MTH 133
TRIGONOMETRY

NATIONAL OPEN UNIVERSITY OF NIGERIA

MTH 133: TRIGONOMETRY

COURSE GUIDE

NATIONAL OPEN UNIVERSITY OF NIGERIA

TABLE OF CONTENT

TRIGONOMETRY

COURSE GUIDE

TABLE OF CONTENTS

(4) WHAT YOU WILL LEARN

(5) WORKING THROUGH THIS COURSE

(6) ASSESSMENT

(7) SUMMARY

MATH 113 -TRIGONOMETRY (1 CREDIT).

0.0 COURSE CONTENT

Trigonometric ratios (sine, cosine and tangent) Trigonometrical ratios of any angle (General angle) Inverse trigonometrical ratios. Trigonometrical identities (sum, and difference formulae, product formula). Applications of trigonometrical ratios - solution of triangles (sine and cosine rules angles of elevation and depression. Bearings.

1.0 **INTRODUCTION**

Trigonometry is a one credit, one module with seven units for mathematics students at the foundational level of their B.Sc or B .Sc (Ed) degree programme. This is part of the course in Algeria and in trigonometry. The second will be offered later.

Trigonometry as the name implies, involves the study or measurement of triangles in relation to their sides and angles. It is interesting to note that trigonometry has a very significant relevance in real life hunting, traveling and is well applied in the field of sciences, engineering, navigation of ships, aero planes and astronomy.

2.0 WHAT YOU WILL LEARN IN THIS COURSE

This course consists of Steven units which introduce you to trigonometry and its applications. During this course, however, you will learn the trigonometric ratios and their reciprocal inverse trigonometrical function graphs of trigonometric functions and applications of trigonometry to real life problems. The course is such that it will give you enough grounding in appreciating and understanding your everyday activities - walking, hunting, traveling, radio waves, flying in the air or sailing in the sea.

2.1 COURSE AIMS

The course aims at giving you a good understanding, , of . .trigonometry and its applications in everyday life > This could be achieved through the following measure:

- Introducing you to the trigonometrical ratios and their reciprocals
- Explaining the graphical solution to trigonometric functions and
- Applying the knowledge of trigonometric ratios to real life problems
- Heights, distance and bearings.

1. COURSE OBJECTIVE

By the time you have successfully completed this course, you should be able to:

- Define the trigonometric ratios and their reciprocals.
- Compute trigonometric ratios of any given angle.
- Identify with the use of tables the trigonometric ratios of given angles.
- Draw the graphs of trigonometric functions. determine the trigonometric ratios of angles from their graphs. state and derive the sine and cosine rules.
- Determine the direction of your movement accurately.
- Discuss intelligently the bearing in a given problem.
- Define the angles of elevation and depression
- Solve problems on trigonometric equations correctly,
- Apply trigonometric ratios to problems on height, distances and bearing correctly,

3.0 WORKING THROUGH THIS COURSE

For the successful completion of this course, you are required to dutifully read the study units, read recommended and other textbooks and materials that will help you understand this course. You will also need to do lots of exercises. Each unit in this course contains at least your self-assessment exercises, and at some stage you will be required to submit your assignment for grading by the tutor. There will be a final examination at the end of this course.

3.1 COURSE MATERIALS

- study units.
- textbooks
- assignments files
- presentation schedule.

3.5 STUDY UNITS

There are seven study units in this course

Units 1 and 2	Trigonometric ratios I and II
Units 3	Inverse of trigonometric ratios
Units 4	Graph of trigonometric ratios
Units 5	Trigonometric Identities and trigonometric equations
Unit 6	Solution of triangle (sine and cosine values) and Angles of elevation and depression.
Unit 7	Bearings.

The first four units concentrate on trigonometric ratios and terms inverse trigonometric functions and graphs, next on trigonometric identities and equations, while the last two units disussing the applications of trigonometric ratios to real life problems.

Each study unit consists of two to three weeks works at the rate of three hours per week. It includes specific objectives direction of study recommended textbooks, summary of the unit and conclusion. At the end of each unit there is an exercise to enable you assess yourself on how far you have understood the contents of the unit and have achieved the stated objectives in the individual units in the course in general.

3.3 TEXTBOOKS

A lot of textbooks are available but the most common and accessible textbooks are:

1. Introductory University Mathematics
   Edited by J.C. Amazigo. Onisha , Afrecana -rep. Publishers Ltd. (1991).

2. Pure Mathematics: A first course. S. I Edition by Backhouse, J. K and Houldsworth, S . P. T. London: Longman

3. Additional mathematics for West Africa (1992) by T. F., Talbert, A Godman and G. Ogum. London: Longman.

4. Pure mathematics for Advanced levels by B. D Bunday, and A. Mulholland. London: Longman PLC.

5. Father Mathematics (1999)
   by E. Egba, G. A. Odili and O. Ugbebor Onitsha: Africana - rep Publishers Ltd.

6. New School Mathematics for Senior Secondary School (2000) by M> David - Osuagwu, C. Anerelu and I. Onyeozili. Onitsha: Africana - rep Publishers.

3.4 ASSIGNMENT FILE

The assignment file contains the exercises and the details of all the work you are to do and submit to your tutor for grading. The marks you make in these assignments will contribute to the final grade you will get in this course.

There will be five assignments to cover the units in this course.

1. Trigonometric ratios and their reciprocals. (Units I and 2.)
2. Inverse trigonometric functions and their graphs (Unit 3)
3. Trigonometric identities and equations (Unit 5)
4. Solutions of triangles- Heights and distances (Unit 6)
5. Bearings

4.2 PRESENTATION SCHEDULE

The presentation schedule included in your course materials gives you the important dates for the completion and submission of your tutor - marked assignments and for attending tutorials at your study center. Do not allow yourself to lag behind.

4.0 ASSESSMENT

Your assessment in this course is:

(i) by tutor-marked assignments which contributes 50% of your total course mark and
(ii) a written examination at the end of your course which contributes the remaining 50% of your course mark.

4.1 TUTOR-MARKED ASSIGNMENTS (TMA)

There are five tutor marked assignments in this course. You are expected to submit all based on the best four out of the five assignments. Each assignment has 12.5% of the total course mark.

When you have completed each assignment, send it together with a tutor-marked assignment form to your tutor. This will reach your tutor before the deadline given in your presentation table.

Note: You are required to read other materials and even the set textbooks for deeper understanding of the course.

3.1. FINAL EXAMINATION AND GRADING

The final examination for math 113 will be a two- hour paper and has a value of 50% of the total course grade. The examination will consist of questions that cover all aspects of the course the time between finishing the last unit and the examination for revision

4.3 COURSE MARKING SCHEME.

The table below shows the break down of the course grade marks.

ASSESSMENT	MARKS
Tutor – Marked Assignments (1.5)	Five assignments which the best four are chosen. Each has 12.5% totaling 50% for the four.
Final examination	50% of overall course marks
Total	100% Of course marks.

4.4 COURSE OVERVIEW.

The table below gives you an idea of how the units and the number of weeks, you are required to cover the units (working at the rate of 3 hours a week.)

Units	Title of work	Weekly Activity	Assessment
	Course Guide		
1	Trigonometric Ratios 1	2	Assignments
2	Trigonometric Ratios 11 : Reciprocal (use of tables)		1
3	Inverse trigonometric ratios	2	2
4	Graph of trigonometric ratios and Graph of their reciprocals	3	3
5	Trigonometric identities & Trigonometric equations	3	3
6	Solution of triangles (sine and cosine rules). Angles of elevation and depression (heights and distances) Bearings	2 2	4
7	Revision	13	5
	Total		

5.0 SUMMARY

This course trigonometry intends to introduce you to trigonometrical functions and their applications. By the end of this course, you should be able to answer questions on trigonometric applications to real life.

Good luck as you study this course.

MTH 133: TRIGONOMETRY

COURSE DEVELOPMENT

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NATIONAL OPEN UNIVERSITY OF NIGERIA

UNIT 1

TRIGONOMETRIC RATIOS I

TABLE OF CONTENTS

1.0 INTRODUCTION

Before starting any discussion in trigonometric ratios, you should be able to:

(i) Identify the sides of a right-angled triangle in relation to a marked angle in the triangle. If this is not the case do not worry. You can quickly go through this now:

In the diagram see. Fig. (i.) showing a

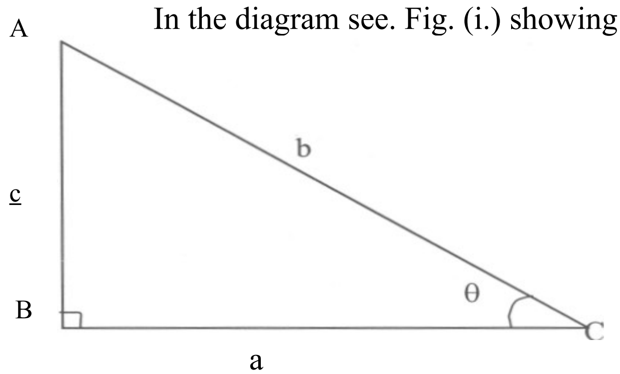


Fig (i)

Right-angled triangle ABC, right angled at B, with angle at C marked θ and the sides marked a, b, c,

AC = b is called the hypothenus
AB = c i.e. the side facing the marked angle θ at C is called the opposite side of the angle at C adjacent side to the angle at C.

(ii) Again, you should recall that the ratios of two numbers "x and y" can either be expressed as $x/_y$ or $^y/_x$. If you have forgotten this, please, refresh your memories for this is important in the unit you are about to study.

2.0 OBJECTIVES

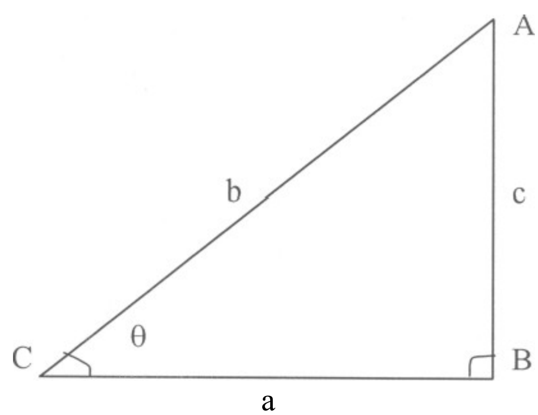
By the end of this unit, you should be able to

- define trigonometric ratios of a given angle.
- State the relationship between the trigonometric ratios
- Locate the quadrant of the trigonometric ratios of given angles
- Find the basic angles of given angles.

3.1.l. TRIGONOMETRIC RATIOS

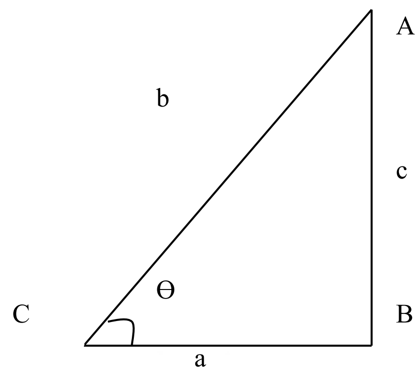
Having refreshed your minds on the sides of a right-angled triangle and the concept of ratios you are now ready to study the trigonometric ratios (sine, cosine and tangent).

This has to do with the ratio of the sides of a right-angled triangle. Here is an example.



In ΔABC, with A < B = 90° and < C = θ and the sides of ΔABC, marked a, b, c, respectively, then $\frac{AB}{AC} = \frac{c}{b} = \frac{\text{opposite side to the angle at C}}{\text{Hypotenuse}}$ is sine θ or simply sin θ.

In ΔABC see Fig 8 1.12



$$\frac{BC}{AC} = \frac{a}{b}$$

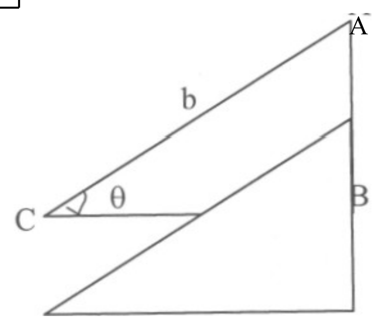


fig: 1.12

$= \frac{\text{adjacent side to the angle C}}{\text{Hypotenuse}}$ is called Cosine θ or simply Cos θ and in fig: 1.13 Below

$$\frac{AB}{BC} = \frac{c}{a}$$

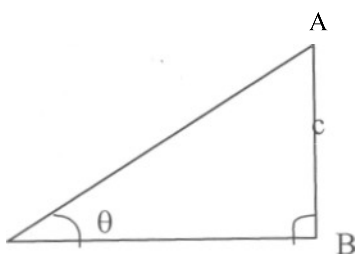


fig: 1.13

= Opposite side, to the angle C is called tangent θ or tan θ from the above ratios,

Adjacent side to the angle C
you can see that

$$\frac{\sin \theta}{\cos\theta} = \frac{\text{opposite side}}{\text{hypothenus}} \div \frac{\text{adjacent side}}{\text{hypothenus}}$$

using the notation of the sides of ΔABC

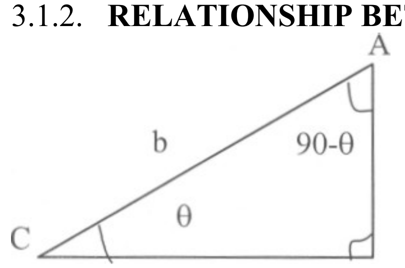
$$\frac{\sin \theta}{\cos\theta} = \left(\frac{c}{b}\right) \div \left(\frac{a}{b}\right)$$

$$\frac{c}{a} = \frac{\text{opposite side}}{\text{adjacent side}}$$

tan = θ

In the above, at an acute angle and with the knowledge that the sum of the interior angles of a triangle is 180º. What do you think will happen to the trigonometric rations? This takes us to the relationships between trigonometric ratios.

3.1.2. **RELATIONSHIP BETWEEN TRIGONOMETRIC RATIOS**.



B

Fig: 1.14

In ΔABC in fig 1.14 with the usual notations < B = 90^0 and < C = θ, therefore < A = 90 - θ. Once more finding the trigonometric ratios in relation to the angle at A.

$$\text{Sin } (90^o - \theta) = \frac{BC}{AC} = \frac{a}{b} = \frac{\text{opposite side to angle A}}{\text{hypothenus}}$$

= COs θ

$$\text{Sin } (90^o - \theta) = \frac{AB}{AC} = \frac{c}{b} = \frac{\text{opposite side to angle A}}{\text{hypothenus}}$$

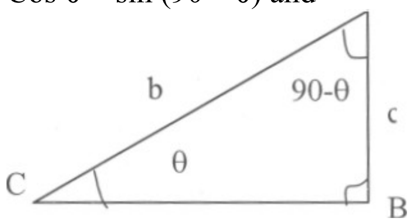
= Sin θ

You might happen wonder what happens to tan (90^0-0), this will be discussed later.

In summary, given ΔABC as shown

Sin θ = Cos (90^0 - θ)

Cos θ = sin (90° -θ) and



Sin θ = tan θ
Cosθ

The conclusion from the summary of these trigonometric ratios is that the sine of an acute angle equals the cosine of its complement and vice versa. Thus sin 30^0 = cos 60^0, cos 50° = sin 40^0 etc.(these angles are called complementary angles because their sum is 90^o 1 .e. 30° + 60° = 90°, 50° + 40 = 90^oetc)

Now go through the examples above carefully and try this exercise.

(1) Find the value of θ in the following

(i) cos θ ° = sin θ (ii) sin 35° = cos θ (iii) sin 12° = Cos θ

(iv) cos 73° = Sin θ . In case you are finding it difficult, the following are the solutions.

Solutions:

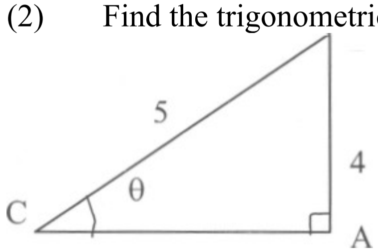
(i) Cos 50° = sin (90° - θ)
=sin (90° -50°) = sin 40° (because 50° + 40° = 90°)

(ii) Sin 35° = cos (90° - 35°)
= Cos 55° (since 35° + 55° = 90°)

(iii) Sin 12° = Cos (90° - 12°)
= Cos 78°

(iv) Cos 73° = Sin (90° - 73°)

(2) Find the trigonometric ratios in their following triangle B



Solution:

Since there is the measurement of a side missing i.e. AC, and the triangle is right - angled Δ, Using Pythagoras theorem to find the missing side

$BC^2 = AB^2 + AC^2$ (Pythagoras theorem) Substituting for the sides

$5^2 = 4^2 + AC^2$

$25 = 16 + AC^2$

$25 - 16 = AC^2$

$9 = AC^2$

$\therefore$ AC = $\sqrt{9}$=3, then since.

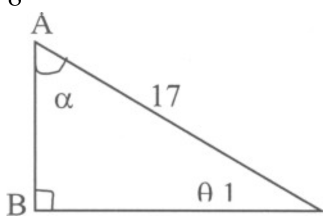
$\sin\theta = \frac{AB}{BC} = \frac{4}{5} = 0.8$

$\cos\theta = \frac{AC}{BC} = \frac{3}{5} = 0.6$

$\tan\theta = \frac{AB}{AC} = \frac{4}{3} = 1.33°$

(3) In the following, angle θ is acute and angle α is acute. Find the following trigonometric ratios.

8



15

(a) sin α
(b) cos α
(c) tan α
(d) cos θ
(e) sin θ
(f) tan θ

Solutions:

(a) $\sin\alpha = \frac{BC}{AC} = \frac{15}{17}$

(b) $\cos\alpha = \frac{AB}{AC} = \frac{8}{17}$

(c) $\tan\alpha = \frac{BC}{AB} = \frac{15}{8}$

(d) $\cos\theta = \frac{BC}{AC} = \frac{15}{17}$

(e) $\sin\theta = \frac{AB}{AC} = \frac{8}{17}$

(f) tang = $\frac{AB}{BC} = \frac{8}{15}$

You can notice from example (3) that since the sum α and θ is 90° (i.e. α + θ 90°)that:

Sin α = Cos θ and Cos α = Sin θ. This again shows that α and θ are complementary angles.

Having known what trigonometric ratios are, you will now proceed to finding trigonometric ratios of any angle.

3.3 TRIGONOMETRIC RATIOS OF ANY ANGLE.

It is possible to determine to some extent the trigonometric ratios of all angles using the acute angles in relation to the right-angled triangle. But since all problems concerning triangles are not only meant for right angle triangles, ~it is then good to extend the concept of the trigonometric ratios to angles of any size (i.e. between 0° and any angle).

To achieve the above, you take a unit circle i.e. a circle of radius I unit, drawn

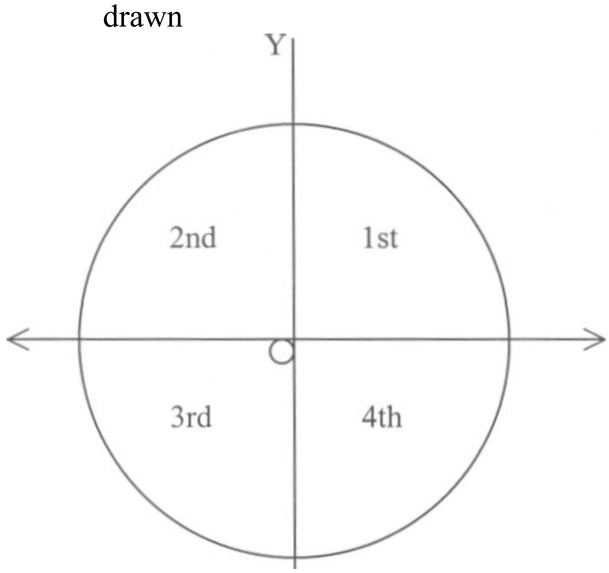
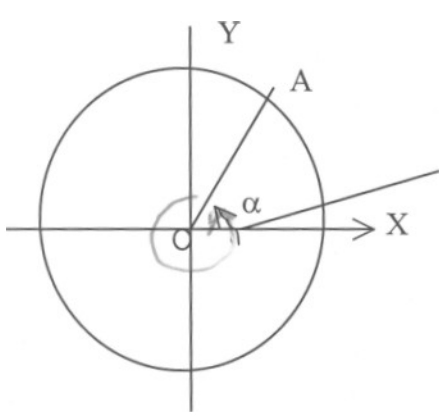


Fig: 2.0

In the Cartesian plane (x and y plane) the circle is divided into four equal parts each of which is called a quadrant (1st, 2^{nd}, 3^{rd}. 4th respectively). Angles are either measured positively in an anti clockwise direction (see fig 2.1)



Positive

Fig: 2.1

Or negatively in a clockwise direction.

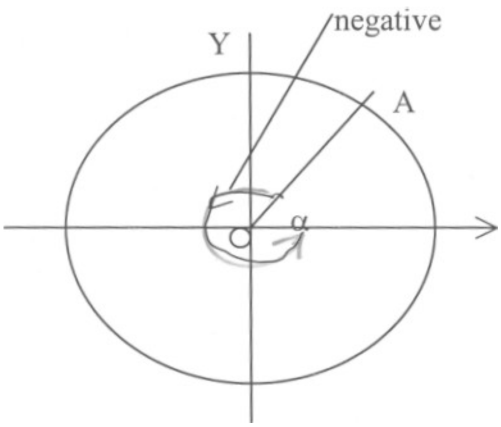
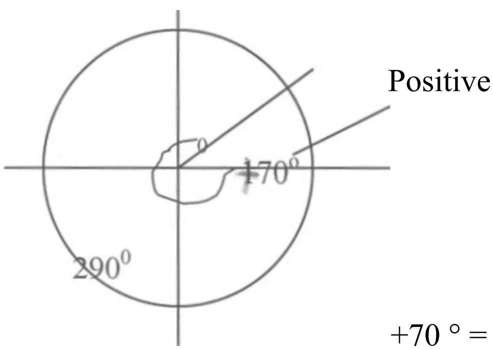
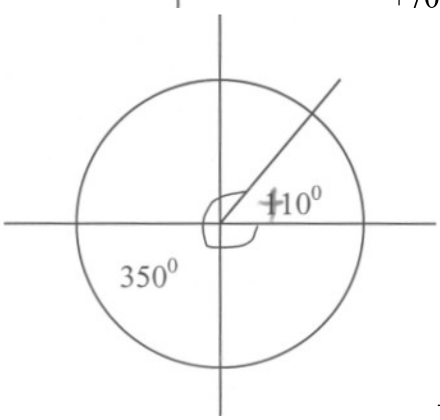


Fig: 2.2

Example. In the diagrams below



+70 ° = - 290°



+ 10° = -350

Note: Since this concerns angles at a point their sum is 360°. But angles of sizes greater than 360° will always lie in any of the four quadrants. This is determined by first trying to find out how many revolutions (one completed revolution = 360°) there are contained in that angle.

For example, (b) 390° contains 1(360°) plus 30° i.e. 390° = 360° + 30^0, 30° is called the basic angle of 390° and since 30° is in the first quadrant, 390° is also in the first quadrant. (a) 600° = 360° + 240°, since 240° is in the third quadrant, 600° is also in the third quadrant.

To find the basic angle of any given angle subtract 360° (1 complete revolution) from the given angle until the remainder is an angle less than 360° , then locate the quadrant in which the remainder falls that becomes the quadrant of the angle.
Now have fun with this exercise,

Exercise:
Find the basic angles of the following and hence indicate the quadrants in which they fell.
(1) 670° (2) 740° (3) 1998°
(4) 2002° (5) 2106° (6) 544°

Are you happy, Now, move to the next step.
To determine the signs whether positive or negative of the angles and their trigonometric ratios in the four quadrants;
First, choose any point P(x, y) on the circle and O is the center of the circle.

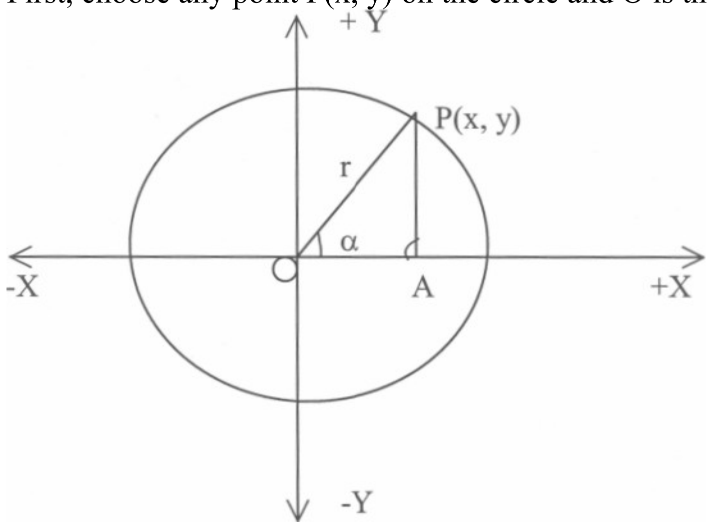


Fig: 2.3

θP = r, is the radius and OP makes an angle of α with the positive x - axis.

Since P is any point, OP is rotated about 0 in the anti clockwise direction, Hence in the
1st quadrant (0° < θ < 90°), using your knowledge of trigonometric ratios.

$\text{Sin } \alpha = \frac{PA}{\theta P} = \frac{+y}{+r}$ = y/r is positive

$\cos \alpha = \frac{\theta A}{\theta P} = \frac{+x}{+r}$ = x/r is also positive

$\tan \alpha = \frac{\theta P}{\theta A} = \frac{+y}{+x}$ = y/x is also positive

Therefore in first quadrant (acute angles) all the trigonometric ratios are positive. 2nd quadrant (90 < α ° < 180°) (obtuse angles)

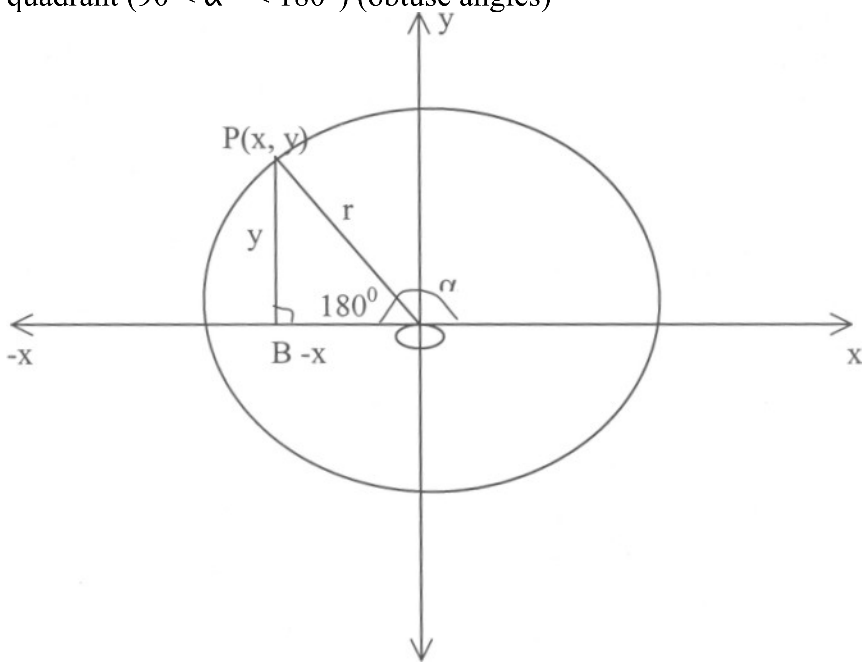


Fig: 2.4

In ΔPBO , < at O is 180 - α, here BO is - x (it lies on the negative x axis) but y and r are positive. The trigonometric ratios are

$\text{Sin } (180 - \alpha) = \frac{PB}{PO} = \frac{+y}{+r}$ = y/r is positive

$\text{Cos } (180 - \alpha) = \frac{BO}{PO} = \frac{-x}{+r}$ = -x/r is negative

$\text{Tan } (180 - \alpha) = \frac{PB}{BO} = \frac{+y}{-r}$ = -y/x is negative

So, only the sine of the obtuse angle is positive, the other trigonometric ratios are negative. Guess what happens in the 3rd quadrant (reflex angles).

