



PHY 220

**PHYSICS
LABORATORY 1**

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MODULE 1

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**UNIT 1 INTRODUCTION TO LABORATORY-I:
MEASUREMENT****CONTENTS**

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1.0 INTRODUCTION

You know that scientists use devices to measure and thereby quantify physical quantities. But even the best of devices yield inexact measurements. We express these measurements as approximate numbers. We distinguish between numbers such as 3.2 cm and 3.20 cm. These are the results of measurements using different devices. While doing computations with these numbers special care is required. You may have wondered why the ratio of two measurements such as 32.1/12 is expressed as 2.7 not as 2.68 or 2.675. The number of digits used in a

measurement has some significance regarding the quality of measuring instruments.

In this unit we will learn about the meaning and usage of approximate numbers. We will also learn about the techniques of computations with these numbers. These techniques are of basic importance in calculating the results of experiments that we will do later. The mastery of these techniques is, therefore, essential at this stage.

In the next unit we will study errors which arise due to defects in measuring instruments, fluctuations in the quantity to be measured and several other reasons. We will also learn how these errors are propagated and how the final results of an experiment are expressed.

2.0 OBJECTIVES

After studying this unit, you should be able to:

- appreciate that all measurements are inexact and are expressed in numbers resulting from approximations or approximate numbers
- distinguish between precision and accuracy
- express a measurement in scientific notation
- add, subtract, multiply and divide approximate numbers.

3.0 MAIN CONTENT

3.1 Errors: Expressing the Results of Measurements

We are familiar with at least two reasons why all measurements are inexact. Firstly, error is caused by the measuring instrument itself, such as the zero error. Secondly, error can be due to limitations of human judgment and perception, such as in aligning the end of a rod to be measured with the zero of the centimetre scale. To better appreciate the inexact nature of measurement let us reflect on the process of measurement of length. Let us obtain a 'perfect' centimetre scale which has clear and equal marking of millimetres. We desire to measure the length of three arrows A , B and C (Fig. 1.1). Let us suppose that we are able to perfectly align the tails of the arrows with zero marking on the scale. Of course, this is impossible to achieve in practice, but let us assume it, to gain an insight into the process of measurement.

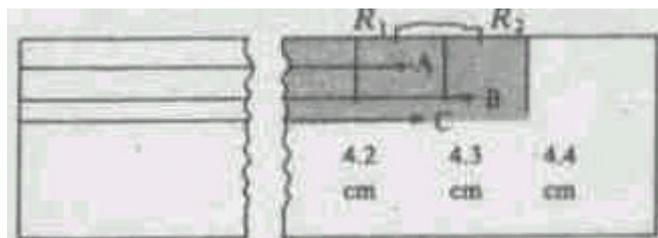


Fig. 1.1: The length of all the three unequal arrows A , B and C is reported as 4.3 cm. The shaded portion on the scale represents the range of error in this measurement. (The scale is highly magnified.)

In order to measure the length of these three arrows we look at the arrow heads. The head of arrow A is closer to the 4.3 cm mark than to the 4.2 cm mark. We will report the length of arrow A as 4.3 cm to the nearest millimetre. Let us now measure the length of arrow B . The head of arrow B is closer to 4.3 cm mark than to 4.4 cm mark. Therefore, we will also report its length as 4.3 cm to the nearest millimetre or simply 4.3 cm. Similarly the length of arrow C would be reported as 4.3 cm. Thus the lengths of all arrows whose tails are aligned with zero marking, and whose heads lie in the range R_1 and R_2 , would be reported as 4.3 cm. We can conclude that a measurement which is reported as 4.3 cm (which is in the middle of R_1R_2) might possibly be in error by 0.05 cm (or one-half of the unit of measure which is 0.1 cm) or less. Thus in the measurement 4.3 cm the last digit 3 is in error. We will, therefore, report measurements in such a manner that only the last digit will have error.

3.1.1 Possible Error and Precision

We have seen that the maximum possible error, barring any mistake in measuring, in a measurement is $1/2$ of the unit of measurement. The possible error is thus due to inherent imprecision in measuring devices. The measurements having less possible error are more precise. Since possible error is proportional to the unit of measure the instruments having smaller units of measure will give more precise measurement. A measurement reported to one hundredth of a centimetre, such as 5.32 cm is more precise than a measurement reported to one tenth of a centimetre, such as 5.3 cm.

The possible error = $1/2$ of the unit of measurement.

SELF ASSESSMENT EXERCISE 1

Consider the following pairs of measurement. Indicate which measurement in each pair is more precise.

- i. 17.9 cm or 19.87 cm

- ii. 16.5 s or 3.21 s
- iii. 20.56 °C or 32.22 °C

3.1.2 Relative Error and Accuracy

So far we have considered measurement of nearly equal lengths with emphasis on precision. Let us now consider measurement of much different lengths. Suppose, two measurements yield 3.2 cm and 98.6 cm using the same metre stick. The possible error in both of these measurements is equal to 0.05 cm but the measurement 98.6 cm is much bigger than measurement 3.2 cm. Would you say that the 98.6 cm is more accurate? How would you compare the accuracy of measurement such as 7.4 s and 98 s? In order to compare such measurements we define relative error as the ratio of possible error to the total measurement. In the Table below we have computed the relative error in some measurement. (The exact method of expressing the relative error will be discussed in section 3.4.)

