\right) = \frac{5}{3}$$

Similarly,

$$\boldsymbol{E(s^2) = \left(\frac{N}{N-1}\right) * \sigma^2} \quad \ldots[1.14]$$

Could also be obtained from table 1.1.1

1.1.3.5 Estimation of Population Mean

If we are interested in estimating a population mean from a simple random sample, we have;

$$\boldsymbol{\hat{\mu} = \bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}} \quad \ldots[1.15]$$

If we are interested in estimating population variance from a simple random sample, we have;

$$\boldsymbol{\hat{V}(\bar{y}) = \frac{s^2}{n}\left(\frac{N-n}{N}\right)} \quad \ldots[1.16]$$

Where,

$$\boldsymbol{s^2 = \frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{n-1}} \quad \ldots[1.17]$$

When the margin of error is two standard errors, we have;

$$\boldsymbol{2\sqrt{\hat{V}}(\hat{y}) = 2\sqrt{\frac{s^2}{n}\left(\frac{N-n}{N}\right)}} \quad \ldots[1.18]$$

1.1.4.0SUMMARY

In this unit, the students are presented with the essentials and applications of the concept of random variables and sampling theory and their estimations. Also, by now the students should be able to identify functions and problems associated with the estimation.

1.1.5.0CONCLUSION

In this unit, the concepts of random variables and sampling theory have been introduced and discussed and the associated estimation explained. The students are made to understand that random variables are of two quantitative (discrete and continuous random) variable types, whose values may be determined through the outcome of a random trials. Discreteand continuous random variables were both explained with examples. Further definition of random variable as it relates to a population is explained. Furthermore, inference is made on how the student may apply the random variable in other statistical analysis like mean, variance, standard deviation, etc.

Sampling theory is introduced in this unit in form of comparison to random variable. Some terminologies and reasons associated with sampling are also discussed. Probability and non-

probability sampling techniques are presented as two sample designs methods that utilise planned randomness. Although, in practice, other more sophisticated probability sampling and estimation methods are commonly used, the unit focused on simple random sampling and population mean techniques as starting points.

1.1.6.0 TUTOR-MARKED ASSIGNMENT

1.) A random variable X is defined to be the difference between the higher value and the lower value when two dice are thrown. If they have the same value, X is defined to be 0. Find the probability distribution for X.

2.) A random variable X is defined to be the larger of the two values when two dice are thrown, or the value if the values are the same. Find the probability distribution for X.

1.1.7.0 REFERENCES /FURTHER READING

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UNIT 2: CO-VARIANCE AND VARIANCE

CONTENTS

1.2.1.0 INTRODUCTION

The previous unit in this module introduced and discussed random variable and associated sampling theories. In other to further equip the students with the adequately understanding of more basic tools needed for regression analysis in the next module

and in general statistical analyses, this unit discusses covariance and variance. The unit explains that, variance and covariance are two measures used in statistics. While variance is an intuitive concept that measures the scatter of the data, covariance on the other hand, is not that intuitive at first but gives a mathematical indication of the degree of change of two random variables together.

1.2.2.0 OBJECTIVE

The main objective of this unit is to provide a broad understanding of the topics Covariance and Variance which is preparatory to the more widely used simple and multiple regression analyses.

1.2.3.0 MAIN CONTENTS

1.2.3.1 Covariance and Variance

Sample covariance is a measure of association between two variables. The sample covariance, Cov(X, Y), is a statistic that enables you to summarize this association with a single number. In general, given n observations on two variables X and Y, the sample covariance between X and Y is given by;

$$\frac{1}{n}\sum_{i=1}^{n}(x_1 - \bar{x})(y_1 - \bar{y}) \qquad \ldots[2.19]$$

Where the bar over the variable signifies the sample mean. Therefore, a positive association would be summarized by a positive sample covariance while a negative sample covariance would summarise a negative association.

1.2.3.2 Some Basic Covariance rules

i. Co-variance Rule 1: If $Y = V + W$, Cov(X, Y) = Cov(X, V) + Cov(X, W)

ii. Co-variance Rule 2: If $Y = bZ$, where b is a constant and Z is a variable, Cov(X, Y) = bCov(X, Z)

iii. Co-Variance Rule 3: If $Y = b$, where b is a constant, Cov(X, Y) = 0

For example, Tables 1.2.0(a) and (b) show years of schooling$S$, and hourly earnings$Y$, for a subset of 20 households in theUnitedStates. We are required to calculate the covariance.

Table 1.2.0(a) *Covariance table*

Observation	S	Y
1	15	17.24
2	16	15.00
3	8	14.91
4	6	4.50
5	15	18.00
6	12	6.29
7	12	19.23
8	18	18.69
9	12	7.21
10	20	42.06
11	17	15.38
12	12	12.70
13	12	26.00
14	9	7.50

15	15	5.00
16	12	21.63
17	16	12.10
18	12	5.55
19	12	7.50
20	14	8.00

Table 1.2.0(b)*Covariance table*

Observation	S	Y	$S-\bar{S}$	$Y-\bar{Y}$	$(S-\bar{S})(Y-\bar{Y})$
1	15	17.24	1.75	3.016	5.277
2	16	15.00	2.75	0.775	2.133
3	8	14.91	-5.25	0.685	-3.599
4	6	4.50	-7.25	-9.725	70.503
5	15	18.00	1.75	3.776	6.607
6	12	6.29	-1.25	-7.935	9.918
7	12	19.23	-1.25	5.006	-6.257
8	18	18.69	4.75	4.466	21.211
9	12	7.21	-1.25	-7.015	8.768
10	20	42.06	6.75	27.836	187.890
11	17	15.38	3.75	1.156	4.333
12	12	12.70	-1.25	-1.525	1.906
13	12	26.00	-1.25	11.776	-14.719

14	9	7.50	-1.45	-6.725	28.579
15	15	5.00	1.75	-9.225	-16.143
16	12	21.63	-1.25	7.406	-9.257
17	16	12.10	2.75	-2.125	-5.842
18	12	5.55	-1.25	-8.675	10.843
19	12	7.50	-1.25	-6.725	8.406
20	14	8.00	0.75	-6.225	-4.668
Total	265	284.49	-	-	305.888
Average	13.250	14.225	-	-	15.294

Note from the above example that the association is positive. This is given by the positive covariance.

1.2.3.3 Population Covariance

If *X* and *Y* are random variables, the expected value of the product of their deviations from their means is defined to be the population covariance σ_{XY}:

$$\sigma_{XY} = E[(X - \mu_X)(Y - \mu_Y)] \quad \ldots[2.20]$$

Where $\mu_X$and μ_Y are the population means of *X* and *Y*, respectively.

As you would expect, if the population covariance is unknown, the sample covariance will

provide an estimate of it, given a sample of observations. However, the estimate will be biased downwards, for

$$E[COV(X,Y)] = \frac{n-1}{n} * \sigma_{XY} \quad \ldots[2.21]$$

The reason is that the sample deviations are measured from the sample means of *X* and *Y* and tend to underestimate the deviations from the true means. Therefore, we can construct an unbiased estimator by multiplying the sample estimate by *n*/(*n*–1).

1.2.3.4 Sample Variance

For a sample of *n* observations, X_1, ..., X_2, the sample variance will be defined as the average squared deviation in the sample:

$$\boldsymbol{Var(X) = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n}} \quad \ldots[2.22]$$

The sample variance, thus defined, is a biased estimator of the population variance. The reason for the underestimation is because it is calculated as the average squared deviation from the sample mean rather than the true mean. This is because the sample mean is automatically in the centre of the sample, the deviations from it will tend to be smaller than those from the population mean. Therefore, sample variance as an unbiased estimate of population variance is given as:

$$\boldsymbol{s^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}} \quad \ldots[2.23]$$

1.2.3.5 Variance Rule

Variance rule 1: If $Y = V + W$, Var(Y) = Var(V) + Var(W) + 2Cov(V, W)

Variance rule 2: If $Y = bZ$, where b is a constant, Var(Y) = b^2Var (Z)

Variance rule 3: If $Y = b$, where b is a constant, Var(Y) = 0.

Variance rule 4: If $Y = V + b$, where b is a constant, Var(Y) = Var(V) since the variance of a constant is 0.

1.2.4.0 SUMMARY

While explaining the variance and covariance, the temptations to make comparison of the two concepts may not be completely overcome. The unit briefly describes variance as the measure of spread in a population while covariance is considered as a measure of variation of two random variables. Furthermore, the unit showed that variance and covariance are dependent on the magnitude of the data values and cannot be compared; therefore, regulated. This means, covariance is dividing by the product of the standard deviations of the two random variables and variance is normalised into the standard deviation by taking the square root of it.

1.2.5.0 CONCLUSION

Variance and covariance concepts as statistical tools are discussed in this unit. How they are estimated were also explained using some basic covariance and variance rules. The existence of population covariance and sample variance estimations were briefly introduced. The introductions and discussions of these two concepts point out that variance can be considered as a special case of covariance.

1.2.6.0 TUTOR-MARKED ASSIGNMENT

1.) In a large bureaucracy the annual salary of each, *Y*, is determined by the formula

$$Y = 10{,}000 + 500S + 200T$$

Where,*S* is the number of years of schooling of the individual and *T* is the length of time, in years, of employment. *X* is the individual's age. Calculate Cov(*X*, *Y*), Cov(*X*, *S*), and Cov(*X*, *T*) for the sample of five individuals shown below and verify that

$$Cov(X,Y) = 500Cov(X,S) + 200Cov(X,T)$$

2.) In a certain country the tax paid by a firm, *T*, is determined by the rule

$$T = -1.2 + 0.2P - 0.1I$$

Where,*P* is profits, and *I* is aninvestment, the third term being the effect of an investment incentive. *S* is sales. All variables are measured in $ million at annual rates. Calculate Cov(*S*, *T*), Cov(*S*, *P*), and Cov(*S*, *I*) for the sample of four firms shown below and verify that

$$Cov(S,T) = 0.2Cov(S,P) - 0.1Cov(S,I)$$

1.2.7.0 REFERENCES /FURTHER READING

Dominick, S., & Derrick, R. (2002).*Theory and problems of statistics and econometrics.*Schaum's Outline Series.

Dougherty, C. (2003). *Numeracy, literacy and earnings: evidence from the National Longitudinal Survey of Youth.*Economics of education review, 22(5), 511-521.

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UNIT 3: CORRELATION CO-EFFICIENT

CONTENTS

1.3.1.0 INTRODUCTION

This unit introduces a statistic called correlation coefficient. The correlation coefficient describes the direction, whether positive or negative and further measures the degree of relationship that exist between two different variables.

1.3.2.0 OBJECTIVE

The main objective of this unit is to provide ways for which the studentmay have a simpler understanding of the topic ‘correlation’.

1.3.3.0 MAIN CONTENTS

Correlation measures the degree of association between two or more variables.

1.3.3.1 Properties of the regression coefficients and hypothesis testing

Like variance and covariance, the correlation coefficient comes in two forms, population and sample. ρ traditionally denotes the population correlation coefficient,

the Greek letter that is the equivalent of "*r*", and pronounced "row", as in row a boat. For variables *X* and *Y* it is defined by

$$\rho_{XY} = \frac{\sigma_{XY}}{\sqrt{\sigma_X^2 \sigma_Y^2}} \qquad \ldots[3.24]$$

If *X* and *Y* are independent, ρ_{XY} will be equal to 0 because the population covariance will be 0. If there is a positive association between them, then we have σ_{XY}, otherwise ρ_{XY} will still be positive.

If there is an exact positive linear relationship, ρ_{XY} will assume its maximum value of 1. Similarly, if there is a negative relationship ρ_{XY} will be negative, with minimum value of –1.

The sample correlation coefficient, r_{XY}, is defined by replacing the population covariance and variances by their unbiased estimators. We have seen that these may be obtained by multiplying the sample variances and co-variances by $\frac{n}{(n-1)}$. Hence,

$$r_{XY} = \frac{\frac{n}{n-1}\text{cov}(XY)}{\sqrt{\frac{n}{n-1}\text{var}(X)\frac{n}{n-1}\text{var}(Y)}} \qquad \ldots[3.25]$$

The factors $\frac{n}{(n-1)}$ could be cancelled out so we can conveniently define the sample correlation by

$$r_{XY} = \frac{COV(XY)}{\sqrt{Var(X)Var(Y)}} \qquad \ldots[3.26]$$

$$= \frac{\sigma_{XY}}{\sigma_X \sigma_Y} \qquad \ldots[3.27]$$

Like ρ, *r* has maximum value 1, which is attained when there is a perfect positive association

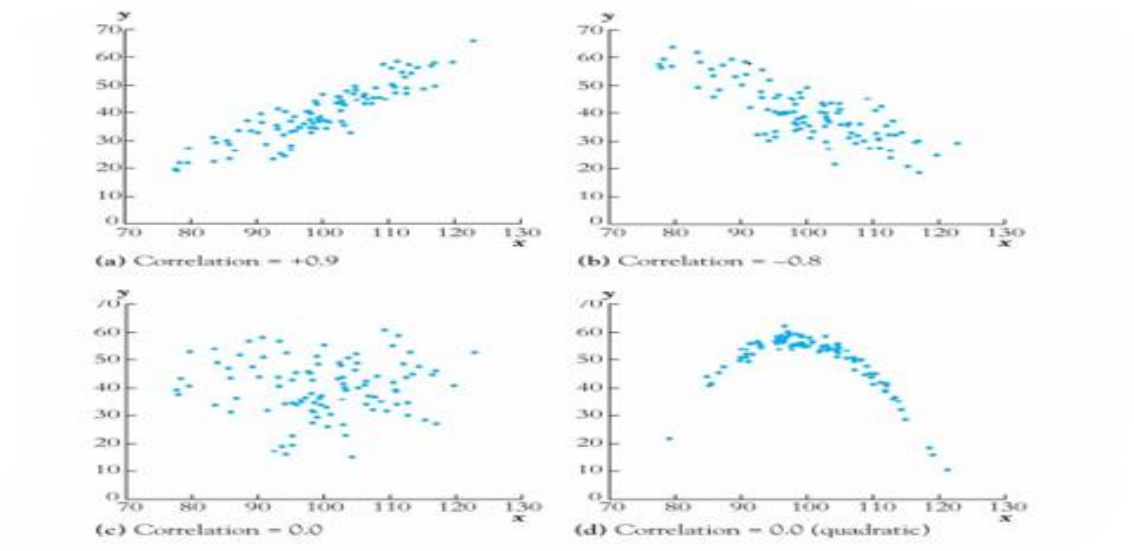
between the sample values of *X* and *Y* (when you plot the scatter diagram, the points lie exactly on an upward-sloping straight line). Similarly, it has minimum value –1, attained when there is a perfect negative association (the points lying exactly on a

downward-sloping straight line). A value of 0 indicates that there is no association between the observations on *X* and *Y* in the sample. Of course the fact that *r* = 0 does not necessarily imply that ρ = 0 or vice versa.

That is;

$$-1 \le corr(X,Y) \le 1$$
$$corr(X,Y) = 1 \text{ means perfect positive linear association}$$
$$corr(X,Y) = -1 \text{ means perfect negative linear association}$$
$$corr(X,Y) = 0 \text{ means no linear association}$$

Figures 1.3(a) to (d) below give more graphical explanations;



Figures 1.3(a) to (d);*Scattered diagrams of correlation coefficient as a measure of linear association*

Example: For illustration, using the education and earning example, the sample correlation coefficient can be estimated. This is shown below:

Table 1.2.1 *Sample correlation coefficient*

Observ.	S	Y	$S-\bar{S}$	$Y-\bar{Y}$	$(S-\bar{S})^2$	$(Y-\bar{Y})^2$	$(S-\bar{S})(Y-\bar{Y})$
1	15	17.24	1.75	3.016	3.063	9.093	5.277
2	16	15.00	2.75	0.775	7.563	0.601	2.133
3	8	14.91	-5.25	0.685	27.563	0.470	-3.599
4	6	4.50	-7.25	-9.725	52.563	94.566	70.503
5	15	18.00	1.75	3.776	3.063	14.254	6.607
6	12	6.29	-1.25	-7.935	1.563	62.956	9.918
7	12	19.23	-1.25	5.006	1.563	25.055	-6.257
8	18	18.69	4.75	4.466	22.563	19.941	21.211
9	12	7.21	-1.25	-7.015	1.563	49.203	8.768
10	20	42.06	6.75	27.836	45.563	774.815	187.890
11	17	15.38	3.75	1.156	14.063	1.335	4.333
12	12	12.70	-1.25	-1.525	1.563	2.324	1.906
13	12	26.00	-1.25	11.776	1.563	138.662	-14.719
14	9	7.50	-1.45	-6.725	18.063	45.219	28.579
15	15	5.00	1.75	-9.225	3.063	85.091	-16.143
16	12	21.63	-1.25	7.406	1.563	54.841	-9.257

17	16	12.10	2.75	-2.125	7.563	4.514	-5.842
18	12	5.55	-1.25	-8.675	1.563	75.247	10.843
19	12	7.50	-1.25	-6.725	1.563	45.219	8.406
20	14	8.00	0.75	-6.225	0.563	38.744	-4.668
Total	265	284.49	-	-	217.750	1,542.150	305.888
Average	13.250	14.225	-	-	10.888	77.108	15.294

From column 6 and 7, you can see that Var (S) is 10.888 and Var (Y) is 77.108, therefore,

$$r_{XY} = \frac{15.294}{\sqrt{10.888\ X\ 77.108}} = \frac{15.294}{28.975} = 0.55$$

1.3.4.0 SUMMARY

This unit briefly introduced correlation coefficient and showed some properties of regression coefficients and hypothesis testing. Four figures and a table were used to further illustrate the correlation coefficient as a measure of linear association and how sample correlation may be estimated.

1.3.5.0 CONCLUSION

The brief discussion on correlation in this unit is to let the students aware that correlation may be approached as a statistical tool that precedes the basic introduction to econometrics. Although correlation measures may be clouded by relationships that exist with other variables, figures and table were however used to show correlation as the linear relationships between two variables.

1.3.6.0 TUTOR-MARKED ASSIGNMENT

1.) Demonstrate that, in general; the sample correlation coefficient is not affected by a change inthe unit of measurement of one of the variables.

2.) Suppose that the observations on two variables *X* and *Y* lie on a straight line

$$Y = b_1 + b_2X$$

Demonstrate that $Cov(X,Y) = b2Var(X)$ and that Var(*Y*) = $b_2^2Var(X)$, and hence that the sample correlation coefficient is equal to 1 if the slope of the line is positive, –1 if it is negative.

1.3.7.0 REFERENCES /FURTHER READING

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MODULE 2: SIMPLE EQUATION REGRESSION MODELS

The general aim of this module is to provide studentswith a thorough understanding of the basic rudiments of simple equation regression models. It shows how a theoretical linear relationship between two variables can be quantified using appropriate data. The principle of least squares regression analysis is explained, and expressions for the coefficients are derived. By the end of this module,students should be able to understand the basic parts of regression analysis. The units to be studied are;

Unit 1: Simple Regression Analyses

Unit 2: Properties of the regression coefficients and hypothesis testing

Unit 3: Multiple regression analysis and Multicollinearity

Unit 4: Transformations of Variables

Unit 5: Dummy Variables

Unit 6: Specification of regression variables: A preliminary skirmish.

UNIT 1: SIMPLE REGRESSION ANALYSES

CONTENTS

2.1.1.0 INTRODUCTION

In this unit, an analytic method to measure the association of one or more independent variables with a dependent variable is discussed. Simple regression analyses is a statistical method which can be used to summarize and study relationships existing between two continuous quantitative variables. This unit further introduces the simple regression as a concept that is better understood after a thorough understanding of the basic correlation procedures.

2.1.2.0 OBJECTIVE

The main objective of this unit is to acquaint students with the rudiments of identifying and differentiating simple equation model from multiple regression.

2.1.3.0 MAIN CONTENTS

2.1.3.1 Simple Regression Analyses

The correlation coefficient may indicate that two variables (bivariate regression model) are associated with one another, but it does not give any idea of the kind of relationship involved. While regression predicts the value of the dependent variable based on the known value of the independent variable. In this module further step is taken for cases which we are willing to hypothesize on, than one variable dependence on another. It must be statedimmediately that one would not expect to find an exact relationship between any two economic variables unless it is true as a matter of definition. In textbook expositions of economic theory, the usual way of dealing with this awkward fact is to write down the relationship as if it were exact and to warn the reader that it is only an approximation. However, in statistical analysis, one acknowledges the fact that the relationship is not exact by explicitly including in it a

random factor known as the **disturbance term**. We shall start with the simplest possible model:

$$Y_i = \beta_1 + \beta_2 X_i + \mu_i \quad \text{...[2.01]}$$

Y_i, the value of the dependent variable in observation *i*, has two components: (1) the non-random (deterministic term) component$\beta_1 + \beta_2 X_i$, X_i being described as the explanatory (or independent/descriptive) variable and the fixed quantities β_1 and β_2 as the parameters of the equation, and (2) the disturbance (stochastic term), μ_i.

Figure 2.0 illustrates how these two components combine to determine *Y*. X_1, X_2, X_3, and X_4, which are four hypothetical values of the explanatory variable. If the relationship between *Y* and *X* were

exact, the corresponding values of *Y* would be represented by the points $Q_1 - Q_4$ on the line. The disturbance term causes the actual values of *Y* to be different. In the diagram, the disturbance term has been assumed to be positive in the first and fourth observations and negative in the other two, with the result that, if one plots the actual values of *Y* against the values of *X*, one obtains the points $P_1 - P_4$.