3rd quadrant 180 < α° < 270° (reflex angles)

Note θP = r (i.e.) the radius is always positive. Reference is made to 180°, so the angle is (180 + α)° or α - 180°

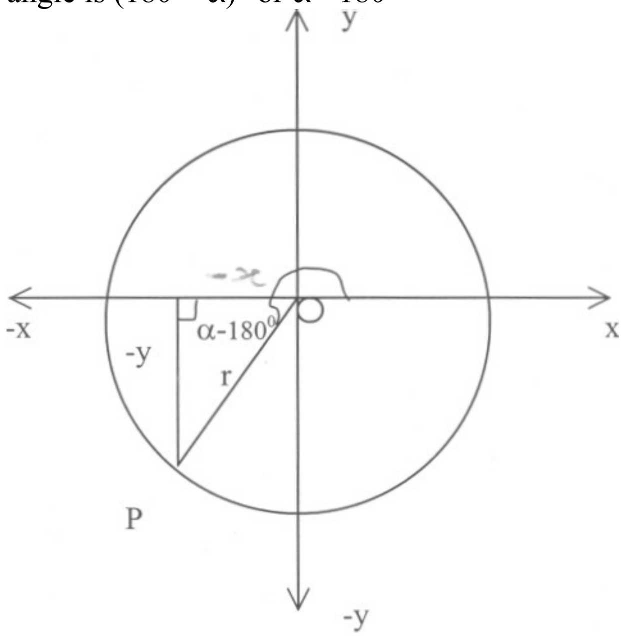


Fig: 2.5

Sin(α -180°) –y/+r = - y/r which is negative

Cos(α -180°) = -x/r = - x/r is negative

tan(α -180°) - - y/-x is positive

so if the angle α lies between 180° and 270° the sine, cosine o9f that angle are negative while the tangent is positive.

4th quadrant 270° <α< 360° (Double Reflex angles) y

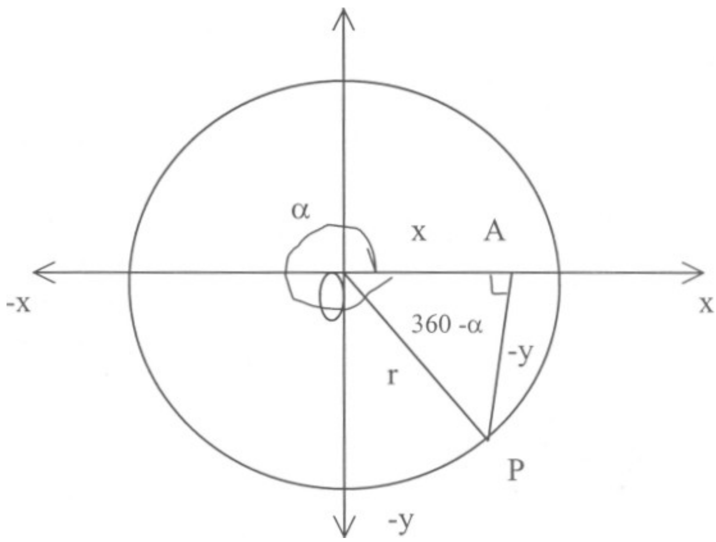


Fig 2.6

Here PA is negative but OA and OP are positive.

Sin (360 – α) = - y/+r = -y/r is negative

Cos (360 – α) +x/+r = x/r is positive

Tan (360 - α) = -y/+x = -- y/x is negative.

Here again sine and tangent of any angle that lies between 270° and 360° are negative the cosine of that angle is positive.

Looking at the figures above, it is seen that the sign of a cosine is similar to the sign of the x - axis(and coordinate) while the sign of a sine is similar to the sign of y - coordinate (i.e. y - axis). The signs can then be written in the four quadrants as shown below see fig: 2.7

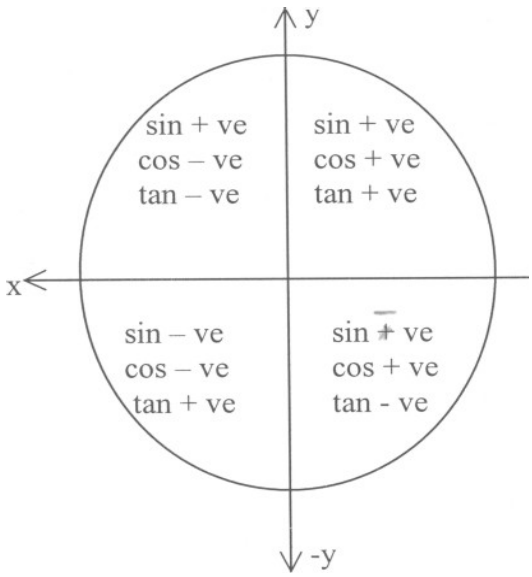


Fig 2.7

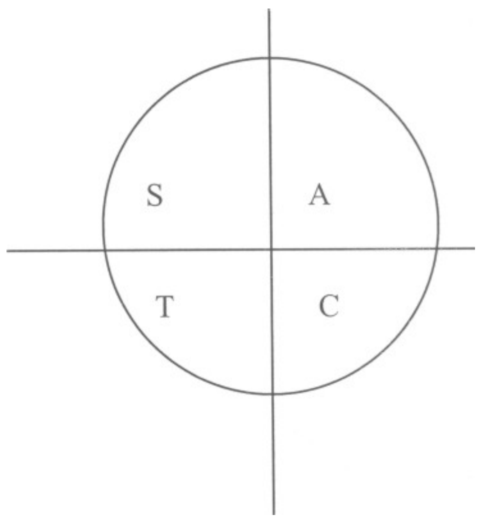


Fig 2.8.

Figure 2.8 is a summary of the signs in their respective quadrants, thus going in the anticlockwise direction, the acronym is;

(1) CASt(from the 4th to 1st to 2nd and then 3rd)

(ii) ACTS (from 1st ⟶ 4^{th} ⟶ 3rd then 2nd)

Clockwise

(iii) All Science Teachers Cooperate (ASTC) (from the 1^{st} ⟶ 2nd ⟹ 3rd -> 4th). The letters in figure 2,8 (marked quadrants) show the trigonometric ratios that are positive.

(iv) SACT (2nd -> 1 st -> 4th -> 3rd)

(v) TASC (3rd --> 2nd -> 1 st -> 4th)

Example:
Indicate the quadrants of the following angles and state whether their trigonometric ratios of each is positive or negative.
(1) 155° (11) 525° (iii) 62° (iv) 310° (v) 233°

Solution:
(1) 155° lies between 90° and 180° and therefore is in the 2nd quadrant. The sine of 155° is positive while the cosine and tangent, of 155° are negative. Thus:
sin 155° is + ve but cos 155° and tan 155° are negative
using the tabular form

No	Angles	Quadrant	Positive trig, ratios	Negative trig. ratios
1	155°	2nd	sin	cos and tan
2	525" =360" +165° the basic angle is 165°	2nd	sin	cos and tan
3	62°	Ist	sin, cos and tan	none
4	310°	4th	cos	sin, and tan
5	233°	3rd	tan	sin and cos

Alternatively, the solution can be thus

1. 155° is in the 2nd quadrant, here only the sin and cosec are positive.
sin(1550) = +sin (180 - 155°) =sin 25°
cos 155° = - cos (180 - 155°) = - cos 25°
tan 155° = - tan (180 - 155) = - tan 25°

2. 525°; the basic angle of 525° is gotten by
525° = 360° + 160° (one complete revolution plus 165°)
.'. 525 = 165 the basic angle lies in the 2nd quadrant and so 525 is in the 2nd quadrant where only the sin is positive
sin 525° = sin 165° = sin (180 - 165) = sin 15°
cos 525° = - cos (180-165) - cos 15°
tan 525° = - tan(180 - 165) = - tan 15°

3. 62°, this is in the first quadrant, where all the trig. Ratios are positive, therefore
   sin 62° = + sin 62°;
   cos 62° = + cos 62° ;
   tan 62° = + tan 62°;

4. 310° is in the 4th quadrant where only the cosine is positive, thence.
   $\sin 310^0 = - \sin (360 - 310) = - \sin 50°$
   cos 310° = + cos (360 - 310) = + cos 50°
   tan 310° = - tan (360 -310) = - tan 50°
5. 233° is in the 3rd quadrant, only tan is positive, so:
   sin 233° = - sin (233 - 180°) = - sin 53°
   cos 233° = -cos (233 - 180°) = - cos 53°
   tan 233° = + tan (233 - 180°) = + tan 53°

Exercise 2.1
Show in which of the quadrant each of the following angles occur and state whether the trigonometric ratio of the angle is positive or negative.

(1)	100^o	(2)	110^o	(3)	123^o	(4)	42^o	(5)	20^o
(6)	231^o	(7)	268^o	(8)	312^o	(9)	591^o	(10)	1999^o

Solutions:

(1)	2^{nd}, only sin + ve	(2)	2^{nd}, only sin + ve
(3)	2ns, only sin + ve	(4)	1^{st} all + ve
(5)	1^{st} all positive	(6)	3^{rd}, only tan + ve
(7)	3^{rd}, only tan +ve	(8)	4^{th}, only cos + ve
(9)	3^{rd}, only tan +ve	(10)	3^{rd}, only tan +ve

4.0 CONCLUSION

In this unit, you have learnt the definition of the trigonometric ratios sine, cosine and tangent and how to find the trigonometric ratios of any given angle. You should have also learnt that the value of any angle depended on its basic angle and its sign depends on the quadrant in which it is found. Thou now understand that the most commonly used trigonometric ratios are the sine , cosine and tangent; and the basic angle θ lies between
O° and 360° i.e. O° <_θ <_ 360°

5.0 SUMMARY

In this unit, you have seen that the trigonometric ratios with respect to a right-angled triangle is

Sin θ = <u>opposite</u> i.e. SOH

Hypothenus

Cos θ = *adjacent* / *Hypothenus* i.e. CAH

Tan θ = *Opposite* / *adjacent* i.e. TOA

hence the acronym SOH CAH TOA which is a combination of the above meaning can be used to remember the trigonometric ratios Again, you saw the relationships between the trigonometric ratios 0 the sine of cosine of an acute angle equals The cosine or sine of its complementary angle. That i.e.

(1) sin θ = cos (90 - θ)
cos θ = sin (90 - θ)
sin (90 + θ) = cos θ
cos(90 + θ) = - sin θ

for obtuse angle

(2) sin(180 -θ) = sin θ
cos (180 = θ) = - cos θ
sin (180+ θ) = sin θ
cos (180 + θ) = - cos θ

(3) sin (θ - 180) = - sin θ
cos (θ - 180) = - cos θ

(4) sin (360 - θ) = - sin θ
cos (360 - θ) = cos θ

6.0 TUTOR MARKED ASSIGNMENT

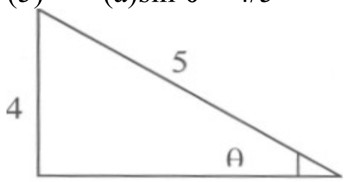
Find the values of y in the following equations

(1) sin y = cos 48°
(2) cos y = sin 280 33^1
(3) sin (90 - y) = cos 72° 31
(4) cos(90 - y) = sin 56° 47^1
(5) find the value of sin 0 and cos 0 if tan 0 = 43

TUTOR MARKED ASSIGNMENT: MARKING SCHEME

(1) Y = 42
(2) Y =61 27

(3) 72 31
(4) 56 47
(5) (a)sin θ = 4/5



(b) cos θ = 35

2 points each.

7.0 REFERENCES

Amazigo, J.C. (ed) (1991): Introductory University Mathematics I: Algebra,

Trigonometry and Complex Numbers. Onitsha: Africana - feb Publishers Ltd

David - Osuagwu, M; Anemelu C and Onyeozilu I. (2000): New School Mathematics for Senior Secondary Schools. Onitsha: Africana - Feb Publishers Ltd

Egbe, E. Odili, G.A and Ugbebor, O. O. (2000): Further Mathematics. Onitsha: Africana - feb Publishers ltd

Vygodsky, M. (1972): Mathematical Handbook: elementary Mathematics. Mosco: M/R Publishers.

This list is not exhaustive, you can use any mathematics textbook no matter the level it is written for, to enable you have a good understanding of the unit. There are a lot of mathematics text in the market and libraries, feel free to use any.

UNIT 2

TRIGONOMETRIC RATIOS II

TABLE OF CONTENTS

1.0 INTRODUCTION

In the previous unit, you learnt about the basic trigonometric ratios - sine, cosine and tangent. You also saw the relationship between the sine and cosine of any angle, nothing was mentioned about the relationship of the tangent except that it is the sine of an angle over its cosine. Also in our discussion, form our definition of ratios only one aspect is treated i.e. Xy or x : y what happens when it is y : x

or y/x. An attempt to answer this question will take us to the unit on the reciprocals of trigonometric ratios - secant, cosecant and cotangent.

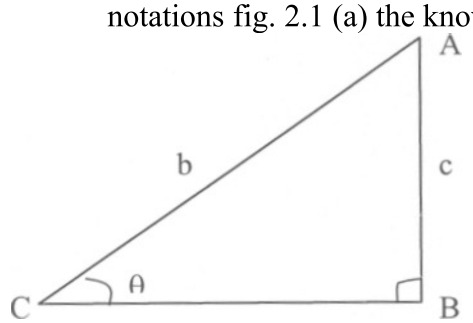
2.0 OBJECTIVES

By the end of this unit, you should be able to:

- define the reciprocals of trigonometric ratios in relation to the right-angled triangle.
- Establish the relationship between the six trigonometric ratios
- Use trigonometric tables to find values of given angles.

3.1 TRIGONOMETRIC RATIOS II

From the previous units, using DABC, right-angled and B and with the usual notations fig. 2.1 (a) the knowledge of the ratio



a

Fig. 2.1

Of two numbers "x and y" expressed as x/y was used to find the sine, cosine and tangent of θ. In this unit, the expressed as y/x will be used thus in fig 2.1 (a)

$\text{Sin } \theta = \frac{AB}{AC} = \frac{c}{b}$

$\text{Cos } \theta = \frac{BC}{AC} = \frac{a}{b}$ and

$\text{Tan } \theta = \frac{AB}{BC} = \frac{c}{a}$.

Now if this relationship is viewed in this order.

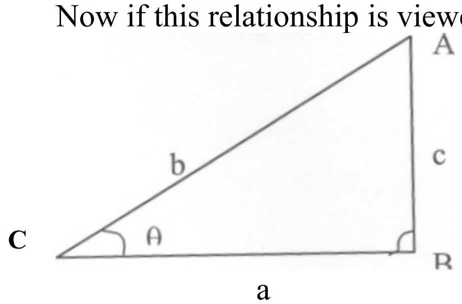


Fig: 2.1 (b)

$\frac{AC}{AB} = \frac{\text{hypothenus}}{\text{opposite}} = \frac{b}{c}$ it is called cosecants or cosecs

$\frac{AC}{BC} = \frac{a}{c} = \frac{\text{adjacent}}{\text{Opposite}}$ is called cotangent opposite of θ or cot θ.

Now study the above ratios carefully, what can you say of their relationship?

This leads us to the following sub-heading

RELATIONSHIPS BETWEEN THE TRIGONOMETRIC RATIOS.

As you can see sin 6 and cosc 6 for example are related in the sense that

$\text{Sin } \theta = \frac{\text{opposite}}{\text{hypothenus}} = \frac{c}{b}$ from fig: 2.1(a)

and $\text{cosec}\theta = \frac{\text{hypothenus}}{\text{opposite}} = \frac{b}{c}$ from fig: 2.1(b) *which means that*

$$\operatorname{cosec}\theta = \frac{1}{\left(\frac{\text{Opposite}}{\text{hypothenus}}\right)}$$

$$= \frac{1}{\sin\theta} = \frac{1}{\left(\frac{c}{b}\right)}$$

$$= \frac{\text{hypothenus}}{\text{opposite}}$$

$$= \frac{b}{c}$$

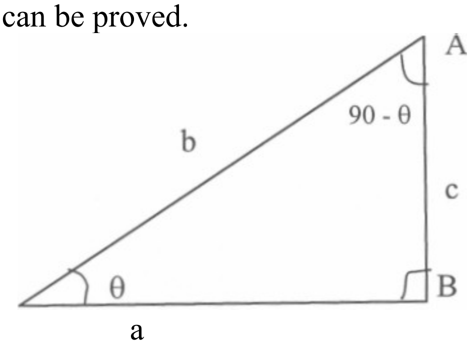
This then means that cosec θ is the reciprocal of sin θ and sin θ is the reciprocal of cosec θ. From the above ratios also, you can see ;

Exercise l:

(i) find the other reciprocals. Now try this. the above example serves as a guide.

(ii) verify that cos θ/ sin θ = cot θ for any triangle. Is this surprising. This is the beauty of the trigonometric ratios

Note from the sum of angles of a triangle giving 180°, the following relations can be proved.



Fig; 2.22

You should recall that in unit 1,

Sin (90º - θ) = cos θ and

Cos (90° - θ) = sin θ now let us, see the tangent.

Tan (90° -θ) = BC/AC in fig: 2.2 i.e.

= a/c = cot θ

Also sec (90^0 – θ) = cosec θ. This brings us to the conclusion that the tangent of an acute ngle is equal to the cotangent of its complement. I.e. cot 30° = tan 60° and tan 10° = cot 80°; also sec 10° = cosec 80 °

Now go through these examples

1. Find the value of 0 in the following
   (a) sec θ = cosec 30° (b) sin 50 = ?
   (c) cot 20° = tan θ (d) sec 40° = cosec θ

Solution:

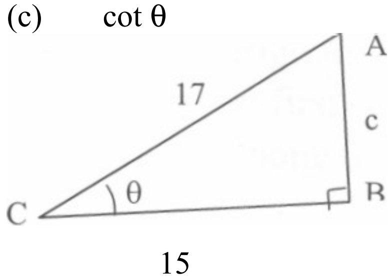
(a) cosec 30° = sec (90 - θ)
cosec 30° = sec (90 - 30°) = sec 60°

(b) 1/sin 50° = cosec 50°

(c) cot 20° = tan (90° - θ)
=tan (90° - 20°) = tan 70°

(d) sec 40° = cosec (90 - θ)
= cosec(90 - 40°) = cosec 50°

2. In the diagram, on the right find the folloiwing:
(a) sec θ
(b) cosecθ
(c) cot θ



(a) $\sec\theta = \frac{\text{hypothenus}}{\text{adjacent}} = \frac{1}{\cos\theta} = \frac{17}{15}$

(b) $\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{\text{hyhothenus}}{\text{opposite}} = \frac{17}{15}$

(C) $\cot\theta = \frac{1}{\tan\theta} = \frac{\text{adjacent}}{\text{opposite}} = \frac{15}{8}$

Now move to the next step, the relationship between trigonometric ratios of other angles.

It has been established that:

(1) the secant of any angle is the reciprocal of the cosine of the angle i.e. sec $\theta = 1/\cos\theta$

(2) cosec $\theta = 1/\sin\theta$ and

(3) cotan $\theta = 1/\tan\theta$

It then means that whatever applies to the trigonometric ratios their reciprocals, so the following are true in the first quadrant i.e.; $O° \leq \theta \leq 90°$ (acute) all the reciprocals trigonometric ratios are positive.
Sec θ -> cosec θ and cotan θ

In the second quadrant $90\ \theta \leq < 180$ (obtuse) since only the sine is positive only its reciprocal the cosecant will also be positive in the third and fourth quadrants respectively only the tangent and cotangent for $180 \leq \theta < 270$ are positive and cosine and secant in $270 \leq 0 < 360$ are positive respectively.

So the following relationships are established

1. sec θ = cosec $(90° - \theta)$
   cosec θ = sec $(90° - \theta)$
   tan θ = cot $(90° - \theta)$
   cot θ = tan $(90° - \theta)$

2. sec $(180 - \theta)$ is negative, θ lies between 90° and 180°
   cosec $(180 - \theta)$ is positive
   cot $(180 -\theta)$ is negative.

3. sec $(\theta - 180)$ is negative, θ lies between 180° and 270°
   cosec $(\theta - 180)$ is negative
   cot $(\theta - 180)$ is positive

4. sec $(360° - \theta)$ is positive, θ lies between 270° and 360 °
   cosec $(360 -\theta)$ is negative
   cotan $(360 - \theta)$ is negative

having seen the relationships between the trigonometric ratios and their reciprocals, let us move on to find angles using the trigonometric tables.

3.2.1 USE OF TRIGONOMETRIC TABLES

In the trigonometric tables for sine, cosine and tangent of angles can be used to find the values of their reciprocals. In the four figure tables available only the tables for sine, cosine and tangent are available so whatever obtains in their case also applies to their reciprocals.

The exact values of the trigonometric ratios obtained using the unit circle may not be accurate due to measurement errors. So to obtain the exact values of the trigonometric ratios, you use the four figure tables or calculators.

The tables to be used here are extract of the Natural sine and cosine of selected angles between 10^0 and 89^0 at the interval of 6^1 or 0.1^0 . The full trigonometric tables will be supplied at the end (are tables for log sine, log cos and log tan)

	0'	6 '	12 '	18,	24'	30'	36'	42'	48'	54'
X°	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9
20"	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	3567
30	0.5000	0.5015	5030	5045	5060	5075	5090	5105	5120	5135
40"	0.6428	6441	6455	6468	6481	6494	6508	6521	6534	6547
50"	0.7660	7672	7683	7694	9705	7716	7727	7738	7749	7760
60"	0.8660	8669	8678	8686	8695	8704	8712	8721	8729	8738
70"	0.9397	9403	9409	9415	9421	9426	9432	9432	9444	9449
80"	0.9848	9851	9854	9857	9860	9863	9866	9869	9871	9874
89°	0.9998	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000

Note that the difference column always at the extreme right - hand corner of the table is omitted

Extracts from natural cosine for cos x° (WAEC, four figure table)

	0	6	12	18	24	30	36	42	48	54
X"	0".0	0".1	0".2	-6'-3-0'4		0".5	0".6	0".7	0".8	0".9
10"	0.9848	9845	9842	9839	9836	9833	9829	9826	9823	9820
20"	0..9397	9391	9385	9379	9373	9367	9361	9354	9348	9342
30"	0.8660	8652	8643	8634	8635	8616	8507	8599	8590	8581
40"	0.7660	76649	7639	7627	7615	7604	7593	7581	7570	7559

50"	0.6428	6414	6401	6399	6374	6361	6347	6334	6320	6307
60"	0.5000	4985	4970	4955	4939	4924	4909	4894	4879	4863
70"	0.3420	3404	3387	3371	3355	3338	3322	3305	3289	3272
80"	0.1736	1719	1702	1685	1668	1650	1633	1616	1599	1583
89"	0.0175	0157	0140	01222	0105	0087	0070	0052	0035	0017

Again the difference column is omitted.

Example:
Find the value of the following angles:
(i) $\sin 20 . 6^0$ (ii) $\cos 30^0 12^1$ (iii) $\sin 70^0 48^1$ (iv) $\cos 40. 7^0$

Solutions:

(I) From the sine table to find $\sin 20 . 6^0$ look at the left hand column marked x get to the number 20 and move across to 0. 6 on the top now their intense gives 0.3518 .'. $\sin 20.6^0 = 0.3518$

(II) For $\cos (30^0 12^1)$, go to the natural cosine table look for 30^0 along the first column (x^0) , either and move across unit 1 you fet to 12^1. The value at this intersection is 0.8643. $\therefore \cos (30^0 12^1) = 0.8643$.

(III) $\sin (70^0 48^1) = 0.9444$

(iv) $\cos (40 . 7^0) = 0.7581$

A times, there might have problems involving minutes or degrees other than the one given in the table. You have to use the difference table when such is the case.
for example
Find (1) $\sin (20^0. 15^1)$ (II) $\cos (50^0 17^1)$
Solutions:

(i) From the sine table (WAEC)
sin (20 12) = 0.3453
plus the difference for 3 = 8 (from the difference column at the extreme right of the sine table)
$\therefore \sin (20\ 15^1) = 0.3461$

Alternatively you can look for $\sin (20° 18^1)$ and then subtract the difference of 3^1 thus
$\sin (20° 18^1) = 0.3469$
Is the difference for $3^1 = -8$

Sin (20^0 15^1) 0.3461

You see that either way the value of sin (20° 15^1) is 0. 3461

Note that the values of then (20° 15^1) is the same in the two methods above but in most cases, the values are not there are slight differences at times.

(ii) From the cosine table
cos (50° 17^1) is nearer cos 50 18
cos (50° 18) = 0. 6388
plus the diff. Forte= + 2 0.6390
∴ cos (500 171) = 0.6390

OR
cos (50° 12^1) = 0. 6401
minus the diff. For 5^1 = -12
cos (50° 17^1) = 0. 6389

Observe that the difference was added to the first method is cos 50° 18^1 and subtracted from the second method i.e. cos 50° 12^1 . This is because the angle increases, the value reduces in cosine. You can have a critical look at the tables for cosine. It is good to note that the values of sine increases form O to 1 while the values of cosine decreases from 1 to O.

The same methods as used in finding the tangent of angles from their tangents tables.

For angles greater than 90°, the same tables are used in finding their trigonometric ratios but firstly, you de4termine the quadrant and sign of the angle and treat accordingly.

Examples:

Find (1) sin 120° (2) sin (- 30)°
(3) cos (- 10°) (4) cos 260°

Solutions:

(1) Sin 120° is in the second quadrant and sine is positive = sin (180 - 1200 = sin 60° and since sin 60 is positive, from the sine table.
Sin 120° = + sin 60° = 0.8660

(2) Sin (-30°) is in the fourth quadrant, where the sine is negative.

∴(- 30°) = - sine (360 - 30) = - sin 330° = - sin 30° from the sine table sin 30° = 0.5000
∴ - sin 300 _ - 0.5000

(3) Cos (- 10) lies in the fourth quadrant, where cosine is positive.
∴cos (-10) = cos (360 - 10)= cos 350° = cos 10°
∴cos (-10) the cosine table is 0.9848
∴cos (- 10) = 0.9848

(4) Cos (260°) is in the 3rd quadrant where cosine is negative = cos (260 - 180°) = cos 80°. From the cosine table cos 80 = 0.1736 and since cosine is negative in the 3rd quadrant cos 260^0 = - cos 80^0 = - 0.1736

3.3.2 USE OF LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

A times you might be faced with problems which require multiplication and direction in solving triangles. Here the use of tables of trigonometric functions becomes time consuming and energy sapping. It is best at this stage to use the tables of the logarithms of trigonometric functions directly.

Examples:
Find (1) log cos 20° 6^1.

Solution:
The use of the tables of cosine will allow you to (1) find cos 20° 6^1 from the table .
(ii) find this value from the common logarithm table
i.e. cos . (20° 6^1.) = 0.9391 (from Natural cosine)

Then log 0.9391 =$\bar{1}$.9727 (from common logarithm)
But using the logarithm table of cosine go straight and find log 20° 6^1.
∴ log cos (20° 6^1) = $\bar{1}$. 9727

Here you can see that applying the log cos table is easier and faster.

(ii) Log sin (24° 13^1)
Log sin (24° 13^1) = $\bar{1}$. 6127,
Plus difference for 1^1 = + 0.0002 (cot from the difference table at the right hand extreme column)
∴ log sin 24° 13^1 = $\bar{1}$. 6029

(iii) log tan 40° 17^1
from the log tangent table;

log tan 40° 17^1= $\bar{1}$. 9269
plus the diff. For 5^1 = + 8
log tan 40° 17^1= $\bar{1}$. 9277
Alternatively, you can look for the logarithm:
log tan 40° 18^1 = $\bar{1}$. 9 284
minus the deff. for 1^1 = - 2
log tan 40° 18^1 = $\bar{1}$. 9282

The two results in this case are not the same. The second result is preferable because the smaller the difference the more accurate the value of the angle being sort for.