Measurement	Unit of measure	Possible error	Relative error
3.2 cm	0.1 cm	0.05 cm	.02
98.6 cm	0.1 cm	0.05 cm	.0005
7.4 s	0.1 s	0.05 s	.007
98s	1 s	0.5 s	.005

Let us compare measurements 3.2 cm and 98.6 cm. Both have equal unit of measure and are therefore equally precise. But the measurement 98.6 cm has less relative error (.0005 compared to 0.02) and is therefore more accurate.

Comparison of measurements 7.4 s and 98 s is more revealing. The measurement 7.4 s is more precise than the measurement 98 s (possible errors 0.05 s and 0.5 s respectively) but less accurate (relative error 0.007 as compared to 0.005).

Relative error is the ratio of possible error to the total measurement.

You will therefore appreciate that a smaller measurement needs to be more precise for the same accuracy. This is why when measuring the dimensions of a room, metre is used as unit of measure while in measuring inter-city distances the unit kilometre is used for the same accuracy.

SELF ASSESSMENT EXERCISE 2

Consider the following pairs of measurements. Indicate which measurement in each pair is more accurate.

- i. 40.0 cm or 8.0 cm
- b. 0.85 m or 0.05 m

3.2 Scientific Notation

In the system of measurement that we use (SI-system) a measurement is expressed in decimal numerals. While measuring interatomic distances, we use very small numbers. On the other hand, while measuring interstellar distances we use very large numbers. In scientific notation these numbers are written as a number between one and ten multiplied by an integral power of ten. For example, the diameter of the sun is 1,390,000,000 metres and the diameter of hydrogen atom is only 0.000000000106 metres. In scientific notation we write the diameter of the sun as 1.39×10^9 m and the diameter of the hydrogen atom as 1.06×10^{-10} m.

SELF ASSESSMENT EXERCISE 3

The mass of a water molecule is 0.000 000 000 000 000 000 03g. Express this in scientific notation.

You have probably guessed that writing numbers in scientific notation will make computations easier. This is because we can apply the laws of exponents readily.

3.3 Significant Digits

We have seen that a measurement reported as 5.32 cm is more precise than 5.3 cm. The number of digits in these measurements is three and two, respectively. This suggests that the number of digits used in reporting a measurement have some significance. All non-zero digits are significant. However, in measurements such as 0.05 m or 0.005 m, none of the zeros is significant. The zeros to the left of the decimal are merely flags pointing to the decimal. The other zeros are placed to help locate the decimal point. Let us investigate this by calculating the possible error and relative error as in the Table below:

Measurement	Unit of measurement	Possible error	Relative error
.5m	.1 m	.05m	.1
.05m	.01 m	.005m	.1
.005 m	.001 m	.0005m	.1
.00005m	.00001 m	.000005m	.1

We can see from this table that the unit of measure and the possible error in all the cases are different. But the relative error is the same. Therefore, we can assert that these zeros are not significant because they do not affect the relative error. We can thus conclude that **a digit is significant if and only if it affects the relative error.**

SELF ASSESSMENT EXERCISE 4

Complete the following Table

S. No.	Measurement	Possible error	Relative error
1.	.2m	.05m	$\frac{.05}{.2} = .25$
2.	.20 m		
3.	.2000m		
4.	25 m		
5.	250m		
6.	25000 m		
7.	102 m		
8.	1002m		

- What can you conclude regarding the significance of 'trailing' zeros in the first three measurements?
- What can you conclude about zeros in the fifth and sixth measurements?
- What can you conclude regarding the significance of zeros between non-zero digits in the seventh and eighth measurements?

SELF ASSESSMENT EXERCISE 5

From the above discussion justify that a measurement possessing greater number of significant digits has greater relative accuracy.

Sometimes we take a sequence of whole number measurements such as 32, 30, 28, 26. All these measurements have two significant digits except the measurement 30. In such special cases zero can be taken as significant without any ambiguity.

SELF ASSESSMENT EXERCISE 6

Comment on the following:

"The distance to the sun from the roof of a house (height 20 m) is 150 million kilometres. Therefore the distance to the sun from the ground is **15 million kilometres** plus 20 m."

3.4 Computations with Approximate Numbers

In section 3.1 we have seen that the reported measurements have error in the last digit. For example, a measurement reported as $3.\bar{2}$, has error in the digit 2, which is indicated by placing a bar (-) over this digit. In computing values of physical quantities from observed experimental data we have to do computations. We will now establish some rules for expressing the results of basic operations with approximate numbers.

3.4.1 Multiplication and Division

Let us consider multiplication first. We want to multiply $1.2\bar{3}$ by $2.\bar{3}$. At each step of the computational process we will put a bar (-) over a significant digit which arises from computation with a digit containing error, as below:

$$\begin{array}{r}
 1 \quad . \quad 2 \quad \bar{3} \\
 \times \quad 2 \quad . \quad \bar{3} \\
 \hline
 . \quad \bar{3} \quad \bar{6} \quad \bar{9} \\
 2 \quad . \quad 4 \quad \bar{6} \quad \times \\
 \hline
 2 \quad . \quad \bar{8} \quad \bar{2} \quad \bar{9}
 \end{array}$$

We see that the product contains three digits which contain errors. Since we report the result in a number having only one digit containing error, we should round off the product to 2.8. Thus the product has two significant digits.