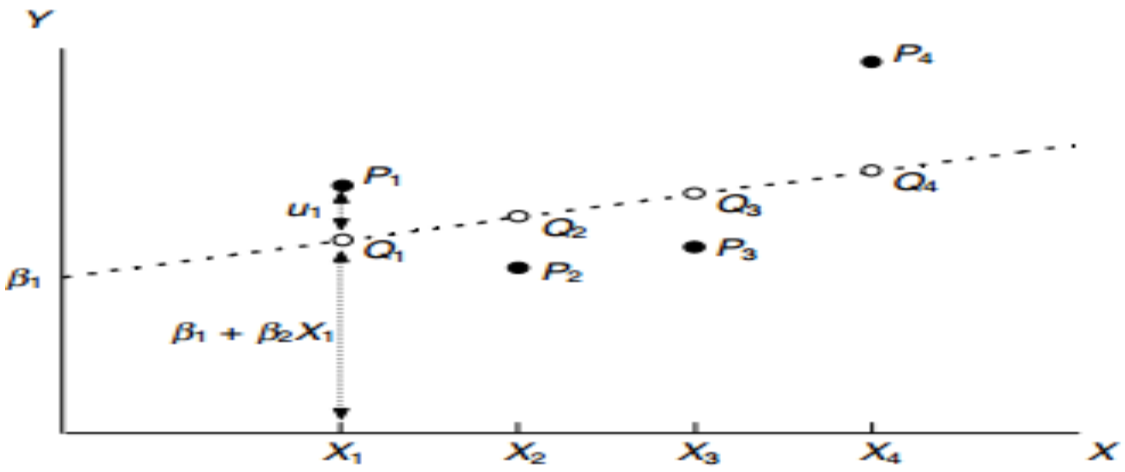

Figure 2.0Illustration of independent component combination to give a dependent variable

In practice, the *P* points are all not what can be seen in Figure 2.0. The actual values of β_1 andβ_2 and hence the location of the *Q* points, are unknown, as these are the values of the disturbance term in the observations. The task of regression analysis is to

obtain estimates of $\beta_1$and β_2, and hence an estimate of the location of the line, given the *P* points. As it is, it's somehow curious. The question "Why then does the disturbance term exist"? would therefore arise. There are several reasons.

2.1.3.2 Reasons for inclusion of Disturbance Term

i. *The omission of explanatory variables*: The relationship between *Y* and *X* is almost certain to be a simplification. In reality, there will be other factors affecting *Y* that have been left out of the non-random dependent component, and their influence will cause the points to lie on the line. It often happens that there are variables that you would like to include in the regression equation but cannot because you are unable to measure them. All of these other factors contribute to the disturbance term.

ii. *Aggregation of variables*: In many cases, the relationship is an attempt to summarise in aggregate somemicroeconomic relationships. For example, the aggregate consumption function is an attempt to summarize a set of individual expenditure decisions. Since the individual relationships are likely to have different parameters, any attempt to relate

aggregate expenditure to aggregate income can only be an approximation. The discrepancy is attributed to the disturbance term.

iii. *Model misspecification*: The model may be misspecifiedregarding its structure. Just to give one of the many possible examples, if the relationship refers to time series data, the value of *Y* may depend not on the actual value of *X* but on the value that had been anticipated in the previous period. If the anticipated and actual values are closely related, there will appear to be a relationship between *Y* and *X*, but it will only be an approximation, and again the disturbance term will pick up the discrepancy.

iv. *Functional misspecification*: The functional relationship between *Y* and *X* may be misspecified mathematically. For example, the true relationship may be non-linear instead of linear. Obviously, one should try to avoid this problem by using an appropriate mathematical specification, but even the most sophisticated specification is likely to be only an approximation, and the discrepancy contributes to the disturbance term.

v. *Measurement error*: If the measurement of one or more of the variables in the relationship is subject to error, the observed values will not appear to conform to an exact relationship, and the discrepancy contributes to the disturbance term.

The disturbance term is the collective outcome of all these factors. Obviously, if you were concerned only with measuring the effect of *X* on *Y*, it would be much more convenient if the disturbance term did not exist. Were it not for its presence, the *P* points in Figure 2.1 would coincide with the *Q* points. Therefore, it would be known that every change in *Y* from observation to observation was due to a change in *X*, and you would be able to calculate β_1 and β_2, exactly. However, part of each change in *Y* is due to a change in μ, and this makes life more difficult. For this reason, μ is sometimes described as **noise**.

2.1.3.3Least Squares Regression

Suppose that you are given the four observations on *X* and *Y* represented in Figure 2.1 and you are asked to obtain estimates of the values of β_1 andβ_2, in [2.01]. As a rough approximation, you could do this by plotting the four *P* points and drawing a line to fit them as best you can, as shown in Figure 2.2 The intersection of the line with the *Y*-axis provides an estimate of the intercept β_1, which will be denoted b_1 and the slope provides an estimate of the slope coefficient β_2, which will be denoted b_2. The fitted line will be written as;

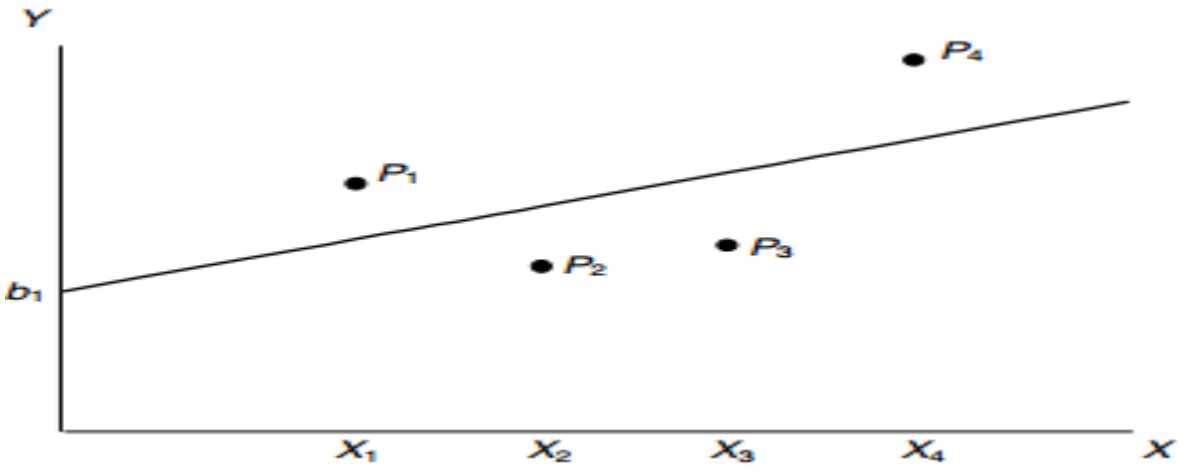

Figure 2.2 Plotting of Observations

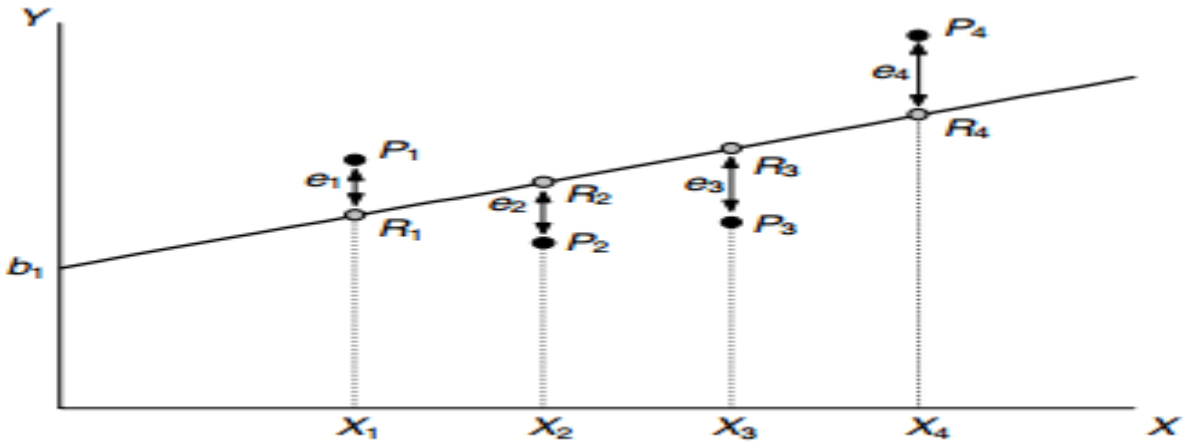

Figure 2.3 fitting Plotted Observations

$$\hat{Y}_i = b_1 + b_2 X_i \qquad \ldots[2.02]$$

The hat mark over Y in [2.02] indicates that it is the fitted value of Y corresponding to X and not the actual value. In Figure 2.3 the fitted points are represented by the points $R_1 - R_4$. One thing that should be accepted from the beginning is that however much care you take in drawing the line; you can never discover the true values of $\beta_1$and β_2. b_1 and b_2 are only estimates, and they may be good or bad. Once in a while your estimates may be absolutely accurate, but this can only be by coincidence and even then you will have no way of knowing that you have hit the target exactly.

This remains the case even when you use more sophisticated techniques. Drawing a regression line by eye is all very well, but it leaves a lot to subjective judgment. Furthermore, as will become obvious, it is not even possible when you have a variable Y depending on two or more explanatory variables instead of only one. The question arises, is there a way of calculating good estimates of

$\beta_1$and$\beta_2$algebraically? The answer is yes! The first step is to define what is known as a residual for each observation. This is the difference between the actual value of Y in any observation and the fitted value given by the regression line, that is, the vertical distance between $P_i$and $R_i$in observation i. Which will be denoted by e_i.

$$e_i = Y_i - \hat{Y}_i \quad ...[2.03]$$

The residuals for the four observations are shown in Figure 2.3

Substituting [2.02] into [2.03], we obtain

$$e_i = Y_i - b_1 - b_2 X_i \quad ...[2.04]$$

and hence the residual in each observation depends on our choice of b_1 and b_2. Obviously, we wish to fit the regression line, that is, choose b_1 and b_2, in such a way as to make the residuals as small as possible. Equally obvious, a line that fits some observations well will fit others badly and vice versa. We need to devise a criterion of fit that takes account of the size of all the residuals simultaneously. There are some possible criteria, some of which work better than others. It is useless minimizing the sum of the residuals, for example. The sum will automatically be equal to 0 if you make b_1 equal to $\bar{Y}$and b_2 equal to 0, obtaining the horizontal line $Y = \bar{Y}$. The positive residuals will then exactly balance the negative ones but other than that, the line will not fit the observations.

One way of overcoming the problem is to minimize ***RSS*** (sum of the squares of the residuals).

$$RSS = e_1^2 + e_2^2 + e_3^2 + e_4^2 \quad ...[2.05]$$

According to this criterion, the smaller one can make *RSS* the better is the fit. If one could reduce *RSS* to 0, one would have a perfect fit, for this would imply that all the residuals are equal to 0. The line would go through all the points, but of course, in general, the disturbance term makes this impossible. There are other quite reasonable solutions, but the least squares criterion yields estimates of b_1 and b_2 that are unbiased and the most efficient of their type, provided that certain conditions are satisfied. For this reason, the least squares technique is far and away the most popular in uncomplicated applications of regression analysis. The form used here is usually referred to as ordinary least squares and abbreviated **OLS**.

Table 2.1

$\boldsymbol{X}$	$\boldsymbol{Y}$	$\boldsymbol{\hat{Y}}$	$\boldsymbol{e}$
1	3	$b_1 + b_2$	$3 - b_1 - b_2$
2	5	$b_1 + 2b_2$	$5 - b_1 - 2b_2$

We shall assume that the true model is;

$$Y_i = \beta_1 + \beta_2 X_i + \mu_i \qquad ...[2.06]$$

And we shall estimate the coefficients b_1 and b_2 of the equation using;

$$\hat{Y}_i = b_1 + b_2 X_i \qquad ...[2.07]$$

When X is equal to 1, according to the regression line $\hat{Y}$ is equal to $(b_1 + b_2)$. When X is equal to 2, $\hat{Y}$ is equal to $(b_1 + 2b_2)$. Therefore, we can set up Table 2.1.0. So the residual for the first observation, e_1, which is given by $(Y_1 - \hat{Y}_1)$, is equal to $(3 - b_1 - b_2)$, and e_2, given by $(Y_2 - \hat{Y}_2)$, is equal to $(5 - b_1 - 2b_2)$. Hence

$$RSS = (3 - b_1 - b_2)^2 + (5 - b_1 - b_2)^2$$

$$= 9 + b_1^2 + b_2^2 - 6b_1 - 6b_2 + 2b_1b_2 + 25 + b_1^2 + 4b_2^2 - 10b_1 - 20b_2 + 4b_1b_2$$

$$= 34 + 2b_1^2 + 5b_2^2 - 16b_1 - 26b_2 + 6b_1b_2$$

...[2.08]

Now we want to choose b_1 and b_2 so as to minimize *RSS*. To do this, we use the calculus and find the values of b_1 and b_2 that satisfy

$$\frac{\partial RSS}{\partial b_1} = 0 \ \ and \ \frac{\partial RSS}{\partial b_2} = 0 \qquad \ldots[2.09]$$

Taking partial differentials of [2.08];

$$\frac{\partial RSS}{\partial b_1} = 4b_1 + 6b_2 - 16 \quad \text{...[2.10]}$$

And

$$\frac{\partial RSS}{\partial b_2} = 10b_2 + 6b_2 - 16 \quad \text{...[2.11]}$$

And so we have

$$2b_1 + 3b_2 - 8 = 0$$

And

$$3 + 5b_2 - 13 = 0$$

Solving these two equations, we obtain b_1 = 1 and b_2 = 2, and hence the regression equation

$$\hat{Y}_i = 1 + 2X_i$$

Just to check that we have come to the right conclusion, we shall calculate the residuals:

$$e_1 = 3 - b_1 - b_2 = 3 - 1 - 2 = 0$$

$$e_2 = 5 - b_1 - 2b_2 = 5 - 1 - 4 = 0$$

Thus both residuals are equal to 0, implying that the line passes exactly through both points.

2.1.3.3.1 Least Squares Regression with One Explanatory Variable

We shall now consider the general case where there are *n* observations on two variables *X* and *Y* and supposing *Y* to depend on *X;* we will fit the equation

$$\hat{Y}_i = b_1 + b_2X_i \quad \text{...[2.12]}$$

The fitted value of the dependent variable in observation *i*.

$\hat{Y}_i$will be ($b_1 + b_2X_i$) and the residual ei will be ($Y_i - b_1 - b_2X_i$). We wish to choose b_1 and b_2 so as to minimize the residual sum of the squares *RSS* given by

$$RSS = e_1^2 + \cdots + e_n^2 = \sum_{i=1}^{n} e_i^2 \quad \text{...[2.13]}$$

We will find that *RSS* is minimised when

$$b_2 = \frac{Cov(X,Y)}{Var(X)} \quad \text{…[2.14]}$$

And

$$b_1 = \bar{Y} - b_2\bar{X} \quad \text{…[2.15]}$$

The derivation of the expressions for b_1 and b_2 will follow the same procedure as the derivation in the preceding example, and you can compare the general version with the examples at each step.

We will begin by expressing the square of the residual in observation *i*regardingb_1, b_2 and the data on *X* and *Y*:

$$e_i^2 = (Y_i - \hat{Y}_i)^2 = (Y_i - b_1 - b_2X_i)^2$$
$$= {Y_i}^2 + b_1^2 + b_2^2X_i^2 - 2b_1Y_i - 2b_2X_iY_i + 2b_1b_2X_i \quad \text{…[2.16]}$$

Summing over all the *n*observations, we can write *RSS* as

$$RSS = (Y_1 - b_1 - b_2X_1)^2 + \cdots + (Y_n - b_1 - b_2X_n)^2$$

$$= \sum_{i=1}^{n} Y_i^2 + nb_1^2 + b_2^2 \sum_{i=1}^{n} X_i^2 - 2b_1 \sum_{i=1}^{n} Y_i - 2b_2 \sum_{i=1}^{n} X_iY_i + 2b_1b_2 \sum_{i=1}^{n} X_i$$
…[2.17]

Note that *RSS* is effectively a quadratic expression in b_1 and b_2, with numerical coefficients

determined by the data on *X* and *Y* in the sample. We can influence the size of *RSS* only through our choice of b_1 and b_2. The data on *X* and *Y*, which determine the locations of the observations in the scatter diagram and are fixed once we have taken the sample. This equation [2.17] is the generalized version of the equations.

The first order conditions for a minimum,

$$\frac{\partial RSS}{\partial b_1} = 0 \ \ and \ \frac{\partial RSS}{\partial b_2} = 0 \quad \text{…[2.18]}$$

Yield the following equations:

$$2nb_1 - 2\sum_{i=1}^{n} Y_i + 2b_2 \sum_{i=1}^{n} X_i = 0$$

$$2b_2 \sum_{i=1}^{n} X_i^2 - 2\sum_{i=1}^{n} Y_i X_i + 2b_1 \sum_{i=1}^{n} X_i = 0 \qquad \ldots[2.19]$$

Noting that

$$\bar{X} = \frac{\mathbf{1}}{\boldsymbol{n}} \sum_{i=1}^{n} X_i \ \ and \ \bar{Y} = \frac{\mathbf{1}}{\boldsymbol{n}} \sum_{i=1}^{n} Y_i \qquad \ldots[2.20]$$

may be rewritten as

$$2nb_1 - 2n\bar{Y} + 2b_2 n\bar{X} = 0 \qquad \ldots[2.21]$$

and hence

$$b_1 = \bar{Y} - b_2\bar{X} \qquad \ldots[2.22]$$

Substituting for b_1 and again noting that $\sum_{i=1}^{n} X_i = n\bar{X}$ we obtain

$$2b_2 \sum_{i=1}^{n} X_i^2 - 2\sum_{i=1}^{n} Y_i X_i + \ (\bar{Y} - b_2\bar{X})n\bar{X} = 0 \qquad \ldots[2.23]$$

Separating the terms involving b_2 and not involving b_2 on opposite sides of the equation, we have

$$2b_2[(\sum_{i=1}^{n} X_i^2) - n\bar{X}^2] = 2\sum_{i=1}^{n} Y_i X_i - 2n\overline{YX} \qquad \ldots[2.24]$$

Dividing both sides by 2*n*,

$$\left[\frac{1}{n}(\sum_{i=1}^{n} X_i^2) - \bar{X}^2\right] b_2 = \frac{1}{n}(\sum_{i=1}^{n} Y_i X_i) - \overline{YX} \qquad \ldots[2.25]$$

Using the alternative expressions for sample variance and covariance, this may be rewritten as;

$$b_2 Var(X) = Cov(X, Y)$$

$$b_2 = \frac{Cov(X,Y)}{Var(X)} \qquad \ldots[2.26]$$

b_2 is from [2.23], b_1 is equally from [2.22]. Those who know about the second-order conditions will have no difficulty confirming that we have minimized *RSS*.

2.1.3.3.2 Alternative Expressions for b_2

From the definitions of Cov(X, Y) and Var(X) one can obtain alternative expressions for b_2

$$b_2 = \frac{Cov(X,Y)}{Var(X)} = \frac{\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})(Y_i-\bar{Y})}{\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2} = \frac{\sum_{i=1}^{n}(X_i-\bar{X})(Y_i-\bar{Y})}{\sum_{i=1}^{n}(X_i-\bar{X})^2}$$

$$= \frac{\sum_{i=1}^{n} xy \sum}{\sum_{i=1}^{n} x^2} \quad \ldots[2.27]$$

where,

$$x = X - \overline{X}$$

$$y = Y - \overline{Y}$$

2.1.4.0 SUMMARY

In this unit, you are expected to have learnt the essentials and applications of the concept of simple regression analyses and its estimation.

2.1.5.0 CONCLUSION

In conclusion, the concept of simple regression analyses and its estimation are explained.

2.1.6.0 TUTOR-MARKED ASSIGNMENT

1.) A researcher obtaineddata on the aggregate expenditure on services *Y*, and aggregate disposable personal income *X*, both measured in N billion at constant prices, for each of the U.S. states and fits the equation

$$\widehat{Y_i} = \widehat{\beta_1} + \widehat{\beta_2} X_t + \mu_i$$

The researcher initially fits the equation using OLS regression analysis. However, suspecting that tax evasion causes both *Y* and *X* to be substantially underestimated, the researcher

adopts

two alternative methods of compensating for the under-reporting:

a.) The researcher adds N90 billion to the data for *Y* in each state and N200 billion to the data for *X*.

b.) The researcher increases the figures for both *Y* and *X* in each state by 10 percent.

2.) Derive from first principles the least squares estimator of $\beta_2 \; and \; \beta_1$ in the model

$$Y_i = \beta_1 + \beta_2 X_i + \mu_i$$

2.1.7.0 REFERENCES /FURTHER READING

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UNIT 2: PROPERTIES OF THE REGRESSION COEFFICIENTS AND HYPOTHESIS TESTING

CONTENTS

2.2.1.0 INTRODUCTION

This unit firstly attempts giving an appropriate explanation to the concept of GAUSS-MARKOV THEOREM before proceeding into the discussion of the properties of regression coefficients and hypothesis testing. However, to properly understand this unit, a brief discussion would be made of some knowledge areas like;

i. Estimators
ii. Assumptions underlying the Classical Linear Regression Model (CLRM)
iii. Properties of Ordinary Least Square (OLS) estimator
iv. Statistical inference (hypothesis testing) like; null and alternative hypotheses.

The basic knowledge of the aforementioned areas are what the students must be equipped with before proceeding in this unit.

- Estimators

  A rule for calculating an estimate of a given quantity based on observed data is referred to as an estimator. Hence in the calculation three quantities are distinguished; the quantity of interest, referred to as an estimand, the result (estimate) and the rule (estimator).

  The properties of estimators are the concerns of Estimation theory. The theory defines and determines properties that can be used under given circumstances to relate different estimators or different rules for creating estimates for the same quantity which are built on the same data.

  A simple example of an estimator is given by a sample mean equation of a population mean shown below.

$$\bar{k} = \frac{1}{N}\sum_{i=1}^{N} k_i$$

  where, $\bar{k}$ is an estimator for the population mean μ.