If the angles are in radius convert to degrees.

Exercise 2.2

Using the four figure table or calculator

1. find the value of each of the following

a.	sin	32° 17°	ans. = 0.5341
b.	sin	$126^0.30^0$	ans. = 0.3382
c.	sin	340° 14^1	ans. = - 0.3382
d.	cos	35^o. 7^o	ans = 0.8121
e.	cos	137^o 16^1	ans = -0.7346
f.	cos	(-40^o)	ans = 0.7660
g.	sin	(-40^o)	ans = -0.6428
h.	tan	120^o	ans = -1.7321
i.	tan	265^o	ans = 11.4301
j.	tan	12^o 46^1	ans = 0.2266

2. Find the quadrant of the following angles and determine whether the trigonometric ratios (reciprocals) are positive or negative.

(a)	100^o	(b)	110^o	(c)	123^o		
(d)	42^o	(e)	20^o	(f)	231^o		
(g)	268^o	(h)	312^o	(i)	591^o	(j)	1999^o.

Solutions:
a. 2nd, only sine and cosine positive
b. 2nd, only sin and cosec + ve
c. 2nd, only sin and cosec + ve
d. 1st all trig ratios positive
e. 1st, all trig. Ratios positive

f. 3rd, only tan and cot positive
g. 4th, cos and sec positive.
h. 3rd only tan and cot positive
i. 3rd, only tan and cotangent positive.

4.0 CONCLUSION

In unit 1 and 2, you have learnt, the definition of the trigonometric ratios and their reciprocals, and how to find the trigonometric ratios of any given angle and the use of trigonometric tables in finding angles. You should hive also learnt that the value of any angle depends on the basic angle and its sign depends on the quadrant in which it is found. However, you need be aware that the most commonly used trigonometric ratios are the sine cosine and tangent and the basic angle 0 lies between O° and 360 i.e. $0 \leq 0 < 360$.

5.0 SUMMARY

In these two units you have seen that the trigonometric ratios and their reciprocals with respect to a right angled triangle is

$$\sin\theta = \frac{\text{opposite}}{\text{hypothenus}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypothenus}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

The acronym SOH CAH TOA meaning

S = sine, O = opposite over, H = hypotenuse
C = cosine, A = adjacent over, H = hypotenuse
T = tangent, o = opposite over, A = adjacent

Can be used to remember the trigonometric ratios their reciprocals are obtained from these.

You have also learnt that:

(i) the sine or cosine or tangent of an acute angle equals the cosine or sine or cotangent of its complementary angle.

Sin $\theta = \cos(90 - \theta)$ $\sin(90 + \theta) = \cos\theta$
Cos $\theta = \sin(90 - \theta 0$ $\cos(90 + \theta) = -\sin\theta$
Tan $\theta = \cot(90 - \theta)$ $\tan(90 + \theta) = -\cot\theta$

This means that you can use the sine table find the cosine of all angles from 90 to 0 at the same interval of 61 or 0° .1°

(ii) the tables of trigonometric functions can also be used in finding the ratios of given angles by bearing in mind the following where0 is acute or obtuse.

(iii) sin (180 - θ) = - sin θ; sin (θ - 180) = - sin θ
cos (180 - θ) = -cos θ ; cos (θ - 180) = -sin θ
tan (180 - θ) = -tan θ; tan (θ - 180) = tan θ

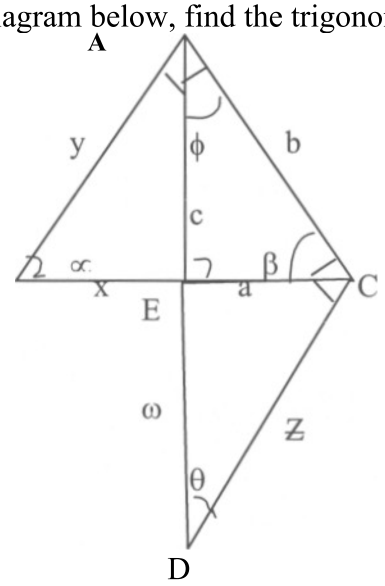
(iv) sin (180 + θ)= - sin θ
cos (180 + θ)= - cos θ
tan (180 + θ)= tan θ

(v) sin (360 - θ) = - sin θ
cos (360 - θ) = cos θ
tan (360 - θ) = -tan θ

In using the table sometimes angles may be expressed in radians, first convert the angles in radians to degrees before finding the trigonometric ratios of the given angles or convert from degrees to radians before finding the trigonometric ratios, if it is in radians

6.0 TUTOR MARKED ASSIGNMENT

In the diagram below, find the trigonometric ratios indicated.



(a) sin θ , cos θ, sec θ, cot θ
(b) cos α, tan α, cosec α, sin α
(c) tan ϕ, cos ϕ, sec ϕ cosec ϕ sin ϕ
(d) sin β cos β tan β, cot β sec β sec β cosec β

2. Express the following interms of the trigonometric ratios of α

(a)	i.	Cos (90 – α)	ii.	Sin (90 + α)
(b)	i.	Cosec (90 – α)	ii.	Sec (90 + α)
(c)	i.	Cos (90 – α)	ii.	Sec (180 – α)
(d)	i.	Sin (360 – α)	ii.	Tan (360 – α)

3. Find the basic angles of the following and their respective quadrants.

(a) 1290^{o} (b) -340^{o} (c) -220^{o}
(d) 19^{o} (e) 125^{o} (f) 214^{o}

4. use trigonometric tables to find the value of the following:

(a) sin 117° (b) cos 11.1 ° (c) tan 275°
(d) sin 204 . 7°(e) cos 121°

5. Use the logarithm table for trig. Functions to find the value of the following.

(a) log cos 34° 17^1 (b) log sin 23° 25^1 (c) log tan 11° 6^1

UNIT 3

INVERSE TRIGONOMETRIC FUNCTIONS OR CIRCULAR FUNCTIONS

TABLE OF CONTENT

1.0 INTRODUCTION

Very often you see relations like y = sin θ R is possible to find the value of y, if θ is known. On the other hand the need might arise to find the value of θ when y is known. What do you think can be done in this case?

In the example above i.e. y = sin θ, sine is a function of an angle and also the angle is a function of sine.

In this unit, you shall learn the inverse trigonometric functions, sometimes called circular functions, the basic relation of the principal value and trigonometric ratios of special angles 0°, 30°, 45°, 60°, 90^0 , 180°, 270° and 360^0

2.0 OBJECTIVES

By the end of the units you should be able to:

- define inverse trigonometric functions
- find accurately the inverse trigonometric functions of given values.
- Determine without tables or calculators the trigonometric ratios of 0°, 30°, 45°, 60°, 90°, 180°, 270 ° and 360°.
- Solve problems involving inverse trigonometric functions and trigonometric ratios of special angles correctly.

3.1 INVERSE TRIGONOMETRIC FUNCTIONS (CIRCULAR FUNCTIONS)

3.1.1. DEFINITION AND NOTATION

The trigonometric ratios of angle are usually expressed as y = sin θ (where y and θ represents any value and angle respectively).
Or y = cos θ

Or y = tan θ.

The above are example when the values of θ is known. When the value of θ is unknown and y is known the above relations can be expressed as:

θ = (Sin^{-1} y) written as arc sin y read as ark sin y

or θ = (Cos^{-1} y), written as arc cosy read as ark cos y

or 0 = (Tan^{-1} y) written as arc tan y read as arktany
Note capital letters are used for the first letters of the trig. ratios. These relations arcsin, arccos and arctan are called the inverse trigonometric functions or circular functions. Thus, in the above
examples, 0 is called the inverse sine or inverse cosine or inverse tangent of y.

Example:

(a) if sin θ = 0.4576, then θ = sin^{-1}(o.4576), meaning that θ is the angle whose sine is 0.4576 or the sine of θ is 0.4576

(b) if cos θ = 0.8594,at then θ = cos^{-1}(0.8954), which implies that θ is the angle whose cosine is 0.8594 or cosine of θ = 0.8954.

(c) if tan θ = 2. 1203, then tan^{-1} (2. 1203) shows that θ is the angle whose tangent is 2.1203 or the tangent of θ is 2.1203.

PROCEDURES FOR FINDING INVERSE TRIGONOMETREIC FUNCTIONS

Having been conversant with the use of the trigonometric tables, the task here becomes easy.

In finding the inverse trigonometric ratio of any angle, first look for the given value on the body of the stated trigonometric table and, read off the angle and minute under which it appeared. If the exact value is not found, the method of interpolation(i.e. finding the value closest to it and finding the difference between this closest value and the original value, then, look for the difference under the minutes in the difference column) can be adopted.

Example 1

Find the value of the following angles

(a) sin^{-1} (0.1780) (b) cos^{-1} (0.2588) (c) tan^{-1} (1.1777)

this question can also be stated thus: Find y if;
(a) siny = 0.1780 (b) cosy= 0.2588 (C) tan y = 1.1777

Solutions:

1(a) From the sine table (Natural sine table) through the body the value 0.1780 (or a value close to it) is located. The value is 0.1771 found under 10° 12'. The difference between 0.1780 and 0.17711 is 9. This is found under 3 in the difference column. So in tabular form

.. 0.1771	= 10°	121
plus difference fog -9	= +	31
0.1780	= 100	15'

$\therefore$ the angle whose sine is 0.1780 is $10° \, 15^1$

Alternatively 0.1788 can be located under 10 18 and difference between 0.1788 and 0.1780 is 8 but 8 cannot be found in the difference column so choose the number nearest to 8 i.e. 9 found under 3^1

$\therefore$ 0.1788 = $10° \, 18^1$

plus difference for 9 = $\underline{+ \quad 3^1}$

0.1779 = $10° \quad 15^1$

.'. the angle whose sine is approximately equal to 0.1780 (0.1779) i.e $10° \, 15^1$

$\theta = 10° \, 151$

b) $\text{Cos}\,\theta = 0.2588$

from the cosine table, the value 0.2588 is found under $75° \, 0^1$

the angle whose cosine is 0.2588 is 75° .. $\theta = 75°$

c) $\tan\theta = 1.\ 1777$,

from the natural tangent table the value closest to 1.1777 is 1.1750 found under $49° \, 36^1$. The difference between the two values is 27 which is found under 4^1

.. 0.1750	= 49°	36^1
plus the difference for 27	= +	4^1
0.1777	= 49°	40^1

In the above examples all angles are acute angles. The inverse trigonometric functions can be extended to any angle.

3.1.2 INVERSE TRIGONOMETRIC FUNCTIONS OF ANY ANGLE.

The inverse trigonometric functions here are extended to include values of given angles between 0° and 360° and beyond

Example 2:
Find the value of 0 between 0° and 360° in the following:
(a) $\sin\theta = 0.8964$ (b) $\cos\theta = -0.6792$ (c) $\tan\theta = 0.2886$

Solutions:
(a) $\sin\theta = 0.8964 \Rightarrow \theta = \sin^{-1}(0.8964)$
.'. $\theta = 63° 41^1$; Since sin θ is positive then the angle must either be in the 1st or 2nd quadrant thus

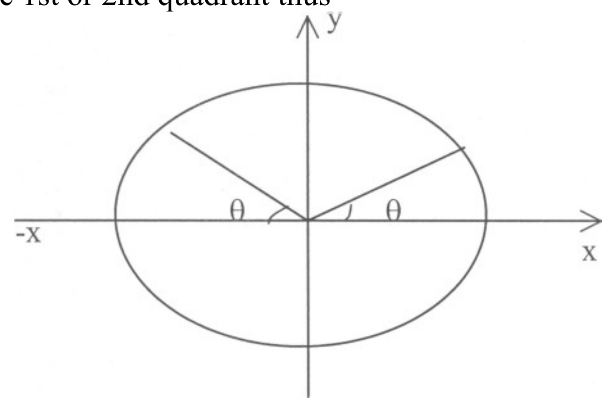


Fig 3.10

In the first quadrant $\theta = 63^o 41^1$ and in the 2nd quadrant
$\theta = 180 - 63^o 41^1 = 116^o 19^1$.

(b) $\cos\theta = 0.6792$

From the cosine tables $\theta = \cos -1\ 0.6792 = 47^o 10^1$ but cosine θ is negative, therefore θ lies either in the 2nd or 3rd quadrant.

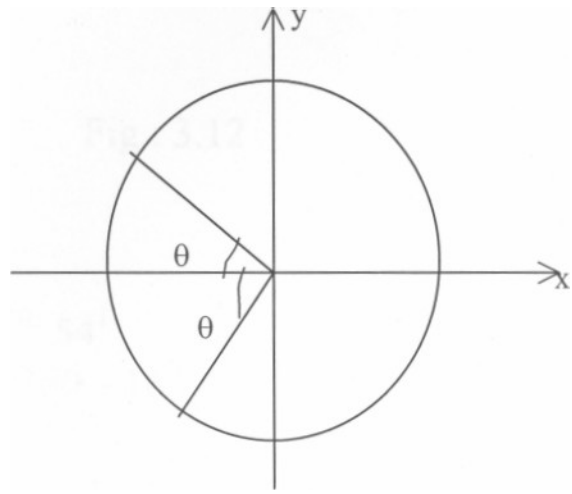


Fig : 3.11

In the 2nd `quadrant
θ = 180 - 47^0 10 = 132 ° 50~
In the 3rd quadrant
θ = 180 + 47° 10^1= 227^0 10^1

(c) tan θ = - 0.2886, here θ = 16^0 6` but since tan θ is negative, θ lies either in the 2nd or 4th quadrants.

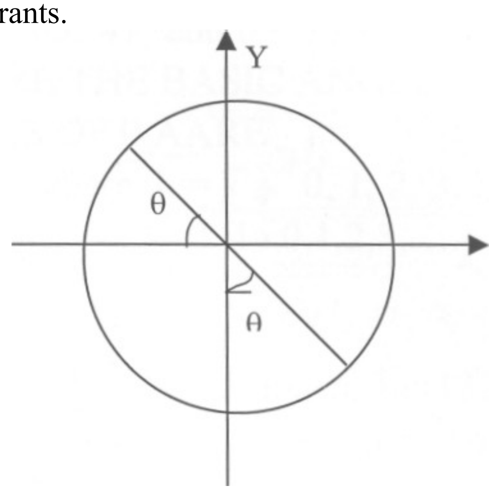


Fig : 3.12

In the 2nd quadrant
θ = 180 - 16°   61= 163^0 54^1
in the 4th quadrant θ = 360 - 16° 6^1
θ = 343° 54^1

Note from the previous units there are several values of θ with the same value but in different quadrants. For example sin 30° = sin 150° = sin 390° = sin 750° etc, hence the inverse trigonometric functions are many valued expressions. This mean that one value of θ is related to an infinite number of values of the function. Hence to obtain all possible angles θ of a given trigonometric ratio either add or subtract (360° K) where K is any integer-positive, negative or zero

Example 3

Find all possible angles of 0 in example 2(a), (b) and (c) above.

(a) given sin θ = 0.8964 and θ = 63° 41^1 and θ = 116° 19^1
These two VALUES OF θ ARE THE BASIC ANGLES
∴ ALL POSSIBLE ANGLES OF θ ARE
63^0 41^1 ± (3 60k°), where k = …-1,0,+1,+2,+3…..
and 116° 19^1 ± (360k°), where k = k = -1,0,1,2,3……

(b) given cos θ = - 0.6792 and θ was found to be 132° 50' and 227° 10' these are the basic angles, So all possible angles of θ, therefore are 132° 50' ± (360k)° for k = . . ., - - 1,0,1,2,...

(c) Given tan θ = - 0.2886, θ equals 163° 54' and 343° 54'
Here all possible angles of 0 are
163^0 54' ± (360k°)
and 243° 54' ± (360k°) for k = ..., -1,0,1,2, ...

Hence to find all possible angles of given angle:
(1) find the basic angles of the given value
(ii) add or subtract (360k°) where k is either a positive negative or zero integer.

3.1.3 PRINCIPAL VALUES OF INVERSE TRIGONOMETRIC FUNCTIONS.

In this section, attention should be found on the value which lies in a specified range for example:

(i) for sin-' y, the range of values are -π/2 (-90°) to π/2 (-90°). This value is called the principal value of the inverse of sine denoted by $\sin^{-1}y$ (smalls). For example if sin -1 1/√2 = 45° or π/4 radians then the principal value of the inverse of sin 1/√2 is $\sin^{-1}$ 1/√2 = 45° or π/4 (since it is within the range).

(ii) If $y = \cos\theta$, then $\theta = \cos^{-1} y$, is the inverse cosine of y. and the principal value of the inverse of cosine is the value of θ in the range 0° to π (180°). This is the same for arc cot θ, and arc sec θ
Example, if $\cos^{-1} 1/\sqrt{2} = 45°$, then arc sec θ the principal value
$\text{Cos}^{-1}(- 1/\sqrt{2}) = -\pi/4(-45°)$ the principal value is $\cos^{-1}/\sqrt{2}) = -\pi/4(135°)$

(iii) The principal value of the inverse of tangent is the value of θ in the range $-\pi/2$ (- 90°) to $+\pi/2$ (- 90°). This is the same for arc cosec θ.

Example of principal values

The principal value of;

(a) $\text{Tan}^{-1}(-1) = -\pi/4 = (45°)$. (b) $\text{Cot}^{-1}\sqrt{3}= \pi/6$ (30°)
(c) $\text{Sec}^{-1}(-2) = 2/3\pi$ (120°)

The relationship between the values of an inverse function and its principal value is given by the formulae below (Vygodsky 1972: 366).

(i) Arc sin x = k π + $(-1)^k$ arc sin x
(ii) Arc cos x = 2kπ ± arc cos x
(iii) Arc tan x = kπ + arc tan x
(iv) Arc cot x = kπ + arc cot x,
where k is any integer positive, negative or zero.

Hence Arc sin, Arc cos, Arc tan denotes arbitary values of inverse trigonometric functions and arcsin, arcos, arctan denotes principal values of given angles.

Example:

(a) Arc sin½ = kπ + $(-1)^k$ arc sin ½
= kπ + $(-1)^k$ x π/6 or k(180°) + (-1) k 30°
fork= 0, Arcsin ½ = 0 x π + (-1)° π/6 = π/6(30)°
k=1, = Arcsin 1/2 = l x π + (-I)' π/6 = π - π/6 = 5π/6 (150°)
k=2, = Arcsin 1/2 = 2 x π + $(-I)^2$ π/6 = 2π + π/6 = 13π/66 (390°)
k=3, = Arcsin 1/2 = 3 x π + $(-I)^3$ π/6 = π3+ π/6 = 17π/6 (510°)
k=-1, = Arcsin 1/2 = -1 x π+ (-I)-' π/6 = - π - π/6 = -7π/6 (-210°)

Note the angles in radians can be converted to degrees (see angles in brackets)

Exercise 3.1

(1) write down the values of
(a) sin-'(- 1/2) (b) cos-'(-1) (c) tan-'(-1)
Ans: (a)-211110°, 330° (b) 180° (c) 135°, 315°
(2) Use tables to evaluate:
(a) tan-' 2 (b) cos-1 (1/4) (c) sin-' (3/5)
Ans: (a) 63° 26' (b) 88° 26' (c) 36° 26'
(3) Find the value of the following angles:
(a) $\sin^{-1}(0.7509)$ (b) $\cos^{-1}(0.9219)$ (c) tan-' (2.574)
Ans: (a) 48° 40' (b) 212° 48' (c) 68° 46'
4. Find all the angles between 0° and 360°.
(a) sin θ = - 0.5120 (b) tan θ = o.9556 (c) cos θ = - 06088
Ans: (a) 210° 48', 329° 12' (b) 43° 42', 223° 42' (c) 127° 30', 230° 53'
5. Find all possible angles in question (4)
Ans: (a) 210° 48'± (360k°) and 329° 12' ± (360k°)
(b) 430 72' ± (3 60k)° and 223' 42'+- (3 60k°)
(c) 127° 30' ± (360k°) and 230° 53'+- (360k°)
6. Find the value of Arc cot√3
Ans: Arc cot√3 = k π+arc cot√3 where k is any integer = k π + π /6
for k = 0, Arc cot√3 = π/6 = 30° (angles in radians)
k = 1, Arc cot√3 =π+ π/6= 6 =210°
k = -l, Arc cot√3 = π + π /6 = -5π/6 = -150° etc.

3.2 TRIGONOMETRIC RATIOS OF COMMON ANGLES

The angles 0°, 30°, 45°, 60°, 90° are called common angles because they are frequently used in mathematics and mechanics in physics.

Although the trigonometric ratios of common angles 0°, 30°, 45°, 60°, 90°, (and multiples of 90° up to 360°) can be found from the trigonometric tables, they can be easily determined and are widely used in trigonometric problems.

3.2.l. THE ANGLE OF 30° AND 60°

Consider an equilateral triangle ABC of sides 2cm. An altitude AD (see fig; 3.2) A

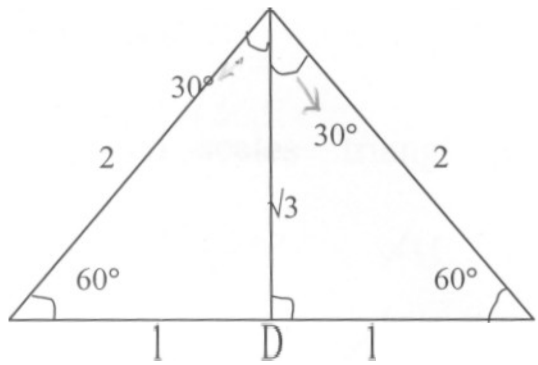


Fig: 3.2

An altitude AD (see fig 3.2)
Bisects < BAC so that < BAD = < CAD = 30°
< ABC = < ACB = 60°
by Pythagoras theorem AD = √3 units.

Hence, the value of the trigonometric ratios of 60° and 30° are

Sin 60°= √3/2 and sin 30° = 1/2

Cos 60° = 1/2 and cos 30° = √3/2

Tan 60° =√3 and tan 30° = I√3

Cot 60° = 1/√3 and cot 30° √3

Sec 60° = 2 and see 30º =-2/√3

Cosec 60° = 2/√3 and cosec 30° = 2

3.2.2. THE ANGLE 45°

Consider a right-angled isosceles triangle ABC with AB= BC = 1 unit,
<B=90° and<A=<C=45°
AC = √2 units (Pythagoras theorem)

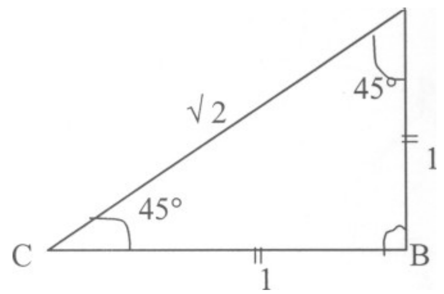


Fig 3.21

Hence the trigonometric ratios of 45° are sin 45° = $1\sqrt{2}$
Cos 45° = $1/\sqrt{2}$
Tan 45° = 1
Cot 45' = 1
Sec 45°=$\sqrt{2}$
Cosec 45° = $\sqrt{2}$

3.2.3. ANGLES 0° AND 90°

It is difficult in practical problems to find angles 0° and 90° in a right - angled triangle as acute angles but with extended trigonometric functions, these angles are considered.

(see fig 3.22 below)

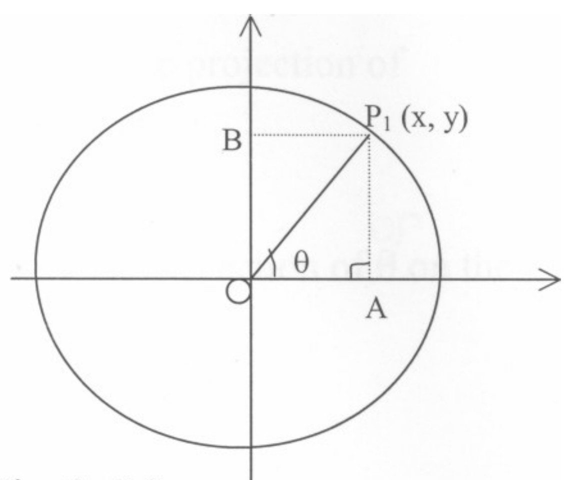


Fig: 3. 2.2.

Using a unit circle let P_1 (x, y) be any point on the circle. If θP, is rotated about 0 in the anti clockwise direction through an acute angle θ, then θ A is the projection of θP, on the X - axis and θB is the projection of θP, on the Y-axis

In $\Delta P_1\theta A$

$\text{Sin}\,\theta = \frac{P_1A}{P_10}$ but $P_10 = 1$ unit (unit radius)

∴ $\text{Sin}\,\theta = P_1A$ = y coordinate of P_1
= projection of OP, on the Y-axis

$\cos\theta = \frac{OA}{OP_1} = \frac{OA}{1} = OA$ but $OA = BP_1$

.. $\cos\theta = BP_1$= x coordinate of P_1
= projection of 0 P_1 on the X-axis

Thus if P is any point on a circle with center 0 and unit radius and OP makes an angle with the X-axis, then the sine and cosine of any angle may be defined thus:

Sin θ = y coordinate or the projection of OP on the y-axis and
cos θ, x coordinate or the projection of OP on the x-axis
Thus for angles 0° and 90°
Sin 90° = y coordinate = 1
cos 90° = x coordinate
= 0 (90° has no projection on the x axis)
$\tan 90° = \frac{\sin 90^0}{\cos 90^0} = \frac{1}{0} = \alpha$ (infinity)

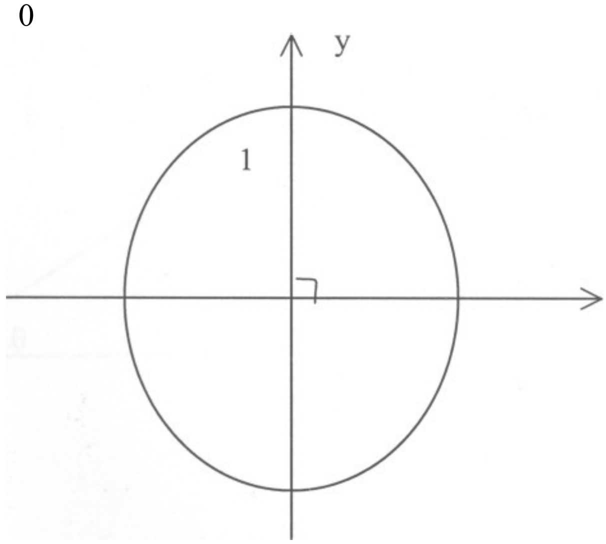


Fig: 3.2.3

Similarly for 0°
Sin 0° = y coordinate
= 0(0° has no projection on y-aixs)
cos 0° = x coordinate

= 1 (0° lies on the x-axis

$\tan 0° = \frac{\sin 0^0}{\cos 0°} = \frac{0}{1} = 0$

Alternatively;

In a right-angled triangle ABC, with < A = 90° and < C = θ which is very small

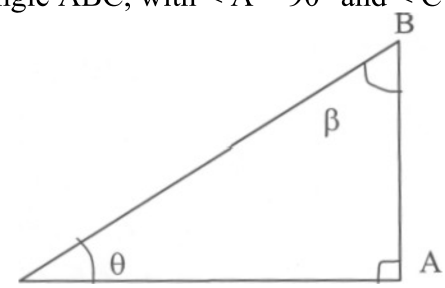


Fig: 3.2.4.