This is also equal to the number of significant digits contained in a factor having the least number of significant digits, namely 2.3. Therefore, we formulate the following rule:

RULE: The product (or quotient) of two measurements should be rounded off to contain as many significant digits as the measurement having fewer number of significant digits.

SELF ASSESSMENT EXERCISE 7

Divide 2.1 by 1.54. Round off the result according to the above rule.

Let us consider the multiplication of the following numbers which have already been rounded off to significant digits.

$$\begin{aligned} &5.2865 \times 3.8 \times 19.62 \\ &= 20.0887 \times 19.62 \\ &= 394.14029 \end{aligned}$$

which must be rounded off to 3.9×10^2 . We could have obtained the same result by rounding off these numbers first as shown below.

$$\begin{aligned} &5.29 \times 3.8 \times 19.6 \\ &= 20.1 \times 19.6 \\ &= 393.9 \text{ which rounds off to } 3.9 \times 10^2. \end{aligned}$$

Here we have rounded off 20.102 (the product of 5.29 and 3.8) to 20.1 before multiplying it with 19.6. We can generalise this as a labour saving rule.

Labour Saving Rule: Before multiplying (or dividing), round off the numbers to one more significant digit than (the number of significant digits) in the least precise factor.

SELF ASSESSMENT EXERCISE 8

Divide 9.5362 by 3.2

3.4.2 Addition and Subtraction

Let us study the process of addition given below:

$$\begin{array}{r}
 2 \ . \ 1 \ 3 \ \bar{5} \\
 2 \ . \ 5 \ \bar{3} \\
 1 \ . \ 0 \ \bar{2} \\
 \hline
 5 \ . \ 6 \ \bar{8} \ \bar{5}
 \end{array}$$

The sum has two error-containing digits. We, therefore, round off the sum to 5.69 so that it contains only one digit containing error. Rounding off is necessary because the sum cannot be more precise than individual measurements. We note that the sum 5.69 has the same unit of measure as the least precise addend. Thus we formulate the following rule.

Rule: While adding (or subtracting) approximate numbers, round off the sum (or difference) to the same unit of measure as the least precise measurement.

SELF ASSESSMENT EXERCISE 9

Subtract 2.11 from 2.1546.

SELF ASSESSMENT EXERCISE 10

Compute the sum of 2.1546 m, 2.11 m and 2.125m.

Hint: In such cases we can use the following labour saving rule.

Labour Saving Rule: Before adding (or subtracting) round off the numbers so that they contain one more digit of precision than the number of precision digits in the least precise.

Thus the addends become 2.155m, 2.11m and 2.125m.

4.0 CONCLUSION

Learners have learnt that all measurements are inexact and are expressed in numbers resulting from approximations or approximate numbers and how to distinguish between precision and accuracy. You have also studied how to express measurements in scientific notations.

5.0 SUMMARY

- Exact measurement is impossible. The result of every measurement is expressed in numbers resulting from approximation such that only

the last digit contains error. In scientific notation, a measurement is expressed as a decimal number between one and ten multiplied by powers of ten.

- Possible error is one-half the unit of measurement. Precision is a function of possible error only.
- Relative error is the ratio of possible error to total measurement. Accuracy is related to relative error. A digit is significant if and only if it affects the relative error.
- Rule for multiplication (or division) The product (or quotient) of two measurements should be rounded off to contain as many significant digits as the measurement having the least number of significant digits.
- Rule for addition (or subtraction) While adding (or subtracting) approximate numbers, round off the sum (or difference) to the same unit of measure as the least precise measurement.

6.0 TUTOR MARKED ASSIGNMENTS

1.
 - a Distinguish between precision and accuracy.
 - b State which of these measurements are more precise in each pair
 - i. 17.7 cm; 17.69
 - ii. 20.56°C; 32.24°CGive reasons for your choice.
 - c Express the followings in scientific notations:
 - i 140000000009
 - ii 0.00000000016
 - iii 4,00000000
 - iv 0.0000035

7.0 REFERENCES/FURTHER READINGS

IGNOU (1997) Physics Laboratory I, Physics PHE-03, New Delhi, India

UNIT 2 INTRODUCTION TO LABORATORY-I: ERROR ANALYSIS

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1.0 INTRODUCTION

In the last unit you studied about errors in measurements due to imprecision of measuring devices. The results of measurements were expressed as approximate numbers. We also learnt about performing basic operations of addition, subtraction, multiplication and division of approximate numbers and expressing results using correct number of significant digits. We assumed that the measuring instruments as well as the observers were perfect. However, as you are aware, there can be defects in measuring instruments and also humans are not perfect. If the environment is not perfectly controlled its changes will affect the object to be measured thereby introducing errors in measurements.

In this unit we will familiarise ourselves with these and other sources of errors. We will also learn how to estimate and possibly eliminate or account for such errors. In most of the physics experiments our objective is to determine relationship among physical quantities. Therefore, we will estimate the errors in the measurement of various physical quantities and make efforts to determine valid relationships as mentioned above. In the next couple of experimental write-ups we will apply our knowledge of errors and its propagation to actual measurements and deduce relationships. We will first concentrate on the

measurements of fundamental quantities such as mass, length and time, and then do experiments involving two or more of these quantities.