- Assumptions underlying Classical Linear Regression Model (CLRM)

  This model is the basis of most econometric theory and provides a description of the method of ordinary least squares (OLS). It also explains

how the OLS, using sample data, can estimate unknown parameters of a regression equation. Furthermore, it makes available opportunity to ask whether statements about the true unknown parameters of the model, based on our estimated values can be made. In doing this, there is need to make a number of assumptions. These assumptions, if satisfied, ensure that the estimators being used are accurate and efficient. Precise predictions about the unknown model parameters can also be make through satisfactory assumptions. Here is a summary of the 10 assumptions in CLRM:

Assumption 1: Linear regression model. The regression model is linear in the parameters,as shown in[01]

$$Y_i = \beta_1 + \beta_2 X_i + \mu_i \qquad \ldots[01]$$

where,

Y is the regressand$Y$ and the regressor$X$ may be nonlinear.

Assumption 2: X values are fixed in repeated sampling. Values taken by the regressor$X$are considered fixed in repeated samples. More precisely, X is assumed to be nonstochastic.

Assumption 3: Zero means value of disturbance μ_i. That is, given the value of X, the mean value of the random disturbance term μ_i is zero. This shows that the uncertain mean value of μ_i is zero, as shown in[02]

$$E(\mu_i \mid X_i) = 0 \qquad \ldots[02]$$

Assumption 4: Equal variance of μ_i. If given the value of X, the variance of μ_i is the same for all observations. Which means that the uncertain variances of μ_i are alike, as shown in[03].

$$\text{var}(\mu_i \mid X_i) = E[\mu_i - E(\mu_i \mid X_i)]^2$$

$$= E(\mu_i^2 \mid X_i) \quad \text{(due to assumption 3)}$$

$$= \sigma^2 \qquad \ldots[03]$$

where,

var is variance.

Assumption 5: No autocorrelation between the disturbances. If given any two X values, X_i and $X_j (i \neq j)$, the autocorrelation between any two μ_i and $\mu_j (i \neq j)$ **is zero,** as shown in[04].

$$\text{cov}(\mu_i, \mu_j \mid X_i, X_j) = E\{[\mu_i - E(\mu_i)] \mid X_i\}\{[\mu_j - E(\mu_j)] \mid X_j\}$$

$$= E(\mu_i \mid X_i)(\mu_j \mid X_j) \quad (\textit{shows } \mu_i \textit{ and } \mu_j \textit{ are uncorrelated})$$

…[04]

$$= 0$$

where,

$(i \textit{ and } j)$ are different observations and cov is covariance.

Assumption 6: Zero covariance between $(\mu_i \textit{ and } X_i)$**.** As earlier expressed;

$$\text{cov}(\mu_i, X_i) = E[\mu_i - E(\mu_i)]X_i - E(X_i)]$$

$$\text{since } E(\mu_i) = 0$$

$$= E[\mu_i (X_i - E(X_i))]$$

$$= E(\mu_i X_i) - E(X_i)E(\mu_i)$$

$$\text{since } E(X_i) \textit{ is nonstochastic and } E(\mu_i) = 0$$

$$E(\mu_i X_i) = 0 \qquad \ldots[05]$$

Assumption 7: The number of observations *n* must be greater than the number of parameters to be estimated. On the other hand, the number of observations *n* must be greater than the number of descriptive (explanatory) variables.

Assumption 8: Variability in *X* values. The *X* values in a given sample must not all be the same. That is, *var(X)* must be a finite positive number.

Assumption 9: The regression model is correctly specified. On the other hand, there is no error in the model used in observed analysis.

Assumption 10: There is no perfect multicollinearity. That is, there are no perfect linear relationships among the descriptive variables.
Multicollinearity as a problem associated with CLRM is discussed in unit 3.

- Properties of Ordinary Least Square (OLS) estimator

The following properties are associated with OLS estimators:

1. Linearity
2. Unbiasedness
3. Efficient: it has the minimum variance
4. Consistency
5. Asymptotic Unbiasedness

The OLS estimator is sometimes referred to as the CLRM and in data analysis the best estimator is refer to as BLUE (best linear unbiased estimator). Therefore, the OLS estimator requires that the descriptive variables are received outside a data group and there is no perfect multicollinearity. Also, OLS is best in the class of linear unbiased estimators when the errors are vector of random variables and successively uncorrelated. Within these conditions, the OLS offers minimum-variance mean-unbiased estimation when the errors have fixed variances. Again, the OLS is a maximum likelihood estimator under the additional assumption that the errors are normally distributed. So, whenever students are planning to use a linear regression model by means of OLS, each time check for the OLS assumptions. In as much as the OLS assumptions are satisfied, the analysis becomes simpler. Through the Gauss-Markov theorem (as will be seen later in this unit) students can directly use OLS for the best results. When the OLS estimator is asymptotically normal and a consistent estimator of the asymptotic covariance matrix is available to carry out hypothesis tests on the coefficients of a linear regression model.

2.2.2.0 OBJECTIVE

The main objective of this unit is to provide basic understanding of the topic, properties of regression coefficients and hypothesis testing. As well as how these properties form the basis for prediction and forecasting analyses. Focus will also be on the use of *regression* analysis to recognise which among the independent variables are related to the dependent variable and to explore the forms of these relationships.

2.2.3.0 MAIN CONTENTS

With the aid of regression analysis, we can obtain estimates of the parameters of a relationship. However, they are only estimates. The next question to ask is, how reliable are they? We shall

answer this first in general terms, investigating the conditions for unbiasedness and the factors governing their variance. Secondly, building on those conditions for unbiasedness and their variances, we shall develop a means of testing whether a regression estimate is compatible with a specific prior hypothesis concerning the true value of a parameter. Hence, we shall derive a confidence interval for the true value,

that is, the set of all hypothetical values not contradicted by the experimental result. We shall also see how to test whether the goodness of fit of a regression equation is better than what might be expectedby pure chance.

2.2.3.1 The Random Components of the Regression Coefficients

The least squares regression coefficient is a special form of arandom variable whose properties depend on those of the disturbance term in the equation. This will be demonstrated first theoretically and then using a controlled experiment. In particular, we will investigate the implications for the regression coefficients of certain assumptions concerning the disturbance term. Throughout the discussion, we shall continue to work with the simple regression model where *Y* depends on *X* according to the relationship $Y_i = \beta_1 + \beta_2 X_i + \mu_i$

And we fit the regression equation $\hat{Y}_i = b_1 + b_2 X_i$ given a sample of *n* observations.

We shall also continue to assume that *X* is a non-stochastic exogenous (not external randomly determined) variable; that is, that its value in each observation may be considered to be predetermined by factors unconnected with the present relationship.

First, note that Y_i has two components. It has non-random component ($\beta_1 + \beta_2 X_i$), which owes nothing to the laws of chance ($\beta_1 \; and \; \beta_2$ may be unknown, but nevertheless they are fixed constants) and it has the random component μ_i. This implies that, when we calculate b_2 according to the usual formula;

$$b_2 = \frac{Cov(X,Y)}{Var(X)} \qquad \ldots[2.28]$$

b_2 would also have a random component $Cov(X,Y)$. $Cov(X,Y)$depends on the values of *Y*, and the values of *Y* depend on the values of μ. If the values of the disturbance term had been different in the *n* observations, we would have obtained different values of *Y*, hence of Cov (*X*, *Y*), and hence of b_2.Thus we have shown that the regression coefficient b_2 obtained from any sample consists of (1) a fixed component, equal to the true value β_2, and (2) a random component dependent on Cov(*X*, μ), which is responsible for its variations around this central tendency. Similarly, one may easily show that b_1 has a fixed component equal to the true value β_1, plus a random component that depends on the random factor μ.

2.2.3.2 Assumptions Concerning the Disturbance Term

It is thus obvious that the properties of the regression coefficients depend critically on the properties of the disturbance term. Indeed the latter has to satisfy four conditions, known as the Gauss–Markov conditions, if ordinary least squares regression analysis is to give the best possible results. If they are not satisfied, the user should be aware of the fact. If remedial action is possible, he or she should be capable of taking it. If it is not possible, he or she should be able to judge how seriously the results may have been affected.

2.2.3.2.1 Gauss–Markov Condition 1: $E(\mu_i) = 0$ for All Observations

The first condition is that the expected value of the disturbance term in any observation should be 0. Sometimes it will be positive, sometimes negative, but it should not have a systematic tendency in either direction. If an intercept is included in the regression equation, it is usually reasonable to assume that this condition is satisfied automatically since the role of the intercept is to pick up any systematic but constant tendency in *Y* not accounted for by the explanatory variables included in the regression equation.

2.2.3.2.2 Gauss–Markov Condition 2: Population Variance of μ_i Constant for All Observations

The second condition is that the population variance of the disturbance term should be constant for all observations. Sometimes the disturbance term will be greater, sometimes smaller, but there should not be any a priori reason for it to be more erratic in some observations than in others. The constant is usually denoted by σ_{μ}^2, often abbreviated to σ^2, and the condition is written as, $\sigma_{\mu i}^2 = \sigma^2 \; for \; all \; i$

Since $E(\mu_i)$is 0, the population variance of $\mu_i$is equal to $E(\mu_i^2)$, so the condition can also be written

$E(\mu_i^2) = \sigma_{\mu}^2 for \; all \; i$, σ_{μ} of course is unknown. One of the tasks of regression analysis is to estimate the standard deviation of the disturbance term. If this condition is not satisfied, the OLS regression coefficients will be inefficient, but you should be able to obtain more reliable results by using a modification of the regression technique.

2.2.3.2.3 Gauss–Markov Condition 3: μ_i Distributed Independently of μ_j $(i \neq j)$

This condition states that there should be no systematic association between the values of the disturbance term in any two observations. For example, just because the disturbance term is large and positive in one observation, there should be no tendency for it to be large and positive in the next (or large and negative, for that matter, or small and positive, or small and negative). The values of the disturbance term should be independent of one another. The condition implies that $\sigma_{\mu i \mu j}$, the population covariance between $\mu_i and \mu_j$, is 0, because;

$$\sigma_{\mu i \mu j} = E[(\mu_i - \mu u)(\mu_j - \mu u)] = E(\mu_i \mu_j) = E(\mu_i)E(\mu_j) = 0 \quad \ldots[2.29]$$

where, u is a value in μ as shown in (u_1) of Figure 2.0

Note that the population means of $\mu_i and \mu_j$ are 0, by the first Gauss–Markov condition, and that $E(\mu_i \mu_j)$ can be decomposed as $E(\mu_i)E(\mu_j)$ if $\mu_i and \mu_j$ are generated independently. If this condition is not satisfied, OLS will again give inefficient estimates.

2.2.3.2.4 Gauss–Markov Condition 4: *u* Distributed Independently of the Explanatory Variables

The final condition comes in two versions, weak and strong. The strong version is that the explanatory variables should be non-stochastic, that is, not have random components. This is very unrealistic for economic variables, and we will eventually switch to the weak version of the condition, where the explanatory variables are allowed to have random components provided that they are distributed independently of the disturbance term. However, the strong version is usually used because it simplifies the analysis of the properties of the estimators.

$$\sigma\, X_i u_i = E[\{X_i - E(X_i)\}\{u_i - \mu u\}] = (X_i - X_i)\, E(u_i) = 0 \quad \ldots[2.30]$$

2.2.3.3 The Normality Assumption

In addition to the Gauss–Markov conditions, one usually assumes that the disturbance term is normally distributed. The reason is that if u is normally distributed, so will be the regression coefficients, and this is useful when performing tests of hypotheses and constructing confidence intervals for β_1 and β_2 using the regression results. The justification for the assumption depends on the Central Limit Theorem; that, if a random variable is the composite result of the effects of a large number of other random variables, it will have an approximately normal distribution even if its components do not, provided that none of them is dominant. The disturbance term u is composed

of a number of factors not appearing explicitly in the regression equation so, even if we know nothing about the distribution of these factors (or even their identity), we are entitled to assume that they are normally distributed.

2.2.3.4 Unbiasedness of the Regression Coefficients

We can show that b_2 must be an unbiased estimator of β_2 if the fourth Gauss–Markov condition is satisfied:

$$E(b_2) = E\left[\beta_2 + \frac{Cov(X,u)}{Var(X)}\right] = \beta_2 + E\left[\frac{Cov(X,u)}{Var(X)}\right] \quad \ldots[2.31]$$

sinceβ_2 is a constant. If we adopt the strong version of the fourth Gauss–Markov condition and assume that X is non-random, we may also take Var(X) as a given constant, and so

$$E(b_2) = \beta_2 + \frac{1}{Var(X)} E[Cov(X,u)] \quad \ldots[2.32]$$

To demonstrate that $E[Cov(X,u)]\ is\ 0$:

$$E[Cov(X,u)] = E\left[\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})(u_i - \bar{u})\right] = \frac{1}{n}\sum_{i=1}^{n} E[(X_i - \bar{X})(u_i - \bar{u})]$$

$$= \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})E[(u_i - \bar{u})] = 0 \quad \ldots[2.33]$$

In the second line, the second expected value rule has been used to bring $(1/n)$ out of the expression as a common factor, and the first rule has been used to break up the expectation of the sum into the sum of the expectations. In the third line, the term involving Xhas been brought out because X is non-stochastic. By virtue of the first Gauss–Markov condition, $E(u_i)$is 0, and hence $E(u)$ is also 0. Therefore $E[Cov(X,u)]$ is 0 and

$$E(b_2) = \beta_2 \qquad \ldots[2.34]$$

In other words, b_2 is an unbiased estimator of β_2. We can obtain the same result with the weak version of the fourth Gauss–Markov condition (allowing X to have a random component but assuming that it is distributed independently of u), unless the random factor in the $n$observations happens to cancel out exactly, which can happen only by coincidence. b_2 will be different from $\beta_2$for any given sample, but in view of unbiased regression coefficient, there will be no systematic tendency for it to be either higher or lower. The same is true for the regression coefficient b_1.

Using [2.22]

$$b_1 = \bar{Y} - b_2\bar{X} \qquad \ldots[2.35]$$

Hence

$$E(b_1) = E(\bar{Y}) - \bar{X}E(b_2) \qquad \ldots[2.36]$$

Since $Y_i$is determined by

$$Y_i = \beta_1 + \beta_2 X_i + \mu_i$$

We have

$$E(Y_i) = \beta_1 + \beta_2 X_i + E(\mu_i) = \beta_1 + \beta_2 X_i \qquad \ldots[2.37]$$

because$E(\mu_i)$is 0 if the first Gauss–Markov condition is satisfied. Hence

$$E(\bar{Y}) = \beta_1 + \beta_2\bar{X} \qquad \ldots[2.38]$$

Substituting this into [2.36], and using the result that $E(b_2) = \beta_2$,

$$E(b_1) = (\beta_1 + \beta_2\bar{X}) - \bar{X}\beta_2 = \beta \quad \ldots[2.39]$$

Thus b_1 is an unbiased estimator of β_1 provided that the Gauss–Markov conditions 1 and 4 are satisfied. Of course in any given sample the random factor will cause b_1 to differ from β_1.

2.2.3.5 Precision of the Regression Coefficients

Now we shall consider$\sigma_{b1}^2$and σ_{b2}^2, the population variances of b_1 and b_2 about their population means.

The following expressions give these

$$\sigma_{b1}^2 = \frac{\sigma_u^2}{n}[1 + \frac{\bar{X}^2}{Var(X)}] \text{ and } \sigma_{b2}^2 = \frac{\sigma_u^2}{nVar(X)} \quad \ldots[2.40]$$

Equation [2.40] has three obvious implications. First, the variances of both b_1 and b_2 are directly inversely proportional to the number of observations in the sample. This makes good sense. The more information you have, the more accurate your estimates are likely to be.

Second, the variances are proportional to the variance of the disturbance term. The bigger the
the variance of the random factor in the relationship, the worse the estimates of the parameters are likely to be.

Third, the variance of the regression coefficients is inversely related to the variance of *X*. What is the reason for this? Remember that (1) the regression coefficients are calculated on the assumption that the observed variations in *Y* are due to variations in *X*, but (2) they are in reality *partly* due to variations in *X* and *partly* to variations in *u*. The smaller the variance of *X*, the greater is likely to be the relative influence of the random factor in determining the variations in *Y*and the more likely is regression analysis give inaccurate estimates.

2.2.3.6 Testing Hypotheses Relating to the Regression Coefficients

Which comes first, theoretical hypothesizing or empirical research? In practice, theorizing and experimentation feed on each other, and questions of this type cannot be answered. For this reason, we will approach the topic of hypothesis testing from both directions. On the one hand, we may suppose that the theory has come first and that the purpose of the experiment is to evaluate its acceptability. This will lead to the execution of significance tests. Alternatively, we may perform the experiment first and then consider what theoretical hypotheses would be consistent with the results. This will lead to the construction of confidence intervals.

Students would already have encountered the logic underlying significance tests and confidence
intervals in an introductory statistics course. Students will thus be familiar with most of the concepts in the following applications to regression analysis. There is, however, one topic that may be new: the use of one-tailed tests. Such tests are used very frequently in regression analysis. Indeed, they are, or they ought to be, more common than the traditional textbook two-tailed tests. It is, therefore, important that you understand the rationale for their use, and this involves a sequence of small analytical steps. None of this should present any difficulty, but be warned that, if students attempt to use a shortcut or, worse, try to reduce the whole business to the mechanical use of a few formulae, you will be asking for trouble.

2.2.3.6.1 Formulation of a Null Hypothesis

We will start by assuming that the theory precedes the experiment and that you have some
thehypothetical relationship in your mind. For example, you may believe that the percentage rate

of price inflation in an economy, p, depends on the percentage rate of wage inflation, w, according to the linear equation

$$p = \beta_1 + \beta_2 w + u \qquad \ldots[2.41]$$

whereβ_1 andβ_2 are parameters and u is a disturbance term. You might further hypothesize that, apart from the effects of the disturbance term, price inflation is equal to wage inflation. Under these circumstances you would say that the hypothesis that you are going to test, known as your *nullhypothesis* and denoted H_0, is that β_2 is equal to 1. We also define an alternative hypothesis, denoted H_1, which represents your conclusion if the experimental test indicates that H_0 is false. In the present case H_1, is simply that β_2 is not equal to 1. The two hypotheses are stated using the notation

$$H_0: \beta_2 = 1$$
$$H_1: \beta_2 \neq 1$$

In this particular case, if we believe that price inflation is equal to wage inflation, we are trying to establish the credibility of H_0 by subjecting it to the strictest possible test and hoping that it emerges intact. In practice, however, it is more usual to set up a null hypothesis and attack it with the objective of establishing the alternative hypothesis as the correct conclusion. For example, consider the simple earnings function

$$EARNINGS = \beta_1 + \beta_2 S + u \qquad \ldots[2.42]$$

Where*EARNINGS* is hourly earnings in dollars and *S* is years of schooling. On very reasonable theoretical grounds, you expect earnings to be dependent on schooling, but your theory is not strong enough to enable you to specify a particular value for β_2. You can nevertheless establish the dependence of earnings on schooling by the inverse procedure in which you take as your null hypothesis the assertion that earnings does *not* depend on schooling, that is, that β_2 is 0. Your alternative hypothesis is that β_2 is not equal to 0, that is, that schooling doesaffect earnings. If you can reject the null hypothesis, you have established the relationship, at least in general terms. Using the conventional notation, your null and alternative hypotheses are $H_0: \beta_2 = 0 \; and H_1: \beta_2 \neq 0$, respectively.

The following discussion uses the simple regression model

$$Y_i = \beta_1 + \beta_2 X_i + \mu_i$$

It will be confined to the slope coefficient, β_2, but exactly the same procedures are applied to the constant term, β_1. We will take the general case, where you have defined a null hypothesis that β_2 is equal to some specific value, say β_2^0 , and the alternative hypothesis is that β_2 is not equal to this value (H_0: $\beta_2 = \beta_2^0$, $H1$: $\beta_2 \neq \beta_2^0$); you may be attempting to attack or defend the null hypothesis as it suits your purpose. We will assume that the four Gauss–Markov conditions are satisfied.

2.2.3.6.2 Developing the Implications of a Hypothesis

If H_0 is correct, values of b_2 obtained using regression analysis in repeated samples will be distributed with meanβ_2^0 and $\frac{\sigma_u^2}{nVar(X)}$, we will now introduce the assumption that *u* has a normal distribution. If this is the case, b_2 will also be normally distributed, In view of the structure of the normal distribution, most values of b_2 will lie within two standard deviations of β_2^0 (if $H0$: $\beta_2 = \beta_2^0$ is true).

2.2.3.7 Compatibility, Freakiness, and the Significance Level

Now, suppose that we take an actual sample of observations on average rates of price inflation and wage inflation over the past five years for a sample of countries and estimate β_2 using regression analysis. If the estimate is close to 1.0, we should almost certainly be satisfied with the null hypothesis, since it and the sample result are compatible with one another. But suppose, on the other hand, that the estimate is a long way from 1.0. Suppose that it is equal to 0.7. This is three standard deviations below 1.0. If the null hypothesis is correct, the probability of being three standard deviations away from the mean, positive or negative, is only 0.0027, which is very low. You could come to either of two conclusions about this worrisome result:

You could continue to maintain that your null hypothesis H_0: β_1 = 1 is correct, and that the experiment has given a freak result. You concede that the probability of such a low value of b_2 is very small, nevertheless it does occur 0.27 percent of the time and you reckon that this is one of those times.

Or you could conclude that the regression result contradicts the hypothesis. You are not convinced by the explanation in (1) because the probability is so small and you think that a much more likely explanation is that β_2 is not really equal to 1. In other words, you adopt the alternative hypothesis H_1: $\beta_2 \neq 1$ instead.