The ratios of θ are:

$\text{Sin } \theta = \frac{AB}{BC}$

$\text{Cos}\theta = \frac{AC}{BC}$

$\tan \theta = \frac{AB}{AC}$

also

$\sin \beta = \frac{AC}{BC}$

$\tan \beta = \frac{AC}{AB}$

When θ gets smaller and smaller, R becomes larger and larger, these are expressed thus as

θ tends to O i.e. θ ---- 0

β tends to 90° i.e. β---- 90°

B-->A and BC-->AC as AB--> 0

$\text{Sin } 0° = \frac{AB}{BC} \rightarrow \frac{0}{AC} = \frac{0}{AC} = 0$

$\text{Cos } 0° = \frac{AC}{BC} \rightarrow \frac{AC}{AC} = 1$

$\text{Tan } 0° = \frac{AB}{AC} \rightarrow \frac{0}{AC} = 0$

$\text{Sin } 90° = \frac{AB}{AC} \rightarrow \frac{AC}{AC} = 1$

$\text{Cos } 90° = \frac{AB}{BC} \quad \frac{0}{AC} = 0$

$\text{Tan } 90° = \frac{AC}{} \rightarrow \frac{AC}{}$

AB 0

Or since 0° and 90° are complementary angles then
Sin θ = cos (90 - 0) = cos 90° =0
Cos 0° = sin (90 -0) = sin 90' = 1

$$\text{Tan } 90^0 = \frac{\sin 90}{\cos 90} = \frac{1}{0} = \alpha$$

Here is the summary of the common trigonometric ratios. The trigonometric ratios of these special angles and that of multiples of 90° are presented in the table 1 below.

Table 1: Trigonometric Ratios of special angles

Angle A°	Sin A	Cos A	Tan A	Cot A	Sec A	Cosec A
0°	0	1	0	α	1	α
30°	½	√3/2	1/√3	√3	2/√3	2
45°	1/√3	1/√2	1	1	√2	√2
60°	√3/2	½	√3	1/√3	2	2/√3
90°	1	0	α	0	α	1
180°	0	-1	0	α	-1	α
270°	-1	0	α	0	α	-1
360°	0	1	0	α	1	α

Example:

Without using tables/calculator find the value of the following:

(1) (i) COs 90° + 1 (ii) $\frac{\sin 60°}{\cos 60}$ (iii) $\frac{2}{\sin 30} - \frac{3}{\tan^2 60} + 1$

(2) if θ = 300 evaluate $\frac{\sin^2 \theta + \tan^2 \theta \text{ x } \cos\theta}{1 - \tan \theta \text{ x } \cos^2 \theta}$

Solutions:

1. (i) cos 90° + 1 from above table, cos 90° = 0
.. Cos90° + 1 = 0 + 1 = 1
.. cos 90° + 1 = 1

(ii) $\frac{\text{S} \sin 60^0}{\cos 60°}$ = tan 60° and tan 60° =√3

OR

Sin 60° = √3/2 and cos 60 = 1/2

. $\therefore \frac{\sin 60^o}{\cos 60^o} = \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} \text{ x } \frac{2}{1} = \sqrt{3}$

$\therefore \dfrac{\sin 60^o}{\cos 60^o} = \sqrt{3}$

(iii) $\dfrac{2}{\sin 30} = \dfrac{3}{\tan^2 60} + 1$ Substituting the values,

$\dfrac{2}{½} - \dfrac{3}{(\sqrt{3})2} + 1 = 2 \times 2 - \dfrac{-3}{3} + 1$

$= 4 – 1 + 1 = 4$

2. $\dfrac{\sin^2 \theta + \tan^2 \theta \times \cos \theta}{1 - \tan^2\theta \times \cos \theta}$, substituting for $\theta = 30°$

sine 30°
cos30°;
x cos 30°

$$= \dfrac{\sin^2 30^o + \dfrac{\sin^2 30^o}{\cos^2 30^o} \times \cos 30^o}{1 - \dfrac{\sin^2 30^o}{\cos 30^o} \times \cos^2 30^o}$$

and sin 30° = 1/2 , cos 30° = 2 and tan 30° = 1 -73
it then becomes;

Alternatively substituted for tan 30 = 1/,13

$$\dfrac{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 x \left(\frac{\sqrt{3}}{2}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)^2 x \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$1 \quad = \dfrac{\frac{1}{4} + \left(\frac{\sqrt{3}}{3} x \frac{}{2}\right)}{1 - \left(\frac{1}{2} x \frac{\sqrt{3}}{2}\right)} = \dfrac{\frac{1}{4} + \frac{\sqrt{3}}{6}}{1 - ¼}$$

$\dfrac{\dfrac{3+2\sqrt{3}}{12}}{3/4}$

$\dfrac{3+2\sqrt{3}}{12} \times \dfrac{4}{3}$

$\dfrac{3+2\sqrt{3}}{9}$

Exercise 3.2

Simplify the following without using tables or calculators

1. (a) $\frac{\sin^3 330° \text{ x } \tan^2 240°}{\text{Cos}^4 30°}$

Ans; -2/3

(b) $\frac{3 - \sin^2 60°}{2 + \text{cost } 60°} + \tan^2 60^o$ Ans; 4

2. If θ = 60°, calculate, without table or calculator

(a) $\frac{\sin \theta + \cos \theta}{1+\cos^2\theta}$ Ans; $\frac{2(\sqrt{3} + 1}{5}$

(b) $\frac{25\cos 3\theta - 2 \sin \theta}{\tan \theta \cos \theta}$ Ans: $\frac{25 - 2\sqrt{3}}{4\sqrt{3}}$

3. (a) $(\sin 135° + \cos 315°)^2$ Ans; 2

(b) $\frac{\tan 240^0}{1+ \tan^2 30°}$ $\frac{\tan 315°}{1+ \tan^2 60°}$ Ans; 2.

4. If sin A = 3/5 and sin B = 5/13. where A and B are acute, find without using tables, the values of

(a) sin A cos B + cos A sin B Ans; 56/65

(b) cos A cos B + sin A sin B Ans; 33/65

(c) $\frac{\tan A - \tan B}{1 +(\tan A)(\text{taan}B)}$ Ans; 16/33

5. If A is in the fourth quadrant and cos A = 5/13 find the value of $\frac{13 \sin A + 5 \sec A}{5\tan A + 6\text{cosec}A}$ without using tables

Ans -2/37

4.0 CONCLUSION

In this unit, you have learnt the inverse trigonometric functions or circular functions, their definitions or meanings and notations, you have also learnt to find the inverse trigonometric functions from trigonometric tables, the principal value of inverse trigonometric angles, the relation between inverse trigonometric functions and their principal values and also the trigonometric ratios of common angles - how they are derived and how to find their ratios with out using tables.

5.0 SUMMARY

In this unit, you have learnt that the inverse of a trigonometric . ratios is the angle whose trigonometric ratios, is given. And these values can be found in the body of the trigonometric ratio table from where the angles are read off.

You have also learnt that to find all possible angles of a given problem first find the basic angles then add or subtract (360k°) to it i.e.

(i) All possible angles = basic angle ±(360k°) where k is any integer, positive, negative or zero.

(ii) The relation between the value of an inverse trigonometric function and its principal value are:
Arcsin x = k(180°) + $(-1)^k$ arc sin x
Arccos x = 360k° ± arcos x
Arctan x = 180k + Arctan x
Arccot x = 180k + acrcot x.

where Arcsin, or Arccos etc represent the values of inverse trigonometric functions and arcsin, arcos etc. represent their principal values.

(iii) The principal values of the following
  (a) arcsin is the value between -90° and + 90°
  (b) arccos is the value between 0° and 1800. This also applies to arccot and arcsec.
  (c) Arctan is the value between -90^0 and + 90^0

(iv) The trigonometric ratios of 0^0, 30^0, 45^0, 60°, 90°,150°, 270° and 360° are presented in the following table.

Angle A°	In degrees & ratios	Sin 0	Cos 0	Tan 0	Cot θ	Sec 0	Cosec 0
<u>Degrees</u>							
0^0	0	0	1	0	α	1	α
30^0	π/6	1/2	√3/2	1/√3	√3	2/√3	2
45^0	π/4	1/√3	I/√2	1	1	√2	√2
60^0	π/3	3/2	1/2	√3	1/√3	2	2/√3
90^0	π/2	1	0	α	0	α	1
180^0	π	0	-1	0	α	-1	α
270^0	3π/2	-1	0	α	0	α	-1
360^0	2π	0	1	0	α	1	α

6.0 TUTOR-MARKED ASSIGNMENTS

1. write an angle in the first quadrant whose tangent is
   (a) 0.8816 (b) 1.9496 (c) 2.0265
2. Find the values of 0 lying between 0° and 360° when
   (a) sin θ ½
   (b) cos θ sin 285°
   (c) tan θ = -1
3. find all the angles between 0° and 720° whose tangent is $1/\sqrt{3}$
4. simplify without tables or calculator the following:
   (a) $\frac{\sin 150° - 5\cos 300° + 7\tan 225°}{\tan 135° + 3\sin 210°}$
   (b) sin 60° cos30' + sin 30° cos 60°
5. if tan θ = 7/24 and θ is reflex, find without tables or calculator the value of,
   (a) sec θ (b) sin θ

7.0 FURTHER READING AND OTHER RESOURCES

7.1 REFERENCES

Amazigo, J. C. (ed) 1991: Introductory University Mathematics 1: Algebra, Trigonometry and Complex Numbers. Onitsha; Africana," Fep Publishers Ltd.

Bunday, B. D, and MulHolland A. (1980): Pure mathematics for Advanced Level London: Butherworth and Co. (Publishers) Ltd.

Vygodsky, M. (1972): Mathematical Handbook: elementary Mathematics. Masco: M/R Publishers.

OTHER MATERIALS:

Any mathematics textbook that, you can lay your hands on, which contain these topics.

UNIT 4

GRAPHS OF TRIGONOMETRIC FUNCTION AND THEIR RECIPROCALS

TABLE OF CONTENTS

1.0 INTRODUCTION

Graphs from elementary mathematics help to establish the relation between an independent variable and another variable a dependent variable. Hope you know what independent variables means?

In this unit, graphs of trigonometric ratios are graphs of y = sin 0 , y = cos 0 and y = tan 0 shall be treated. Also the graphs of their reciprocals. Here the relation between the values of a variable angle and the corresponding trigonometric function can be seen by means of graph. These graphs are applied in physics - radio waves, sound waves, light waves, alternating current, simple harmonic motions etc.

2.0 OBJECTIVES

By the end of this unit, the students should be able to:

- Draw the graphs of trigonometric functions and their reciprocals accurately.
- Read values of any given angle form the graphs correctly.
- Determine the periodicity (period) and amplitude of given trigonometric ratios.

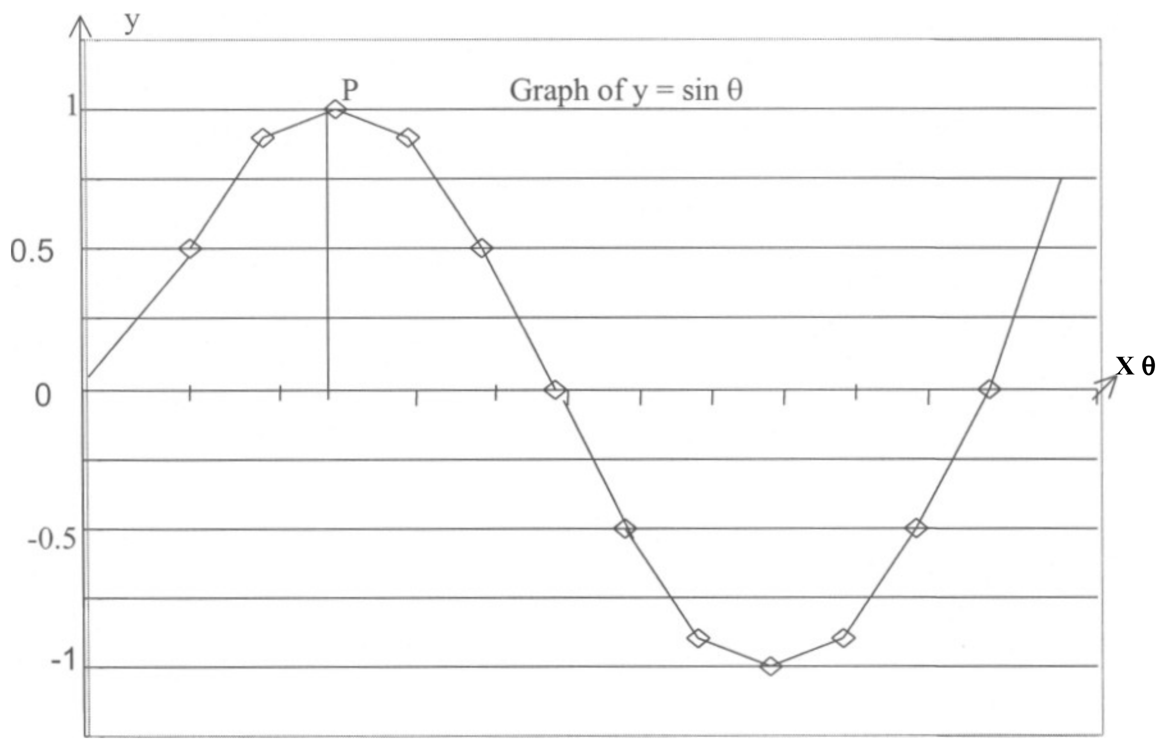


Fig: 4.1: Graph of y = sin θ

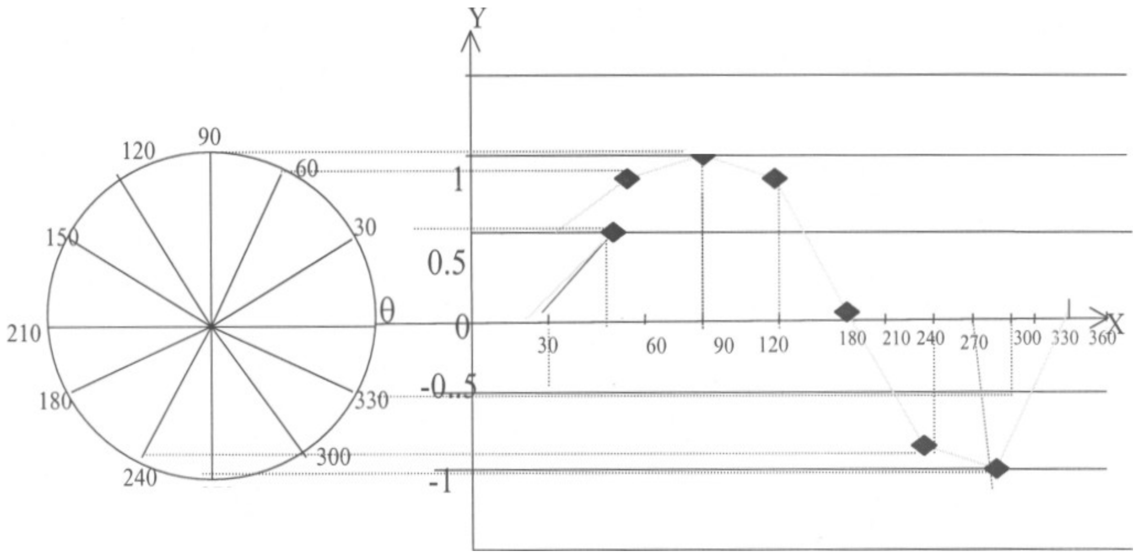


Fig: 4.1. Graph of y = sin θ (by projection)

3.1. GRAPHS OF TRIGONOMETRIC FUNCTIONS

In drawing the graphs of trigonometric ratios, abscissa (x-axis) is taken as the independent variable and the ordinate (y-axis) as the dependent variable. This is so because the values of arc dependent on the values of x. The following explains this

3.1. GRAPH OF y = sin θ

The graph of y = sin 0 will show the relationship between 0 and sin 0. This graph can be drawn in two possible ways.
Method 1: from table values

Steps:

(a) assign different values to 0° at intervals of 30° to 360 ° i.e. 0 = 0°, 30°, 60°, 90° ...,360°
(b) find the corresponding values of sin 0 , this is used to from the table of values
(c) choose a suitable scale then plot the values
(d) join the plotted points with either the free hand or a broom stick to get a smooth curve. This curve is then the graph of y = sin θ.

Thus following the steps, the table of values approximated to 2 decimal places is.

Table l: Table of Value for y = sin θ for θ ≤ 0° ≤ 360

θ	0°	30°	60°	90°	120° 00	150° 00	180° 00	210° 00	240⁰	270°	300°	330°	360°
Sin	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0

Scale:

Let 1cm represent 1 unit on the θ axis i. e. the horizontal or x-axis.

Let 4cm represent 1 unit on the sin θ axis i.e. the vertical or y-axis.

The points arte then plotted on a graph sheet and is joined by a broom stick (see over leaf graph of y = sin θ

This means that the length from the height point on the graph to the x-axis is

1.

Method 2: Projection

Steps:

(a) Construct a unit circle and mark out correctly the angles of θ = 0°, 30°, 60°, ... to 360° (see fig 4.2).
(b) Draw the x and y axis as in other graphs
(c) Draw a horizontal line through the center of the circle to meet the x-axis.
(d) On the x-axis at 30° interval, mark out the angles 0°, 30°, 60°, .. to 360°.
(e) Draw dotted horizontal lines from the angles of sectors marked on the unit circle to meet the vertical lines from their corresponding values at the x-axis at a point.
(f) Join these points, then the graph of y = sin θ is obtained see fig 4.2.

Properties of the graph of y = sin θ

1. The graph of y = sin θ or the sine curve is a continuous function i.e. it has no gaps between the values => no break
2. The value of sin θ increases from θ at 0° to that 90° and then decreases to - 1 at 270 and back to 0° at 360°.
3. The sine curve repeats itself at intervals of 360° [or comes to coincidence with itself upon a translation along the axis of abscissa(x-axis)by some amount]. It is called a period (or cycle) of the function. In this or cycle is 360°.
4. The height of the graph P D (amplitude) in the sine curve is l.

3.1.2 GRAPH OF y = cos θ for 0 ≤ θ ≤ 3600

The graph of y = cos θ is similar to the sine curve i.e. graph of y = sin θ. Here again the table of values for θ and cos θ is shown below at the intervals of 300

Table 2: Table of Values for y = cos θ0

θ	0º	30º	60º	90º	120º	150°	180º	210º	240º	270º	300º	330º	360º
Cos θ	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0	0.5	0.87	1

These points are plotted as in the sine curve and joined to give the cosine curve thus

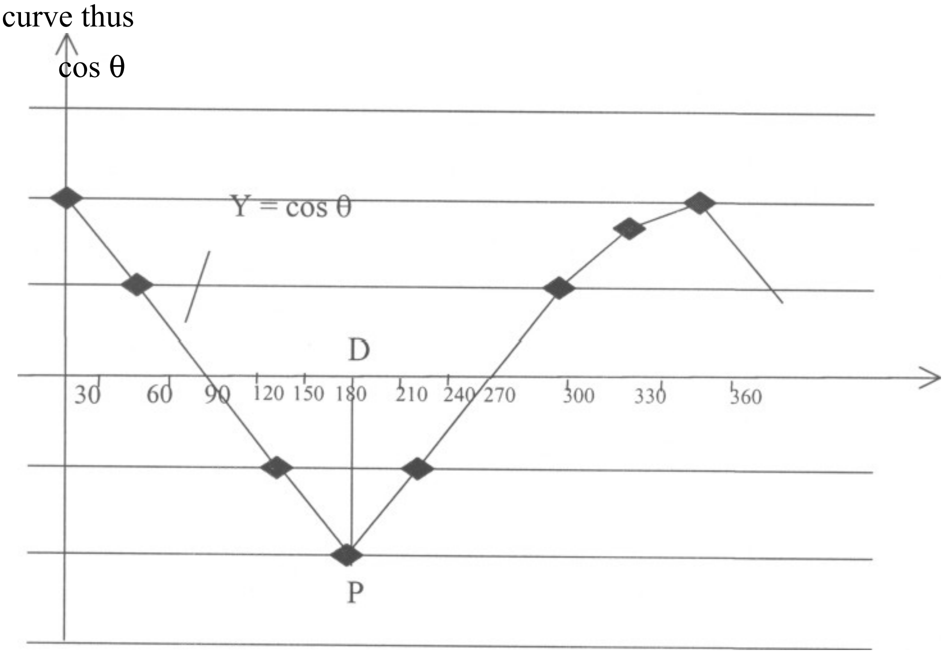


Fig 4.3 Graph of y = cos θ

Method 2: Projection A

The graph of cos θ may be drawn in a similar way to that of sine. In this case the values of cos θ= sin (90 - θ).

The universally used method of plotting graph is the method by the use of table of values. So the cosine curve will not be shown by the projection method here properties of the cosine curve.

1. The cosine curve is continuous
2. The minimum value is continuous the minimum values i.e. - 1. So like the sine curve, it lies between -1 and 1.
3. The graph repeats itself at the interval of 360° and the function is also called a periodic function with periodical 360°
4. The length of graph of $y = \cos \theta$ (amplitude) is 1.

Note the curves of the sine and cosines are identical because they have the same wavelength. The differences are that:

(1) The sine curve goes from 0 to 1 while the cosine curve goes from 1 to 0 and
(2) Since $\cos\theta = \sin (90 - \theta)$, the difference between the curves is 90°

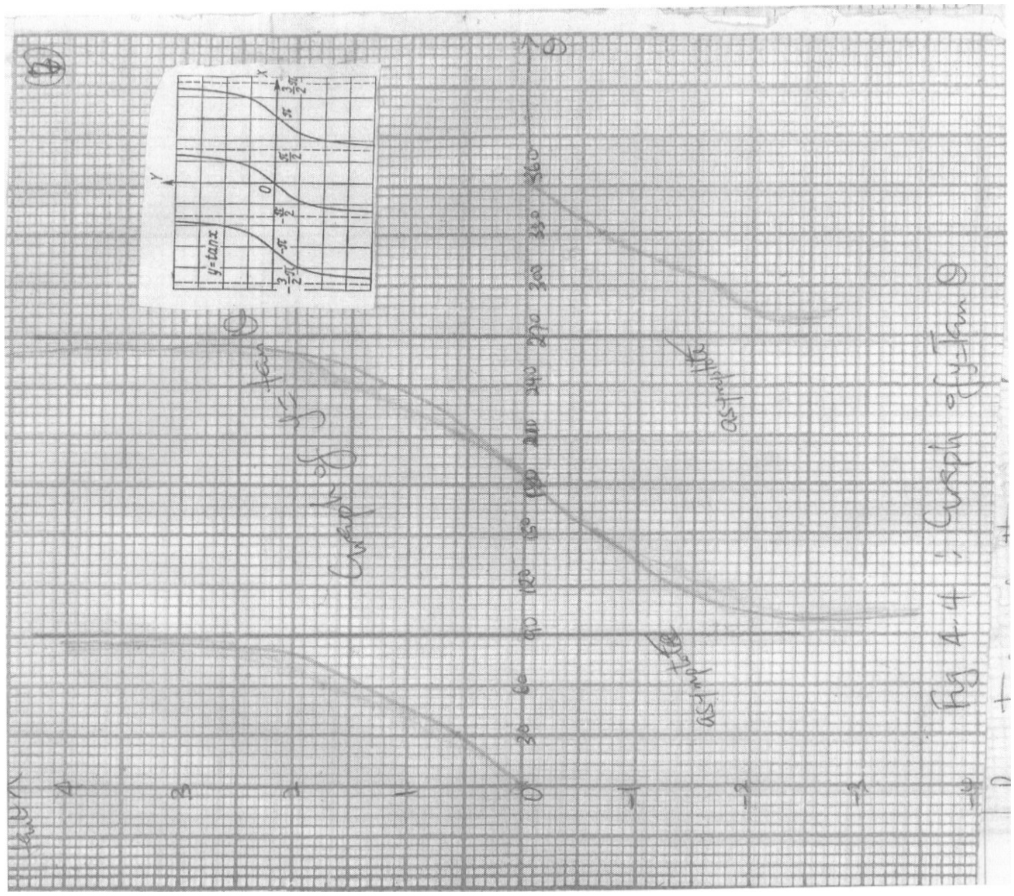
3.1.3 THE GRAPH OF y = tan θ

The graph of y = tan 0 is treated as in the case of the sine and cosine curves thus the table of values is shown in tables 3

Table 3: Table of values for y = tan 0, 00 <_ 0<_ 360°

θ	0º	30º	60º	90º	120º	150°	180º	210º	240º	270º	300º	330º	360º
Tan θ	0	0.58	1.73	α	-1.73	-0.58	0	0.58	1.73	α	-1.73	-0.58	0

Scale:
Chose suitable scales: here the scales chosen are: 1 cm for I unit at the θ-axis (x-axis)
2cm for 1 unit at the tan θ axis (y-axis)
the graph of y = tan θ is shown in fig 4.3 below.



Properties of the graph of y = tan θ

1. The tangent curve is discontinues because tan θ is not defined at 90° and 270° respectively i.e. tan 90^o = tan 270^o = α
2. The graph of y = tan θ has 3 parts namely 0° -> 90°, 90° → 270^0, → 360^0
3. The tangent curve indefinitely approaches the vertical lines at 90° and 270° but never touches them. Such lines (at 90° and 270°) are called asymptotes(here the curve approaches straight line parallel to the y-axis and distance from it by ± 90°, ±270°, ±450° etc. but never reaches these straight lines. Put in another form, the lines at 90° and 270° are said to be asymptotic curves.

3.2. GRAPHS OF RECIPROCAL OF TRIGONOMETRIC FUNCTIONS

3.2.1. GRAPH OF y = cot θ

This is the reciprocal of the graph of y = tan θ and is shown below.