2.0 OBJECTIVES

After studying this unit, you should be able to:

- distinguish between random errors and systematic errors;
- eliminate to some extent the systematic errors;
- compute errors in the measurement of various physical quantities;
- analyse data by calculation and by plotting graphs to determine functional relationship; and
- interpret the slope of a graph and to determine the value of certain physical quantities from the slope of a straight line graph.

3.0 MAIN CONTENT

3.1 Types of Errors

Every measuring instrument has a limitation in that it cannot measure physical quantities smaller than a certain value known as the least count of instrument. For example, a metre scale can measure only up to 1mm (smallest division of the scale). Vernier callipers can generally measure up to 0.1 mm whereas a spherometer and screw gauge can measure lengths up to 0.01mm. Similarly a thermometer usually has the least count of half a degree. In addition to these limitations which are inherent in a measuring device, there are other sources of error. These arise due to changes in environment, faults in observational techniques, malfunctioning of measuring devices etc. The errors in any measurements can be classified into two broad headings namely - Systematic Errors and Random Errors.

Let us now study the causes of such errors, and see how they are eliminated or minimised,

3.1.1 Systematic Errors

The systematic errors, also called determinant errors, are due to causes which can be identified. Therefore, these errors, in principle, can be eliminated. Errors of this type result in measured values which are consistently too high or consistently too low. Let us discuss these errors one by one.

(i) Zero Error

In the case of vernier callipers, for example, when the jaws are in contact, the zero of the vernier may not coincide with the zero of the main scale. The magnitude and sign of the 'zero error' can be determined for the scale readings. We can easily eliminate this error from the measurement by subtracting or adding the zero error.

(ii) Back Lash Error

While measuring a physical quantity there may be an error due to wear and tear in the instruments like screw gauge or spherometer due to defective fittings. Such an error is called back lash error and can be minimised in a particular set of measurements by rotating the screw head in only one direction.

(iii) End Correction

Sometimes the zero marking of the metre scale may be worn out. Unless we are careful, this will lead to incorrect measurements. We must therefore compensate for this by shifting our reference point.

(iv) Errors due to Changes in the Instrument Parameters

Usually, in experiments involving electrical quantities, the value of the electrical quantities change during the course of the experiment due to heating or other causes. For example, the value of the resistance of a wire will increase because of current passing through it. This will lead to errors which are generally difficult to calculate and compensate for. To some extent this can be avoided by not allowing current to flow through the circuit while observations are not being taken.

(v) Defective Calibration

Occasionally instruments may not be properly calibrated leading to errors in the results of measurement. This type of error is not easily detected and compensated for. This is a manufacturer's defect and if possible the instrument should be calibrated against standard equipment.

(vi) Faulty Observation

This could be due to causes like parallax in reading a metre scale. These errors are eliminable by using proper techniques.

3.1.2 Random Errors

You must have noticed that many times repeated measurements of the same quantity do not yield the same value. The readings obtained show a scatter of values. Some of those values are high while others are low. This fluctuation is due to random errors whose possible sources are:

(i) Observational

These arise due to errors in judgement of an observer when reading a scale to the smallest division.

(ii) Environmental

These arise due to causes like unpredictable fluctuations in line voltage, variation in temperature etc. They could also be due to mechanical vibrations and wear and tear of the systems. There could also be a random spread of readings due to friction say, wear and tear of mechanical parts of a system.

SELF ASSESSMENT EXERCISE 1

Which of the figures 2.1 (a) or (b) show random errors only.

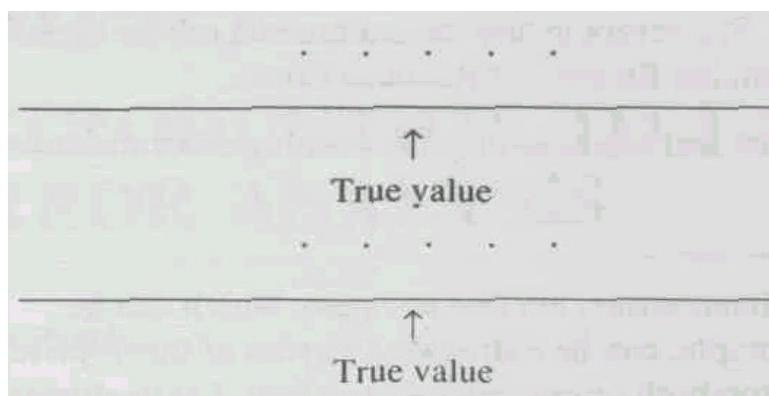


Fig. 2.1. Set of measurements. Each point indicates the result of a measurement

Unlike systematic errors, random errors can be quantified by statistical analysis. Let us now learn to determine the size of such error.