We can summarize this decision rule mathematically by saying that we will reject the null
hypothesis if

$$z > 1.96 \; or \; z < -1.96 \qquad \ldots[2.43]$$

wherez is the number of standard deviations between the regression estimate and the hypothetical value of β_2:

$$z = \frac{distance\ between\ regression\ estimate\ and\ hypothetical\ value}{standard\ deviation\ of b_2} = \frac{b_2 - \beta_2^0}{s.d.(b_2)} \qquad \ldots[2.44]$$

The null hypothesis will not be rejected if

$$-1.96 \leq z \leq 1.96$$

This condition can be expressedregarding$b2$ and β_2^0 by substituting for z from

$$-1.96 \leq \frac{b_2 - \beta_2^0}{s.d.(b_2)} \leq 1.96 \qquad \ldots[2.45]$$

Multiplying through by the standard deviation of b_2, one obtains

$$-1.96\, s.d.(b_2) \leq b_2 - \beta_2^0 \leq 1.96\, s.d.(b_2) \qquad \ldots[2.46]$$

from which one obtains

$$\beta_2^0 - 1.96\, s.d.(b_2) \leq b_2 \leq \beta_2^0 + 1.96\, s.d.(b_2) \qquad \ldots[2.47]$$

[2.47] gives the set of values of $b_2$which will not lead to the rejection of a specific

null hypothesis $2 = \beta_2^0$. It is known as the **acceptance region**for b_2, at the 5 percent significance level.

2.2.3.8 What Happens if the Standard Deviation of b_2 is Not Known

So far we have assumed that the standard deviation of b_2 is known, which is most unlikely in practice. It has to be estimated by the standard error of b_2. This causes two modifications to the test procedure. First, *z* is now defined using s.e.(b_2) instead of s.d.(b_2) and it is referred to as the *t* statistic:

$$t = \frac{b_2 - \beta_2^0}{s.d.(b_2)} \qquad \ldots[2.48]$$

Second, the critical levels of *t* depend on upon what is known as a *t*distribution instead of a normal distribution. We will not go into the reasons for this, or even describe the *t* distribution mathematically. But enough to say that it is a partner of the normal distribution. Its exact shape depends on the number of degrees of freedom in the regression and approximates the normal distribution increasingly closely as the number of degrees of freedom increases. You will certainly have encountered the *t* distribution in your introductory statistics course.

The estimation of each parameter in a regression equation consumes one degree of freedom in the sample. Hence the number of degrees of freedom is equal to the number of observations in the sample minus the number of parameters estimated. The parameters are constant (assuming that this is specified in the regression model) and the coefficients of the explanatory variables. In the present case of simple regression analysis, only two parameters, $\beta_1 + \beta_2$, are estimated and hence the number of degrees of freedom is *n* – 2. It should be emphasized that a more general expression will be required when we come to multiple regression analysis.

The critical value of *t*, which we will denotet_{crit}, replaces the number 1.96 in [2.43], so the

condition that a regression estimate should not lead to the rejection of a null hypothesis H_0: $\beta_2 = \beta_2^0$ is

$$-t_{crit} \leq \frac{b_2 - \beta_2^0}{s.d.(b_2)} \leq t_{crit} \qquad \ldots[2.49]$$

Hence we have the decision rule:

rejectH_0 if $\left|\frac{b_2-\beta_2^0}{s.d.(b_2)}\right|>t_{crit}$,

do not reject if $\left|\frac{b_2-\beta_2^0}{s.d.(b_2)}\right| < t_{crit}$

Where $\left|\frac{b_2-\beta_2^0}{s.d.(b_2)}\right|$ is the absolute value (numerical value, neglecting the sign) of *t*.

2.2.4.0 SUMMARY

In this unit, basic understanding of the properties of regression coefficients and hypotheses testing has been explained. As introductory step to understanding this unit, some knowledge areas like estimators, assumptions underlying CLRM and properties of OLS estimators were briefly discussed. These knowledge areas acquainted the students of what is expected to be known for

better understanding of the different aspect of regression Coefficients, hypotheses testing and the assumptions associated with studying these topics in this unit.

2.2.5.0 CONCLUSION

Assumptions and basic knowledge areas needed for the students to be acquainted with the properties of regression coefficients and hypotheses testing have been introduced and discussed in this unit. Most especially, the respective Gauss-Markov conditions in the assumptions concerning disturbance term of regression analyses were discussed in a manner that the students would be able to properly understand. Also, formulation of null hypothesis and developing implications of a hypothesis were discussed as testing hypotheses relating to the regression coefficients.

2.2.6.0 TUTOR-MARKED ASSIGNMENT

1.) Where performance on a game of skill is measured numerically, the improvement that comes with practice is called a learning curve. This is especially obvious with some arcade-type games. The first time players try a new one; they are likely to score very little. With more attempts, their scores should gradually improve as they become

accustomed to the game, although, obviously, there will be variations caused by the luck factor. Suppose that the learning curve determines their scores

$$Y_i = 500 + 100X_i + \mu_i$$

where, Y is the score, X is the number of times that they have played before, and μ is a disturbance term.

The following table gives the results of the first 20 games of a new player. X automatically goes from 0 to 19; μ was set equal to 400 times the numbers generated by a normally distributed random variable with 0 mean and unit variance, and X and μ determined Yaccording to the learning curve.

Observation	***X***	μ	***Y***
1	0	-236	264
2	1	-96	504
3	2	-332	368
4	3	12	812
5	4	-152	748
6	5	-876	124
7	6	412	1,512
8	7	96	1,296
9	8	1,012	2,312
10	9	-52	1,348
11	10	636	2,136
12	11	-368	1,232
13	12	-284	1,416
14	13	-100	1,700
15	14	676	2,576
16	15	60	2,060

17	16	8	2,108
18	17	-44	2,156
19	18	-364	1,936
20	19	568	2,968

Regressing *Y* on *X*, one obtains the equation (standard errors in parentheses):

$$\hat{Y} = 369 + 116.8X$$

S.E.E (190) (17.1)

Why is the constant in this equation not equal to 500 and the coefficient of *X* not equal to 100?

What is the meaning of the standard errors?

2.) The experiment is repeated with nine other new players (the disturbance term being generated by 400 times a different set of 20 random numbers in each case), and the regression results for all ten players are shown in the following table. Why do the constant, the coefficient of *X*, and the standard errors vary from sample to sample?

Player	Constant	Standard error of constant	Coefficient of X	Standard error of coefficient of X
1	369	190	116.8	17.1
2	699	184	90.1	16.5
3	531	169	78.5	15.2
4	555	158	99.5	14.2
5	407	120	122.6	10.8
6	427	194	104.3	17.5
7	412	175	123.8	15.8
8	613	192	95.8	17.3

9	234	146	130.1	13.1
10	485	146	109.6	13.1

The variance of *X* is equal to 33.25,and the population variance of μ is equal to 160,000. Using appropriate equation, show that the standard deviation of the probability density function of the coefficient of *X* is equal to 15.5. Are the standard errors in the table good estimates of this standard deviation?

2.2.7.0 REFERENCES /FURTHER READING

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UNIT 3 MULTIPLE REGRESSION ANALYSIS AND MULTICOLLINEARITY

CONTENTS

2.3.1.0 INTRODUCTION

The multiple regression analysis is an extension of simple regression analysis. It covers cases in which the dependent variable is hypothesized to depend on more than one descriptive variable. Most of the multiple regression analysis is a direct extension of the simple regression model but hasonly two new dimensions. First, when evaluating the influence of a given descriptive variable on the dependent variable, we would now have to face the problem of discriminating between its effects and the effects of the other descriptive variables. Second, we shall have to tackle the problem of model specification. Often some variables might be thought to influence the behaviour of the dependent variable; though, they might be unconnected. We shall have to decide which should be included in the regression equation and which should be omitted. However, the arrangement of flow for the multiple regression analysis is to firstly, carry out derivation of formula, then estimation procedures using values, followed by presentation of results and lastly interpretations. As an extension of unit 2, we shall discuss multicollinearity being a problem associated with CLRM.

2.3.2.0 OBJECTIVE

The main objective of this unit is to provide broad understanding of the topic; multiple regression analysis and appropriate alleviation measures associated with multicollinearity problems. This understanding includes the knowledge of the properties, principles behind the derivation of and how to interpret multiple regression coefficients.

2.3.3.0 MAIN CONTENTS

2.3.3.1The Multiple Regression Coefficients Derivation

In the simple regression case, the values of the regression coefficients were chosen to make the fit as good as possible in the hope of obtaining most satisfactory estimates of

the true unknown parameters. Our earlier stated definition of goodness of fit; is the minimization of *RSS*, which is the sum of squares of the residuals:

$$RSS = \sum_{i=1}^{n} e_i^2 \quad \ldots[2.50]$$

Where $e_i$is again, the residual in observation i, the difference between the actual value $Y_i$in that observation and the value $\hat{Y}_i$predicted by the regression equation:

$$\hat{Y}_i = b_1 + b_2X_{2i} + b_3X_{3i} + e_i \quad \ldots[2.51]$$

$$e_i = Y_i - \hat{Y}_i - Y_i - b_2X_{2i} - b_3X_{3i} \quad \ldots[2.52]$$

It could be observed that the X variables now have two subscripts. The first identifies the X variable and the second identifies the observation.

Applying [2.52] into [2.50];

$$RSS = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (Y_i - b_1 - b_2X_{2i} - b_3X_{3i})^2 \quad \ldots[2.53]$$

From first-order conditions for a minimum;

$$\frac{\partial RSS}{\partial b_1} = 0, \frac{\partial RSS}{\partial b_2} = 0 \; and \; \frac{\partial RSS}{\partial b_3} = 0$$

[2.52] will give the following equations:

$$\frac{\partial RSS}{\partial b_1} = -2\sum_{i=1}^{n}(Y_i - b_1 - b_2X_{2i}i - b_3X_{3i}) = 0 \quad \ldots[2.54]$$

$$\frac{\partial RSS}{\partial b_2} = -2\sum_{i=1}^{n}(Y_i - b_1 - b_2X_{2i}i - b_3X_{3i}) = 0 \quad \ldots[2.55]$$

$$\frac{\partial RSS}{\partial b_3} = -2\sum_{i=1}^{n}(Y_i - b_1 - b_2X_{2i} - b_3X_{3i}) = 0 \quad \ldots[2.56]$$

Resulting in three equations from the three unknowns, b_1, b_2, and b_3.

The first can easily be rearranged to express $b_1$regardingb_2, b_3, and the data on Y, X_2, and X_3:

$$b_1 = \bar{Y} - b_2\overline{X_2} - b_3\overline{X_3} \quad \ldots[2.57]$$

From [3.57] and working through [3.55] to [3.56], the following expression for $b_2$is obtained:

$$b_2 = \frac{Cov(X_2, Y)Var(X_3) - Cov(X_3, Y)Cov(X_3, X_2)}{Var(X_2)Var(X_3) - [Cov(X_3, X_2)]^2}$$

$$= \frac{[\sum(X_2, Y)(X_3)] - [\sum(X_3, Y)(X_3, X_2)]}{[\sum(X_2)\sum(X_3) - \sum[(X_3, X_2)]^2}$$

…[2.58]

Similarly, theexpressionofb_3 can be obtained by switching X_2 and X_3 in [2.58].

Clearly, the principles behind the derivation of the regression coefficients have been shown to be the same for multiple regression as that of the simple regression. But, it should also be observed that the expressions are however different and so should not try to use expressions derived for simple regression in a multiple regression situations.

A generalized framework for the multiple regression model is

$$Y_i = \beta_1 + \beta_2 X_{2i} + ...\beta_k X_{ki} + \mu_i \quad \text{…[2.59]}$$

We may write [2.59] for three variables as,

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \mu_i \quad \text{…[2.60]}$$

where$Y$ is the dependent variable, X_2 and X_k (kth term) the regressors, μ the stochastic disturbance term and i the ith (tth, if in time series) observation. Also β_1, β_2 and β_k are the partial regression coefficients but β_1 is the intercept term which gives the mean effect on Y of all the variables excluded from the model. That is, in the case of [2.50], when X_2 and X_k are set equal to zero.

Zero mean value of μ_i in [2.60] is;

$$E(\mu_i \mid X_{2i}, X_{3i}) = 0 \quad \text{…[2.61]}$$

2.3.3.1 Multiple Regression Coefficients Interpretation

Discriminate between the effects of the explanatory variables and making allowance for the fact that they may be correlated is enabled in multiple regression analysis. The regression coefficient of each X variable provides an estimate of its influence on Y.

There are two ways in which this can be demonstrated. First is the case where there are only two explanatory variables; to demonstrate that the estimators are unbiased if the model is correctly specified and the Gauss–Markov conditions are fulfilled.

The second method is to run a simple regression of *Y* on one of the *X* variables, having first purged both *Y* and the *X* variable of the components that could be accounted for by the other explanatory variables. The estimate of the slope coefficient and its standard error thus obtained are the same as in the multiple regression. It follows that a scatter diagram plotting the purged *Y* against the purged *X* variable will provide a valid graphical representation of their relationship that can be obtained in no other way.

$$EARNINGS = \beta_1 + \beta_2 S + \beta_3 ASVABC + u \qquad \ldots[2.62]$$

If thegraphical illustration is particularly interested in, in the relationship between earnings and schooling; a direct plot of *EARNINGS* on *S* would give a distorted view of the relationship. This is because *ASVABC* is positively correlated with *S* and having some consequences as *S* increases. These are [1] *EARNINGS* will likely increase, because β_2 is positive; [2] *ASVABC* will tend to increase, because *S* and *ASVABC* are positively correlated; and [3] *EARNINGS* will receive a lift due to the increase in *ASVABC* and the fact that β_3 is positive. That is, the variations in *EARNINGS* will overstate the apparent influence of *S* because in part they will be due to associated variations in *ASVABC*. And the outcome of this is that in a simple regression the estimator of $\beta_2$will be biased. The graphical illustration is shown in Figure 3.1.

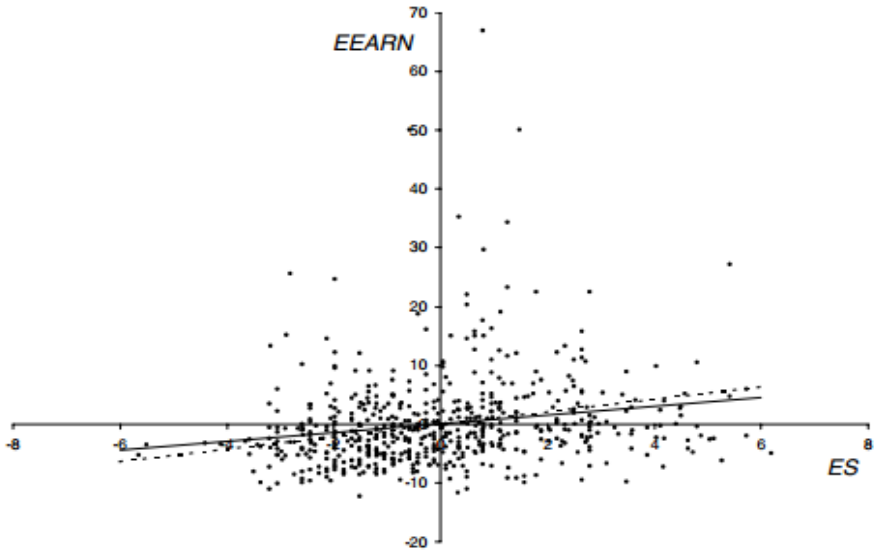

Figure 3.1: Regression of EARNINGS residuals on S residuals

2.3.3.2 Properties of the Multiple Regression Coefficients

Concerning simple regression analysis, the regression coefficients should be thought of as different categories of random variables whose random components are related to the existence of the disturbance term in the model. Each regression coefficient is calculated as a function of the values of *Y* and the explanatory variables in the sample. *Y,* in turn, is determined by the explanatory variables and the disturbance term. It follows that the regression coefficients are indeed determined by the values of the explanatory variables and the disturbance term, in which their properties depend on critically upon the properties of the disturbance term.

In continuation of the assumption that the Gauss–Markov conditions are satisfied, which are:

(*i*) that the expected value of *u*in any observation is 0

(*ii*) that the population variance of its distribution is the same for all observations

(*iii*) that the population covariance of its values in any two observations is 0, and

(*iv*) that it is distributed independently of any explanatory variable.

The first three conditions are the same as for simple regression analysis but (*iv*) is a generalization of (*i*) to (*iii*).

Furthermore, there are two practical requirements to be met.

(i) There must be enough data to fit the regression line. That is, there must be at least as many (independent) observations as there are parameters to be estimated.

(ii) There must not be an exact linear relationship among the explanatory variables.

2.3.3.3 *t* Tests and Confidence Intervals

The *t* tests on the regression coefficients are performed in the same way as for simple regression analysis. Particular attention should, however,be taken when looking up the critical level of *t* at any given significance level. It depends on the number of degrees of freedom ($n - k$); the number of observations n minus the number of parameters estimated k.

The confidence intervals are also obtained in the same manner as in simple regression analysis and equally based on the number of degrees of freedom ($n - k$).

2.3.3.4 Consistency

Once the fourth Gauss–Markov condition is satisfied, OLS yields consistent estimates in the multiple regression models, as is the case in thesimple regression model. One condition for consistency is that when n becomes large, the population variance of the estimator of each regression coefficient tends to 0, and the distribution falls to a spike. The other condition for consistency is since the estimator is unbiased, the spike would be located at the true value.

2.3.4.0 MULTICOLLINEARITY

In most situations, the available data for use in multiple regression analysis would not provide significant solutions to problems at hand. The reason being that the standard errors are very high, or the *t* test ratios are very low.Which means the confidence intervals for such parameters are very wide. A situation of this nature occurs when the explanatory variables show little variation and high intercorrelations. Multicollinearity is the aspect of the situation where the explanatory variables are highly intercorrelated.

Let's look at multicollinearity in a model with two explanatory variables. It would be observed that the higher the correlation between the explanatory variables, the larger the population variances of the distributions of their coefficients and the greater the possibility of attaining irregular estimates of the coefficients.

You should, however, bear in mind that a high correlation does not necessarily lead to poor estimates. If all the other elements determining the variances of the regression coefficients are properly in the number of observations and the sample variances of the explanatory variables are large and the variance of the disturbance term small, good estimates could still be obtained. Multicollinearity, therefore, must be caused by a mixtureof a high correlation and one or more of the other elements being inappropriate. This is a matter of degree and not kind of element of which any regression will suffer from it to some extent unless all the explanatory variables are

uncorrelated. But the consequence is only taken into consideration when it is obviously going to have aserious effect on the regression results.

It is a common problem in time series regressions, particularly where the data consists of a series of observations on the variables over a number of time periods. Which may give rise to multicollinearity if two or more of the explanatory variables are highly correlated in a strong time trend.

Using Table 3.1 as an example let's consider first the case of exact multicollinearity where the explanatory variables are perfectly correlated.

Table 3.1

X_2	X_3	Y	*Change in* X_2	*Change in* X_3	*Approximate change in Y*
10	19	$51+u_1$	1	1	5
11	21	$56+ u_2$	1	1	5
12	23	$61+ u_3$	1	1	5
13	25	$66+ u_4$	1	1	5
14	27	$71+ u_5$	1	1	5

15	29	76+ u_6	1	1	5

Let [2.40] be the true relationship, that is;

$$Y = 2 + 3X_2 + X_3 + u \qquad \ldots[2.63]$$

Suppose that there is a linear relationship between $X_2$and X_3:

$$X_3 = 2X_2 - 1 \qquad \ldots[2.64]$$

and suppose that X_2 increases by one unit in each observation. X_3 will increase by two units, and Y by approximately five units as indicated in Table 3.1. Applying the linear relationship between X_2 andX_3 in manipulating [2.40] will result in different conclusions for Y.

In such a situation it is impossible for regression analysis, or any other technique for that matter, to distinguish between these possibilities. You would not even be able to calculate the regression

coefficients because both the numerator and the denominator of the regression coefficients would collapse to 0. This willbe demonstrated with the general two-variable case. Suppose

$$Y = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + u \qquad \ldots[2.65]$$

And

$$X_3 = \lambda + \mu X_2 \qquad \ldots[2.66]$$

Substituting for X_3 in [3.58] gives

$$\frac{\boldsymbol{Cov(X_2, Y)Var(\lambda + \mu X_2) - Cov([\lambda + \mu X_2], Y)Cov(X_2, [\lambda + \mu X_2])}}{\boldsymbol{Var(X_2)Var(\lambda + \mu X_2) - [Cov(X_2, [\lambda + \mu X_2])]^2}}$$

$$= \frac{\boldsymbol{Cov(X_2,Y)Var(\lambda+\mu X_2) - Cov(\mu X_2,Y)Cov(X_2,\mu X_2)}}{\boldsymbol{Var(X_2)Var(\mu X_2) - [Cov(X_2,\mu X_2])]^2}} \ldots[2.67]$$

From Variance Rule 4, the additive $\boldsymbol{\lambda}$ in the variances can be dropped. A similar rule could be developed for covariances, since an additive $\boldsymbol{\lambda}$ does not affect them either.

Therefore,

$$b_2 = \frac{Cov(X_2,Y)u^2Var(X_2) - uCov(\mu X_2,Y)uCov(X_2,\mu X_2)}{Var(X_2)u^2Var(X_2) - [uCov(X_2,\mu X_2])]^2} \quad \ldots[2.68]$$

$$= \frac{u^2Cov(X_2,Y)Var(X_2) - u^2Cov(X_2,Y)Var(X_2,)}{u^2Var(X_2)Var(X_2) - [uVar(X_2])]^2} = \frac{0}{0} \quad \ldots[2.69]$$

Which is unusual for there to be an exact relationship among the explanatory variables in a regression. So, when this occurs, it is typical because there is a logical error in the specification.

2.3.4.1 Multicollinearity in Models with More Than Two Explanatory Variables

The previous discussion of multicollinearitywas restricted to the case where there are two

explanatory variables. In models with a greater number of explanatory variables, multicollinearity may be caused by an approximately linear relationship among them. It may be difficult to discriminate between the effects of one variable and those of a linear combination of the remainder. In the model with two explanatory variables, an approximately linear relationship automatically means a high correlation, but when there are three or more, this is not necessarily the case. A linear relationship does not inevitably imply high pairwise correlations between any of the variables. The effects of multicollinearity are the same as in the case with two explanatory variables and as in that

case, the problem may not be serious if the population variance of the disturbance term is small, the number of observations large and the variances of the explanatory variables are equally large.