Table 4; Table 3: Table of values for y = cot θ, $0^0 \leq 0 \leq 360^0$

θ	0º	30º	60º	90º	120º	150°	180º	210º	240º	270º	300º	330º	360º
Cot θ	α	1.73	0.58	0	-0.58	-1.73	α	1.73	0.58	0	-0.58	-1.73	α

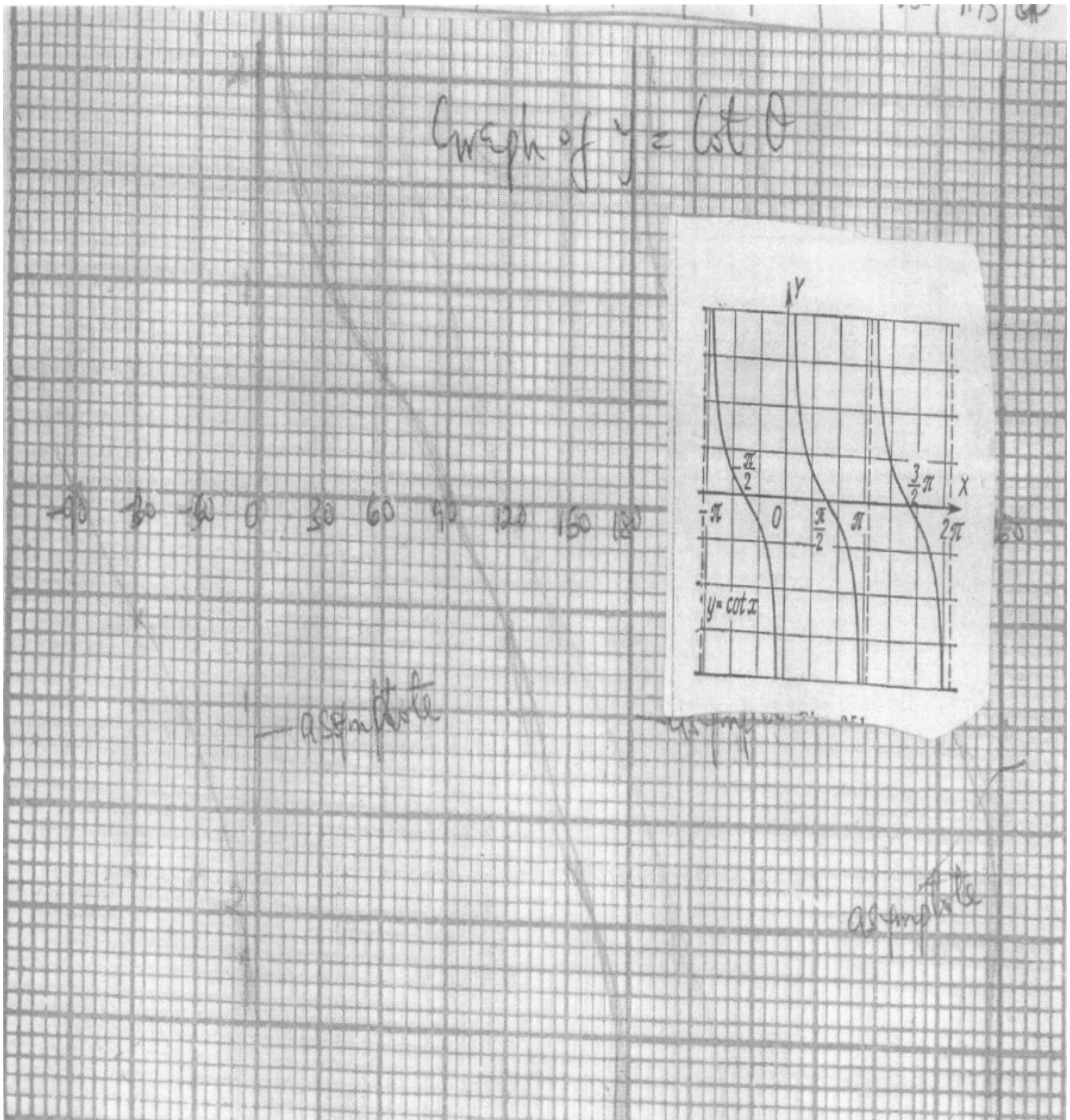
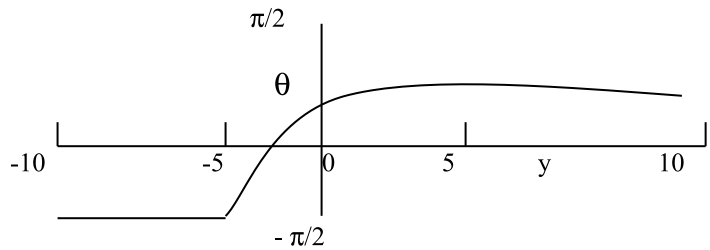
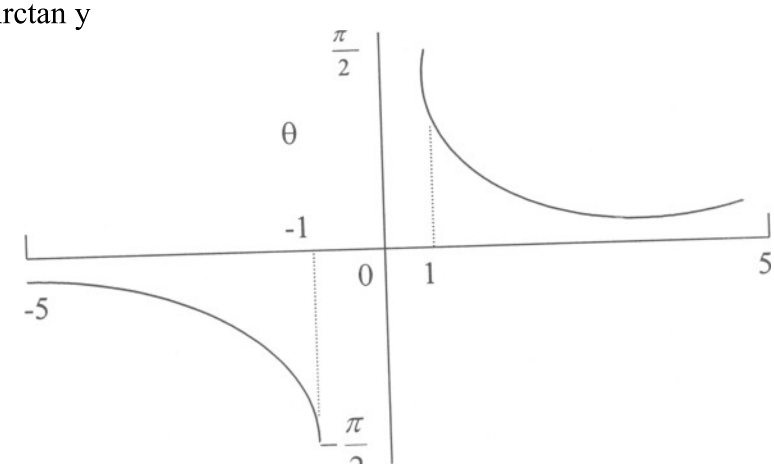


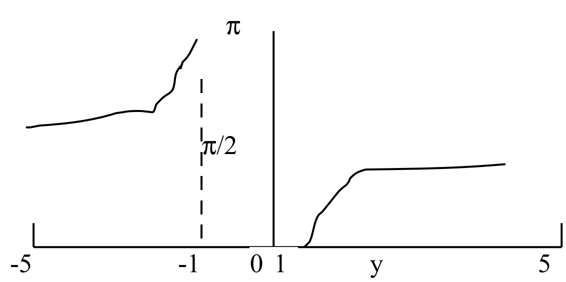
Fig 4.5 : Graph of y = Cot θ



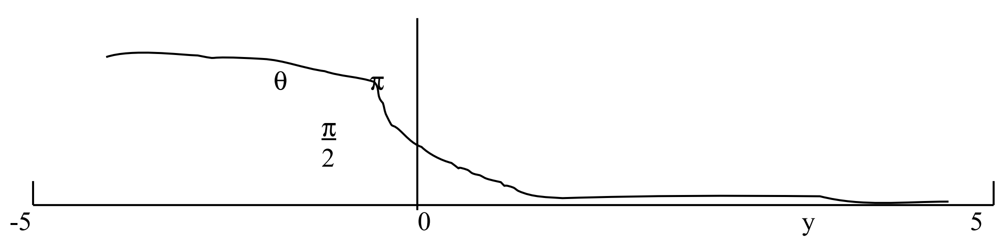
B. θ = Arctan y



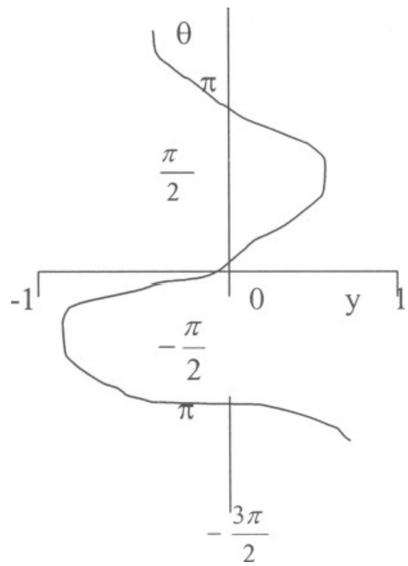
C 0 = Arccosec y



D θ = Arcsec y



E . θ = Arcot y



F. 0 = arcsin y. Range: Unrestricted.

3.2.3. APPLICATIONS OF GRAPHS OF TRIGONOMETRIC

Functions: composite functions.

Example:

(i) (a) Draw the graph of y = sin 20 for values of θ between θ and 360°
(b) use your graph to find the value of the following when θ
(ii) 25° (iii) 35° (iv) 50°

Solution:
(i) make a table of values thus for sin 2θ°

θ	0°	30°	45°	60°	90°	120°	135°	150°	180°
SIN 2θ	0	0.87	1.0	0.87	0	-0.87	-1	-0.87	0

Note when θ = 30° sin 20 = sin (2x 30) = sin 60° = 0.87 etc.
(ii) Chose a suitable scale for clarity

Here the scale of 1 cm to 30° on the 0 axis and 4 cm to 1 unit on the sin 20, axis since no value of sin 20 exceeded 1.

(iii) Plot the points. Here use graph sheet for a clearer picture of the graph.
(a) the angles being sort for are then marked out on the θ (x-axis) and a vertical line drawn from it to the graphs wherever it

touches the graph, draw a horizontal line to the y -axis (sin 2θ) axis then read off the values or its approximations

(b) θ = 25° means that sin 2θ = sin 2 x25° = sin 50°. Then 50° lies between 30° and 60° so its value will be between the values of 30°(0.5) and 60° (0.87). This value is approximately 0.85 (see graph below)

(c) θ = 35° means that sin 20 = sin 2x 35° = sin 70° 70° lies between 60° and 90°. So its value will be between the values of 60° and 90° i.e. (0.87 and 0). From the graph it is approximately 0.49

(d) θ = 50° -----> 2θ = sin 2x50 = 100°
.'. sin 100 is approximately -0.25 from the graph

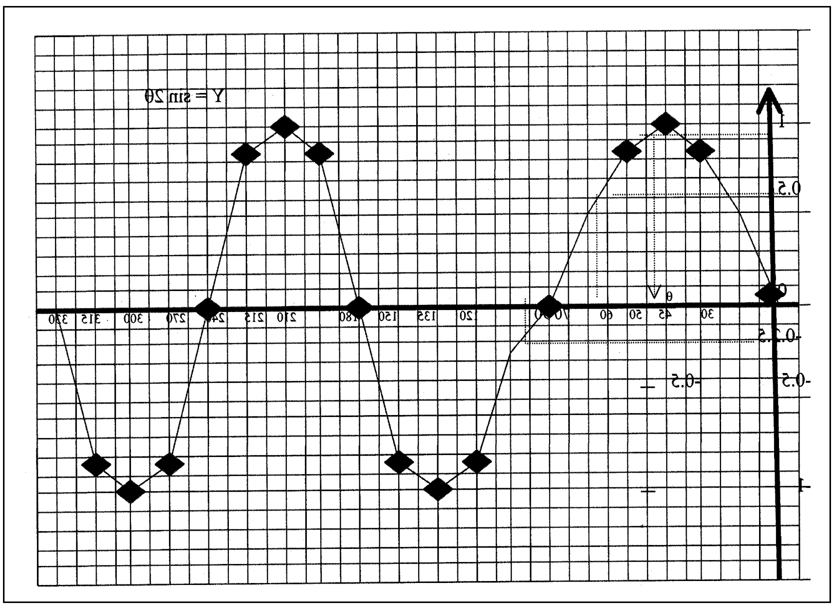


Fig: 4.6

2. Draw the graph of y = 3 - 25sin x for values of x between 0 and 360°

Solution:

The table blow shows values of y = 3, y = sin x and y = 25sin x and finally y = 3 - 2 sinx

Table of values for x from 0° to 360°

x^0	0^0	30^0	60^0	90^0	120^0	150^0	180^0	210^0	240^0	270^0	300^0	330^0	360^0

Sinx	0	0.5	0.87	1.0	0.87	0.5	0	-0.5	-0.57	-1	-0.87	-0.5	0
3	3	3	3	3	3	3	3	3	3	3	3	3	3
2sinx	0	1.0	1.74	2.0	1.74	1.0	0	-1.0	-1.74	-2.0	-1.74	-1.0	0
Y =3-2sinx	3	2.0	1.26	1.0	1.26	2.0	3	4.0	4.74	5.0	-4.74	-4.0	3

Observe that we first found the values of 2sin x (for the given values of x) before subtracting them from 3 as seen in the lastly 6 row of the table of values above.

With suitable scales the values of x i.e. plotted against the values of y = 3 - 2sin x as other graphs.

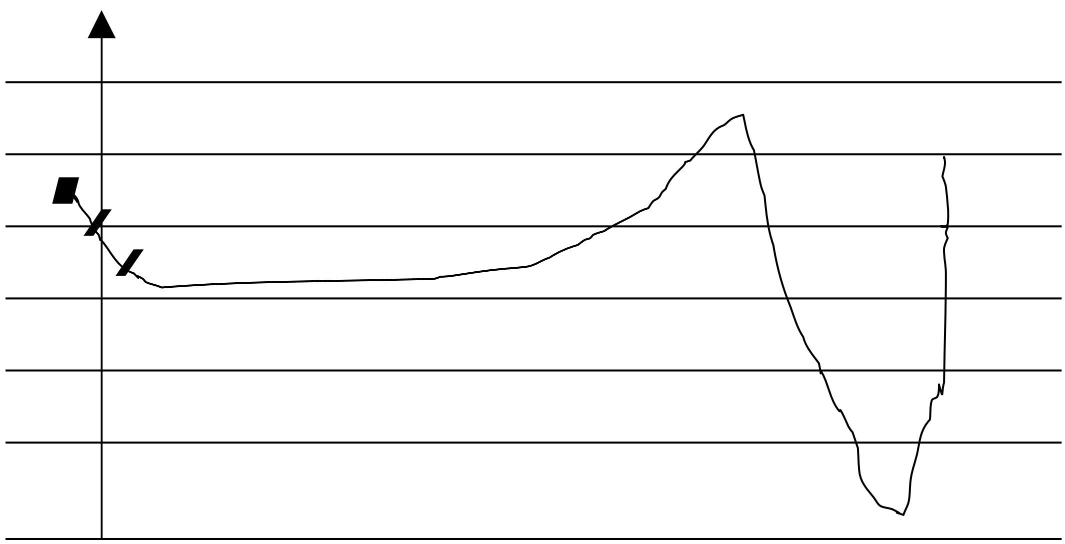


Fig 4.7

Try these ones

Exercise : 4.2

(1) (a) Construct a table for y = cos x - 3 sinx for values of x from 0° to 180° at 39° interval

(b) use a scale of 23cm to 30° on the x -axis and 2cm to 1 unit on the y-axis to draw your graph.

(2) Draw the graph of y = sinx + cosx for the interval $0° \leq x \leq 360°$ use your graph to find

(a) the maximum values of y = sinx + cos x

(b) the minimum values of y = sinx + cos x

Exercise 4.3

1. Draw the graph of the following for values of 0° from 0° to 360° inclusive.
   (a) y = cos θ (b) y = -sin
   (c) y = 1-cos θ (d) y = -2sin 2θ
2. without plotting the graph, find the;
   (i) amplitude (ii) periodicity of the following functions.
   (a) y =5 sin 7θ (b) y = 5 sin (θ+ 360°)
   (c) y = cos5β (d) y = -2 cos 2x

Solution:

(a)	i.	5	ii.	360/7
(b)	i.	5	ii.	360°
(c)	i	1	ii.	360/5
(d)	i	2	ii.	180

3 Copy and complete the table below for y = cos 2θ + 2sin θ for 0° ≤ θ ≤ 360
in the interval o f 300

Table: y = cos 2θ+ 2sin θ for 0° ≤ θ ≤ 360 in the interval of 30°

θ	00	30^0	60^0	90^0	120^0	150^0	180^0	210^0	240^0	270^0	300^0	330^0	360^0
sinθ	0	0.5	0.57	1.0	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0
2sinθ	0	1	1.73	2	1.74	1.0	0	-1	-1.73	2.4	-1.73	-1.0	0
Cos2θ	1	0.5	-0.5	-1	-0.5	0.5	1.0	0.5	-0.5	-1	-0.5	0.5	1
Y=2sinθ+cos2θ	1	1.5	1.223	1.0	1.24	1.5	1.0	-0.5	-2.23	-3	-2.24	-0.5	1

The periodicity of the cosine, secant and cosecant of an angel x are also 360° but the periodicity of the tangent and cotangent of an angle x are both 180° (this is because tan (x ± R180°) = tan x.

Note the table below shows the (1) amplitude (height) (ii) periodicity of some functions.

Angle	Function	Amplitude	Periodicity
θ	Y = sinθ	1	360°
	Y = 2sinθ	2	360°
	Y= 5 sinθ	5	360

2θ	Y = sin2	1	$360/2=180^0$
	Y =2sinθ	2	180°
	Y = 5sinθ	5	180°
$n\theta$	Y = sin nθ	1	360/ n
	Y = 2sin nθ	2	360/ n
	Y = 5 sin nθ	5	360/ n

Note that for any graph of y = A sin θ, the amplitude is /A / i.e. where A is any constant (coefficient of sin θ) and a periodicity of 360°, while if the graph is that of y = A sin n0 the amplitude is still /A/, provided A is any constant and its periodicity is 360/n where n is any constant. This also applies to cosine, secant and cosecant.
Amplitude is always a positive number.

4.0 CONCLUSION

Having treated the graph of trigonometric functions and their reciprocals and also in this unit, you have seen that the treatment of graphs here are the same with the treatment of graphs of algebraic functions, the only difference is in the values assigned to the
independent variable (x) which in this case are angles. The processes are the same thus

(1) table of values
(2) choice of scales
(3) plotting of the points and joining it is believed that the treatment of graphs
of trigonometric functions, will enable you see the interrelatedness of function waves, motions etc.

5.0 SUMMARY

In this unit, we have attempted to draw the graphs of trigonometric functions, their reciprocals and inverse functions. The properties of these graphs of trigonometric functions were highlighted such as:

(i) The sine and cosine curves are continuous functions while the tangent and cotangent are discontinuous functions.

(ii) The periodicity of a function is the interval at which the graph repeats itself and such functions are called periodic functions example, the sine, cosine, tangent etc are periodic functions.

(iii) The periodicity for the sin, cos, sec and cosec is 360° while that of the tan and cot is 180°

(iv) The amplitude or the length of a graph is the distance between the highest point and the x-axis of the function

(v) The sine and cosine curves lies between -1 and 1 and they have the similar shape because cos θ - sin (90 - θ)

7.0 REFERENCE

Backhouse, J. K. and Houldsworth, S. P. T. (1971): Pure Mathematics: A - First Course. S. 1. Edition London: Longman Group Ltd.

Buthday, B. D. and Mulholland H. (1980): Pure Mathematics For Advanced Level. London: Butterworth and Co (publishers) Ltd.

Talbert, J> F> Godman, A and Ogum, G. E. O. (1992) Additional Mathematics for West Africa London: Longman Group UK Ltd.

The West African Examinations Council (1997) Mathematical and statistical Tables and Formulae.

Vygodsky, M. (1972). Mathematical Handbook: Elementary Mathematics. Moscow: M/R Publishers

UNIT 5

TRIGONOMETRIC IDENTITIES AND EQUATIONS

TABLE OF CONTENT

1.0 INTRODUCTION

In the earlier units on trigonometric ratios, their reciprocals and inverse trigonometric functions there ware lots of important relations between trigonometric functions. For example;

$\frac{\sin\theta}{\cos\theta} = \tan\theta$; $\frac{1}{\sin\theta} = \text{cosec}\,\theta$

$\frac{\cos\theta}{\sin\theta} = \cot\theta$; $\quad \frac{1}{\cos\theta} = \sec\theta$

If these relations are true for any given value of θ such relations are called trigonometric identities, provided the functions are defined.

This unit will focus on trigonometric identities, which should form the basis for proving other identities, Compound angles, difference and product formulae, multiple and half angles and finally trigonometric equations, which are embedded in them.

2.0 OBJECTIVES

By the end of this unit, the students should be able to:

- Define trigonometric identities correctly
- Prove given trigonometric identities correctly
- Simply and solve problems involving trigonometric identities and equations.
- Express sum and difference of two given angles in trigonometric identities
- Express multiple and half angles of given identities
- Factorize trigonometric expressions

3.1 TRIGONOMETRIC IDENTITIES (FUNDAMENTAL IDENTITIES)

3.1.1. TRIGONOMETRIC IDENTITIES (RIGHT-ANGLED TRIANGLE)

Trigonometric identities are relations, which are true for any given value of given a right-angled triangle ABC, right-angled at B and angle C = θ with the usual notations see Fig 5.1.

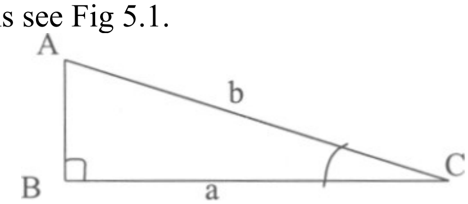


Fig: 5.1.

$\text{Sin}\,\theta = \frac{c}{b}$ -----> b sin c _____________ (1)

$\text{Cos}\,\theta = \frac{a}{B}$ -------> b cos θ = a _____________ (2)

By Pythagoras theorem $a^2 + c^2 = b^2$, so substituting the values of a and c from (1) and (2) we obtain;

$(b \cos \theta)^2 + (b \sin \theta)^2 = b^2$, simplifying
$b^2 COS^2 \theta + b^2 \sin \theta = b^2$,
dividing through by b^2, we have

$$\sin^2\theta + \cos^2\theta = 1 \text{ OR } \cos^2\theta + \sin^2 = 1$$

Also

$$\sin^2\theta = 1 - \cos^2 \theta \text{ and}$$

$$\cos^2 \theta = 1 - \sin^2\theta$$

Example:

1. Without tables or calculators find
   (a) $\sin^2 60° + \cos^2 60°$ (b) $\sin^2 330° + \cos^2 330°$

Solution:

(a) $\sin^2 60^o = \sqrt{3}/2$ and $\cos^2 60° = ½$ and substituting into $\sin^2 60° + \cos^2 60°$, gives;

$$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= \frac{4}{4} = 1$$

(b) $\sin 330° = -\sin 30° = -1/2$ and $\cos 330° = \cos 30 = \sqrt{3}/2$ substituting into the given expression $\sin^2 330° + \cos^2 330°$ to obtain;

since this relation $\sin^2\theta + \cos^2\theta = 1$, holds true for all values of θ, it is then a trigonometric identity.

From the above trigonometric identity $\sin^2 + \cos^2 = 1$, the following trigonometric identities can be deduced:

$\sin^2 \theta + \cos^2 \theta = 1$

Divide through by cos θ, this becomes

$$\frac{\text{Sin}^2\theta}{\text{Cos}^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

but $\frac{\sin^2\theta}{\cos^2\theta} = \tan\theta$ and $\frac{1}{\cos\theta} = \sec\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + 1 = \frac{1}{\cos\theta} \text{ gives } \left(\frac{\sin\theta}{\cos\theta}\right)^2 + 1 = \text{Sec}^2\theta$$

$= \text{Tan}^2\theta + 1 = \sec^2\theta$
$= \sec^2\theta - 1 = \tan^2\theta$

Again, if we divide $\sin^2\theta + \cos^2\theta = 1$ by $\sin^2\theta$, it becomes

$$\frac{\text{Sin}^2\theta}{\sin^2\theta} + \frac{\cos\theta}{\sin^2\theta} \quad \frac{1}{\sin^2\theta}, \text{ but}$$

$\frac{\cos\theta}{\sin\theta} = \cot\theta$ and $\frac{1}{\sin\theta} = \text{cosec}\,\theta$

$1 + \cot^2\theta = \text{cosec}^2\theta$
$\text{Cosec}^2\theta - 1 = \cot^2\theta$

Other relations which can be deduced are

(1) $\tan\theta \times \cot\theta = 1$
(2) $\cos\theta \times \sec\theta = 1$
(3) $\sin\theta \times \text{cosec}\,\theta = 1$
(4)

$\frac{1-\cos^2\theta}{\sin^2\theta}$ Note that $1 - \cos^2\theta = \sin^2\theta$

$$\frac{1-\cos^2\theta}{\sin^2\theta} = \frac{\sin^2\theta}{\sin^2\theta}$$

.'. $1- \cos^2 \theta = 1$

Hence from these $\sin^2\theta$ examples it can be deduced that knowing the value one of the trigonometric functions of an acute angle is possible to find the value of the others.

3.1.2. TRIGONOMETRIC EQUATIONS.

Trigonometric equation is an equation involving an unknown quantity under the sign of a trigonometric function.

Techniques for solving trigonometric equations.:

(1) take care to see that the transformed equation is equivalent to the original
equation.

(8) reduce the given equation to an equation involving only one trigonometric ratio where involving only one trigonometric ratio where possible. This is about the simplest way of solving a trigonometric equation, example $3+ 2 \cos \theta = 4 \sin^2\theta$, it is convenient to express this equation in terms of $\cos \theta$ since $\sin^2\theta = 1 - \cos$ i.e.
$3+ 2 \cos \theta = 4 (1- \cos^2\theta)$ the solve the equation as a quadratic equation in one variable ($\cos^2\theta$)

(3) when the terms of the equation have been squared or you have performed some transformed that do not guarantee equivalence, check all the solutions to avoid less of roots.

Example:

1. Solve the equation, giving values of θ from 0 to 360 inclusive.
(a) $3 - 3 \cos \theta = 2 \sin^2 \theta$ (b) $\cos^2 \theta + \sin \theta + 1 = 0$

Solution

(a) $3-3\cos\theta = 2\sin^2 \theta$, here the members of the equation can be expressed as

$\cos \theta$ since $\sin \theta = 1 – \cos^2 \theta$

$\therefore\ 3 - 3\cos \theta = 2(1 - \cos^2\theta)$
$= 3-3\cos\theta = 2-2\cos^2\theta$
$= 2\cos^2\theta - 3\cos \theta +1 = 0.$

This is a quadratic equation in cos θ and thus can be solved by any of the methods of quadratic equation.

By factorization $2\cos\theta - 3\cos\theta + 1 = 0$ gives;
$(2\cos\theta - 1)(\cos\theta - 1) = 0$

either $2\cos\theta - 1 = 0$ OR

$\cos\theta - 1 =$

if $2\cos\theta - 1 = 0$ =:> $\cos = \frac{1}{2}$ and cos θ is +ve
$\therefore$ $\theta = \cos^{-1} \frac{1}{2}$ $= 60° \text{ or } 300°$
if $\cos\theta = 1$, $\theta = \cos^{-1} 1 = 0 \text{ or } 360°$
$\therefore$ the values of θ which satisfy the equation within the given range of $0 \leq \theta \leq 360°$ are $\theta = 0°$. 60°, 300° and 360°.

(b) $\text{Cos}^2\,\theta + \sin\theta + 1 = 0$

It is easier to transform $\text{Cos}^2\,\theta$ to $1\text{-}\sin\theta$ to form an equation of powers of a $\sin^2\theta$. Thus;

$(1 - \sin^2\theta) + \sin\theta + 1 = 0$
$1\text{-}\sin^2\theta + \sin\theta + 1 = 0$
=:> $1\text{-}\sin^2\theta + \sin\theta + 2 = 0$

Factorizing: $(\sin\theta - 2)(\sin\theta + l) = 0$
.'. either $\sin\theta - 2 = 0$ or $\sin\theta + 1 = 0$
if $\sin\theta - 2 = 0 \Rightarrow \sin\theta = 2$ and $\theta = \sin^{-1} 2$

This value of θ does not satisfy the given equation because sinθ lies between -1 and 1 to satisfy the given equation. So $\theta = \sin^{-1} 2$ is not a solution if $\sin\theta + 1 = 0$.

$\text{Sin} = - 1 \Rightarrow \theta = \sin^{-1}(-1) = 270°$
$\therefore$ $\theta = 270°$ is the root of the equation becomes it falls within the range $0 \leq \theta \leq 360°$

2. Find all the solutions of the equation in the interval $0 < \theta \leq 360°$
$16\cos^2\theta + 2\sin\theta = 13$

Solution

$\text{Cos}^2\theta = 1 \sin^2\theta$, this will be substituted into the equation to give;
$16(1\text{-} \sin^2\theta) + 2\sin\theta = 13$

$16 – 16 \sin^2 \theta + 2\sin \theta = 13 = 0 =:> 16\sin^2 \theta - 2\sin \theta - 16 – 13 = 0 => 16\sin^2 \theta$ $-2\sin - 3 = 0$

Factorising gives $(8\sin\theta + 3)(2\sin \theta - 1) = 0$

.'. either $8\sin\theta + 3 = 0$ OR
$2\sin\theta - 1 = 0$

so, if $8\sin 0 + 3 = 0 =:> \sin\theta = -3/8$
$\theta = \sin^{-1}(-3/8) = \sin^{-1}(-0.375)$
From the tables 0 = -22°. This lies either in the third or fourth quadrant since sinθ is negative

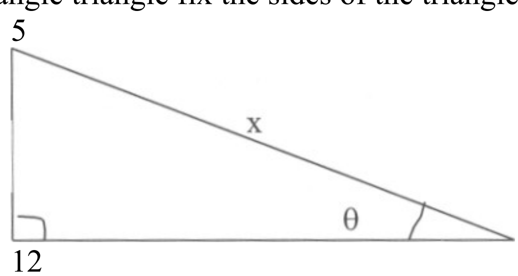
∴ $\theta = 180 + 22° 2'$ or $360° - 22° 2'$
$= 202° 2'$ or $337° - 58'$
if $2\sin\theta = 1 = ===> \sin \theta = ½ =:> \theta = \sin^{-1} (½)$
.'. θ = 30° since sinθ is positive, θ

is either in the first or second quadrant

.'. θ = 30° or 180° - 30°
= 30 ° or 150°
∴ the solution of the equations for $0 \leq \theta \leq 360°$
is θ = 30, 150°, 202° 21' and 337° 58' .