3.2 Determining the Size of Error

When we measure a quantity it is important to take several readings. It may be preferable that readings are taken by independent observer. This has the advantage that bias of a single observer is eliminated. The value obtained will indicate whether the data is scale limited or random. An error analysis can be made to determine the size of error from these

readings. A typical set of values of a measurement are given below in Table 1. The quantity to be measured as a "true" value is independent of our measuring process. But the imperfection of our measuring process prevents us from obtaining that value every time. Which one of the values listed in Table 1 would be "true" value ? It is impossible to tell that from the measurements because of this spread. Under the circumstances the average A value can be quoted. To get the average value we simply add up all the measurements and divide the sum by the total number of measurements. As you can see from the Table I the average is 3.68. Also notice that most of the data in Table 1 deviates from the average. Therefore, a measure of spread of values would be the average deviation. To obtain average \bar{d} we first take the difference of each data from the average to get individual deviations d . These deviations are then added and their sum is divided by the number of observations to obtain \bar{d} . As you can see from Table 1 the average deviation in this case is 0.009.

Table 1

S. No.	Data	Deviation (\bar{d})
1.	3.69	0.01
2.	3.67	0.01
3.	3.68	0.0
4.	3.69	0.01
5.	3.68	0.0
6.	3.69	0.01
7.	3.66	0.02
8.	3.67	0.01
$A = 3.68$		$d = 0.009$

As you are aware, repeated measurement of the same quantity yield results with better precision. A measure of this is the precision index S whose definition (without proof) is

$$S = \frac{\bar{d}}{\sqrt{n}}$$

where \bar{d} is the average deviation and n is the number of observations. The precision index S is a measure of uncertainty of average. Using the data of Table 1, the precision index is

$$S = \frac{\bar{d}}{\sqrt{n}} = \frac{0.009}{\sqrt{8}} = 0.003$$

Thus the final result can be expressed as $A \pm S$. In this case the result of random data analysis gives 3.68 ± 0.003 . We can see that this error is

much less than the possible error which is ± 0.005 . Thus in such cases we will consider the possible error only.

SELF ASSESSMENT EXERCISE 2

The measurement of the length of a table yields the following data.

$$l_1 = 135.0\text{cm} \quad l_2 = 136.5\text{cm} \quad l_3 = 134.0\text{cm}$$

$$l_4 = 134.5\text{cm}$$

Calculate (a) the average value and (b) precision index. How does the precision index compare with possible error? How will you express the final result?

3.3 Propagation of Error

We have so far learnt how to determine the error in the measurement of a quantity which can be measured directly. In actual practice, however, we determine values of a quantity from the measurements of two or more independent quantities. In such cases the error in the value of the quantity to be determined will depend on the errors in other independent quantities. In other words the error will 'propagate'. The actual analysis of propagation of error is beyond the scope of this course. We shall, therefore, quote some rules which can be used in our laboratory.

3.3.1 Error Propagation in Addition and Subtraction

What will be the error, in quantity E defined by $E = x + y + z$? Let us take the differential of this quantity, we get $dE = dx + dy + dz$. If the error is small compared to the measurement we can replace the differential by 'delta' to get

$$\delta E = \delta x + \delta y + \delta z$$

which is simply the sum of errors in x , y and z . It, therefore, is the maximum error in E . Statistical analysis shows that a better approximation is

$$\delta E = \sqrt{(\delta x)^2 + (\delta y)^2 + (\delta z)^2}$$

We only consider the magnitude of errors in the above calculation. Therefore, the error in the quantity $(x + y - z)$ will also be the same.

Solved Examples: Let the measured value of two lengths be

$$L_1 + \delta L_1 = 1.746 \pm 0.010 \text{ m}$$

$$L_2 + \delta L_2 = 1.507 \pm 0.010 \text{ m}$$

The error in the quantity

$$L = L_1 + L_2 \text{ will be } \delta L = \sqrt{(0.010 \text{ m})^2 + (0.010 \text{ m})^2} = 0.014 \text{ m}$$

3.3.2 Error Propagation in Multiplication and Division

If a quantity $E = A \times B$ and the result of measurement of A & B is $A \pm \delta A$ and $B \pm \delta B$, then what will be the error δE in E ? Here if we take differentials we get

$$dE = B dA + A dB$$

Dividing by $E = AB$ and changing differentials by 'deltas' we get

$$\frac{\delta E}{E} = \frac{\delta A}{A} + \frac{\delta B}{B}$$

SELF ASSESSMENT EXERCISE 3

Take logarithm of $E = AB$ and then differentiate to show that

$$\frac{\delta E}{E} = \frac{\delta A}{A} + \frac{\delta B}{B}$$

which is generally known as the logarithmic error.

The statistical analysis, however, gives the following better result of the fractional error in E .

$$\frac{\delta E}{E} = \sqrt{\left(\frac{\delta A}{A}\right)^2 + \left(\frac{\delta B}{B}\right)^2}$$

Rule 1: When independent measurements are multiplied or divided the fractional error in the result is the square root of the sum of squares of fractional errors in individual quantities.

Solved Example: In an experiment, velocity was obtained from the measurements of distance and time. If the distance measured is $S + \delta S = 0.63 \pm 0.02 \text{ m}$

$$\frac{\delta S}{S} = 0.03$$

and time is $T \pm \delta T = 1.71 \pm 0.10$

$$\frac{\delta T}{T} = 0.06$$

Then the velocity (V)

$$V = \frac{S}{T} = 0.368 \text{ m/s}$$

The fractional error in V is given by

$$\frac{\delta V}{V} = \sqrt{\left(\frac{\delta S}{S}\right)^2 + \left(\frac{\delta T}{T}\right)^2} = 0.07$$

$$\delta V = 0.368 \text{ m/s} \times 0.07 = 0.02 \text{ m/s}$$

Thus the final result becomes $V + \delta V = 0.37 \pm 0.02 \text{ m/s}$.