2.3.4.2 Ways to alleviate multicollinearity problems

Two categories exist to alleviate multicollinearity problems:

i. The direct attempts to improve the four conditions responsible for the reliability of the regression estimates, and

ii. The indirect methods.

First, you may try to reduce σ_u^2. The disturbance term is the joint effect of all the variables

influencing *Y* that you have not included explicitly in the regression equation. If you can think of an important variable that you have omitted, and is therefore contributing to *u*, you will reduce the population variance of the disturbance term if you add it to the regression equation.

Second, consider *n*, the number of observations. If you are working with cross-section data (individuals, households, enterprises, etc.) and you are undertaking a survey, you could increase the size of the sample by negotiating a bigger budget. Alternatively, you could make a fixed budget go further by using a technique known as clustering.

A further way of dealing with the problem of multicollinearity is to use minor information, if available, concerning the coefficient of one of the variables.

$$Y = \beta_1 + \beta_2 X + \beta_3 P + u \qquad \ldots[2.70]$$

For example, suppose that *Y* in equation is the aggregate demand for a category of consumer expenditure, *X* is aggregate disposable personal income, and *P* is a price index for the category. To fit a model of this type, you would use time series data. If *X* and *P* possess strong time trends and are therefore highly correlated, which is often the case with time series variables, multicollinearity is likely to be a problem. Suppose, however, that you also have cross-section data on *Y* and *X* derived from a separate household survey. These variables will be denotedY' and X' to indicate that the data are household data, not aggregate data. Assuming that all the households in the survey were paying roughly the same price for the commodity, one would fit the simple regression

$$\hat{Y} = b_1 + b_2 X \qquad \ldots[2.71]$$

Now substitute b_2 for β_2 in the time series model

$$Y = \beta_1 + b_2 X + \beta_3 P + u \qquad \ldots[2.72]$$

Subtract $b_2 X$ from both sides,

$$Y - b_2X = \beta_1 + \beta_3P + u \qquad \ldots[2.73]$$

And regress $Z = Y - b_2X$ on price. This is a simple regression, so multicollinearity has been eliminated.

There are, however, two possible problems with this technique.
First, the estimate of β_3 depends on the accuracy of the estimate of b_2', and this of course is subject to sampling error.
Second, you are assuming that the income coefficient has the same meaning in time series and cross-section contexts, and this may not be the case.
For many commodities, the short-run and long-run effects of changes in income may differ because expenditure patterns are subject to inertia. A change in income can affect expenditure both directly, by altering the budget constraint, and indirectly, through causing a change in lifestyle, and the indirect effect is much slower than the direct one. As a first approximation, it is commonly argued that time series regressions, particularly those using short sample periods, estimate short-run effects while cross-section regressions estimate long-run ones.

For the indirect methods to alleviate multicollinearity problems. If the correlated variables are similar conceptually, it may be reasonable to combine them into some overall index.

2.3.7.0 SUMMARY

In this unit, we discussed the multiple regression model, a model in which there is more than one descriptive variable but a direct extension of the simple regression model having two new dimensions. We introduced the arrangement of flow for the multiple regression analysis and started with the derivation of formula by showing the linkage of the simple regression model and the multiple regression. However, we briefly discussed on interpretation and properties of multiple regression coefficients. For more understanding, the student may use the reference materials to look up estimation procedures using values and presentation of results, aspects of arrangement of flow for the multiple regression analysis not discussed. Finally, the concept of multicollinearity as an existence of linear relationship in the midst of regressors was equally discussed in this unit. Students are shown two ways to alleviate multicollinearity being a problem associated with CLRM.

First, when evaluating the influence of a given descriptive variable on the dependent variable, we would now have to face the problem of discriminating between its effects and the effects of the other descriptive variables. Second, we shall have to tackle the problem of model specification. Often some variables might be thought to influence the behaviour of the dependent variable; though, they might be unconnected. We shall have to decide which should be included in the regression equation and which should be omitted. However, the arrangement of flow for the multiple regression analysis is to firstly, carry out derivation of formula, then estimation procedures using values, followed by presentation of results and lastly interpretations.

2.3.6.0 CONCLUSION

The features of multiple regression analyses and multicollinearity introduced in this unit are extension of unit 2. Here, we pointed out some of the complications arising from the introduction of several descriptive variables. In the discussions, we explained that when we go beyond the two-variable model and consider multiple regression models we add the assumption that there is no perfect multicollinearity (assumption 10 of CLRM). That is, there are no perfect linear relationships among the descriptive variables when two or more of these variables move together and difficult to determine their separate influences.

2.3.7.0 TUTOR-MARKED ASSIGNMENT

1.) The following earnings functions were fitted separately for males and females (standard errors in parentheses):

Males

$$\widehat{EARNINGS} = -3.6121 + 0.7499S + 0.1558ASVABC$$

S.E.E (2.8420) (0.2434) (0.0600)

Females

$$\widehat{EARNINGS} = -5.9010 + 0.8803S + 0.1088ASVABC$$

S.E.E (2.6315) (0.1910) (0.0577)

3.) Explain why the standard errors of the coefficients of *S* and *ASVABC* are greater for the male subsample than for the female subsample, and why the difference in the standard errors are relatively large for *S*.

2.3.8.0 REFERENCES /FURTHER READING

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UNIT 4: TRANSFORMATIONS OFVARIABLES

CONTENTS

2.4.1.0 INTRODUCTION

In model transformation, the functional form of an equation or model determines the estimation techniques and interpretation of results obtained from it. Transforming a variable involves using mathematical procedure to modify its measured values. Single equation (or any other form of equation) may be in different forms. There are two kinds of transformations and generally, models can be of the form;

i. Linear transformation; this preserves the linear relationships between variables (parameters and variables are linear). That is the correlation between *x*and *y* (say) would be unchanged after a linear transformation.

Examples of a linear transformation to variable *x* would be multiplying *x* by a constant, dividing *x* by a constant, or adding a constant to *x*.

ii. Nonlinear transformation; A nonlinear transformation changes (increases or decreases) linear relationships between variables and, thus, changes the correlation between variables.

Examples of a nonlinear transformation of variable *x* would be taking the square root of *x* or the reciprocal of *x*. By extension, nonlinear transformation is a non-linear model that can be made linear. For example;

$Y_t = AX_t^{\beta} e^{\mu_t}$ ...is an example of production function that can be made linear by taking logarithms, that is; $\ln Y_t = \ln A + \beta \ln X_t + u_t$.

In regression, however, a transformation to achieve linearity is a special kind of nonlinear transformation. It is a nonlinear transformation that increases the linear relationship between two variables.

2.4.2.0 OBJECTIVE

The main objective of this unit is to show that regression analysis can be extended to fit nonlinear models through transformation of nonlinear model that can be made linear.

2.4.3.0 MAIN CONTENT

A limitation out of other limitations of linear regression analysis is that it is contained in its very name, in that it can be used to fit only linear equations where every explanatory term, except the constant, is written in the form of a coefficient multiplied by variable:

$$y = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 \quad \ldots[2.74]$$

Y equations such as the two below are non-linear

$$Y = \beta_1 + \beta_2 \frac{1}{X} \quad \ldots[2.75]$$

And

$$Y = \beta_1 X^{\beta_2} \quad \ldots[2.76]$$

Nevertheless, both [2.75] and [2.76] have been suggested as suitable forms for Engel curves, (the relationship between the demand for a particular commodity, *Y* and income, *X*). As an illustration, given data on *Y* and *X*, how could one estimate the parameters $\beta_1 \; and \; \beta_2$ in these equations? Actually, in both cases, with a little preparation one can actually use linear regression analysis.

Here, first, note that [2.74] is linear in two ways. The right side is linear in variables because the variables are included exactly as defined, rather than as functions. It, therefore, consists of a weighted sum of the variables, the parameters being the weights. The right side is also linear in the parameters since it consists of a weighted sum of these as well, the *X* variables being the weights in this respect.

For the purpose of linear regression analysis, only the second type of linearity is important.

Nonlinearity in the variables can always be sidestepped by using appropriate definitions.

For example, suppose that the relationship was of the form

$$Y = \beta_1 + \beta_2 X_2^2 + \beta_3 \sqrt{X_3} + \beta_4 \log X_4 + \cdots \quad \ldots[2.77]$$

By defining $Z_2 = X_2^2$, $Z_3 = \sqrt{X_3}$, $Z_4 = \log X_4$ etc, the relationship can be rewritten

$$Y = \beta_1 + \beta_2 Z_2 + \beta_3 Z_3 + \beta_4 Z_4 + \ldots \quad \ldots[2.78]$$

and it is now linear in variables as well as in parameters. This type of transformation is only beautifying, and you will usually see the regression equation presented with the variables written in their nonlinear form. This avoids the need for explanation and extra notation.

But [2.76] is nonlinear in both parameters and variables and cannot be handled by a mere redefinition. That is, even if attempted, the equation cannot be made linear by defining $Z = X^{\beta_2}$ and replacing X^{β_2} with Z; since you do not know β_2, you have no way of calculating sample data for Z.

However, you could define $Z = \frac{1}{X}$, the equation now becomes

$$Y = \beta_1 + \beta_2 Z \quad \ldots[2.79]$$

and this is linear, which is the regress of Y on Z. The constant term in the regression will be an estimate of β_1 and the coefficient of Z will be an estimate of β_2.

2.4.4.0 SUMMARY

This unit discussed transformation of variables. Two kinds of transformations were discussed, the linear transformation in which the linear relationships between the parameters and variables are preserved after transformation. As well as the nonlinear

transformation in which there is increase or decrease in the linear relationship of the variables involved.

2.4.5.0 CONCLUSION

In this unit the concept of transformation of variables is discussed to show that regression analysis can be extended to fit nonlinear models through transformation of non-linear models. That nonlinearity in the variables can always be sidestepped by using appropriate definitions. Example of these definitions is taking logarithm of the nonlinear model and application of the least squares principle when the model cannot be linearised.

2.4.6.0 REFERENCES /FURTHER READING

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UNIT 5: DUMMY VARIABLES

CONTENTS

2.5.1.0 INTRODUCTION

It sometimes happens that some descriptive variables do exist in our regression equation,and/or the factors that you would like to introduce into a regression model are qualitative (racial, sex or age differences) in nature and therefore not measurable in numerical terms. In such circumstances, dummy variables are utilised.

2.5.2.0 OBJECTIVE

The main objective of this unit is to provide basic understanding of the topic 'Dummy Variable' through the use of imitation variables existing or being introduced into a regression equation to solve some variables that are qualitative or immeasurable in numerical terms.

2.5.3.0 MAIN CONTENTS

The inherent assumption for the application of dummy variables is that the regression lines for the different groups differ only in the intercept term but have the same slope coefficients. For example; (1). You are investigating the relationship between schooling *x* and earnings *y*, and you have both males and females in your sample. You would like to see if the sex of the respondent makes a difference.

(2). You are investigating the relationship between income and expenditure in Cameroun, and your sample includes both English-speaking and French-speaking households. You would like to find out whether the ethnic difference is relevant.

(3). You have data on the growth rate of GDP per capita and foreign aid per capital for a sample of developing countries, of which some are democracies and some are not. You would like to investigate whether the impact of foreign aid on growth is affected by the type of government.

A solution to these examples would be to run separate regressions for the two categories and see if the coefficients are different. Alternatively, you could run a single regression using all the observations together, measuring the effect of the qualitative factor with what is known as a dummy variable. This effect has the two important advantages of providing a simple way of testing whether the effect of the qualitative factor is significant

The qualitative variable has four categories, and we need to develop a more elaborate set

of dummy variables. The standard procedure is to choose one category as the reference category to which the basic equation applies, and then to define dummy variables for each of the other categories. In general, it is good practice to select the dominant or most normal category, if there is one, as the reference category.

Accordingly, we will define dummy variables for the other three types. *TECH* will be the dummy variable for the technical schools: *TECH* is equal to 1 if the observation relates to a technical school, 0 otherwise. Similarly, we will define dummy variables *WORKER* and *VOC* for the skilled workers' schools and the vocational schools. The regression model is now

$$COST = \beta_1 + \delta TTECH + \delta WWORKER + \delta VVOC + \beta_2 N + u \quad \ldots[2.80]$$

Where$\delta T, \delta W, and\ \delta V$are coefficients that represent the extra overhead costs of the technical, skilled workers', and vocational schools, relative to the cost of a general school. Note that you do not include a dummy variable for the reference category, and that is the reason that the reference category is usually described as the omitted

category. Note that we do not make any prior assumption about the size, or even the sign, of the δ coefficients.

2.5.3.1 The Dummy Variable Trap

What would happen if you included a dummy variable for the reference category? There would be two consequences.

i. Were it is possible to compute regression coefficients, you would not be able to give them an interpretation. The coefficient b_1 is a basic estimate of the intercept, and the coefficients of the dummies are the estimates of the increase in the intercept from this basic level, but now there is no definition of what is basic, so the interpretation collapses.

ii. The other consequence is that the numerical procedure for calculating the regression coefficients will break down, and the computer will simply send you an error message (or possibly, in sophisticated applications, drop one of the dummies for you). Suppose that there are *m* dummy categories, and you define dummy variables *D*1... *Dm*.

Then, in observation *i*, $\sum_{j=1}^{m} D_{ji} = 1$ because one of the dummy variables will be equal to 1 and all the others will be equal to 0. But the intercept β_1 is really the product of the parameter β_1 and a special variable whose value is 1 in all observations. Hence, for all observations, the sum of the dummy variables is equal to this special variable, and one has an exact linear relationship among the variables in the regression model. As a consequence the model is subject to a special case of exact multicollinearity, making it impossible to compute regression coefficients.

2.5.3.2 Change of Reference Category

The skilled workers' schools are considerably less academic than the others, even the technical schools. Suppose that we wish to investigate whether their costs are significantly different from the others. The easiest way to do this is to make them the omitted category (reference category). Then the coefficients of the dummy variables become estimates of the differences between the overhead costs of the other types of school and those of the skilled workers' schools. Since skilled workers' schools are

now the reference category, we need a dummy variable, which will be called *GEN*, for the general academic schools. The model becomes

$$COST = \beta_1 + \delta TTECH + \delta VVOC + \delta GGEN + \beta_2 N + u \quad \ldots[2.81]$$

where$\delta T, \delta V, and\ \delta G$are the extra costs of technical, vocational, and general schools relative to skilled workers' schools.

2.5.3.3 Slope Dummy Variables

We have so far assumed that the qualitative variables we have introduced into the regression model are responsible only for shifts in the intercept of the regression line. We have implicitly assumed that the slope of the regression line is the same for each category of the qualitative variables.

This is not necessarily a plausible assumption, and we will now see how to relax it, and test it, using the device known as a slope dummy variable (also sometimes known as an interactive dummy variable).

The assumption that the marginal cost per student is the same for occupational and regular schools is unrealistic. Because occupational schools incur expenditure on training materials related to the number of students, and the staff-student ratio has to be higher in occupational schools because workshop groups cannot be, or at least should not be, as large as academic classes. We can relax the assumption by introducing the slope dummy variable, *NOCC*, defined as the product of *N* and *OCC*:

$$COST = \beta_1 + \delta OCC + \beta_2 N + \lambda NOCC + u \quad \ldots[2.82]$$

If this is rewritten

$$COST = \beta_1 + \delta OCC + (\beta_2 + \lambda OCC)N + u, \quad \ldots[2.83]$$

it can be seen that the effect of the slope dummy variable is to allow the coefficient of N for occupational schools to be λ greater than that for regular schools. If *OCC* is 0, so is *NOCC* and the equation becomes

$$COST = \beta_1 + \beta_2 N + u \quad \ldots[2.84]$$

If *OCC* is 1, *NOCC* is equal to *N* and the equation becomes

$$COST = \beta_1 + \delta + (\beta_2 + \lambda)N + u \quad \ldots[2.85]$$

$\lambda$is thus the incremental marginal cost associated with occupational schools, in the same way that λ is the incremental overhead cost associated with them.

2.5.5.0 SUMMARY

In this unit, the essentials and applications of the concept of dummy variable estimation was introduced. By way of illustrative example to the students, an investigation is made which showed that in the application of dummy variables the regression lines for the different groups differ only in the intercept term but have the same slope coefficients. We also discussed two traps that may occur in the application of dummy variable. First, change in reference category, in which a reference variable is created out of the variables in consideration for analysis. Second, the slope dummyvariables; which considered the slope of the regression line not being the same for each category of the qualitative variables.

2.5.4.0 CONCLUSION

This unit discussed the use of dummy variable estimation to solve some variables that are existing or being introduced into a regression equation which are qualitative or immeasurable in numerical terms. In this discussion, students are made aware that the inherent assumption for the application of dummy variables is that the regression lines for the different groups differ only in the intercept term but have the same slope coefficients.

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UNIT 6: SPECIFICATION OFREGRESSION VARIABLES:A PRELIMINARY SKIRMISH

CONTENTS

2.6.1.0 INTRODUCTION

The construction of an economic model involves the specification of the relationships that constitute it, the specification of the variables that participate in each relationship and the mathematical function representing each relationship.

2.6.2.0 OBJECTIVE

The main objective of this unit is to provide a general understanding of the topic 'Specification of Regression Variable'. This include the creating an opportunity for the students to know that *model specification denotes the determination of which independent variables may be included in or omitted from a regression equation.*

2.6.3.0 MAIN CONTENTS

2.6.3.1 Model Specification

The knowledge of exactly which descriptive variables ought to be included in the equation helps when we undertake regression analysis, our task would equally be limited to calculating estimates of their coefficients, confidence intervals for these estimates and so on. In practice, however, we can never be sure that we have specified the equation properly. Economic theory ought to provide a guide, but thetheory is never flawless. Unaware, we might be including some variables that ought not to be in the model and we might be leaving out others that ought to be incorporated.

Existing properties of the regression estimates of the coefficients depend significantly on the validity of the specification of the model. The consequences of misspecification of the variables in a relationship are stated below.

i. When a variable that ought to be included is left out, the regression estimates are in general (but not always) biased. The standard errors of the coefficients and the corresponding *t* tests are in general invalid. Another serious consequence of omitting a variable that ought to be included in the regression is that the standard errors of the coefficients and the test statistics are in general invalidated. This means of course that you are not in principle able to test any hypotheses with your regression results.

ii. On the other hand, if you include a variable that ought not to be in the equation, the regression coefficients are in general (but not always) inefficient but not biased. The standard errors are in general valid but, because the regression estimation is inefficient, they will be needlessly large.

2.6.4.0 SUMMARY

In this unit, the specification of regression variables at a preliminary skirmish is discussed. In general, an introductory opportunity is created for the students to know that the specification of a regression model should be based primarily on theoretical considerations rather than what may be obtainable in practice.

2.6.5.0 CONCLUSION

The specification of regression variables at a preliminary skirmish was explained in this unit. And briefly made the students aware that this is one of the foundational econometrics topic that prepares the readers for intermediate econometrics. That is, specification of model is the first and most critical of the stages in regression analysis which students may use to identify functions and solve problems associated with all other topics discussed in this module.

2.6.6.0 REFERENCES /FURTHER READING

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MODULE 3: HETEROSCEDASTICITY

CONTENTS

3.1.1.0 INTRODUCTION

The general aim of this module is to provide you with a thorough understanding of the violation of one of the classical assumptions, equal variances (homoscedastic). The properties of the estimators of the regression coefficients depend on the properties of the disturbance term in the regression model. In this module, we shall be looking at some of the problems that arise when violations of the Gauss–Markov conditions, the assumptions relating to the disturbance term, are not satisfied. Basic understanding of heteroscedasticity (unequal-variances) will be likewise explained.

3.1.2.0 OBJECTIVE

The main objective of this unit is to provide a platform for the students to understand that in statistic, heteroscedasticity is a collection of random variables and the absence of it is homoscedasticity.

3.1.3.0 MAIN CONTENTS

3.1.3.1 Heteroscedasticity and Its Effects

Gauss–Markov second conditions listed in the previous module states; that the variance of the disturbance term in each observation should be constant. This sounds peculiar and needs a bit of explanation. The disturbance term in each observation has only one value, so what can be meant by its "variance"?

The focus point of discussion here is, its potential behaviour before the sample is generated. So when the model is written as*;*

$$Y = \beta_1 + \beta_2 X + u \qquad \ldots[3.01]$$

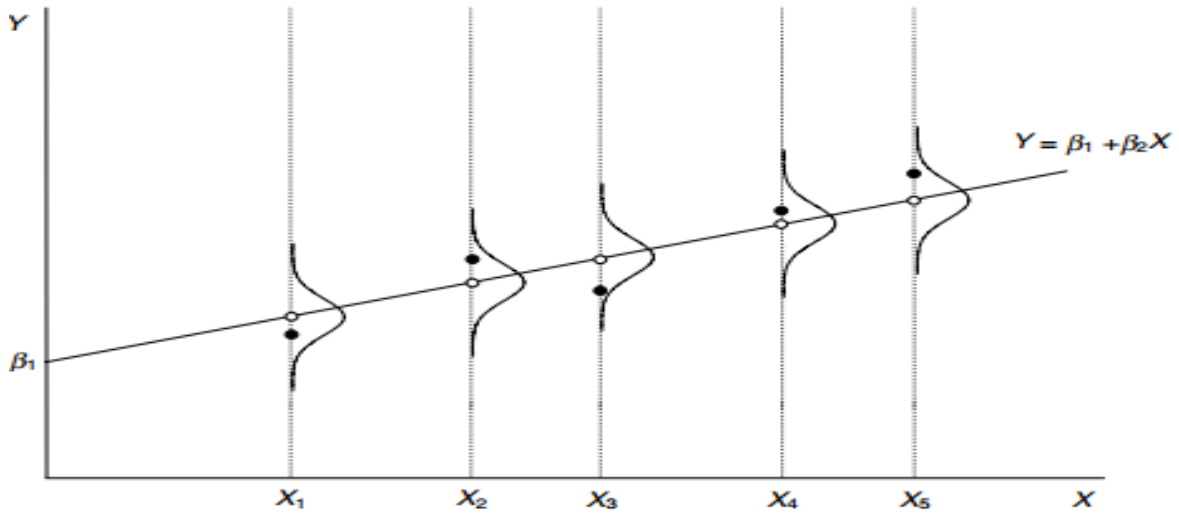

Figure 1.1 Homoscedasticity

[3.01]has in it the first two Gauss–Markov conditions stating that the disturbance terms u_1, ..., $u_n$in the *n*observations are drawn from probability distributions that have 0 mean and the same variance. Their actual values in the sample will sometimes be positive, sometimes negative, sometimes relatively far from 0, sometimes relatively close, but there will be no a priori reason to anticipate a particularly erratic value in any given observation. To put it another way, the probability of *u*reaching a given positive or negative value will be the same in all observations. This condition is known as homoscedasticity, which means "same dispersion".