3. Find without table, the value of sec θ, sinθ if tan θ = -5/12

Solution;
Using a right angle triangle fix the sides of the triangle using the knowledge



Fig; 5.1

That tan θ is $\frac{Opposite}{adjacent}$. Finding x i.e. x ≈ the hypotenuse side by
Pythagoras theorem gives $5^2 + 12^2 = x^2$

25+144=x^2y=$\cos^2\theta$ + 2sin θ for 0°≤ θ ≤ 360 in the interval of 30°

$169 = x^2$

.'. $x = \sqrt{169} = \pm 13$

.'. sin θ = 5/±13 and cos θ = ±12/±3, but θ is obtuse, hence sin θ = 5/13 and

cosθ = -12/13

.'. sin θ = 5/13 and sec θ = $1/\cos\theta$

gives; $\sec\theta = \frac{-1}{\frac{12}{13}} - \frac{-13}{12}$

4. Prove the following identities:

$\sec^2\theta + \text{cosec}^2\,\theta = \sec\theta\,\text{cosec}^2\,\theta$

Solution:

In problems of this sort, start from whatever expressions (either left hand side or right hand side) to show that it is equal to the other (right hand side or left hand side) whichever is simpler Thus starting from the left hand side (LHS)

$$\text{Sec}^2\,\theta + \text{cosec}^2\,\theta = \frac{1}{\text{Cos}^2\theta} + \frac{1}{\sin^2\theta}$$

Simplifying gives $\frac{\sin^2\theta + \cos\theta}{\text{Cos}^2\theta\,\sin^2\theta}$

But $\sin^2\theta + \cos^2\theta = 1$

$$\therefore \quad \frac{\sin^2\theta + \cos\theta}{\text{Cos}^2\theta\,\sin^2\theta} = \frac{1}{\text{Cos}^2\theta\,\sin^2\theta}$$

$$= \frac{1}{\text{Cos}^2\theta} + \frac{1}{\sin^2\theta}$$

$$= \sec^2\theta \times \text{cosec}^2\theta = \sec^2\theta \times \text{cosec}^2\theta = \text{RHS}$$

Note that in examples 1 and 2 we concentrated only on angles in the first revolution or basic angles. This is because in many applications of trigonometry they are the ones usually required

3.2. COMPOUND ANGLES

3.2.1. (A) ADDITION FORMULAE

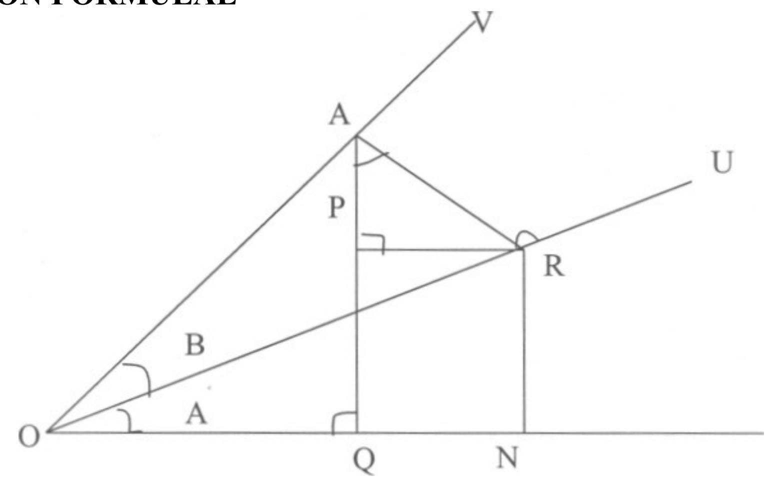


FIG 5.2.

In fig 5.2 above

< PAR = 90° - <ARP

<PRO = <RON (alt <s PR| |ON)

$$\text{Sin}(A+B) = \frac{AQ}{\theta A} = \frac{PQ + AP}{\theta A}$$

$$\frac{RN + AP}{\theta A} = \frac{RN}{\theta A} + \frac{AP}{\theta A}$$

$$\frac{RN}{OR}\,\frac{OR}{OA} + \frac{AP}{AR}\,\frac{AR}{OA}$$

$$= \text{Sin A Cos B} + \text{Cos A Sin B}$$

$$\therefore\ \text{Sin}(A+B) = \text{Sin A Cos B} + \text{Cos A Sin B}$$

Similarly from the same fig 5.2

$$\text{Cos}(A+B) = \frac{0Q}{\theta A} = \frac{\theta N - QN}{\theta A}$$

$$= \frac{ON - PR}{OA} = \frac{ON}{OA} - \frac{PR}{OA}$$

$$\frac{ON}{OR}\,\frac{OR}{OA} - \frac{PR}{MR}\,\frac{MR}{OA}$$

= CosACosB - SinASinB

~

.'. Cos(A +B) = CosACosB - SinASinB

$$\text{Tan}(A+B) = \frac{\text{Sin}(A+B)}{\text{Cos}(A+B)} \qquad \text{sin ce tan}\,\theta = \frac{\text{Sin}\,\theta}{\text{Cos}\,\theta}$$

$$\frac{\text{Sin A Cos B + CosASin B}}{\text{CosACosB} - \text{SinASinB}}$$

Dividing both the numerator and denominator by CosACosB,

$$\text{Tan}(A+B) = \frac{\frac{\text{SinACosB}}{\text{CosACosB}} + \frac{\text{CosASinB}}{\text{CosACosB}}}{\frac{\text{CosACoB}}{\text{CosACosB}} + \frac{\text{SinASinB}}{\text{CosACosB}}}$$

Simplifying gives; $\frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$.'.\ \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

3.2.1b. DIFFERENCE FORMULAE

The difference formulae can be obtained from the addition formula for replacing B with (-B) in each case thus ;

(a) Sin(A - B) = SinACosB - CosASinB

(b) Cos(A - B) = CosACosB + SinASinB

(c) $\text{Tan}(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Example:

Without using tables or calculators find the values of the following leaving your answers in surd form.

(i) Cos(45° - 30') (ii) sin(60 + 45°) (ii) tan 75°

Solutions;

(i) Cos (45° - 30°) is in the form of cos (A -B) and by the addition/difference formula it is

(Cos(A - B) = CosACosB + SinASinB expanding Cos(45 - 30) thus, where A = 45 and B = 30 gives

Cos 45Cos30 + Sin45° Sin30 so substituting the values for

Cos 45 = 1/√2 and sin 45 = 1/√2

Cos 30 = √3/2 and Sin 30 = ½ in

Cos 45Cos30 Sin45Sin30 gives
(1/√2) (√3/2) + (1/√2)(1/2)

= √3/2√2 + 1/2√2

$$\frac{\sqrt{3} + 1}{2\sqrt{2}} = \frac{1 + \sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}\,(1 + \sqrt{3})}{4}$$

(ii) Sin (60 +45) = sin60 Cos45 + Cos60Sin45, here
Sin(A +B) = SinACosB + CosASinB as applied
If A = 60 and B = 45
and substituting the values of Sin 60° = √3/2 , Cos60 = ½ ,
Sin45° = Cos° = 1/√2 to obtain;

(√3/2)(1/2) + (1/2)(1/√2)

$$\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$\frac{1 + \sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}\,(1 + \sqrt{3}}{4}$$

(iii) tan 75° = tan (45° + 30°) Applying the formula

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}, \text{ where}$$

A = 45 and B = 30 and tan 45° = 1 and tan 30 = 1/√3

gives

$$\frac{1 + \frac{1}{\sqrt{3}}}{1 - 1\ \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3} + 1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

and simplifying by rationalizing the denominator gives;

$$\frac{(\sqrt{3}+1)(3+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{3+2\sqrt{3}+1}{3-3+\sqrt{3}-1}$$

$$= \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$$

Exercise 5.1

1. Prove the following identities
   - (a) $\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$
   - (b) $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$
   - (c) $2\cos^2\theta - 1 = 1 - 2\sin^{20} = \cos^2\theta - \sin^2\theta$
   - (d) $\operatorname{cosec}\theta + \tan\theta\sec\theta = \operatorname{cosec}\theta\sec^2\theta$

2. Solve the following equations, giving values of θ from 0° to 360° inclusive
   - (a) $\sec^2\theta = 3\tan\theta - 1$ Ans: 45°, 63^0 26`, 225°, 243^0 26'
   - (b) $3\cos^2\theta = 7\cos\theta + 6$ Ans: 131° 49', 228° 12'
   - (c) $2\sin\theta = 1$ Ans: 30°, 150°.

3. Find without tables/calculators the values of
   - (a) Sin θ, tan θ, if cos θ =45 and θ is acute
     Ans: sin θ = 3/5 and tan θ = ¾
   - (b) cos θ, cot θ, if sin θ = 15/17 and θ is acute
     Ans: cos θ = 8/17. cot θ = 8/15
   - (c) sen θ, sec θ, if cot θ = 20/21 and θ is reflex;
     Ans: sin θ = - 21/29 sec θ = -29/20

4. If sinA = 4/5 and cosB = 12/13, where A is obtuse and B is acute, find without tables/calculators the values of
   - (a) sin (A – B) Ans: 63/65
   - (b) tan (A – B) Ans: -63/16
   - (c) tan (A + B) Ans: -33/56

3.2.2. MULTIPLE AND HALF ANGLE

3.2.2a. MULTIPLE ANGLES (DOUBLE ANGLE)

This is an extension of the addition formula; In each case, putting B = A we obtain for sin (A+ B) = sin (A + A) = sin 2A since sin(A + B) = sinAcosB + cosAsinB and replacing B with A gives:

sin(A + A) = sinAcosA + cosAsinA
$\therefore$ sin 2A = 2sinAcosA and
cos(A +A) = cosAcosA - SinAsinA
cos(2A) = Cos^2 A - $\sin^2$ A but $\sin^2$ A= 1 - $\cos^2$ A
substituting gives cos(2A) = $\cos^2$A - 1 + $\cos^2$A
.'. $\cos^2$A = 2$\cos^2$A - 1 and $\cos^2$ = 1 –$\sin^2$ A s
so $\cos^2$A = 1- $\sin^2$A - $\sin^2$ A = 1- 2sin A
.'. $\cos^2$A = $\cos^2$A- $\sin^2$A
=2$\cos^2$A - 1

$$\tan 2A = \frac{\tan A + \tan A}{1 - \tan A \tan A}$$

$$\frac{2\tan A}{1-\tan^2 A}$$

3.2.2b. HALF ANGLES

By substituting half angles example A/2 or B/2 into the double angles, the formulae above become

(a) $\sin\left(\frac{A}{2} + \frac{A}{2}\right) = \sin A = 2\sin\frac{A}{2}\,\text{Cos}\,\frac{A}{2}$

(b) $\text{COS}\left(\frac{A}{2} + \frac{A}{2}\right) = \text{COSA} = \text{COS}^2\frac{A}{2} - \sin^2\frac{A}{2}$

$= 2\cos^2\frac{A}{2} - 1$

$= 1 - 2\sin^2\frac{A}{2}$

(c) $\tan\left(\frac{A}{2} + \frac{A}{2}\right) = \tan A = \frac{2\tan A/2}{1-\tan^2 A/2}$

(d) $\sin^2\frac{A}{2} = \frac{1 - \cos A}{2}$, this comes from $\sin^2 A == \frac{1-\cos A}{2}$,

(e) $\cos^2 \frac{A}{2} = \frac{1 + \cos A}{2}$

(f) $\tan^2 \frac{A}{2} = \frac{1 - \cos A}{1 + \cos A}$

3.3. SUM AND DIFFERENCE FORMULAE (FACTOR FORMULAE)

(a) sin(A + B) = sinAcosB + cosAsinB ___________ (1)
sin(A- B) = sinAcosB –CosAsinB ___________ (2)
adding both (1) and (2)
sin(A +B) + sin (A -B) = 2sinAcosB
subtracting both (1) and (2)
sin(A + B) - sin(A -B) = 2cosAsinB

(b) cos(A +B) = cosAcosB – sinAsinB ___________ (3)
cos(A - B) = cosAcosB + sinAsinB ____________ (4)
adding both (3) and (4)
cos(A+B) + cos(A -B) = 2cosAcosB
and substracting both (3) and (4) gives:
cos(A+ B) - cos(A - B) = -2sinAsinB
which can be rewritten as
cos(A -B) - cos(A+B) = 2sinAsinB
(this is to avoid the minus sign gotten in the first one).

3.3.1 PRODUCT FORMULAE

From the above sum and difference formulae, another interesting identities emerged
Sin(A+B) + sin(A - B) = 2sinAcosB if A+B is equal to x i.e. A+B = x and A - B = y, this implies that adding both gives

$$\text{Sin}(A+B) + \sin(A-B) = 2\sin\frac{x+y}{2}\ \text{Cos}\frac{x-y}{2}$$

$$\text{Sin}x + \sin y = 2\sin\frac{x+y}{2}\ \text{Cos}\frac{x-y}{2}$$

$$\text{Sin}x - \sin y = 2\cos\frac{x+y}{2}\ \sin\frac{x-y}{2}$$

$$\cos x + \text{Cos}\, y = 2\cos\frac{x+y}{2}\ \cos\frac{x-y}{2}$$

$$\text{Cos}\, x - \text{Cos}\, y = 2\sin\frac{x+y}{2}\ \sin\frac{x-y}{2}$$

$$\text{Cos}y - \text{Cos}x = 2\sin\frac{x+y}{2}\ \sin\frac{x-y}{2}$$

These formulae can also be stated in this form

CosAcosB = ½ {cos(A + B) + cos(A -B) }

SinAsinB= ½ {sin(A -B) - cos(A+B)}

SinAcosB = ½ { sin(A + B) + sin(A -B) }

These are called the product formulae.

Example

Find the value of the following angles without tables or calculators.
(a) cos 75° cos l5° (b) sin75° + sin 15' (c) cos83° - cos 17°

Solution

(a) to solve the given problem, apply the product at formula which states that cosAcosB = ½ {cos(A + B) + cos(A -B) } so taking A = 75° and B = 15° substituting gives

cos 75°cosl5° = ½ { cos(75 +15) + cos(75 -15) }
= ½ { cos90 + cos 60}

and the values of cos90' and 60° without tables or calculator are:
cos 90 = 0 and 60= ½

and substituting

∴ cos75°cos 15° = ½ (0 + 1/2)
= ½ x ½ = ¼

(b) For sin75° + sin 15° to solve this apply the sum and difference formula which states that sinx + siny = $2\sin\frac{x + y}{2}$ $\cos\frac{x - y}{2}$
so taking x = 75' and y ==15º and substituting into the formula gives:

$\sin\frac{75° + \sin 15°}{2} = \cos\frac{75 - 15}{2}$

$=2\sin\frac{90}{2}\ \cos\frac{60}{2}$

$= 2\sin45°\cos30°$

the values of $\sin45° = 1/\sqrt{2}$ $\cos30° = \sqrt{3}/2$, are known, so substituting back

$\sin75° + \sin 15° = 2\,(1/\sqrt{2})(\sqrt{3}/2)$
$= \sqrt{3}/\sqrt{2}$
$= \frac{\sqrt{3} \times \sqrt{2}}{2} = \frac{\sqrt{6}}{2}$

(c) $\cos 83° - \cos 17$, since $\cos A - \cos B = -2\sin\frac{A + B}{2}\sin\frac{A - B}{2}$, then substituting for
$A = 83°$ and $B = 17°$ into the above formula, we have

$\text{Cos}83° - \cos 17° = -2\sin\frac{83 + 17}{2}\sin\frac{83 - 17}{2}$
$= -2\sin 100/2 \sin 66/2$
$= -2\sin 50° \sin 33°$.

2. Solve the equation $\sin 5x + \sin 3x - 0$ for values of x from $- 180°$ to $180°$ inclusive.

Solution

Applying the formula. $\text{Sin}A+ \sin B = 2\sin\frac{A + B}{2}\cos\frac{A - B}{2}$, and substituting for $A = 5x$
and $B =3x$ gives;
$\text{Sin}5x + \text{Sin}3x = 2\sin\frac{5x+3x}{2}\cos\frac{5x-3x}{2}$
$= 2\sin\frac{8x}{2}\cos\frac{2x}{2}$
$= 2\sin 4x \text{ cox}$

but $\sin 5x + \sin3x = 0$,
this implies that $2\sin 4x\cos x = 0$ but 2 cannot be zero
∴ either so, $\sin4x \cos x = 0$ or $\cos x = 0$
since x lies in the range of $-180°$ to $180°$
∴ 4x will lie in the range of $4(-180)$ to $4(180) = -720$ to 720
so if $\cos x = 0$ => $x = 90°$ or $- 90°$
∴ $x = - 90°, 90°$

if sin 4x = 0 sin-10 = 4x

since these are values at which when sinx 4x = 0, -180 and 180 the 0 between - 180 and 180.

4x = -720°, -180°,0°, 180°, 720° (including the intervals)

∴ x = - 180°, -45°, 0°, 45°, 180° (dividing through by 4)

so the value of x which satisfies the equation are;

x = -180°, -90°, -45°, 0°, 45°, 90°1, 180°.

Exercise 5.2

1. Solve the equation sin(x+ 17°) cos(x -12°) = 0. 7 for values of x from 0° to 360 inclusive.

   Ans: x = 30° 37', 54^0 23', 210^0 37', 234^0 23'

2. Prove the identities
   (a) $\frac{\cos B + \cos C}{\sin B - \sin C} = \cot \frac{B - C}{2}$
   (b) sin x - sin(x+60) + sin (x+120° =0
   (c) cosx + cos(x+120) + cos(x+240°) =0

3. Solve for the following equations, for values of x from 0° to 360° inclusive.
   (a) cosx + cos 5x = 0
   Ans: 30°, 90°, 150°, 240°, 270°, 330°, 45°, 135°, 225°, 315°.
   (b) sin3x + cos2x = 0 (hint cos2x = sin(90° - 2x)
   Ans: 54°, 126°, 198°, 270^0, 342^0 .

4. Express the following in factors.
   (a) sin2y - sin2x
   Ans: 2cos(y + x) siny -x)
   (b) sin(x + 30) + sin(x -30°)
   Ans √3sinx
   (c) cos(0° - x) + cosy
   Ans: 2cos(45° - ½ x – ½ y)
   (d) sin2(x + 40°) + sin2(x -40°)
   2sin2x cos80

4.0 CONCLUSION

In this unit, you have seen the beauty of the relations of trigonometric identities and how easy they are applied in solving trigonometric functions problems. From the addition formulae, we were able to define the sum and difference and product formulae by simple manipulation of one o f the angles and by the operations of addition and subtraction. This made trigonometric identities fun.

5.0 SUMMARY

In this unit, the following trigonometric functions identities were deduced from the fundamental identities i.e.

$\text{Sin}^2\theta + \cos^2\theta = 1$
$1 - \sin^2 \theta = \cos^2\theta$
$1 - \cos^2\theta = \sin^2\theta$
$1 + \tan^2\theta = \sec^2\theta$
$1 + \cot^2 \theta = \text{cosec}^2 \theta$
$(\tan\theta) (\cot \theta) = 1$
$(\cos \theta) (\sec 0) = 1$
$(\sin \theta) (\text{cosec}\ \theta) = 1$

From the addition formulae, (addition and subtraction).

$\text{Sin}(A+B) = \sin A\cos B + \cos A\sin B$
$\text{Sin}(A - B) = \sin A\cos B - \cos A\sin B$
$\text{Cos}(A+B) = \cos A\cos B - \sin A\sin B$
$\text{Cos}(A-B) = \cos A\cos B - \sin A\sin B$
$\text{Tan}\ (A+B) = \frac{\tan A + \tan B}{1\tan A \tan B}$ and
$\text{Tan}(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

The following multiple angles (double angles), half angles, sum and difference and product formulae were deduced.

$\text{Sin}2A = 2\sin A\cos A$
$\quad = 1 - 2\sin^2 A$

$\text{Tan}\ 2A = \frac{2\tan A}{1 - \tan^2 A}$

$Cos2A = cos^2 A - sin^2 A = 2cos^2 A - 1$

(Multiple angles (double angles)

Half Angles

$SinA = 2sin \frac{A}{2} Cos \frac{A}{2}$

$COSA = COS^2 \frac{A}{2} - I$

$= 1 - 2Sin^2 \frac{A}{2}$

$Tan A = \frac{2 tan A}{1-tan^2 A}$

Sum and Difference formulae (factor formulae)

2cosAsinB = sin(A+B) - sin(A -B)

2sinAcosB= sin(A+B - sin(A -B)

2cosAcosB = Cos (A+B) + cos(A-B)

2sinAsinB = cos(A-B) - cos(A+B)

And finally the

Product formulae

CosAcosB = ½ {cos(A + B) + cos(A -B) }

SinAsinB = ½ {sin(A -B) - cos(A+B)}

SinAcosB = ½ { sin(A + B) + sin(A -B) }

6.0 TUTOR-MARKED ASSIGNMENT

1. find the values of the following without tables or calculators, leaving your answers in surd form.

   (a) (i) tan 105° (ii) cos 15° (iii) cos 345° (iv) sin 165°

   Ans: (i) $-2-\sqrt{3}$ (ii) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (iii) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (iv) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

   (b) if cosA 4/5 and cos B 12/13 (A and B are both acute) Find the values of
      (i) sin (A+ B) Ans; 56/65
      (ii) cos(A -B) Ans; 63/65
      (iii) tan(A+B) Ans; 56/33

2. Solve the equations for $0 \leq \theta \leq 360°$
   (a) $\sin 2\theta = \tan \theta$ Ans: $\theta = 0°, 45°, 135°, 180°, 225°, 340°, 360°$
   (b) $\cos 2\theta = 2\cos\theta$ Ans: $\theta = 11.47°$ or $248.53°$

3. Find without tables or calculators, the values of
   (a) 2sin15° cos15° Ans: ½
   (b) $\frac{540°}{8} \cos \frac{540°}{8}$ Ans: $\frac{1}{2\sqrt{2}}$
   (c) $\dfrac{2 \tan \frac{540^o}{8}}{1-\tan^2 \frac{540^o}{8}}$ Ans;-1
   (d) $\sin^2 22\frac{1}{2}^0 - \cos^2 22\frac{1}{2}°$ Ans: $- -1/\sqrt{2}$

7.0 FURTHER READING AND OTHER RESOURCES

REFERENCES

Amazigo, J.C. (ed) (1991) : Introductory University Mathematics 1: Algebra, trigonometry. Complex. Numbers. Onitsha : Africa - fep Publishers. Ltd.

Backhouse, J. K. and Houldsworth, S. P. T. (1971): Pure Mathematics: A First Course. S. I. Edition London: Longman Group Ltd.

Talbert, J. F. Godman, A and Ogum, G. E. O. (1992) Additional Mathematics for West Africa London: Longman Group UK Ltd.

Vygodsky, M. (1972). Mathematical Handbook: Elementary Mathematics. Moscow: M/R Publishers

UNIT 6

SOLUTION OF TRIANGLES AND HEIGHTS AND DISTANCES

TABLES OF CONTENT

1.0 INTRODUCTION

This unit is a follow-up of the previous units on trigonometric functions and their relations with the right-angled triangle. In this unit, we shall discuss, the solution of triangles (all types of triangles) in which case finding all the angles and sides of a triangle when the following are known either;

(1) 2 sides and an included or non included angle or angles and a side and
(2) the three side or three angles are given .

Several methods are used in the solution of triangles but here, we shall consider two important ratios - the sine and cosine rules, which make use of the definitions of sine and cosine of an angle.

In providing solutions, the following are to be remembered

(1) the sum of the interior angles of a triangle is 180°
(2) the angles are in proportion to their sides,

these information help in the fixing of the shape of a triangle(the angle opposite the greater side is bigger than the angle opposite the smaller side).

The angles of elevation and depressions are defined and is used in the calculation of heights and distances in practical problems

2.0 OBJECTIVES

By the end of this unit, the students should be able to:

- derive the sine and cosine rules
- apply the sine and cosine rules correctly to solutions of triangles
- deduce the correct area of triangle using trigonometric ratios is ½ a b Sin C or ½ b c sin A or ½ ac sin B
- define angles of elevation and depression
- apply trigonometric ratios to angles of elevation and depression in finding heights and distances.

3.1 SOLUTION OF TRIANGLES

3.1.1 SINE RULE

So many of you must have used the sine rule without knowing its proof. Here is a proof of the Sine Rule

The Sine Rule is for any triangle (acute or obtuse angled). Hence,

Given any triangle ABC with the usual notations see fig 6.1 (a) and (b) below.

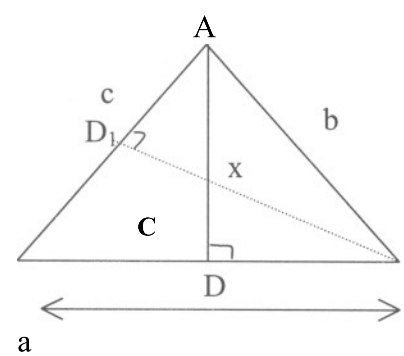


Fig 6.1 (a)

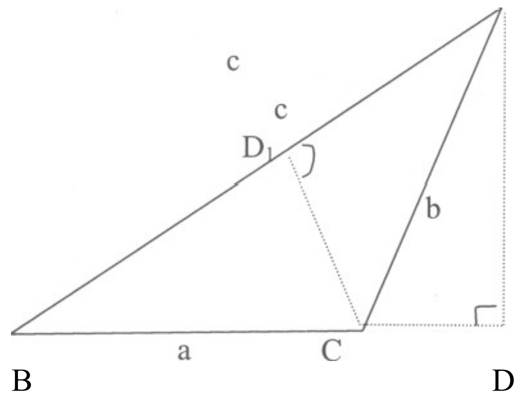


Fig 6.1 (b)

Fig 6.1 9a) is an acute angled triangle and fig. 6.1 (b) is an obtuse - angled triangle:

Given; ΔABC as shown in figs 6.1 (a and b) above

Required to prove (R.T.P) ; $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (sin rule)

Construction: Draw AD perpendicular to BC (fig 6.1 a)

Or Draw AD perpendicular to BC produced (fig 6.1 b)

Proof: in ΔABD in fig 6.1a. __________________ (1)

Sin B = x/c, this means that

C sin B = x __________________ (2)

Also in ΔADC in fig 6.1 a.
Sin C = x/b => b sin C = x, this means that in both (1) and (2)

X = c sin B = b sin C and dividing both sides by Sin B Sin C gives.

$\frac{c}{\sin C} = \frac{b}{\sin B}$ ________________ (3)

In fig 6.1 b the obtuse angled triangle In ΔADC

Sin B = x/c => c sin B= x _________________(4)

$\text{Sin}(180 - C) = \frac{x}{B}$ but sin (180 - C) = sin C (the sin θ of an obtuse angle is equal to the sine of its supplement i.e. both angles sum up to 180°)

$$\therefore \text{Sin } C = x/b \Rightarrow b \sin C = x \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_} \quad (5)$$

dividing both sides c Sin B = b Sin C = x by Sin B Sin C gives

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

So in both the acute and obtuse angled triangles the same results were obtained if the perpendicular is dropped from C to AB, it can also be proved that (you can try this)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

This is the Sine Rule. Here A is angle at A and a is the side opposite the angle at A, the same applies to B and b and C and c.

Note that this sine rule can be written as

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The sine rule is used to solve a triangle when;

(i) any two sides and any one side is given and
(ii) any two sides and a non-included angle is given.

Example 1

1. Solve the following triangles ABC which have;
   (a) $< A = 25° 25'$, $< B = 62° 51'$ and a = 3. 82cm.
   (b) $<A = 112° 2'$, a = 5.23 cm and b = 7.65cm
   (c) $<C = 125° 43'$, a = 4.2cm and c = 8.2cm.

Solutions:

Remember to make a sketch of the triangle putting into consideration the conditions.