3.3.3 Error Propagation in Other Mathematical Operations

Errors in exponential quantity: Let us first consider a special case where a quantity appears with an exponent. For example $S = A^2 = A \times A$. Here the two numbers multiplied together are identical and hence **not independent**. The rule mentioned above does not apply. Detailed analysis shows that logarithmic error gives a good estimate. Taking the logarithm of above equation we get

$$\log S = 2 \log A$$

on differentiation and changing differentials to 'deltas' we get

$$\frac{\delta S}{S} = 2 \frac{\delta A}{A}$$

Therefore, the fractional error in A^2 would be twice the error in A , the fractional error in A^3 will be 3 times the fractional error in A , and the fractional error in \sqrt{A} will be 1/2 the fractional error in A .

Rule: The fractional error in the quantity A^n is given by n times the fractional error in A .

Example: Suppose two measurements of mass are $M_1 \pm \delta M_1 = 0.743 \pm 0.005 \text{ kg}$ and $M_2 \pm \delta M_2 = 0.384 \pm 0.005 \text{ kg}$. Determine the value of $M = 2M_1 + 5M_2$ along with δM .

What will be the error in $(M_1 + M_2)^2$ and $(M_1 - M_2)^3$.

Hint: The error in $2M_1$ is $2\delta M_1$ and in $5M_2$ is $5\delta M_2$.

Thus error in $2M_1 + 5M_2$ is $\delta M = \sqrt{(2\delta M_1)^2 + (5\delta M_2)^2}$.

Error in $(M_1 + M_2)^2 = 2\sqrt{(\delta M_1)^2 + (\delta M_2)^2}$;

Error in $(M_1 - M_2)^3 = 3\sqrt{(\delta M_1)^2 + (\delta M_2)^2}$

Similarly in other mathematical operations and deducing results from graphs (about this you will learn in the next subsection) the following rule is used.

Rule: The error in the result is found by determining how much change occurs in the result when the maximum error occurs in the data.

Example: Let us compute the error in the sine of $30^\circ \pm 0.5^\circ$. Using the logarithmic tables we get:

$$\sin 30^\circ = 0.5, \sin 30.5^\circ = 0.508, \sin 29.5^\circ = 0.492$$

The difference between $\sin 30^\circ$ and $\sin 30.5^\circ$ is 0.008, and the difference between $\sin 30^\circ$ and $\sin 29.5^\circ$ is also 0.008. Thus the error in $\sin 30^\circ$ would be ± 0.008 .

SELF ASSESSMENT EXERCISE 4

Determine the error in the sine of 90° , when the error in the angle is 0.5° . Compare your result with that of the example above.

3.3.4 Error Propagation in Graphing

Very often we can better visualise the functional relationship between two physical quantities by plotting a graph between them. This is another useful way of handling experimental data because the values of some quantities can be obtained from the slope. While plotting a graph we will use the following guidelines:

1. A brief title may be given at the top.
2. Label the axes with the names of the physical quantities being presented along with units. It is customary to plot the independent variable (the quantity which is varied during the experiment at one's will) on the x -axis and the dependent variable, on the y -axis (the dependent variable is the one that varies as a result of change in the independent variable). We

would write the name of the variable represented on each axis along with units in which they are measured.

- We should choose the range of the scales on the axis so that the points are suitably spread out on the graph paper and not cramped into one corner. Check for the minimum and maximum values of the data that has to be plotted. We may then round off these two numbers to slightly less than the minimum and a slightly more than the maximum. Their difference may be divided by the number of divisions on the graph paper. For example, if we are to plot 5.2 and 17.7 it would be convenient to allow the scale to run from 5 to 20 rather than from 0 to 18.

Each set of data points is indicated by a point within a circle on the graph paper and the error is shown by using bars above and below this point as shown in Fig. 2.2. The graphed data show that velocity V is the linear function of time T . We recall that the general equation of a straight line is $y = mx + c$ where m is the slope of line and c the vertical intercept in the value of y when $x = 0$. From the graph we can thus write $V = aT + V_0$. By comparing the above equation we can conclude that the slope of the graph gives the acceleration and the intercept gives the velocity V_0 at $T = 0$. From the graph $V_0 = 0.32 \text{ m/s}$. To determine the slope we consider two points on the straight line which are well separated. Then

$$a = \text{Slope} = \frac{V}{T} = \frac{2.35 - 0.40(\text{m/s})}{10.0 - 0.5(\text{s})} = 0.20 \text{ ms}^{-2}$$