Figure 1.1 is a depiction of homoscedasticity. For a simple illustration, the sample in Figure 1.1 contains only five observations. Let us start with the first observation, where *X* has the value X_1. If

there were no disturbance term in the model, the observation would be represented by the circle vertically above X_1 on the line $Y = \beta_1 + \beta_2 X$. The effect of the disturbance term is to shift the observation upwards or downwards vertically. The potentialdistribution of the disturbance term, before the observation has been generated, is shown by the normal distribution centered on the circle. The actual value of the disturbance term for this observation turned out to be negative, the observation being represented by the darkened indicator. The potential distribution of the disturbance term, and the actual outcome, are shown in a similar way for the other four observations. Although homoscedasticity is often taken for granted in regression analysis, in some contexts it may be more reasonable to suppose that the potential distribution of the disturbance term is different for different observations in the sample. This is illustrated in Figure 1.2 where the variance of the potential distribution of the disturbance term is increasing as *X* increases. This does not mean that thedisturbance term will necessarilyhave a particularly large (positive or negative) value in anobservation where *X* is large, but it does mean that the a priori probabilityof having an erratic value will be relatively high. This is an example of heteroscedasticity, which means "differing dispersion".

Mathematically, homoscedasticity and heteroscedasticity may be defined:

Homoscedasticity: $\sigma_{ui}^2 = \sigma_u^2$ same for all observations

Heteroscedasticity: σ_{ui}^2not the same for all observations

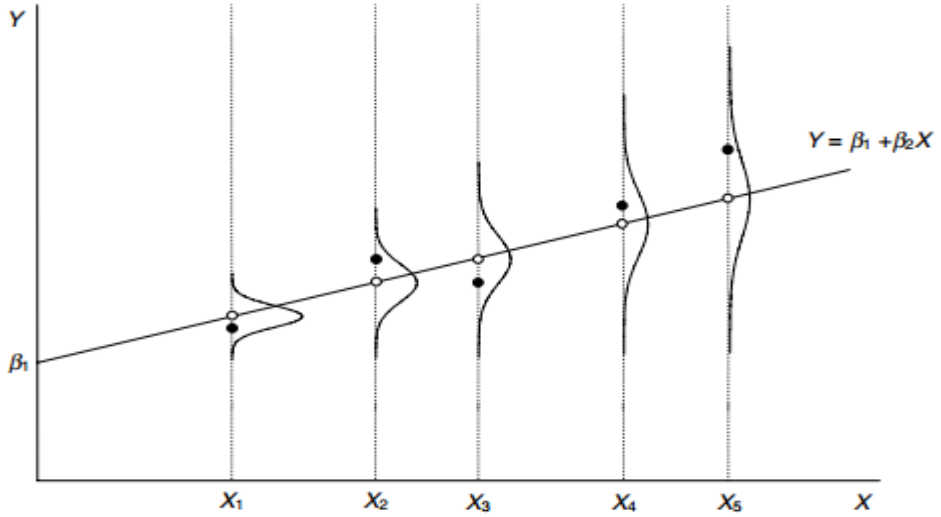

Figure 1.2 Heteroscedasticity

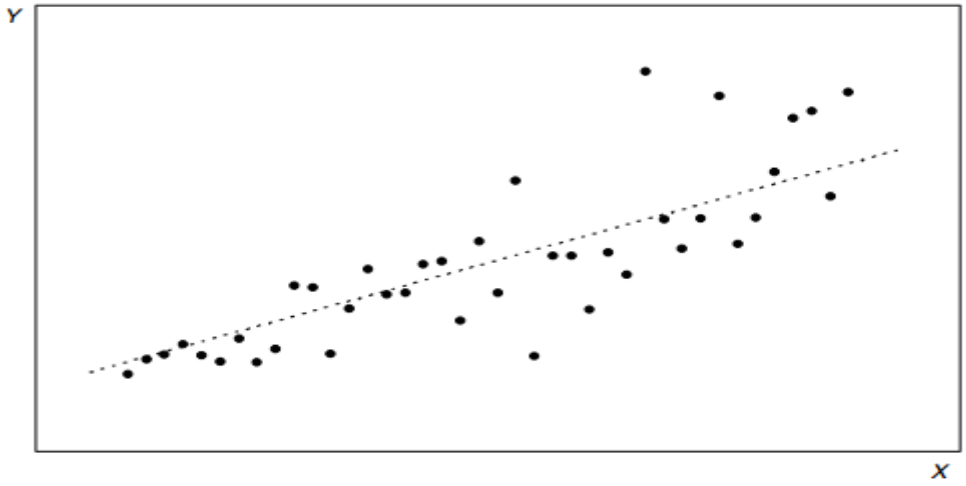

Figure 1.3 Model with a heteroscedastic disturbance term

Figure 1.3shows how a typical scatter diagram would look if Y were an increasing function of X and the heteroscedasticity were of the type shown in Figure 1.2. It could be seen that, although the observations are not necessarily further away from the non-stochastic component of the relationship, represented by the line $Y = \beta_1 + \beta_2 X$, there is a tendency for their dispersion to increase as X increases. Thus this particular Gauss–Markov condition does not seem to have been used anywhere in the analysis so

far, so it might look almost irrelevant. In particular, the proofs of the unbiasedness of the OLS regression coefficients did not use this condition. There are however two explanations for the presence of heteroscedasticity.

The first explanation has to do with making the variances of the regression coefficients as small as possible, so that in a probabilistic sense, maximum precision is achieved. If there is no heteroscedasticity and if the other Gauss–Markov conditions are satisfied, the OLS regression coefficients have the lowest variances of all the unbiased estimators that are linear functions of the observations of *Y*. If heteroscedasticity is present, the OLS estimators are inefficient because there are still other estimators that have smaller variances and are still unbiased.

The other reason is that the estimators of the standard errors of the regression coefficients will be wrong. This is because their computation is based on the assumption that the distribution of the disturbance term is homoscedastic. Otherwise, they are biased. As a consequence, the *t*tests and also the usual *F*tests will be invalid. It is therefore quite likely that the standard errors will be underestimated, so the *t*statistics will be overestimated which will have a misleading impression of the precision of the regression coefficients. The coefficient may appear significantly different from 0, at a given significance level, when in fact, it is not. The inefficiency property can be explained quite easily assuming that heteroscedasticity of the type displayed in Figures 1.2 and 1.3 is present.

Which is an observation where the potential distribution of the disturbance term has a small standard deviation, similar to that of Figure 1.1.

3.1.3.2 Likely Sources of Heteroscedasticity

For heteroscedasticity, it is likely to be a problem when the values of the variables in the sample vary substantially in different observations. Given that $Y = \beta_1 + \beta_2 X + u$, the variations in the omitted variables and the measurement errors that are jointly responsible for the disturbance term (u) would be somewhat small when *Y*and *X*are small and large when they are large. This is simply because economic variables in such a true relationship tend to move in size together.

3.1.3.3 Detection of Heteroscedasticity

There seems to be no limit to the different possible types of heteroscedasticity, and consequently, a large number of different tests appropriate for different conditions have been suggested. The attention here would, however,be focused on three tests that hypothesize a relationship between the variance of the disturbance term and the size of the explanatory variable(s). These would be the Spearman rank correlation, Goldfeld–Quandt, and Glejser tests.

3.1.3.4 The Spearman Rank Correlation Test

This test assumes that the variance of the disturbance term is either increasing or decreasing as *X*increases and that there will be a correlation between the absolute size of the residuals and the size of *X*in an OLS regression. The data on *X*and the absolute values of the residuals are both ranked, and the rank correlation coefficient is defined as

$$r_{x,e} = 1 - \frac{6\sum_{i=1}^{n} D_i^2}{n(n^2-1)} \qquad \ldots[3.02]$$

where$D_i$is the difference between the rank of *X*and the rank of *e*in observation *i*. Under the assumption that the population correlation coefficient is 0, the rank correlation

coefficient has a normal distribution with 0 mean and variance $\frac{1}{(n-1)}$ in large samples. Theappropriate test statistic is therefore $r_{x,e}\sqrt{n-1}$and the null hypothesis of homoscedasticity will be rejected at the 5 percent level if its absolute value is greater than 1.96 and at the 1 percent level if its absolute value is greater than 2.58, using two-tailed tests. If there is more than one explanatory variable in the model, the test may be performed with any one of them.

Example

Table 1.1

Manufacturing Value Added, GDP, and Population for a Sample of Countries, 1994

Country	*MANU*	*GDP*	*POP*	*MANU/POP*	*GDP/POP*
Belgium	44517	232006	10.093	4411	22987
Canada	112617	547203	29.109	3869	18798
Chile	13096	50919	13.994	936	3639
Denmark	25927	151266	5.207	4979	29050
Finland	21581	97624	5.085	4244	19199
France	256316	1330998	57.856	4430	23005
Greece	9392	98861	10.413	902	9494
Hong Kong	11758	130823	6.044	1945	21645
Hungary	7227	41506	10.162	711	4084
Ireland	17572	52662	3.536	4970	14893
Israel	11349	74121	5.362	2117	13823
Italy	145013	1016286	57.177	2536	17774
Korea, S.	161318	380820	44.501	3625	8558
Kuwait	2797	24848	1.754	1595	14167
Malaysia	18874	72505	19.695	958	3681
Mexico	55073	420788	89.564	615	4698
Netherlands	48595	334286	15.382	3159	21732
Norway	13484	122926	4.314	3126	28495
Portugal	17025	87352	9.824	1733	8892
Singapore	20648	71039	3.268	6318	21738
Slovakia	2720	13746	5.325	511	2581
Slovenia	4520	14386	1.925	2348	7473
Spain	80104	483652	39.577	2024	12221
Sweden	34806	198432	8.751	3977	22675
Switzerland	57503	261388	7.104	8094	36794
Syria	3317	44753	13.840	240	3234
Turkey	31115	135961	59.903	519	2270
UK	244397	1024609	58.005	4213	17664

Source: UNIDO Yearbook 1997

Note: *MANU* and *GDP* are measured in U.S. $ million. *POP* is measured in million. *MANU/POP* and *GDP/POP* are measured in U.S. $.

Using the data in Table 1.1 above, an OLS regression of manufacturing output on GDP yields the following result (standard errors in parentheses):

$$\text{MANU} = 604 + 0.194\,\text{GDPR}^2 = 0.8$$

S.E.E. (5700) (0.013)

This implies that manufacturing accounts for $194,000 out of every $1 million increase in GDP in the cross-section. The residuals from the regression and GDP are both ranked in Table 1.2 and Di and D_i^2 are computed.

Table 1.2

GDP	*Rank*	|e|	*Rank*	*D*	D^2	*GDP*	*Rank*	|e|	*Rank*	*D*	D^2
13746	1	547	2	−1	1	130823	15	14185	23	−8	64
14386	2	1130	4	−2	4	135961	16	4176	12	4	16
24848	3	2620	8	−5	25	151266	17	3976	11	6	36
41506	4	1417	5	−1	1	198432	18	4233	14	4	16
44753	5	5955	15	−10	100	232006	19	1025	3	16	256
50919	6	2629	9	−3	9	261388	20	6270	17	3	9
52662	7	6768	19	−12	144	334286	21	16758	24	−3	9
71039	8	6284	18	−10	100	380820	22	86952	28	−6	36
72505	9	4227	13	−4	16	420788	23	27034	25	−2	4
74121	10	3611	10	0	0	483652	24	14180	22	2	4
87352	11	499	1	10	100	547203	25	6024	16	9	81
97624	12	2067	6	6	36	1016286	26	52439	27	−1	1
98861	13	10360	20	−7	49	1024609	27	45333	26	1	1
122926	14	10929	21	−7	49	1330998	28	2093	7	21	441

The sum of the latter came to 1608. The rank correlation coefficient is thus

$$1 - \frac{6 \times 1608}{28 \times 783} = 0.56$$

and the test statistic is $0.56\sqrt{27} = 2.91$. This is above 2.58 and hence the null hypothesis of homoscedasticity is rejected at the 1 percent level.

3.1.3.5 The Goldfeld–Quandt Test

Goldfeld and Quandt (1965) are so far attributed with the most common formal test for heteroscedasticity. The test assumes that σ_{u_i} the standard deviation of the probability distribution of the disturbance term in observation *i*, is about the size of X_i. It also assumes that the disturbance term is distributed and satisfies the other Gauss–Markov conditions. The sizeof *X* ordersthe *n*observationsinthesampleand separate regressions are carried out for the first *n′*and the last *n′* observations, the middle (*n*–2*n′*) observations being dropped completely. If heteroscedasticity is present, and if the assumption regarding its nature is correct, the variance of *u*in the last *n′* observations will be more than that in the first *n′* and this will be reflected in the *RSS* in the two

sub-regressions. Representing these by *RSS1* and *RSS2* for the sub-regressions with the first n' and the last n' observations, respectively.The ratio *RSS2/RSS1* will be distributed as an *F*statistic with $(n' - k)$ and $(n' - k)$ degrees of freedom, where *k*is the number of parameters in the equation, under the

null hypothesis of homoscedasticity. The power of the test depends on the choice of n'about *n*. As a result of some experiments undertaken by Goldfeld and Quandt, they recommend that in general, n' should be about 11 when *n*is 30 and about 22 when *n*is 60. Which clearly shows that *n*' should be about $\frac{3}{8}$ of *n*.

If there is more than one explanatory variable in the model, the observations should be ordered by that which is hypothesized to be associated with the null hypothesis for the test is that *RSS2* is not significantly greater than *RSS1,* and the alternative hypothesis is that it is significantly greater. If *RSS2* turns out to be smaller than *RSS1*, the null hypothesis should not be rejected; it only means that there would not be any point in computing the test statistic*RSS2/RSS1*. However, the Goldfeld–Quandt test can also be used for the case where the standard deviation of the disturbance term is hypothesized to be inversely proportional to X_i. The procedure is the same as before, but the test statistic is now *RSS1/RSS2*, and it will again be distributed as an*F*-statistic with $(n' - k)$ and $(n' - k)$ degrees of freedom under the null hypothesis of homoscedasticity.

3.1.3.6 The Glejser Test

This test permits you to search the nature of the heteroscedasticity a little more closely. Here, the assumption that σ_{u_i} is a relative quantity to X_i is relaxed,and you can then investigate whether some other efficient form may be more suitable, for example

$$\sigma_{u_i} = \beta_1 + \beta_2 X_i^{\gamma} \qquad \ldots[3.03]$$

To use the procedure, you regress *Y*on *X*using OLS and then fit the absolute values of the

residuals, $|e|$ to the function for a given value of γ. You may fit several such functions, varying the choice of γ. In each case the null hypothesis of homoscedasticity will be rejected if the estimate of β_2 is significantly different from 0.

If more than one function gives rise to a significant estimate of β_2, that with the best fit may be a guide to the nature of the heteroscedasticity.

3.1.3.6 Solution to Heteroscedasticity Problem

Suppose that the true relationship is

$$Y_i = \beta_1 + \beta_2 X_i + u_i \qquad \ldots[3.04]$$

Let the standard deviation of the disturbance term in observation *i*be σ_{u_i}. If you happened to now σ_{u_i} for each observation, you could eliminate the heteroscedasticity by dividing each observation by its value of σ. The model becomes

$$\frac{\mathrm{Y_i}}{\sigma_{u_i}} = \beta_1 \frac{1}{\sigma_{u_i}} + \beta_2 \frac{\mathrm{X_i}}{\sigma_{u_i}} + \frac{u_i}{\sigma_{u_i}} \ldots[3.05]$$

The disturbance term $\frac{u_i}{\sigma_{u_i}}$becomes homoscedastic because the population variance of $\frac{u_i}{\sigma_i}$ is

$$\mathrm{E}\left\{\left(\frac{u_i}{\sigma_{u_i}}\right)^2\right\} = \frac{1}{\sigma_{u_i}^2}\mathrm{E}(\mathrm{u_i^2}) = \frac{1}{\sigma_{u_i}^2}\sigma_{u_i}^2 1 \qquad \ldots[3.06]$$

That is, every observation will have a disturbance term drawn from a distribution with population variance 1, and the model will be homoscedastic. The revised model may be rewritten as;

$$Y_i^{'} = \beta_1 h_i + \beta_2 X_i^{'} + u_i^{'} \qquad \ldots[3.07]$$

where $Y_i^{'} = \frac{Y_i}{\sigma_{u_i}}$, $X_i^{'} = \frac{X_i}{\sigma_{u_i}}$, *h*is a new variable whose value in observation *i*is $\frac{1}{\sigma_{u_i}}$ and $u_i^{'} = \frac{u_i}{\sigma_{u_i}}$

Note that there should not be a constant term in the equation. By regressing Y' on *h*and X',

you will obtain efficient estimates of β_1 and β_2 with unbiased standard errors.

3.1.3.7 Consequences of Heteroscedasticity

The seriousness of the consequences of heteroscedasticity will depend on the nature of the occurred heteroscedasticity, and there are no general rules. In the case of the heteroscedasticity, where the standard deviation of the disturbance term is proportional to *X*and the values of *X*are integers from 5 to 44. Here, the population variance of the OLS estimator of the slope coefficient is approximately double that of the estimator, where the heteroscedasticity has been eliminated by dividing through

by*X*. Further, the standard errors of the OLS estimators are underestimated, giving a misleading impression of the precision of the OLS coefficients.

3.1.4.0 SUMMARY

This unit begins with a general discussion ofheteroscedasticity and its meaning. Also discussed are the reasons why the distribution of a disturbance term may be subject to heteroscedasticity, and the consequences of the heteroscedasticityproblem for OLS estimators. We continued with presentation of several tests for heteroscedasticity and methods of alleviating the problem.

3.1.5.0 CONCLUSION

The discussion in this unit concludes with an awareness to the students that the concept of heteroscedasticity and associated problems are indications of how an apparent case of heteroscedasticity may be caused by model misspecification.

3.1.6.0 TUTOR-MARKED ASSIGNMENT

A researcher investigating whether government expenditure tends to crowd out investment fits the regression (standard errors in parentheses):

$$\hat{I} = 18.10 - 1.07G + 0.3Y \qquad R^2 = 0.99$$

S.E.E. (7.79) (0.14) (0.02)

She sorts the observations by increasing size of *Y* and runs the regression again for the 11
countries with smallest *Y* and the 11 countries with largest *Y*. *RSS* for these regressions is 321
and 28101, respectively. Perform a Goldfeld–Quandt test for heteroscedasticity.

3.1.7.0 REFERENCES /FURTHER READING

Maddala, G. S., &Lahiri, K. (1992).*Introduction to econometrics* (Vol. 2). New York

Carter, H. R., Griffiths, W. E., & Judge, G. (2001).*Undergraduate econometrics.*2nd Ed. New York: John Wiley and Sons.

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MODULE 4: ECONOMETRIC MODELLING AND AUTOCORRELATION

The general aim of this module is to provide you with a thorough understanding of the basic rudiments of econometric modelling. Stochastic Regression and Measurement Errors, autocorrelation, econometric modelling and models using time series data are explained. By the end of this module, you would have been able to understand the components of the module stated below. The units to be studied are;

Unit 1: Stochastic Regression and Measurement Errors

Unit 2: Autocorrelation

Unit 3: Econometric Modelling and Models Using Time Series Data

UNIT 1: STOCHASTIC REGRESSORS ANDMEASUREMENT ERRORS

CONTENTS

4.1.1.0 INTRODUCTION

The least squares regression model assumed that the explanatory variables arenonstochastic, that is, that they do not have random components. Although relaxing this assumption does not in itself undermine the OLS regression technique, it is typically an unrealistic assumption, so it is important

you know the consequences of relaxing it. We shall see that in some contexts we can continue to use OLS, but in others, for example when one or more explanatory variables are subject to measurement error, it is a biased and inconsistent estimator.

4.1.2.0 OBJECTIVE

The main objective of this unit is to provide a general understanding of the topic ‘stochastic regressors and measurement errors and point out that random element in a regression model is not the only disturbance term but that the variables themselves do have random components.

4.1.3.0 MAIN CONTENTS

4.1.3.1 Stochastic Regressors

Based on the adopted assumption that the regressors, which is the explanatory variables in the regression model are nonstochastic, their values in the sample are therefore fixed and unaffected by the way the sample is generated. Perhaps the best example of a nonstochastic variable is time, which, as we will see when we come to time series analysis, is sometimes included in the regression model as a proxy for

variables that are difficult to measure, such as technical progress or changes in tastes. Nonstochastic explanatory variables are unusual in regression analysis.