(a) in Δ ABC, since two angles A and B and one side a is given we need to find < C and sides b and c.;

substituting the values of <A and <B into the equation gives;

$$25° \, 52' + 62° \, 15' + \angle C = 180$$
$$\therefore \; < C = 180 - (25° \, 52' +- 62° \, 15')$$
$$=180 – 88^{o} \, 7'$$
$$= 91^{o} \, 53'$$

$$\therefore < C \quad 91° \, 53'$$

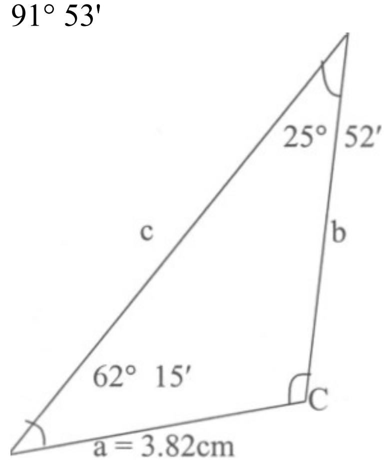


Fig 6: 2.

To get the sides b and c using the sine rule $\frac{a}{\text{Sin A}} = \frac{b}{\sin B} = \frac{c}{\sin C}$, any two of these equations can be used thus:

$$\frac{3.82}{\sin(25°45')} \quad \frac{b}{\sin(62°15')}$$

$$= b \sin 25° \, 52' = 3.82\sin(62° \; 15')$$

$$\therefore \qquad b = \frac{3.82 \sin(62°15')}{\sin 25°52'} = \frac{3.82 \sin(62.25)}{\sin(25.87)}$$

You can use your calculator or logarithm tables here for easy calculations. In the example calculations was used.

$$b = \frac{3.380652755}{0.4363307212} \qquad = 7.75 \text{ cm}$$
$$\therefore b = 7.75 \text{ cm}$$

for c, any of the two equations can also be applied.

$$\frac{a}{\sin A} \quad \frac{c}{\sin C}$$

$$\frac{3.82}{\sin 25.870} \qquad \frac{c}{\sin 91{:}88}$$

$= c \sin 25\,.\,87° = 3.82 \sin 91{:}88°$

$$\Longrightarrow \frac{3.82 \sin 91° :88'}{\sin 25.87}$$

$$C = \frac{3.817943802}{0.4363307212}$$

C = 8.75cm

∴ < c =91.88°,b=7.75 cm and c=8.75cm.

Note this values are in agreement with the condition that "greater angles face greater sides.

(b) < A = 112° 2' and a = 5.23 cm and b = 7.65cm observe here that the side a = 5.23 facing angle A is smaller than the side b = 7.65cm and it is not possible to have a triangle with two obtuse angles, hence side should be greater than b and since this is not the case. The triangle has be opposite the larger angle (try to solve this triangle, what did you observed?)

(c) <C = 125° 43°, a = 4.2 cm and c = 8.2cm.

Solution

There is a possible solution here since < C which is larger has a side c greater than side a.

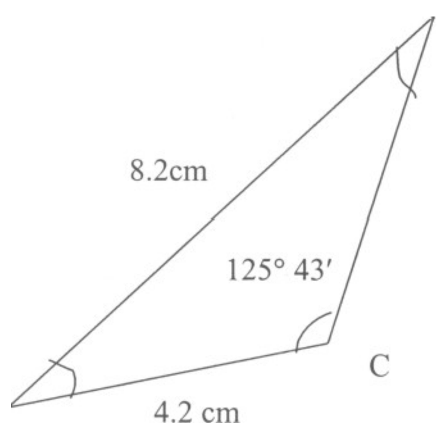


Fig. 6.3

Using the sine rule

$$\frac{a}{\sin A} \quad \frac{c}{\sin C}$$

and substituting the given a = 4.2cm and c = 8.2cm gives

$$\frac{4.2}{\sin A} = \frac{8.2}{\sin 125°43'}$$

$\therefore$ 8.2 sinA = 4.2sin 125° 43'

$$\text{Sin } A = \frac{4.2\sin 125°43'}{8.2} = \frac{4.2\sin.72}{8.2}$$

$$\text{Sin}A = \frac{3.409895089}{8.2} = 0.4158408645$$

$\therefore$ A = sin-' 0. 4155408645 = 24.57°

$\therefore$ < B = 180° - (125°. 72 + 24.57)

= 180 - 150. 72 = 29.71 °

$\therefore$ <B = 29. 71

to get b.

$$\frac{b}{\sin B} \quad \frac{c}{\sin C}$$

$$\frac{b}{\sin 29.71} \quad \frac{8.2}{\sin 125^0.72}$$

b sin 125.72 = 8.2sin 29.71.

$$b = \frac{8.\sin 29.71}{\sin 125.72}$$

$$b = \frac{4.064004179}{0.811797831}$$

b = 5.0056 cm = 5.006cm

.'. <A = 24. 57°, <B = 29.71, b = 5.006cm

3.12. THE COSINE RULE

Like the sine rule, the proof of the cosine rule which is an extension of the Pythagoras theorem will be given here.

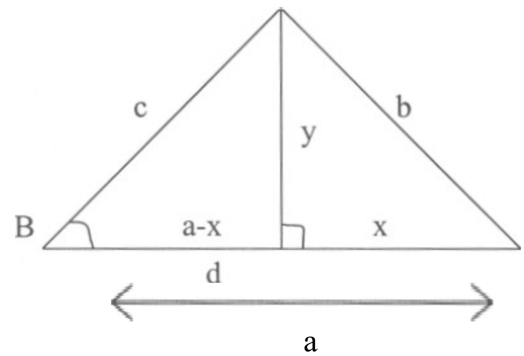


Fig 6.3a.

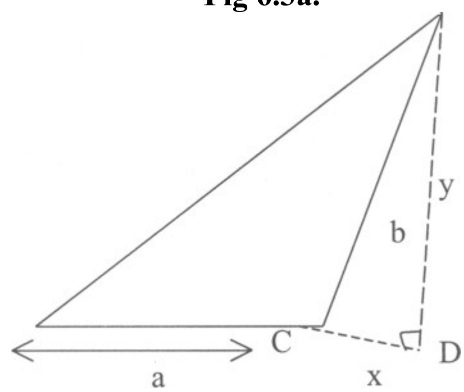


Fig. 6.3.b.

Given: DABC with the usual notations see fig : 6.3a. (acute - angled) and 6.3b. (obtuse angled triangles.

Required to Prove: $c^2 = a^2 + b^2 - 2a\,b \cos C$ (cosine rule)

Construction: Draw a perpendicular from A to B C (fig 6.3a) and from A to BC
produced (fig 6.3b.)

Proof: in fig:6.3a in ΔABC.

$b^2 = x^2 + y^2$ (1) (Pythagoras theorem) in ΔABC, $c^{22} = (a - x)^2 + y^2$,by simplification,

$c^2 - a^2 - 2ax + x^2 + y^2$
but $x^2 + y^2 = b^2$ in (1)
substituting $c^2 = a^2 - 2ax + b^2$ (2)

In ΔABC, $\cos C = x/b \Rightarrow b \cos C = x$

so substituting for x in (2) gives

$c^2 = a^2 + b^2 - 2ab \cos C$
.'. $C^2 - a^2 + b^2 - 2ab\cos C$

Proof: in fig 6.3b. (Obtuse angled triangle)

In ΔADC, $b^2 = X^2 + y^2$ (ii) (Pythagoras theorem.)

In ΔABD, $C^2 = (a + x)^2 + y^2$(Pythagoras theorem.)

Simplifying gives, $c^2 = a^2 + 2ax + Y?$ (2)
But $x^2 + y^2 = b^2$ (1)

So substituting in (2) for b^2 gives $c^2 = a^2 + b^2 + 2ax$

In ΔADC, $\cos(180 - c) = x/b$, but cos (I80 - C)
Since < C is obtuse is - cos C therefore;
$-\cos C = x/b$
====> $-b\cos C = x$

So substituting for the value of x in (2) , you obtain:

$C^2 = a^2 + b^2 + 2a(-b \cos C)$
$C^2 = a^2 + b^{22} - 2ab \cos C$.

This is the same result as for the acute angled triangle. And this is the Cosine Rule:

$$C^2 = a^2 + b^{22} - 2ab \cos C$$

Also by renaming the angles and /or re-drawing the perpendiculars the following cosine rules can also be proved (try it)

$$a^2 = b^2 + c^2 - 2ax \cos A$$

OR

$$b^2 = a^2 + c^2 - 2ac \cos B$$

The cosine rule is applied in the solution of triangles when the following are given:

(1) two sides and an included angle or
(2) three sides.

Example: 2

1. Solve triangle ABC, with;
   (a) A= 42°. 83°, b = 7.23 cm and C = 5.46cm.
   (b) <B = 150 .3°, a = 8.91 cm and c = 5.26 cm.
   (c) C = 4.05, a = 2.25cm and b = 6.24 cm

Solution:

Note always remember to make a sketch of the triangle, noting that the angles should represent its type (acute or obtuse) and the sides should be proportional.

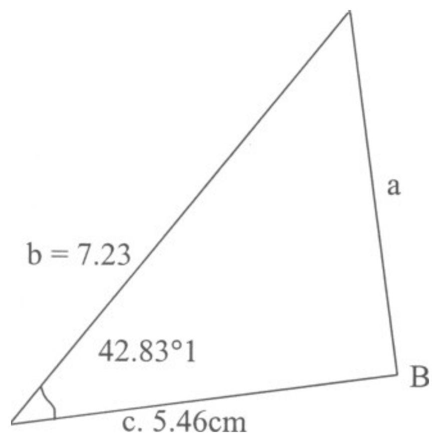


Fig 6.4.

Since we are looking for the side a the cosine rule is to be used since b and c are given using:

$A^2 = b^2 + c^2 - 2bc \cos A$ and substituting b = 7.23 cm,

C = 5.46cm and A = 42.83°

which were given into the formula and simplifying gives:

A^2 = $(7.23)^2 + (5.46)^2 - 2\ (7.23)(5.46) \cos 42.83$
= 52.2729 + 29.8116 - (14.46)(5.46) (0.7334)
= 52.2729 + 29.8116 - 57.9015
= 82.0845 - 57.9015
= 82.0845 - 57.9015 = 24.183

a2 = 24.183

a =$\sqrt{24.183}$ = 4.9176

.'. a = 4.92cm

Here the positive square root of a was taken because we are dealing with lengths.

To find the angles either use the sine or cosine rule whichever is easier for you

(Now we try both rules)

(a) using the sine rule

$\frac{\text{Sin } C}{c} = \frac{\sin A}{a}$ this means that

$a \sin C = c \sin A$ and then

$\sin C = \frac{c \sin A}{a}$

so substituting for the values of c = 5.46cm, A = 42.83° and a = 4.92 cm into the equation gives;

$\sin C = \frac{5.46 \sin 42.s3}{4.92} = 0.754403732$

.'. <C = $\sin^{-1}$ (0.7544) = 48.98° = 49°

(b) Using the cosine rule:

$C^2 = b^2 + a^2 - 2ab\cos C$

From here, even one can make cos C the subject of the formula thus:

$\text{COs } C = \frac{a^2 + b^2 - C^2}{2ab}$

and substituting the values of

a = 4.92cm, b = 7.23cm and c = 5.46cm, we obtain

$\cos C = \frac{(4.92) + (7.23) - (56)}{2(4.92)(7.23)}$

$= \frac{24.2064 + 52.2729 - 29.8116}{71.1432}$

$\cos C = \underline{46.6677} = 0.6559685255$

71.1432

.'. <C = $\cos^{-1}$ 0.6559685255

= 49.40687

= 49.01°

So the two methods (a) and (b) gave the same value for but it is usually easier to sue the sine rule in finding the missing angles.

Then <^B = 180° - (<â + <c) (sum <s of Δ)

= 180 - (42.83 + 49.01)

= 180 - 91.84 < ^B = 88.16°,

therefore the missing parts are:

a = 4.92cm, < ^B = 88.16° and < = 49.01°

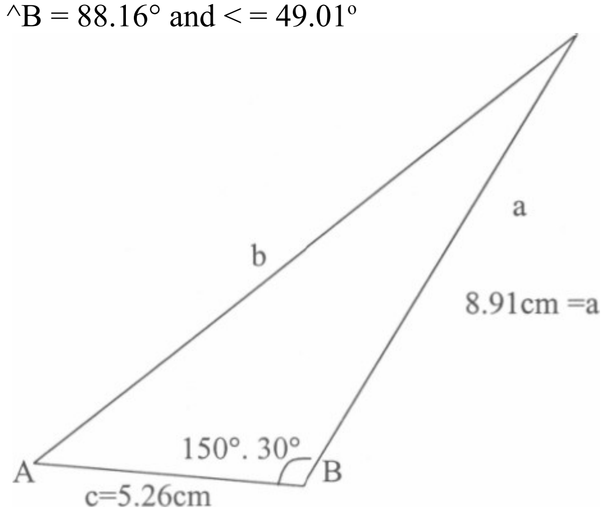


Fig: 6.4

Using the cosine rule

$b^2 = a^2 + c^2 - 2ac \cos B$

and substituting values of:

a=8.91cm, c=5.26cmand<^B

into the formula, we obtain

$b^2 = (8.91)^2 + (5.26)^2 - 2(8.91)(5.26) \cos 150.$

= 79.3881 + 27.6676 - 93.7332 x cos (180 - 150)

=79.3881 + 27.6676 + 93.7332 cos 29.7

= 79.3881 + 27.6676 + 81.4196

b^2 = 188.4753 and b = $\sqrt{188.4753}$

b = 13.73 cm

Note cos 150.3° = - cos 29.7 and when substituted into the formula it becomes

$b^2 = a^2 + c^2 + 2ac \cos B$ (where B is acute)

To find angle A using the sine rule

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{8.91} = \frac{\sin 150.3}{13.73} \text{ cross multiply}$$

Simplifying = 13.73 sinA = 8.91 sin 150.3 gives;

$$\sin A \ \frac{8.91\sin150.3}{13.73} = 0.3215248897$$

$\therefore$ sinA = 0.3215248897

and < A = sin-1 0.3215248897 = 18 .76°

Angle B will then be equal to:

180-(<A+<) (sum < of 0)

<B= 180 - (18.76+ 150.3)

= 180 - 169.06

= 10.94°

.'. b = 13. 73cm, < A = 18.76 and < ^B = 10.94°

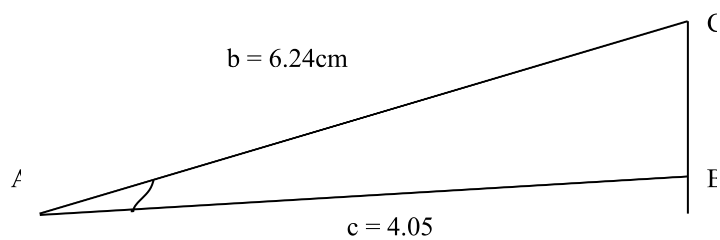


Fig 6.5

Solution:

The three sides of the triangle are given to obtain the angles, the cosine formula is used t thus:

$$\text{Cos A } \frac{b^2 + c^2 - a^2}{2bc}$$

substituting for a = 2.25, b = 6.24 and c = 4.05 into the above formula, we obtain;

$$\text{Cos A} = \frac{(6\text{-}24)^2 + (4.05)^2 - (2.25)^2}{2(6.24)(4.05)} \text{ and}$$

$$\text{Simplifying} = \frac{38.9376 + 16.4 - 5.062}{50.544}$$

$$\text{SO cos A} = \frac{55.3401 - 5.0625}{50.544\ 5} = \frac{50.2776}{0.544}$$

.'. cos A = 0.9947293447

and < A = $\cos^{-1}$ (0.9947) = 5.88°

again using cosine rule to obtain thus:

Cos A $\frac{b^2 + c_2 - a^2}{2bc}$ substituting the values of a, b, c as above into the formula gives

$$\text{Cos} = \frac{(2.25)^2 + (6.24)^2 - (4.05)^2}{2(2.25)(6.24)}$$

$$= \frac{5.0625 + 38.9376 - 16.4025}{28.08}$$

∴ cos C = 0.9828205128

so <C = $\cos^{-1}$ 0.982828205128

<C = 10.64°

By the knowledge of the angle sum of a triangle B is then calculated thus

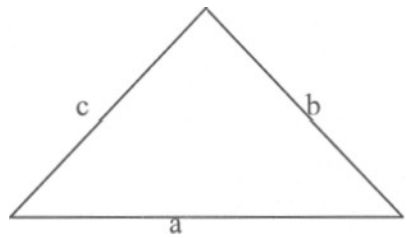
<A+<B+<c= 180

∴ <B = 180 -< A - <C

= 180 - 5.88 - 10.64<B = 163°. 48°

Note from our earlier discussion on the proofs of the sine and cosine rules the quickest/easiest method used in the solution of triangles depends on the information given, which are summarized below:

1. Given three sides of the triangle as in example 2(C)



$$\text{Cos } A = \frac{b^2 + c^2 - a^2}{2bc}$$

OR

$$\text{Cos } B = \frac{a^2 + c^2 - b^2}{2ac}$$

OR

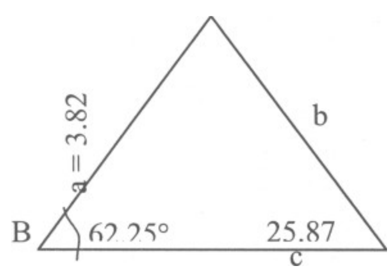
$$\text{Cos } C = \frac{a^2 + b^2 - c^2}{2ab}$$

Can be used to find any of the two angles and the third angles is found, by the use of the sum of the interior angles of a triangle i.e.

$$<A+<B+<C=180^\circ$$

2. Given two angles and one side
In example 1(a), two angles and one side is given, the sine rule is used here, to find the length of the sides thus:

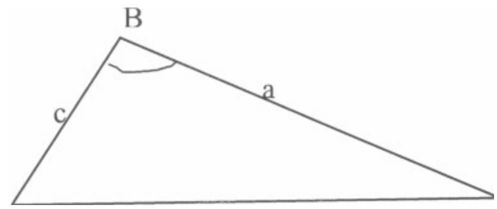
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$



because < C can easily be found by 180 - 62.25 -25.87 = 91 .88°

or $\frac{a}{\sin A} = \frac{c}{\text{Sin } C}$ to find the side C.

3. Given two sides and an included angle in this figure,



< B is given which lies between the two given sides a and c.

The cosine rule is used a in example 2(a) above to find the side to b then the sine rule is used to find either < A or < C then the third angle can be found by the sum of angles of triangle theorem

4,. Given two sides and a non - included angles that is the angle does not lies between the two given sides.
Two cases are treated here.

Case 1:

When the given angle is acute two possible triangles can be drawn as follows:

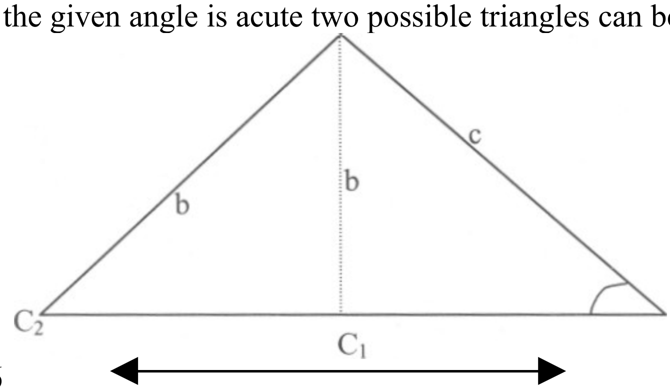


Fig6.6

If the side opposite the given angles B in this case is less than the other given angle C i.e. b < c as in Fig 6.6. above. This means that < C will have two values(acute and obtuse)

Case II

When the side opposite the given angle is greater as in Fig 6.7 below, only one triangle is possible (b > c and ^B is acute)

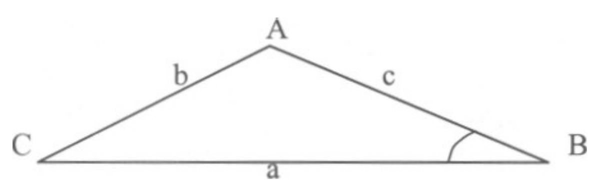
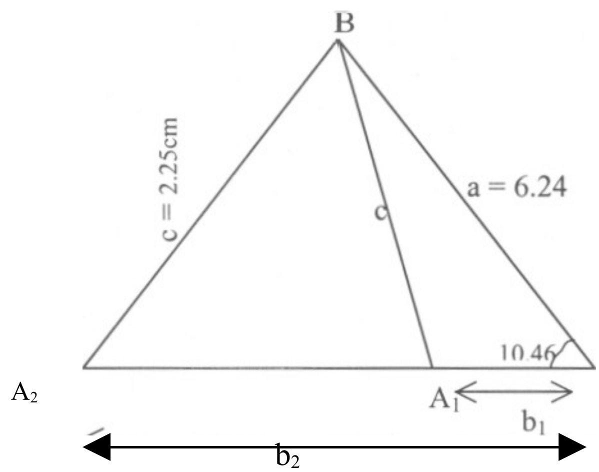


Fig: 6.7

In both cases the sine rule is used to find one of the angles and the side can be found by either the sine or cosine rule

Example:

Solve the triangle A BC with
C = 10.46°, c = 2.25cm and a = 6.24cm.



Solution

In the figure above, there are two possible triangles since c < a i. e.

$$2.25 < 6.24$$

using the sine rule to find < A

$$\frac{\sin A}{a} \quad \frac{\sin C}{c}$$

$$= \sin A = \frac{a \sin C}{c}$$

$$= \frac{6.24 \sin 10.46}{2.25} = 0.5034960036$$

sin A = 0.5034960036
$A = \sin^{-1}(0.5034960036)$
A= 30.23 °

But A is either acute or obtuse.

.'. A = 30.23° or (180 - 30.23°)

= 30.23° or 149.77°
so when < A = 30.23°
< B = 180 - 30.23 -10.46 (sum s < of Δ)

180 - 40.69 = 139.31° and when <A=30.23°
< B = 180 - (149.77 + 10.46) = 180 - 160.23 = 190 . 77°

then using the sine rule again to obtain the sides by bl and b2

$$\frac{b_2}{\sin B} \quad \frac{c}{\sin C} \qquad \text{where } <B = 139.31$$

$$\frac{b_2}{\sin 13\,9.31} \; - \; \frac{2.25}{\sin 110.46}$$

$$b_2 = \frac{2.25 \sin l39.31}{\sin 10.46} = \frac{1.466923668}{0.1815490397}$$

$b_2 = 8.08$cm

Also, finding b_l when < B = 19.77°, using the sine rule

$$\frac{b1}{\sin B} = \frac{c}{\sin C} \qquad (<B = 19.77)$$

$$b_l = \frac{c \sin B}{\sin C}$$

and substituting the values of c, ^B and

$$b_l = \frac{2.25 \sin 19.77}{\sin 10.46}$$

$$= \frac{0.7610519671}{0.1815490397} = 4.192$$

.'. b_l = 4. 192cm 4.19cm (2 dec. places)
∴ Δ ABC has either

A = 30.23, B = 139.31 and b_2= 8.08cm

OR

A = 149.77°, "B = 19.77and b1 = 4.19cm.

The two possible answers are then

< A = 30.23, <B = 139.31, and b_2= 8.08cm
<A = 149.77°, <B = 19.77°, and b_1 = 4.19cm

5. When the given angles is obtuse No triangle is formed if the side opposite the given angle is lee than the other given side. Here is an example

< A = 125°43', a = 4.2cm and c = 8.2cm.

Here < A is obtuse and side a should be greater than side c given, but since A=4.2cm<c=8.2cm i.e. a<c

No triangle is formed, therefore no solution for clarity, attempt to solve this triangle, what
are your observations.

Exercise 6.1.

Solve the following ΔABC completely

(1) <A=62015', <B=25°52' and b=3.82cm.
Ans: <C = 91° 53' a = 7.75cm and c = 8.75cm

(2) < C 17.6°, b = 6.52cm and c = 8.91 cm
Ans: <A 149.62, <B =12°.78° a = 14.9cm

(3) <A= 105.08°, b = 5.24cm and c = 5.25 cm
Ans: <B = 37.42°, <C = 37.5°, a = 8.33cm

(4) a = 3.49cm, b =7.36cm and c = 5.25cm
Ans; A = 25.88°, ^B = 1130. 18', = 40.970

6.2 HEIGHTS AND DISTANCE

6.2.1 ANGLES OF ELEVATION AND DEPRESSION

DEFINITIONS:

The angles of elevation and depression are better explained by the following examples:
A lady stays at a particular spot a ground floor outside a house to discuss with her friend, at the first floor of a one story building probably next to hers she first looks horizontally towards the storey building then looks up to her friend see fig: 6.8

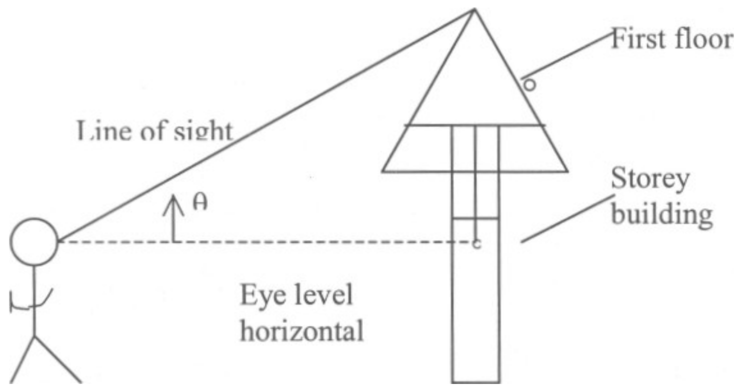


Fig:6.9

The angle the eye level (horizontal) makes with the line of sight is called the angle of elevation 6 in fig 6.8. So we can define the angle of elevation as the angle that lies between the observers eye level (horizontal plane) and the line of sight when the observer tries to see something above him/her.

Similarly, the angle of depression is the angle between the observer's eye level and the ground when the observer is above the ground see fig. 6.9

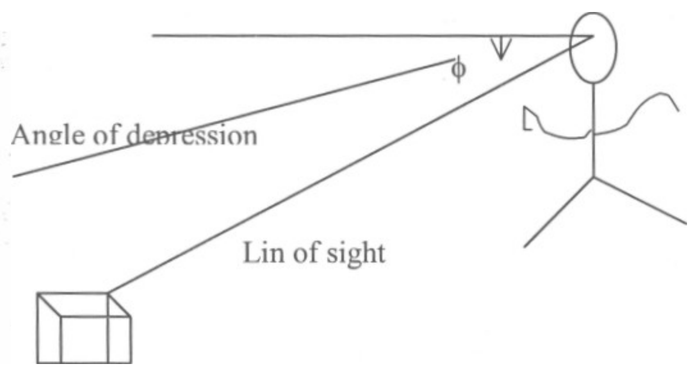


Fig: 6.9

As in the illustration above the person in the one storey building looking down to discuss with the lay downstairs

So

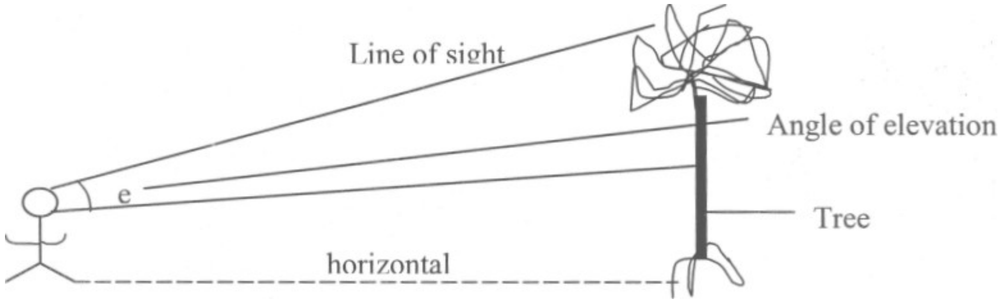


Fig 6.10a

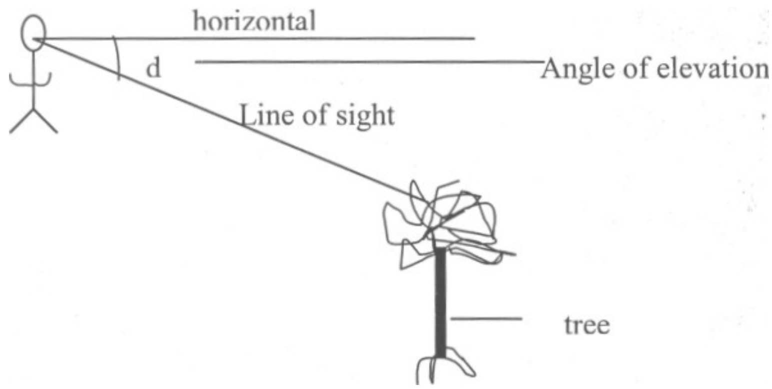


Fig: 6.10 b.