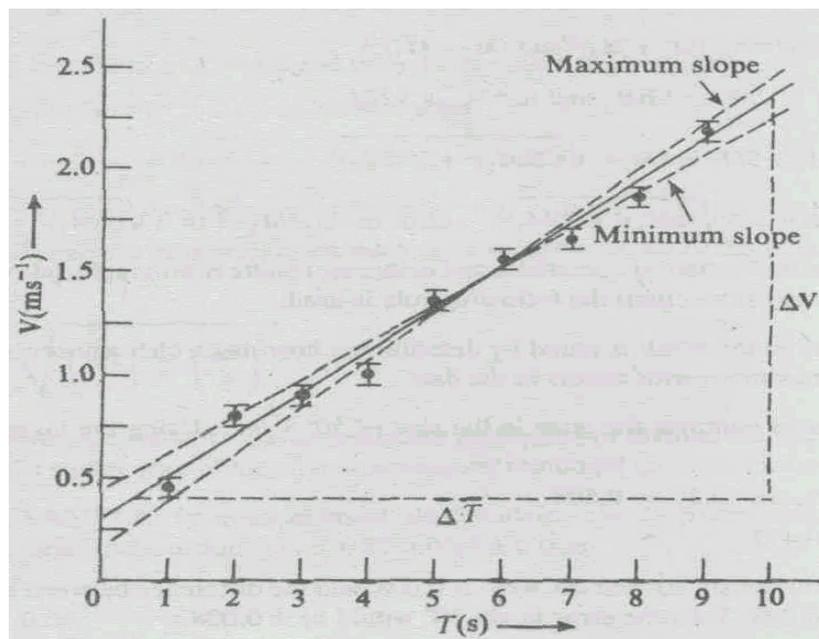


Fig. 2.2: Graph between velocity and time

In the above example, we have plotted the variable V which is a linear function of T in a linear graph paper. In some experiments we may get data where the relationship between the measured variables is not linear. Suppose a man gets salary of ₦200 on the 1st of every month and he decides that each day he will spend half the money he has with him on that day. Then the amount of money, which the man will have over a period of first seven days of any month, will be given as in the Table 2.

Table 2

Day of any month	Money left with the man (M) (₦)
1 st	200.00
2 nd	100.00
3 rd	50.00
4 th	25.00
5 th	12.50
6 th	6.25
7 th	3.12
8 th	1.56

Let us plot these data on a linear graph paper. The graph will be of type shown in Fig. 2.3. Look at the graph carefully. You will find that seven of the ten experimental points are clustered together near the bottom right-hand corner of the graph. The shape of the curve we have drawn also involves a bit of guesswork. Therefore, we have to find some method so that these data can be plotted in a better way.

Try to recollect what you did in school when you came across data like this which range over a few orders of magnitude or having big gaps between the points. We will tell you. In such cases you took the logarithm of the data and then plot those data in a linear graph paper. When you did this, you must have found that the result was a straight line. So, let us take the logarithm of the data of Table 2 and tabulate them as shown in Table 3.

Table 3

Day of any month	$\log M$
1st	2.301
2nd	2.000
3rd	1.699
4th	1.397
5th	1.097
6th	0.796
7th	0.494
8th	0.193

Now plot $\log M$ against days as shown in Fig.2.4. You obtain a graph in which points are more clearly spaced evenly and hence you can more easily draw a straight line through the points.