A rationale for making the nonstochastic assumption has been one of simplifying the analysis of the properties of the regression estimators. For example, we saw that in the regression model

$$Y = \beta_1 + \beta_2 X + u \qquad \ldots[4.01]$$

the OLS estimator of the slope coefficient may be decomposed as follows:

$$b_2 = \frac{Cov(X,Y)}{Var(X)} = \beta_2 + \frac{Cov(X,u)}{Var(X)} \qquad \ldots[4.02]$$

Here, if X is nonstochastic, so is $Var(X)$,
and the expected value of the error term can be written $E[Cov(X,u)]/Var(X)$.
Also if X is nonstochastic, $E[Cov(X,u)]$ is 0.
Which easily helps us to prove that b_2 is an unbiased estimator of β_2.

The desirable properties of the OLS estimators remain unchanged even if the descriptive variables have stochastic components, provided that these components are distributed independently of the disturbance term, and provided that their distributions do not depend on the

parameters $\beta_1, \beta_2 \; or \; \sigma_u$. Let us demonstrate the unbiasedness and consistency properties and as typical, taking an efficient approach.

4.1.3.2 Unbiasedness

Once X is stochastic, $Var(X)$ cannot be treated as a scalar, so we cannot rewrite $E[Cov(X,u)/Var(X)]$ as $E[Cov(X,u)]/Var(X)$. Hence the previous proof of unbiasedness is blocked. However, we can find another route by decomposing the error term:

$$\frac{Cov(X,u)}{Var(X)} = \frac{\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})(u_i-\bar{u})}{Var(X)} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{(X_i-\bar{X})}{Var(X)}\right)(u_i-\bar{u}) = \frac{1}{n}\sum_{i=1}^{n} f(X_i)(u_i-\bar{u})$$

$\ldots[4.03]$

where $f(\mathrm{X}_i) = \frac{(X_i - \overline{X})}{Var(X)}$. Now, if X and $u$are independently distributed,

$E[f(X_i)(u_i - \bar{u})]$

may be decomposed as the product of $E[f(\mathrm{x}_i)]$ and $E[(u_i - \bar{u})]$. Hence

$$E[f(\mathrm{X}_i)(u_i - \bar{u})] = E[f(\mathrm{X}_i)E(u_i - \bar{u})] = E[f(\mathrm{X}_i)] \times 0 \quad \ldots[4.04]$$

since by assumption $E(u_i)$ is 0 in each observation. This implies,of course, that $E(\bar{u})$ is also 0.

Hence, when we take the expectation of $\frac{1}{n}\sum_{i=1}^{n} f(X_i)(u_i - \bar{u})$, each term within the summation has expected value 0. Thus the error term as a whole has expected value 0 and b_2 is an unbiased estimator of β_2.

4.1.3.3 Consistency

Generally stated, $plim\ (A/B)$ is equal to $plim(A)/plim(B)$, where A and B are any two stochastic quantities, on condition that both $plim(A)$ and $plim(B)$ exist and that $plim(B)$ is nonzero ("$plim$" is the limiting value as the sample size becomes large). As also stated, sample expressions tend to their population counterparts as the sample size becomes large, so $plimCov(X,u)$ is the population covariance of X and u and $plimVar(X)$ is X_2, the population variance of X. If X and u are independent, the population covariance of X and u is 0 and we can write that:

$$plimb_2 = \beta_2 + \frac{plim\ Cov(X,u)}{plim\ Var(X)} = \beta_2 + \frac{0}{\sigma_x^2} = \beta_2 \quad \ldots[4.05]$$

4.1.3.4 The Consequences of Measurement Errors

As it is in other human activities, it habitually happens in economics that, when investigating a relationship, the variables involved could be measured defectively. For example, surveys often contain errors caused by the person being interviewed not remembering properly or not understanding the question correctly. However, misreporting is not the only source of inaccuracy. It sometimes happens that you have defined a variable in your model in a certain way, but the available data correspond to a slightly different definition.

4.1.3.5 Measurement Errors in the Descriptive Variable(s)

To keep the analysis simple, we will confine it to the simple regression model. Let us suppose that a variable *Y* depends on a variable *Z* according to the relationship

$$Y_i = \beta_1 + \beta_2 Z_i + v_i \qquad \ldots[4.06]$$

where*v* is a disturbance term with mean 0 and variance σ_y^2, distributed independently of *Z*. We shall suppose that *Z* cannot be measured absolutely accurately, and we shall use *X* to denote its measured value. In observation $i, X_i$is equal to the true value, Z_i, plus the measurement error, w_i:

$$X_i = Z_i + w_i \qquad \ldots[4.07]$$

We shall suppose that *w* has mean 0 and variance σ_w^2 , that *Z* has population varianceσ_z^2, and that *w* is distributed independently of *Z* and *v*.

[4.07] into [4.06], will yield

$$Y_i = \beta_1 + \beta_2(X_i - w_i) + v_i = \beta_1 + \beta_2 X_i + v_i - \beta_2 w_i \qquad \ldots[4.08]$$

Two random components are present in [4.08], the original disturbance term *v* and the measurement error (multiplied by$-\beta_2$). Together they form a composite disturbance term, which we shall call *u*:

$$u_i = vi - \beta_2 w_i \qquad \ldots[4.09]$$

Therefore, [4.08] becomes

$$Y_i = \beta_1 + \beta_2 X_i + u_i \qquad \ldots[4.10]$$

You have your data on *Y* (which, for the time being, we shall assume has been measured accurately) and *X*, and you unsuspectingly regress *Y* on *X*.

As usual, the regression coefficient *b* is given by

$$b_2 = \frac{Cov(X,Y)}{Var(X)} = \beta_2 + \frac{Cov(X,u)}{Var(X)} \qquad \ldots[4.11]$$

Looking at the error term, we can see that it is going to behave badly. By [4.07] and [4.09], both $X_i$and $u_i$depend on w_i. The population covariance between X and u is nonzero and, so b_2 is an inconsistent estimator of β_2. Even if you had a very large sample, your estimate would be inaccurate. In the limit it would underestimate β_2 by an amount

$$\frac{\sigma_w^2}{\sigma_z^2+\sigma_w^2}\beta_2 \qquad \ldots[4.12]$$

4.1.3.6 Measurement Errors in the Dependent Variable

These measurement errors in the dependent variable do not matter as much. In practice, they can be thought of as contributing to the disturbance term. They are undesirable, because anything that increases the noise in the model will tend to make the regression estimates less accurate, but they will not cause the regression estimates to be biased.

By assumption, let the true value of the dependent variable be *Q*, and the true relationship be

$$Q_i = \beta_1 + \beta_2 X_i + v_i, \qquad \ldots[4.13]$$

where*v* is a disturbance term. If $Y_i$is the measured value of the dependent variable in observation *i*, and $r_i$is the measurement error,

$$Y_i = Q_i + r_i \qquad \ldots[4.14]$$

which may be rewritten

$$Y_i = \beta_1 + \beta_2 X_i + u_i \qquad \ldots[4.15]$$

where*u* is the composite disturbance term (*v* + *r*)

The only difference from the usual model is that the disturbance term in [4.15] has two

components: the original disturbance term and the error in measuring *Y*. The important thing is that the explanatory variable *X* has not been affected. Hence OLS still yields unbiased estimates provided that *X* is nonstochastic or that it is distributed

independently of *v* and *r*. The population variance of the slope coefficient will be given by

$$\sigma_{b_2}^2 = \frac{\sigma_u^2}{n\sigma_x^2} = \frac{\sigma_v^2+\sigma_r^2}{n\sigma_x^2} \qquad \ldots[4.16]$$

and so will be greater than it would have been in the absence of measurement error, reducing the precision of the estimator. The standard errors remain valid but will be larger than they would have been in the absence of the measurement error, reflecting the loss of precision.

4.1.4.0 SUMMARY

In this unit, in other for the students to have understanding of the topic stochastic regressors and measurement errors, we explained conditions under which OLS estimator remain unbiased when the variable in a regression model possessing random components. A demonstration of the unbiasedness and consistency properties was also approached. Equally, the consequences of measurement errors, errors in descriptive and dependents variables were discussed.

4.1.5.0 CONCLUSION

The unit concludes that under general conditions the regression model remain unchanged even if the descriptive variables have stochastic components. Provided that these components are distributed independently of the disturbance term and considering measurement errors in the descriptive and dependent variables.

4.1.6.0 TUTOR-MARKED ASSIGNMENT

In a certain industry, firms relate their stocks of finished goods, *Y*, to their expected annual

sales, X^e, according to a linear relationship

$$Y = \beta_1 + \beta_2 X^e$$

Actual sales,*X*, differ from expected sales by a random quantity *u* that is distributed with mean 0 and constant variance:

$$X = X^e + u$$

*u*is distributed independently of X^e. An investigator has data on *Y* and *X* (but not on X^e) for a cross-section of firms in the industry. Describe the problems that would be encountered if OLS were used to estimate $\beta_1$and β_2, regressing *Y* on *X*.

4.1.7.0 REFERENCES /FURTHER READING

Dougherty, C. (2007). *Introduction to econometrics*. Oxford University Press, USA.

Dominick, S., & Derrick, R. (2002).*Theory and problems of statistics and econometrics.*Schaum's Outline Series.

N Gujaratti, D. (2004). *Basic econometrics*. McGraw-Hill, New York.

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UNIT 2: AUTOCORRELATION

CONTENTS

4.2.1.0 INTRODUCTION

Autocorrelation is the correlation between the error terms arising in time series data. Such correlation in the error terms often arises from the correlation of the omitted variables that the error term captures. Furthermore, the assumption in the third Gauss–Markov condition is that the value taken by the disturbance term in any observation and determined independently of its values in all the other observations, is satisfied, and hence that the population covariance of u_i and$u_j$is 0 for $i \neq j$. When the condition is not satisfied, the disturbance term is said to be subject to autocorrelation, often called serial correlation or cross-autocorrelation.

4.2.2.0 OBJECTIVE

The main objective of this unit is to provide a basic understanding that autocorrelation may arise as a consequence of the exclusion of a significant variable or the mathematical misspecification of regression model.

4.2.3.0 MAIN CONTENTS

The significances of autocorrelation for OLS are to some extent comparable to those of heteroscedasticity. The regression coefficients remain unbiased, but OLS is inefficient because one can find an alternative unbiased estimator with smaller variance. The other main concern, which should not be mixed up with the first, is that the standard errors are estimated wrongly, probably being biased downwards. Finally, although in general autocorrelation does not cause OLS estimates to be biased, there is an important special case where it does.

4.2.3.1 Possible Causes of Autocorrelation

There is two forms autocorrelation occurrence, which could either be positive and negative. Persistent effects of excluded variables are probably the most frequent cause of positive autocorrelation, the usual type of economic analysis. In Figure 4.1, *Y* depends on *X* and some minor variables not included explicitly in the specification. The disturbance term in the model is generated by the combined effects of these excluded variables. In the first observation, the excluded variables have a net positive effect and the disturbance term is positive. If the excluded variables change slowly, their positive effect will persist, and the disturbance term will remain positive. In time the balance will change, and the net effect of the excluded variables becomes negative. Here, the persistence effect works the other way, and the disturbance term remains negative for a few observations. The duration and amplitude of each positive and negative sequence are essentially random, but overall there will be a tendency for positive values of the disturbance term to be followed by positive ones and for negative values to be followed by negative ones. However, a factor to note is that autocorrelation is on the whole more likely to be a problem for shorter intervals between observations.

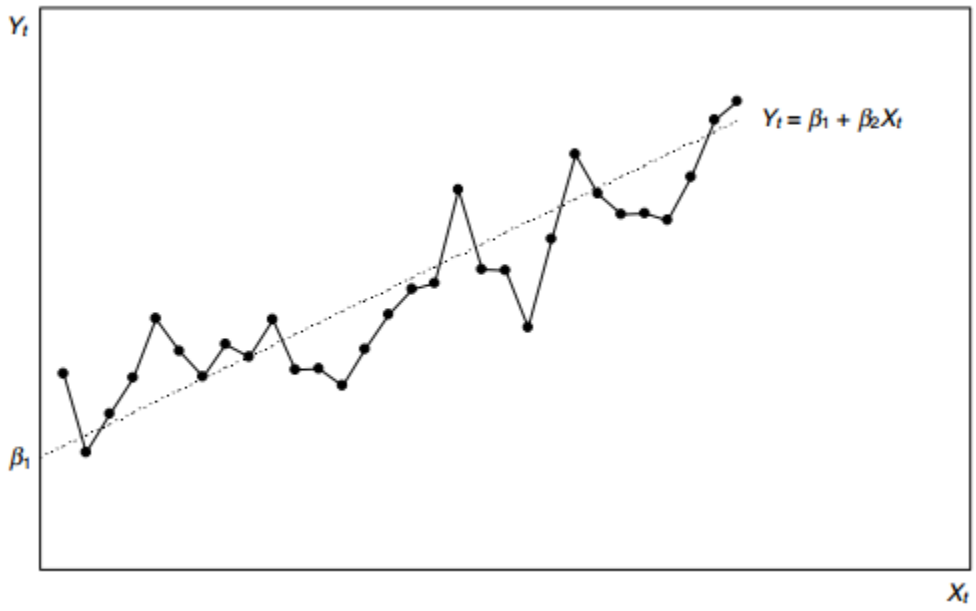

Figure 4.1 Positive Autocorrelation

Negative autocorrelation means that the correlation between successive values of the disturbance term is negative. A positive value in one observation is more likely to be followed by a negative value than a positive value in the next, and vice versa; this is shown by an illustrative scatter diagram in Figure 4.2. A line joining successive observations to one another would cross the line relating *Y* to *X* with greater frequency than one would expect if the values of the disturbance term were independent of each other. Economic examples of negative autocorrelation are relatively uncommon, but sometimes it is induced by manipulations used to transform the original specification of a model into a form suitable for regression analysis.

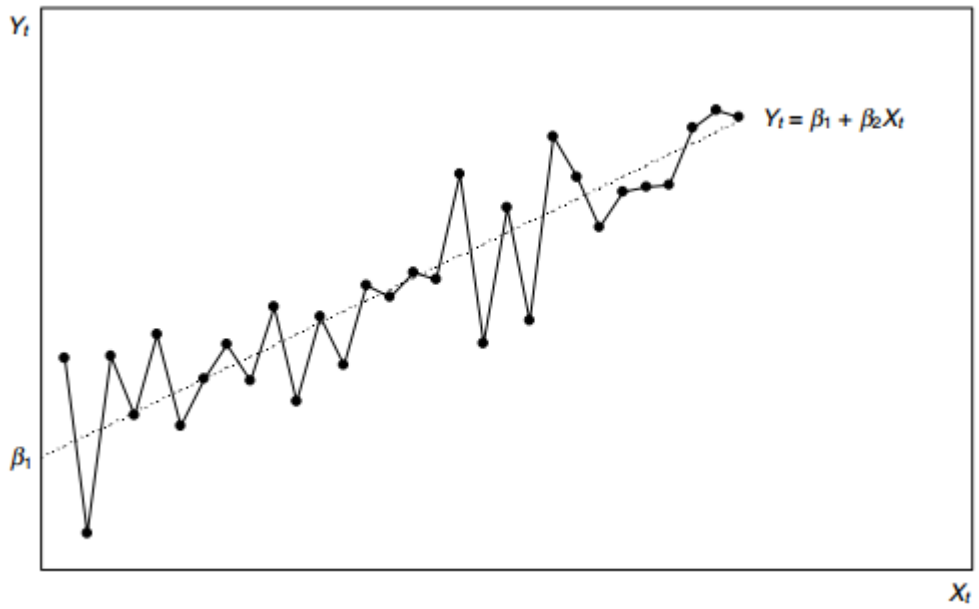

Figure 4.2 Negative Autocorrelation

When an error term $\cup_t$ at time period *t* is correlated with error terms in time series, the correlation between $\cup_t$ and $\cup_{t-k}$ is called an autocorrelation of order *k*. The correlation between $\cup_t$ and $\cup_{t-1}$ is the first-order autocorrelation and is usually denoted by ρ_1 The correlation between $\cup_t$ and $\cup_{t-2}$ is called the second order autocorrelation and is denoted by ρ_2, and so on. There are (*n* - 1) such autocorrelations if we have *n* observations. However, we cannot hope to estimate all of these from our data. Hence we often assume that these (*n* - 1) autocorrelations can be represented in terms of one or two parameters.

4.2.3.2 Detection of First-Order Autocorrelation: the Durbin–Watson Test

We will mostly be concerned with first-order autoregressive autocorrelation, often denoted AR (1). AR (1) appears to be the most common type of autocorrelation approximation. It is described as positive or negative according to the sign of ρ. Note that if ρ is 0, there is no autocorrelation occurrence.

There are two major things that will be discussed in this unit, which are:

1. Test for the presence of serial correlation.
2.Estimate the regression equation when the errors are serially correlated.

Durbin-Watson Test (DW)

The simplest and most commonly used model is one where the errors $\cup_t$and $\cup_{t-1}$ have a correlation ρ. For this model one can think of testing hypotheses about ρ on the basis of ρ, the

correlation between the least squares residuals $\cup_t$and $\cup_{t-1}$. A commonly used statistic for this purpose which is related to ρ is the DW statistic, which will be denote by d. It is defined as

$$d = \frac{\sum_2^n (\cup_t - \cup_{t-1})^2}{\sum_1^n U_t^2} \qquad \ldots[4.17]$$

Where U_t is the estimated residual for period t. DW can be re-written as

$$d = \frac{\sum U_t^2}{\sum U_t^2} + \frac{\sum U_{t-1}^2}{\sum U_t^2} - \frac{2\sum U_t U_{t-1}}{\sum U_t^2} \qquad \ldots[4.18]$$

Since $\sum U_t^2$ and $\sum U_{t-1}^2$ are approximately equal if the sample is large, we have $d = 2(1-\rho)$. If

$\rho = +1, then\ d = 0\ and\ if\ \rho = -1, then\ d = 4. We\ have\ d = 2\ if\ \rho = 0. if\ d\ is\ close\ to\ or\ 4, the\ residuals\ are\ highly\ correlated.$

The sampling distribution of d depends on the values of the explanatory variables and hence DW derived upper $d(u)$ limits and lower $d(l)$ limits for the significance levels for d**.** There are tables to test the hypothesis of zero autocorrelation against the hypothesis of first-order positive autocorrelation. (For negative autocorrelation we interchange $(l)\ and\ d(u)$**)**, hence;

If $d < d(l)$., we reject the null hypothesis of no autocorrelation.

If $d > d(u)$.we do not reject the null hypothesis.

If $d(l) < d < d(u)$the test is inconclusive.

The upper bound of the DW statistic is a good approximation to its distribution when the regressors are slowly changing. DW argue that economic time series are slowly changing, and hence one can use $d(u)$as the correct significance point.

The significance points in the DW tables are tabulated for testing ρ = 0 against ρ> 0. If d > 2 and we wish to test the hypothesis ρ = 0 against ρ<0, we consider 4…d and refer to the DW tables as if we are testing for positive autocorrelation. Although we have said that $d \xrightarrow{yields} 2(1-\rho)$ this approximation is valid only in large samples. The mean of d when $\rho = 0$has been shown to be given approximately by

$$E(d) = 2 + \frac{2(k-1)}{n-k} \qquad \ldots[4.19]$$

where*k* is the number of regression parameters estimated (including the constant term), and *n* is the sample size. Thus, even for zero serial correlation, the statistic is biased upward from 2. If *k* = 5 and *n*= 15, the bias is as large as 0.8.

4.2.5.0 SUMMARY

The unit explained the concept of autocorrelation at first order, its possible causes and detection (with particular interest on the first-order autoregressive autocorrelation, denoted by AR (1)) using Durbin-Watson test.

4.2.4.0 CONCLUSION

In this unit, autocorrelation is statistically explained as a random process that measures the linear correlation between values of the process at different times, as a function of time or of the time lag. The significances of autocorrelation for OLS are shown to be comparable to those of heteroscedasticity and have two forms of occurrences, which could either be positive or negative. Students are advised to use the further reading materials to look at more autocorrelation techniques and study more on the correlation between the error terms arising in time series data as indicated in the introduction section of this unit.

4.2.6.0 REFERENCES /FURTHER READING

Maddala, G. S., &Lahiri, K. (1992).*Introduction to econometrics* (Vol. 2). New York.

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UNIT 3: ECONOMETRIC MODELLING AND MODEL USING TIME-SERIES DATA

CONTENTS

4.3.1.0 INTRODUCTION

The modelling of expectations using time series data is often an important and difficult task of the applied economist. This is especially true in macroeconomics, in that investment, saving, and the demand for assets are all sensitive to expectations about the future. Unfortunately, there is no satisfactory way of measuring expectations directly for macroeconomic purposes. Consequently, macroeconomic models tend not to give particularly accurate forecasts, and this makes economic management difficult.

4.3.2.0 OBJECTIVE

The main objective of this unit is to introduce the application of regression analysis to time series data, starting with static models and then continuing to dynamic models with lagged variables used as descriptive variables.

4.3.3.0 MAIN CONTENTS

4.3.3.1 The Adaptive Expectations Model

As a makeshift solution, some models use an indirect technique known as the adaptive expectations process. This involves a simple learning process in which, in each period, the actual value of the variable is compared with the value that had been expected. If

the actual value is greater, The expected value is adjusted upwards for the next period.If it is lower, the expected value is

adjusted downwards. The size of the adjustment is hypothesized to be proportional to the discrepancy between the actual and expected value.

If *X* is the variable in question, and X_t^e is the value expected in time period *t* given the information available at time period *t*–1,

$$X_{t+1}^e - X_t^e = \lambda(\mathrm{X_t} - X_t^e)(0 \leq \lambda \leq 1) \qquad \ldots[4.20]$$

This canbe rewritten

$$X_{t+1}^e = \lambda \mathrm{X_t} + (1-\lambda)X_t^e(0 \leq \lambda \leq 1) \qquad \ldots[4.21]$$

Which states that the expected value of *X* in the next period is a weighted average of the actual value of *X* in the current period and the value that had been expected. The larger the value of λ, the quicker the expected value adjusts to previous actual outcomes.