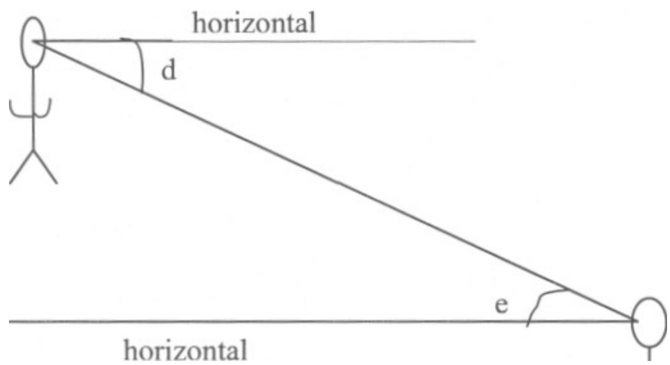


Fig: 6.10c.

From the above figures it can be easily seen that angle of elevation and angle of depression are alternate angles (two horizontal lines are). These two angles are frequently used in the application of trigonometric functions to triangles.

UNIT 7

BEARING

TABLE OF CONTENTS

1.0 INTRODUCTION

Very often people talk about finding their bearings. Have you ever taken out time to find out what this means/ has it any relation with mathematics? Why should one bother about his/her location?

The answer to these questions shall be provided in this unit many a time, when walking along the road or traveling by air or 'ship, the compass is displayed in front of the Geologists, Surveyors Pilots or Navigators or sailors. This instrument makes them have a focus on their journey or gives sense of direction.

In this unit, the place of the cardinal point or compass in relation to the location of places shall be discussed. This will form the basis of our solution to practical problems. The bearing is a means of locating the angular inclination between two or more objects in different positions.

Note: In trigonometry (positive) angles are measured in the anticlockwise direction but in bearing, the angles are measured in the clockwise direction, that is from the first quadrant to the fourth quadrant to the third, the second and back to the first depending on the location of the object whose bearing is sought.

In the treatment of bearing, the following should be borne in mind.

(i) all measurement of angles start from the North pole (clockwise direction) previously the letter N or S comes before any angular measure but this is no longer conventional rather we now use the three digit number referred to as "true bearing" Example instead of N 30 E, we now write 030°. This is for easy location of the quadrant where the place or object or point is.

(ii) Write the angle derivation starting from the north to the desired line

2.0 OBJECTIVES

By the end of this unit, you should be able to

- explain clearly the term bearing
- locate the bearing of given places
- apply trigonometric ratios to problems involving bearings.

3.1 BEARING

The compass has eight cardinal points namely, North (N), South (S), East(E), West(W),North East(NE), North West(NW), South East(SE) and South West(SW). the diagram is in figure 7.1 below

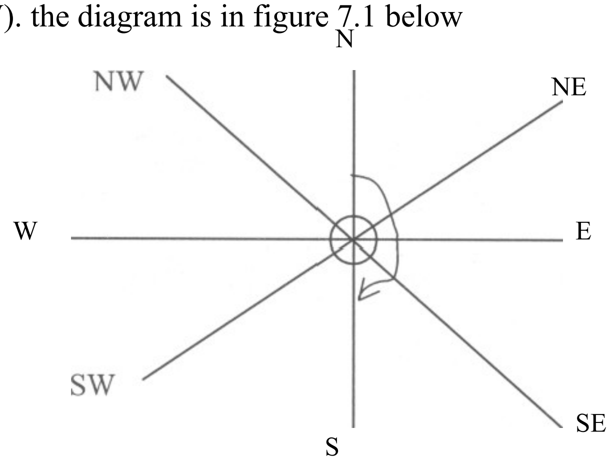


Fig 7.1

Using fig. 7.1 as an example the bearing of S from O (center) is obtained by measuring the angle from the North in the clockwise direction to the line joining O and S i.e to OSSO the bearing of S from O is 180° in this case (see arrow)

Example

In the diagram below find the following

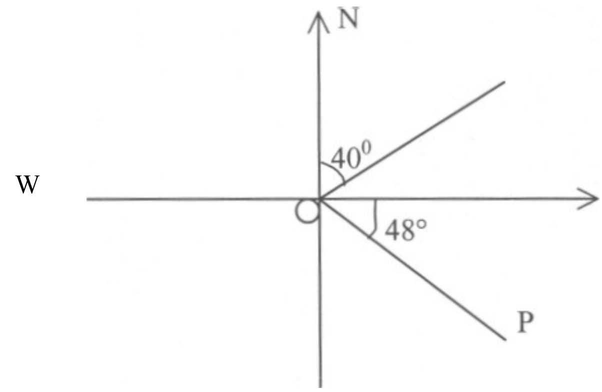


Fig 7.2a.

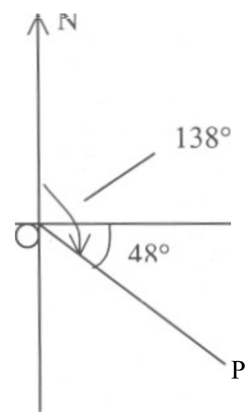


Fig 7.2b.

(a) the bearing of P from O (b) the bearing of A from O.

Solution:

(a) the bearing of P from O is the angle OP makers with the North Pole measured in the clockwise sense so here it is 90 + 48 = 138° i.e. the angle measured from North to the line OP in the clockwise direction. (see fig 7.2b.)

(b) The bearing of A from O is 40° written as cardinal points, it is written N 40° E

In which case the letter N will be written first and E or W after the angle.

In cardinal points the bearing of P from O might be written as S 42° E = 138°.

Note it is always better to use the three digit number (true bearing) for easy identification of the quadrant where the place or point is located.

2. State the bearing of each of the following directions
(a) N, (b) E (c) SE (d) S (e) W and (f) NW

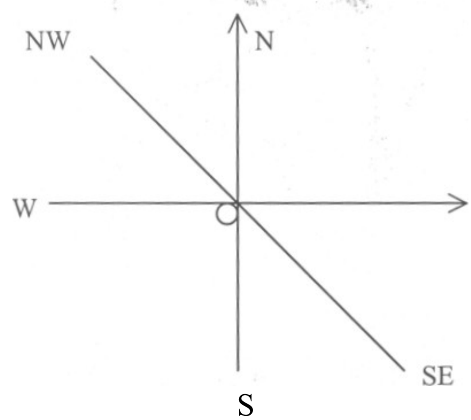


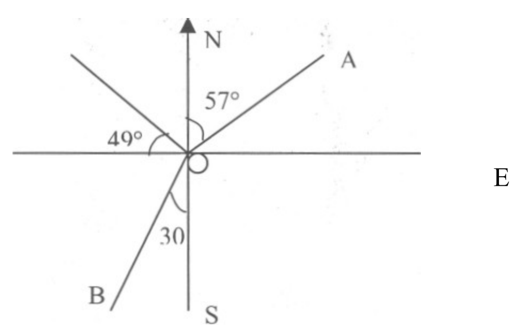
Fig. 7.3

Solution:

Since bearings are measured in the clockwise direction from the North the bearing are as follows:

(a) N = 000° (it has no inclination)
(b) E = 090° (90° from the North Pole)
(c) SE = 90 + 45° = 135° (SE is half way between the fast and the South)
(d) S = 180°
(e) W = 270°
(f) NW = 315° 9270° + 45°) again because NW is half way between the North and the West.

3. In the figure on below what is the bearing of



(a) A from O
(b) B from O
(c) C from O
(d) O from A

Solution

(a) the bearing of,&from O is 057°
(b) the bearing of B from O = 180° + 30 = 210°
(c) the bearing from C from O = (270 + 49) = 319°
(d) the bearing of O from A - this is got by first drawing a cardinal point at A see diagram below. A now lies on the East - West line, the alternate angle is located

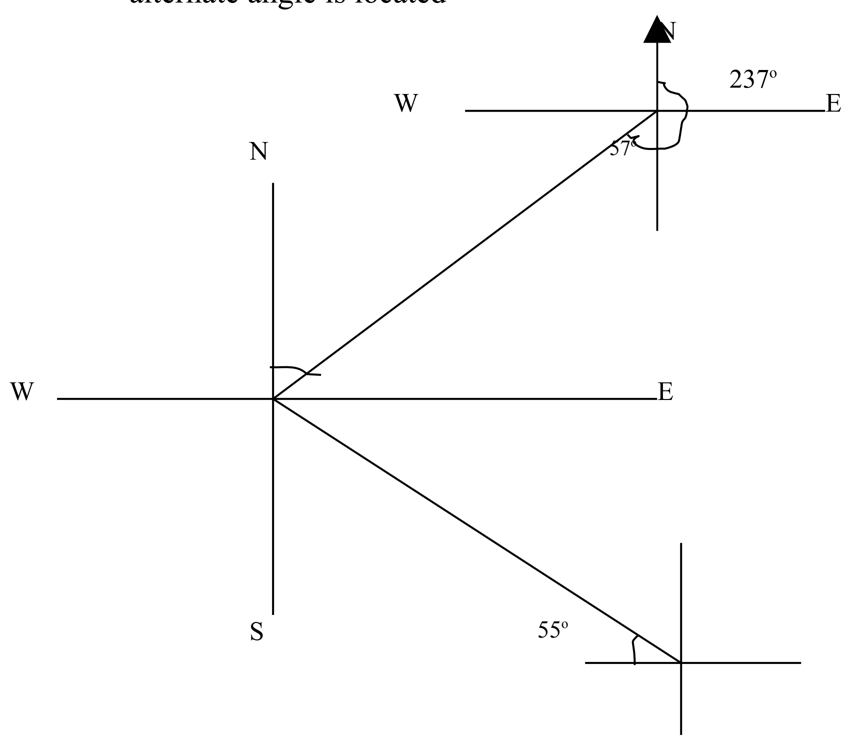


Fig 7.4

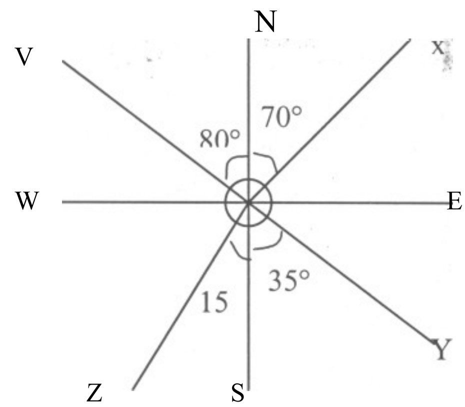
Therefore the bearing of O from A is 180 + 57° = 237° (starting from the North pole at A to the line 0 A (see arrow above)

Also finding the bearing of O from D , the same procedure is followed, hence the bearing of O from D = 270 + 55° = 325°

Exercise 7.1.

Now try this as an exercise.

In the figure below find the following:



(a) the bearing of X from O = 070
(b) the bearing of Y from O = 45^o
(c) the bearing of Z from O = 195^o
(d) the bearing of V from O = 195^o
(e) the bearing of O from X = 250^0
(f) the bearing of O from Y = 3250^o
(g) the bearing of O from Z = 015^o
(h) the bearing of O from V.= 100^o

The answers to the above exercise are written in red to serve as a check on your progress.

Now substituting the values a = 15km because it is facing angle A, B = 12km and C = 9km or b = 9km and c = 12km, the most important side has been determined and that is the side facing the angle A

.'. Cos A=

$$\frac{12^2 + 9^2 - 15^2}{2x12x9}$$

$$\frac{144+81-225}{216}$$

$$\frac{225 - 225}{216} = \frac{0}{216} = 0$$

====> CosA=0

.'. A = cos-1 0 = 90°

.'. A=<XYZ=90°

Hence the bearing of Z from Y is then calculated from the North Pole in Y to the line joining Y and Z i.e. Y Z 90° + 60 = 150°

4. The calculate the bearing of X from Z the sine rule is first applied to find part the angle of Z thus:

$$\frac{15}{\sin A} = \frac{12}{\sin Z}$$

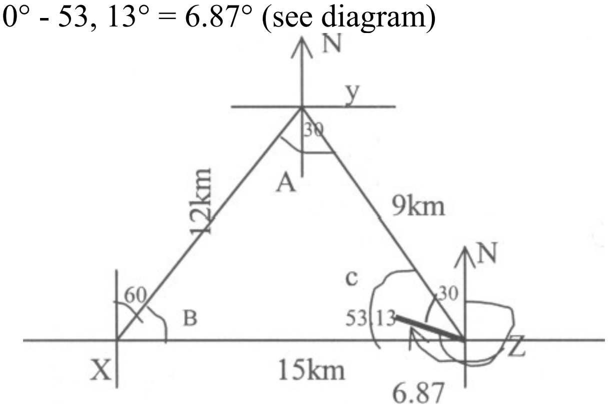
'. $\sin Z = \frac{12 \sin A}{15}$, but A = 90° *from (b)*

$$\sin Z = \frac{12 \sin 90}{15} = \frac{12}{15} = 0.8$$

$Z = \sin^{-1}(0.8) = 53.13°$,

Then the bearing of X from Z (see diagram) Remember the angle at Z should be

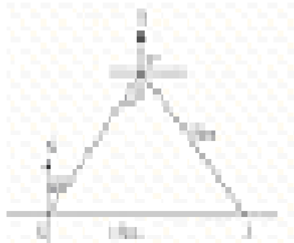
90 - 30° - 53, 13° = 6.87° (see diagram)



Practical Example On Bearing

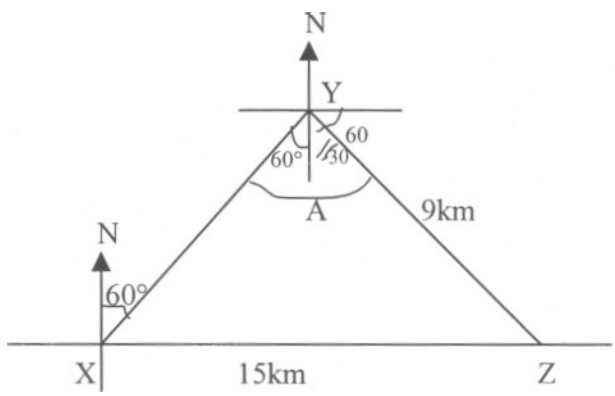
1. Femi traveled a distance of 12km from X on a bearing of 060° to Y. He then travels a distance of 9km to a point Z and Z is 15km from X.
(a) Draw the diagram showing the position of X, Y and Z.
(b) What is the bearing of Z from Y.
(c) Calculate the bearing of X from Z.

Solution

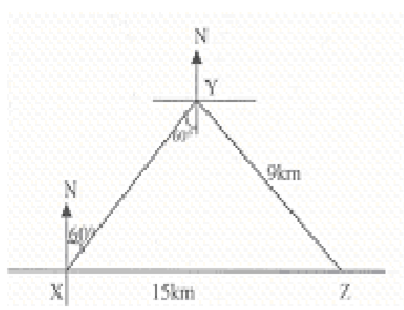


In drawing the diagram above, Femi moved from X to Y so the angle is between the North in X to Y i.e. from the North pole in X to the line XY and the distance stated from Y to Z is on a different bearing. To find the bearing draw the four cardinal points in Y and read off.

(b) A repeat of the diagram is made here,

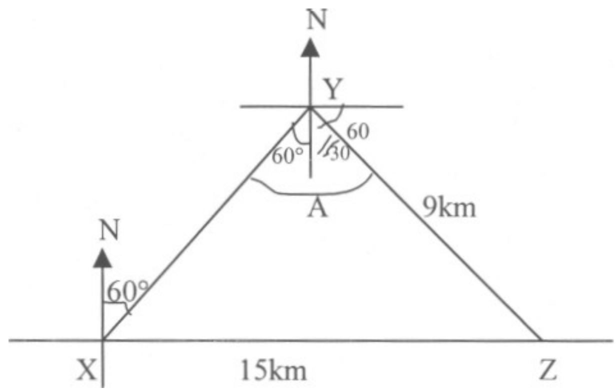


Solution



In drawing the diagram above, Femi moved from X to Y so the angle is between the North in X to Y i.e. from the North pole in X to the line XY and the distance stated from Y to Z is on a different bearing. To find the bearing draw the four cardinal points in Y and read off.

(b) A repeat of the diagram is made here,



Let the < X Y Z in Δ X Y Z be denoted by A and since the three sides of the triangle are known, the cosine formula is used -

$$a^2 = b^2+c^2 - 2bc\cos A$$

∴ $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

∴ X = 20sin30 sin 105

$\dfrac{20 \times 0.5}{0.9659} = 10.3528$km

∴ x = 10.4km (3 sig figs). So the distance between the two village is 10.4km

(b) Let the distance between Lokoja and the villager be represented by y km

Again applying the sine rule, we obtain.

$$\frac{20}{\sin 105} = \frac{y}{\sin 450}$$

∴ $y = \dfrac{20\sin 45^o}{\sin 105}$

$= \dfrac{20 \times 0.7071}{0.9659}$

y = 14.64km

the distance between Lokoja and the village Z is 14.6km (3 sig figs)

Exercise 7.2

1. Two men Abudullahi and Olufemi set off from a navel base in Lokoja prospecting
for fish. Abudullahi moves 20km on a bearing of 205° from Olufemi and Olufemi moves 15km on a bearing of 060°. Calculate correct to the nearest

   (a) distance of Olufemi from Abudullahi 33km
   (b) bearing of Olufemi from Abudullahi 40°

2. A man moves from a point A in Onitsha on a bearing of 060° to another,as point
B, 400m away. He then moves from the point B on a bearing 120° to another point Z in the same town which is 250m away.

   i.e. 270° + 6.870
   = 276. 87°

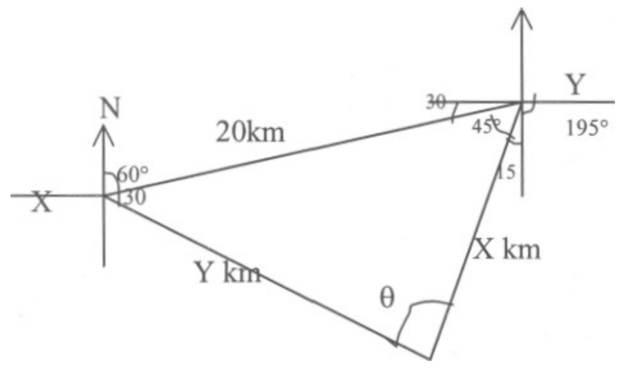
   Alternatively it can be calculated as 270° (from the three quadrants) plus
   (90 - 53.13 -30º because of the remaining angle Z

   = 270 + 6.87 = 276.870

3. A man traveled from Lokoja on a bearing of 060° to a village Y which is 20km
away. From this village X he moves to another village Z on a bearing of 195°. If
the village Z is directly east of Lokoja, calculate correct to 3 significant figures
the distance of (a) Y from Z (b) Z from Lokoja

Solution

Let Lokoja be represented by X The diagram shows, a sketch of the journey made by this man.



(a) Let the distance between the villages Y and Z be denoted with X km, let angle at Z = θ

In ΔXYZ,
$\theta + 456° + 30 = 180$ (< sum of Δ)
$\theta = 180 - 75 = 105^0$

$\therefore \quad <Z = \theta = 105°$

Applying the sine rule

$$\frac{20}{\sin Z} = \frac{x}{\sin 30}$$

$$\frac{20}{\sin 105} = \frac{x}{\sin 30}$$

Calculate:
(a) the distance between A and Z (AZ)
(b) the bearing of A from B . correct to 3 sig figs

Ans: (a) 568m(b) 240°

4.0 CONCLUSION

In this unit an attempt has been made to bring to life the applications to real life of the trigonometrical functions that have been studies in this course. You will see from this unit that bearing/trigonometric functions are in everyday usage though we use them without reference to the name given to it in mathematics by mathematicians. It is expected that at this juncture you can - orientate yourself by looking out for the other beauties of this course in your everyday affair.

5.0 SUMMARY

In this unit the application of bearing have been treated and it was discovered that;

(1) pole (reference pole)
(2) the angles are measured in the clockwise direction as against the angles in the other trigonometric functions
(3) the three - digit number is used in writing out the angles often referred to as true bearing and this is conventional
(4) that the cardinal points are not being used in bearings
(5) the real life applications of trigonometric functions through bearing were also illustrated.

7.0 REFERENCES AND OTHER RESOURCES

Egbe, E. ; Odili, G. A and Ugbebor O. O. (1999) Further Mathematics. Onitsha : African- fep Publishers Ltd

David - Osuagwu, M., Anemelu, C. and Onyeozuli 1. (2000).New School Mathematics for Senior Secondary Schools. Onitsha: Africans - fep. Publishers Ltd.

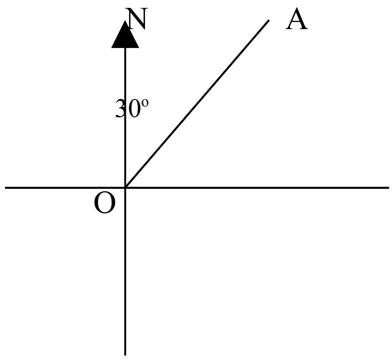
OTHER RESOURCES

Any mathematics text book that deals with trigonometry is also good for you.

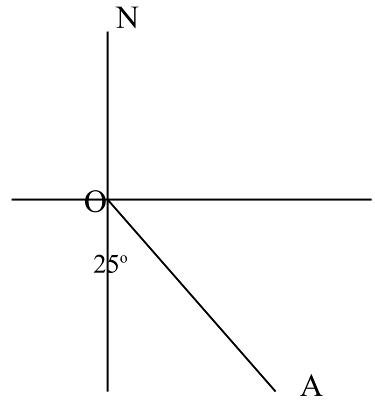
5.0 TUTOR - MARKED ASSIGNMENT

1. Find the bearing of A from O in the diagrams below: N

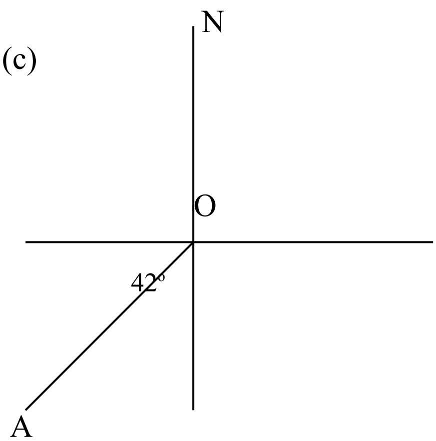
(a)



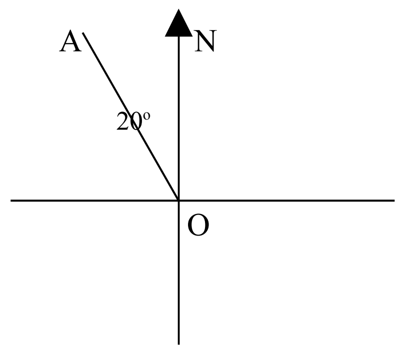
(b)



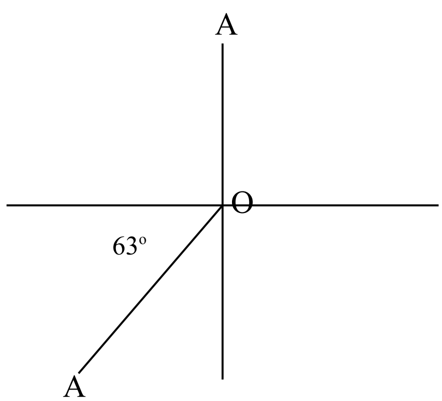
(c)



(d)



(e)



2. A traveler moves from a town A on a bearing of 055° to a town B 200kni away. He then moves from B on a bearing of 155° to a town C 400km from B find correct to the nearest whole number.

   (a) the distance between A and C

(b) the bearing of A from C.

3. The bearing of a lighthouse from a ship 10km from it is 105°. The ship sails due East to a point and stops. If the bearing of the light house from the ship is now 300°, calculate correct to the nearest whole number.

4.0 CONCLUSION

In this unit, we have proved the two most important relations in the solution of triangle. These are the sine and cosine rules. The formula for the area of a triangle and parallelogram were derived. Also we saw the interrelations between the sine and cosine rules in solving triangles and the rules that must be observed before any solution would be possible.

The importance of trigonometry in solving problems on heights and distances were illustrated through the angles of elevations and depressions.

5.0 SUMMARY

In this unit, you have learnt that to solve triangles the following information must be given before the application of the sine or cosine rules.

1. (a) three sides or (b) two sides and an included sides or
   (c) two angles and a side or (d) two sides and a non - included angle

   In the case of (d) care should be taken to find the possible triangles that might be formed when the given angle is acute.

2. When the side opposite a bigger angle is less than the other side there will be no solution,

3. The angle of elevation is formed when you look up to see an object and the angle of depression when you look down to view an object.

4. the area of triangle and parallelogram were found using trigonometric ratios.

5. sine rule states that:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

where a, b, c, are the sides of a triangle and A, B, C its angles. This rule can also be written as:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

6. Cosine rule states that:

$a^2 = b^2 + c^2 - 2bc \text{ Cos } A$

$b^2 = a^2 + c^2 - 2ac \text{ Cos } B$

$c^2 = a^2 + b^2 - 2ab \text{ Cos } C$

(only one of these is used at a time).

It can also be stated as:

$$\text{Cos } A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\text{Cos } B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\text{Cos } C = \frac{a^2 + b^2 - c^2}{2ab}$$

With these, you can feel relaxed and enjoy this all-important unit

6.0 TUTOR –MARKED

1. Solve the following triangles ABC,
   (a) $< A = 6o°$, b = 96cm and c = 64cm
   (b) $<B = 146.\ 33°$,c = 35cm $<$ = 5.17°

2 In Δ ABC (a) b = 6cm, c = 4cm and $<B = 60°$, find sin sin C and
   (b) a = 20cm, b = 14cm and $< A = 30°$ find sin B and <^ B

3. A passerby 1.8metres tall stood 40metres away from an Iroko tree about 26metres high and saw a bird at the topmost branch of the tree. What is the angle of depression of the bird from this passerby assuming the bird saw him also.

4. The angle of elevation of a tower from a point A is 45', and also at a point B in a horizontal line to the foot of the tower D and 50metres away to it is 75. Find;
   (i) the height of tower (ii) the distance of A from the tower.

5. Find the area of Δ ABC given that A = 45°, and c = 4.2cm.

7.0 FURTHER READING AND OTHER RESOURCES

REFERENCE.

Amazigo, J. C. (ed) (1991) introductory University Mathematics 1: Algebra, Trigonometry Complex Numbers. Onitsha. African -fep. Publications Ltd.

A Panel of Experienced mathematics teachers. Secondary Mathematics 3 and 4 (1980) Singapor: fep Publishers. Ltd.

Osuagwu, M. , Anemelu, C and Onyeozili, 1. (2000) New school Mathematics for Senior secondary Schools: Onitsha: Africa - fep Publishers Ltd.

Egbe, E. ; Odili, G. A and Ugbebor O. O. (1999) Further Mathematics. Onitsha : African- fep Publishers Ltd