For example, suppose that you hypothesize that a dependent variable, Y_t, is related to the expected value of the descriptive variable, *X*, in year *t*+1, X_{t+1}^e :

$$Y_t = \beta_1 + \beta_2 X_{t+1}^e + u_t \qquad \ldots[4.22]$$

expresses$Y_t$in terms of X_{t+1}^e, which is unobservable and must somehow be replaced by observable variables, that is, by actual current and lagged values of *X*, and perhaps lagged values of *Y*. We start by substituting for X_{t+1}^e,

$$Y_t = \beta_1 + \beta_2(\lambda X_{t+1}^e + (1-\lambda)X_t^e) + u_t = \beta_1 + \beta_2 \lambda X_{t+1}^e + \beta_2(1-\lambda)X_t^e) + u_t$$
$$\ldots[4.23]$$

Of course, we still have unobservable variable X_t^e as an descriptive variable, but if it is true for time period *t*, it is also true for time period *t*–1:

$$X_t^e = \lambda \mathrm{X_t} + (1-\lambda)X_t^e \qquad \ldots[4.24]$$

Substituting for X_t^e, in [4.23] we now have

$$Y_t =$$
$$\beta_1 + \beta_2\lambda X_t + \beta_2(1-\lambda)X_{t-1} + \beta_2\lambda(1-\lambda)^2 X_{t-2} + \cdots +$$
$$\beta_2\lambda(1-\lambda)^{s-1}X_{t-s+1} + \beta_2(1-\lambda)^s X^e_{t-s+1} + u_t$$
…[4.25]

Now it is reasonable to suppose that λ lies between 0 and 1, in which case (1 – λ) will also lie between 0 and 1. Thus $(1-\lambda)^s$becomes progressively smaller as *s* increases. Eventually, there will be a point where the term $\beta_2(1-\lambda)^s X^e_{t-s+1}$ is so small that it can be neglected and we have a model in which all the variables are observable.

A lag structure with geometricallydeclining weights, such as this one, is described as having a Koyck distribution. It is highly sparing regarding its constraint, requiring only one parameter more than the static version. Since it is nonlinear in the parameters, OLS should not be used to fit it, for two reasons. First, multicollinearity would almost certainly make the estimates of the coefficients so erratic that they would be worthless – it is precisely this problem that caused us to search for another way of specifying a lag structure. Second, the point estimates of the coefficients would yield conflicting estimates of the parameters.

4.3.4.0 SUMMARY

In this unit, an indirect technique known as the adaptive expectations process is explained as a makeshift solution used in some models. This involves simple learning process for which, in each period, the size of adjustment is proportional to the discrepancy between the actual and expected value.

4.3.5.0 CONCLUSION

The unit introduces the application of regression analysis to time series data, starting with static models and then proceeding to dynamic models with lagged variables used as descriptive variables. In this unit, the adaptive expectations process was mainly used for explanation but the multicollinearity problem in time series models, especially dynamic ones with lagged descriptive variables was also explained. The students may use the reference materials for more understanding and further readings.

4.3.6.0 TUTOR-MARKED ASSIGNMENT

1.) The results of linear and logarithmic regressions of consumer expenditure on food, *FOOD*, on *DPI* and a relative price index series for food, *PRELFOOD*, using the Demand Functions data set, are summarized below. Provide an economic interpretation of the coefficients and perform appropriate statistical tests.

$$\widehat{FOOD} = 232.6 + 0.089DPI + 0.534PRELFOOD \qquad R^2 = 0.989$$

S.E.E.(31.9) (0.002) (0.332)

$$\widehat{LGFOOD} = 2.66 + 0.61LGDPI - 0.30LGPRELFOOD \qquad R^2 = 0.993$$

S.E.E(0.28) (0.01) (0.07)

2.) Sometimes a time trend is included in a regression as an explanatory variable, acting as a proxy for some gradual change not associated with income or price. Changing tastes might be an example. However, in the present case, the addition of a time trend might give rise to a problem of multicollinearity because it will be highly correlated with the income series and perhaps also the price series. Calculate the correlations between the *TIME* variable in the data set, *LGDPI*, and the logarithm of expenditure on your category. Regress the logarithm of expenditure on your category on *LGDPI*, the logarithm of the relative price series and *TIME* (not the logarithm of *TIME*). Provide an interpretation of the regression coefficients, perform appropriate statistical tests, and compare the regression results with those of the same regression without *TIME*.

4.3.7.0 REFERENCES /FURTHER READING

Dougherty, C. (2007). *Introduction to econometrics*. Oxford University Press, USA.

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MODULE 5: SIMULTANEOUS EQUATION, BINARY CHOICE, AND MAXIMUM LIKELIHOOD ESTIMATION

The general aim of this module is to provide you with a thorough understanding of the basic rudiments of Simultaneous Equation, Binary Choice, and Maximum Likelihood Estimation. By the end of this module, you should be able to understand the components of the module stated below. The units to be studied are;

Unit 1: Simultaneous Equation

Unit 2: Binary Choice and Limited Dependent Models with Maximum Likelihood Estimation

UNIT 1: SIMULTANEOUS EQUATIONSESTIMATION

CONTENTS

5.1.1.0 INTRODUCTION

In the engagement of OLS to estimate the factors of an equation that is set in a simultaneous equations model, it is likely that the estimates will be biased and erratic which would invariably make the statistical tests invalid and inconsistent.

5.1.2.0 OBJECTIVE

The main objective of this unit is to demonstrate to the students that in practice most economic relationships interact with others in a system of simultaneous equations and when this is the case the application of OLS to a single relationship in isolation yields biased estimates.

5.1.3.0 MAIN CONTENTS

5.1.3.1 Simultaneous Equations Models: Structural and Reduced Form Equations

As explained earlier in other modules, measurement error is not the only probable cause why the fourth Gauss–Markov condition may not be satisfied. Simultaneous equations bias is another. To illustrate this; suppose there is an investigation on the determinants of price inflation and wage inflation. For ease, it would be better to start with a very simple model that supposes that *p*, the annual rate of growth of prices, is related to *w*, the annual rate of growth of wages, it being assumed that increases in wage costs force prices upwards:

That is;

$$p = \beta_1 + \beta_2 w + u_p \qquad \ldots[5.01]$$

Here, *w* is related to *p*and *U*, the rate of unemployment, workers protecting their real wages by demanding increases in wages as prices rise, but their ability to do so being the weaker, the higher the rate of unemployment ($\alpha_3 < 0$). Which is stated as:

$$w = \alpha_1 + \alpha_2 p + \alpha_3 U + u_w \qquad \ldots[5.02]$$

where, $u_p \; and \; u_w$are disturbance terms

Clearly, this simultaneous equations model involves a certain amount ofcomplexity:*w*determines p in the first equation [5.01], and in turn,p helps to determine *w* in the second [5.02]. For better clarity in resolving this complexity, we need to make a distinction between endogenousand exogenous variables. Endogenous variables are variables whose values are determined by the interaction of the relationships in the model. Exogenous ones are those whose values are determined

externally. Thus in the present case,*p* and *w*are both endogenous, and *U* is exogenous. The exogenous variables and the disturbance terms ultimately determine the values of the endogenous variables, once the complexity is cleared. The mathematical relationships expressing the endogenous variables regarding the exogenous variables and disturbance terms are known as the reduced form equations. The original equations that we wrote down when specifying the model are

described as the structural equations. We will derive the reduced form equations for *p* and *w*. To obtain that for *p*, we take the structural equation for *p* and substitute for *w* from the second equation:

$$p = \beta_1 + \beta_2 w + u_p = \beta_1 + \beta_2(\alpha_1 + \alpha_2 p + \alpha_3 U + u_w) + u_p \quad \ldots[5.03]$$

Hence,

$$(1 - \alpha 2\,\beta_2)p = \beta_1 + \alpha_1\beta_2 + \alpha_3\beta_2 U + u_p + \beta_2 u_w \quad \ldots[5.04]$$

and so we have the reduced form equation for *p*;

$$p = \frac{\beta_1 + \alpha_1\beta_2 + \alpha_3\beta_2 U + up + \beta_2 u_w}{(1 - \alpha_2\beta_2)} \quad \ldots[5.05]$$

Similarly we obtain the reduced form equation for *w*:

$$w = \alpha_1 + \alpha_2 p + \alpha_3 U + u_w = \alpha_1 + \alpha_2(\beta_1 + \beta_2 w + up) + \alpha_3 U + u_w \quad \ldots[5.06]$$

Hence

$$(1 - \alpha_2\beta_2)w = \alpha_1 + \alpha_2\beta_1 + \alpha_3 U + u_w + \alpha_2 up \quad \ldots[5.07]$$

and so

$$w = \frac{\alpha_1 + \alpha_2\beta_1 + \alpha_3 U + u_w + \alpha_2 u_p}{1 - \alpha_2\beta_2} \quad \ldots[5.08]$$

5.1.3.2 Simultaneous Equations Bias

In almost all simultaneous equations models, the reduced form equations express the endogenous variables regarding all of the exogenous variables and all of the

disturbance terms. You can see that this is the case with the price inflation/wage inflation model. In this model, there is only one exogenous variable, *U*.

*w*depends on it directly; *p* does not depend on it directly but does so indirectly because *w* determines it. Similarly, both *p* and *w*depend on u_p, *p* directly and *w* indirectly. And both depend on u_w, *w* directly and *p* indirectly.

The dependence of *w* on $u_p$means that OLS would yield inconsistent estimates if used to fit
equation [5.01], the structural equation for *p*. *w* is a stochastic regressor and its random component is not distributed independently of the disturbance term u_p. Similarly the dependence of *p* on u_w

means that OLS would yield inconsistent estimates if used to fit [5.02]. Since [5.01] is a simple regression equation, it is easy to analyze the large-sample bias in the OLS estimator of β_2.

5.1.5.0 SUMMARY

In this unit, we started by explaining structural and reduced form of equations which was illustrated by showing that measurement error is not the only probable cause why the fourth Gauss–Markov condition may not be satisfied for which the biasness of simultaneous equations is an example. We then went further to explain the simultaneous equations bias.

5.1.4.0 CONCLUSION

In this unit, Simultaneous equations estimation is discussed. The structural and reduced form of equations *as it relates to Simultaneous equations model bias is also explained. Clearly, for deeper understanding of the equation models, the students should* make a distinction between endogenousand exogenous variables *in Simultaneous equations estimation.*

5.1.6.0 TUTOR-MARKED ASSIGNMENT

1.) Simple macroeconomic model consists of a consumption function and an income identity:

$$C = \beta_1 + \beta_2 Y + u$$

$$Y = C + I$$

where*C* is aggregate consumption, *I* isaggregate investment, *Y* is aggregate income, and *u* is a disturbance term. On the assumption that *I* is exogenous, derive the reduced form equations for *C* and *Y*.

2.) From the model above, demonstrate that OLS would yield inconsistent results if used to fit the consumption function, and investigate the direction of the bias in the slope coefficient.

5.1.7.0 REFERENCES /FURTHER READING

Maddala, G. S., &Lahiri, K. (1992).*Introduction to econometrics* (Vol. 2). New York.
Dougherty, C. (2007). *Introduction to econometrics*. Oxford University Press, USA.

Unit 2: Binary Choice and LimitedDependent Models andMaximum Likelihood Estimation

CONTENTS

5.2.1.0 INTRODUCTION

Most times economists are known to be interested in the factors behind the decision-making of individuals or enterprises. Examples are:

- Why do some people go to college while others do not?
- Why do some women enter the labour force while others do not?
- Why do some people buy houses while others rent?
- Why do some people migrate while others stay put?

Models have been developed to proffer solutions to these interest, and they are known as abinary choice or qualitative response models. The outcome will be denoted by *Y*, and assigned a value of 1 if the event occurs and 0 otherwise. Models with more than two possible outcomes have also been developed, but let us restrict our scope to abinary choice. The linear probability model apart, binary choice models are fitted using maximum likelihood estimation.

5.2.2.0 OBJECTIVE

The main objective of this unit is to provide the students with a clear understanding that apart from the linear probability model, binary choice models are fitted using maximum likelihood estimation.

5.2.3.0 MAIN CONTENTS

5.2.3.1 The Linear Probability Model

The simplest binary choice model is the linear probability model where, as the name implies, the probability of the event occurring, *p*, is assumed to be a linear function of a set of descriptive variable(s). That is:

$$p_i = p(Y_i = 1) = \beta_1 + \beta_2 X \qquad \ldots[5.09]$$

For one descriptive variable, the relationship is as shown in Figure 5.1. Of course,*p* is unobservable, and as expected there is only one data *Y*, on the outcome. In the linear probability model, this is used as a dummy variable for the dependent variable.

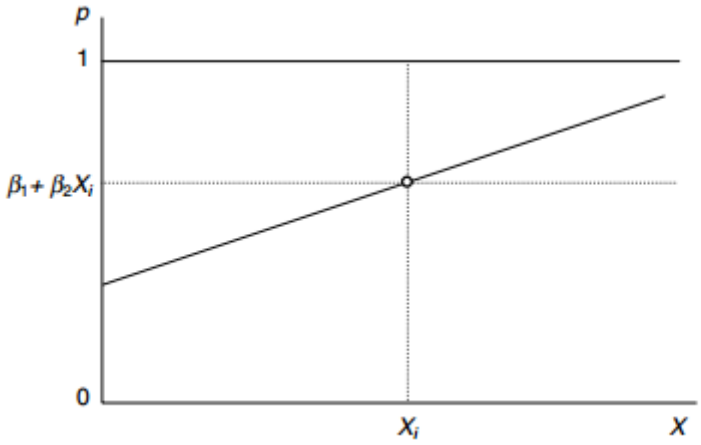


Figure 5.1. Linear Probability Model

Regrettably, the linear probability model though simple still has some serious defects. First, there are problems with the disturbance term. As usual, the value of the dependent variable $Y_i$in observation *i*,has a nonstochastic component and a random component. The nonstochastic component depends on $X_i$and the parameters and is the expected value of $Y_i$given X_i, $E(Y_i \mid X_i)$. The random component is the disturbance term.

$$Y_i = E(Y_i \mid X_i) + u_i \qquad \ldots[5.10]$$

It is simple to compute the nonstochastic component in observation *i* because *Y* can take only two values. It is 1 with probability $p_i$and 0 with probability (1 – p_i):

$$E(Y_i) = 1 \times p_i + 0 \times (1 - p_i) = p_i = \beta_1 + \beta_2 X_i \qquad \ldots[5.11]$$

The expected value in observation *i* is therefore $\beta_1 + \beta_2 X_i$.This means that we can rewrite the model as;

$$Y_i = \beta_1 + \beta_2 X_i + u_i \qquad \ldots[5.12]$$

Probability function is thus also the nonstochastic component of the relationship between *Y* and *X*. It follows that, for the outcome variable Y_1 to be equal to 1, as represented by the point *A* in Figure 5.2, the disturbance term must be equal to $(1 - \beta_1 - \beta_2 X_i)$. For the outcome to be 0, as represented by the point *B*, the disturbance term must be $(-\beta_1 - \beta_2 X_i)$. Thus the distribution of the disturbance term consists of just two specific values.

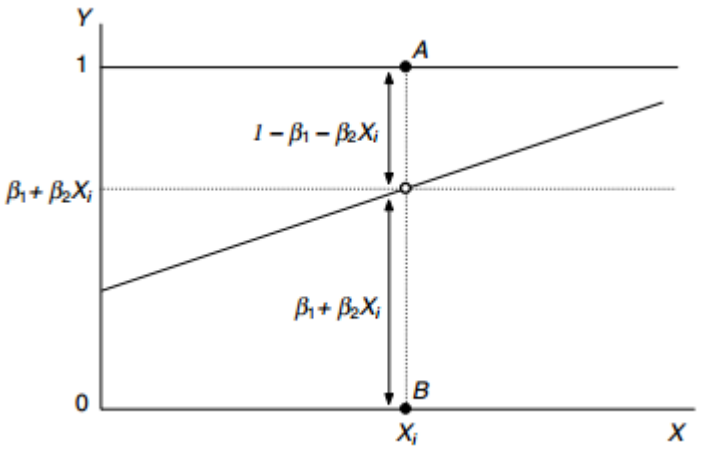


Figure 5.2. Linear Probability Model

Which means that the standard errors and the usual test statistics are invalidated. For good measure, the two possible values of the disturbance term change with *X*, so the distribution is heteroscedastic as well. It can be shown that the population variance of $ui$is $(\beta_1 + \beta_2 X_i)$ $(1 - \beta_1 - \beta_2 X_i)$, and this varies with X_i.

The other problem is that the predicted probability may be greater than 1 or less than 0 for extreme values of *X*. The first problem is dealt with by fitting the model with a technique known as maximum likelihood estimation.

The second problem involves elaborating the model as follows. Define a variable *Z* that is a linear function of the descriptive variables. In the present case, since we have only one descriptive variable, this function is;

$$Z_i = \beta_1 + \beta_2 X_i \quad \ldots[5.13]$$

5.2.3.2 Goodness of Fit and Statistical Tests

Even though numerous measures have been proposed for comparing alternative model specifications, there is still no measure of goodness of fit equivalent to R^2 in maximum likelihood estimation. Denoting the actual outcome in observation $i\ as\ Y_i, with\ Y_i =$ 1 if the event occurs and 0 if it does not, and denoting the predicted probability of the event occurring$\widehat{P_i}$, the measures include the following:

i. the number of outcomes correctly predicted, taking the prediction in observation $i\ as\ 1\ if\ \widehat{P_i}\ i$ is greater than 0.5 and 0 if it is less;

ii. the sum of the squared residuals $\sum_{i=1}^{n}(Y_i - \widehat{P_i})^2$ t

iii. the correlation between the outcomes and predicted probabilities, $r_{\widehat{P_i}Y_i}$

iv. the pseudo-R^2 in the logit output,

Every of these measures has its shortcomings, and it is recommended to consider more than one and compare their results. Nevertheless, the standard significance tests are similar to those for the standard regression model. The significance of an individual coefficient can be evaluated via its *t* statistic. However, since the standard error is valid only asymptotically (in large samples), the same goes for the *t* statistic, and since the *t* distribution converges to the normal distribution in large samples, the critical values of the latter should be used. The counterpart of the *F* test of the explanatory power of the model (H_0: all the slope coefficients are 0, H_1: at least one is nonzero) is a chi-squared test with the chi-squared statistic in the logit output distributed under H_0 with degrees of freedom equal to the number of explanatory variables.

5.2.4.0 SUMMARY

In this unit, we started with the linear probability model being the simplest binary choice model where the probability of the event occurring is assumed to be a linear function of a set of descriptive variables. We then proceeded to goodness of fit and statistical tests using maximum likelihood estimation as a method of estimating the parameters of a model given observations, by finding the parameter values that maximise the likelihood of making the observations given the parameters.

5.2.5.0 CONCLUSION

Although numerous measures have been proposed for comparing alternative model specifications, there is still no measure of goodness of fit equivalent to maximum likelihood estimation. The students should be of the opinion that every of the estimation measure has its shortcomings and it is recommended to consider more than one and compare their results.

5.2.6.0TUTOR-MARKED ASSIGNMENT

A researcher, using a sample of 2,868 individuals from the NLSY (National Longitudinal Survey of Young Men), is investigating how the probability of a respondent obtaining a bachelor's degree from a four-year college is related to the respondent's score on *ASVABC*. 26.7 percent of the respondents earned bachelor's degrees. *ASVABC* ranged from 22 to 65, with mean value 50.2, and most scores were in the range 40 to 60. Defining a variable *BACH* to be equal to 1 if the respondent has a bachelor's degree (or higher degree) and 0 otherwise, the researcher fitted the OLS regression (standard errors in parentheses):

$$\widehat{BACH} = -0.864 + 0.023ASVABC \qquad R^2 = 0.21$$

S.E.E. (0.042) (0.001)

The researcher also fitted the following logit regression:

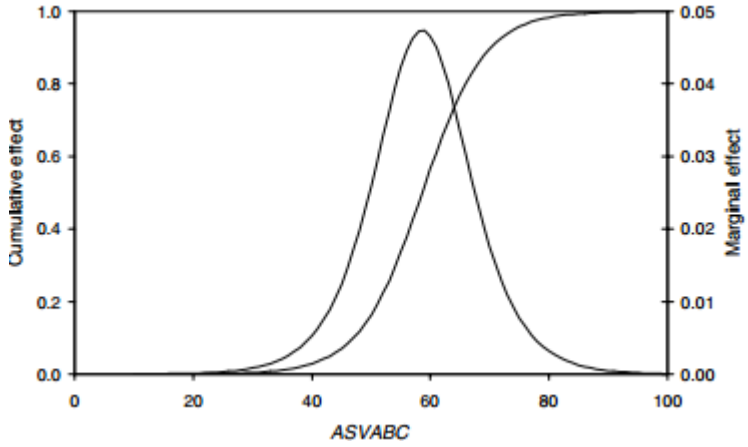
$$z = -11.103 + 0.189ASVABC$$

S.E.E. (0.487) (0.009)

where$Z$ is the variable in the logit function. Using this regression, the researcher plotted the probability and marginal effect functions shown in the diagram below.

a.) Give an interpretation of the OLS regression and explain why OLS is not a satisfactory estimation method for this kind of model.

b.) With reference to the diagram below, discuss the variation of the marginal effect of the *ASVABC* score implicit in the logit regression and compare it with that in the OLS regression.



c.) Sketch the probability and marginal effect diagrams for the OLS regression and compare them with those for the logit regression. (In your discussion, make use of the information in the first paragraph of this question.)

5.2.7.0 REFERENCES /FURTHER READING

Dominick, S., & Derrick, R. (2002).*Theory and problems of statistics and econometrics.*Schaum's Outline Series.

Dougherty, C. (2007). *Introduction to econometrics*. Oxford University Press, USA.

Stock, J. H., & Watson, M. W. (2015).*Introduction to econometrics*. Pearson.