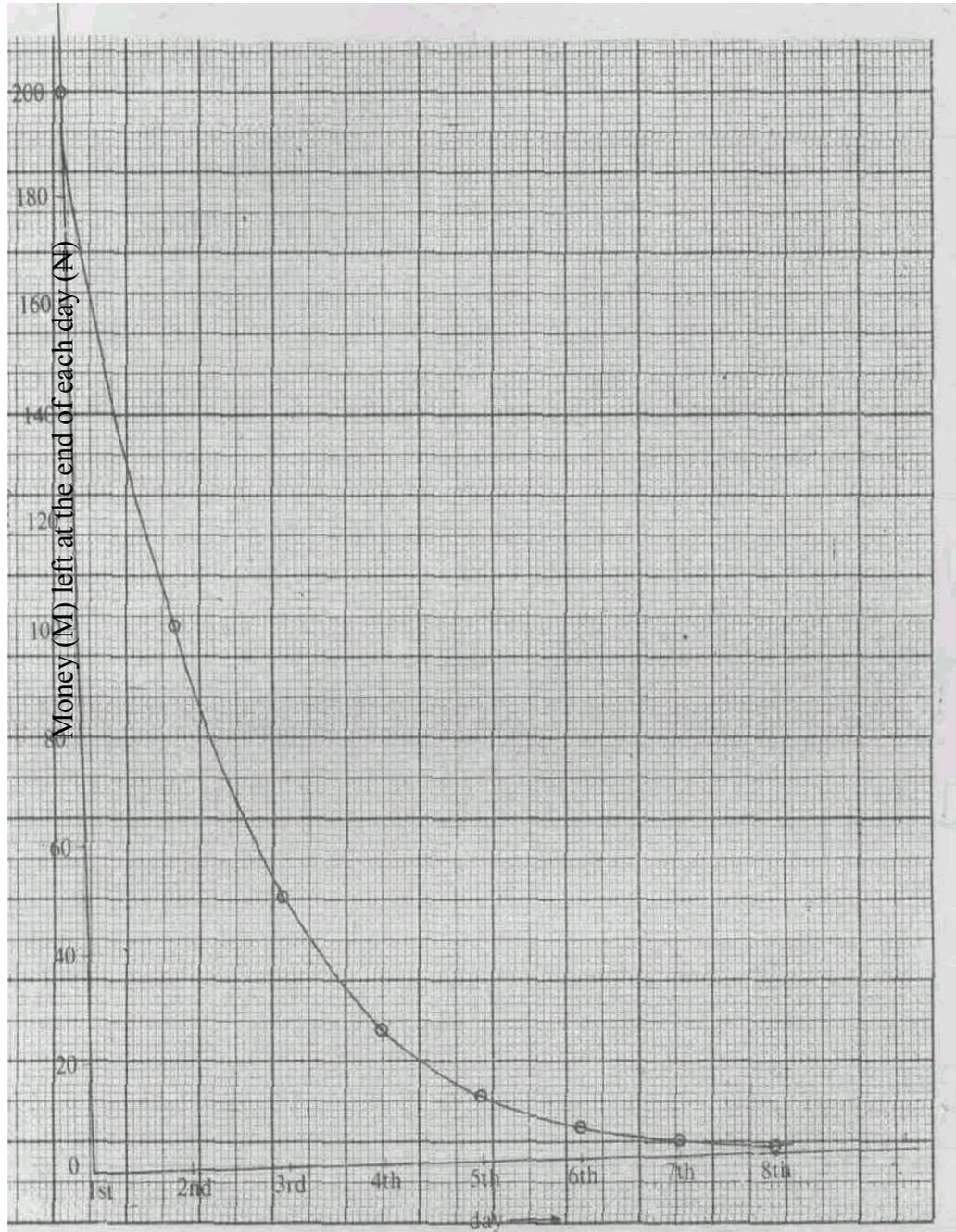


Fig. 2.3: Graphical representation of Table 2

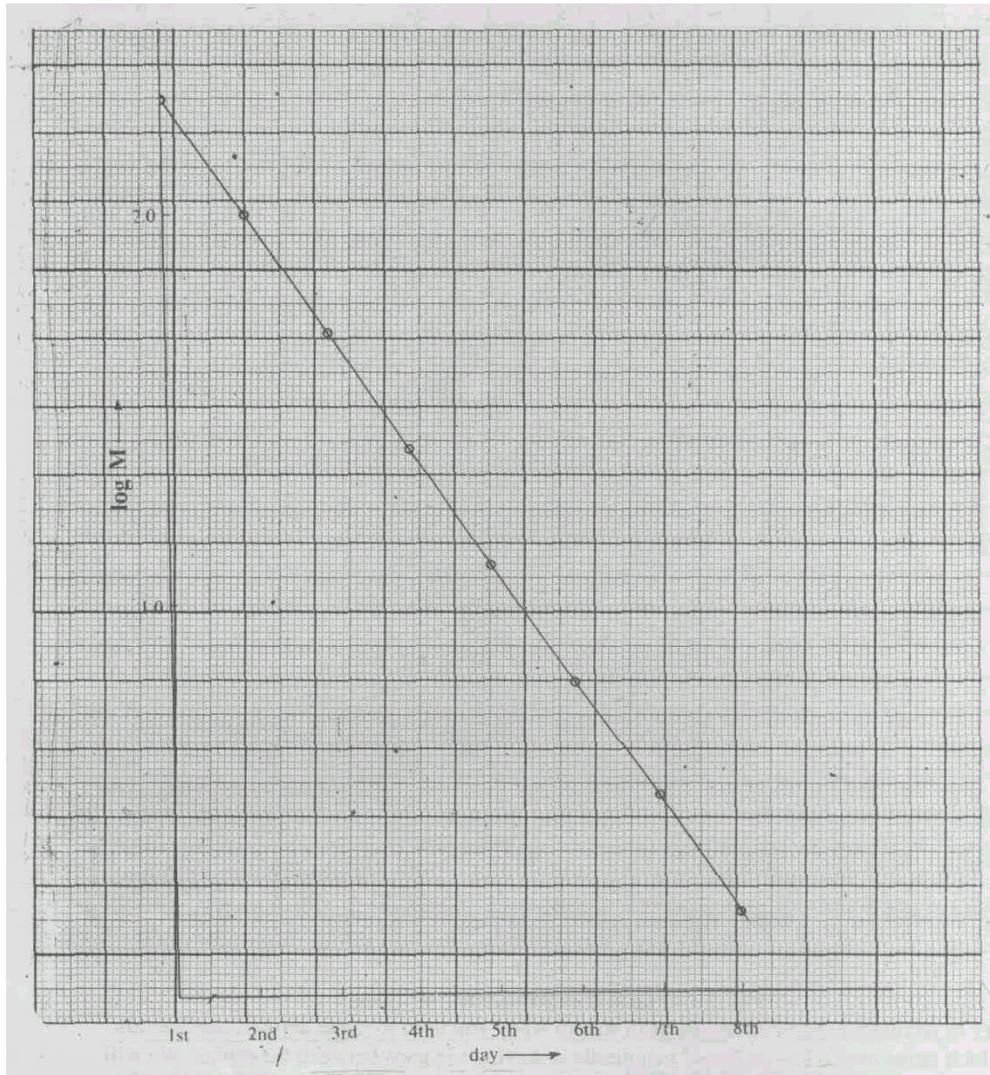


Fig. 2.4: Graphical representation of Table 3

You might have realised that working out the log values for each data is tedious and it also introduces another step, which may introduce error between the data and the graph. Therefore, to plot such data we use a graph paper called semi-logarithmic or log-linear graph paper where the lines on one axis have been drawn in a logarithmic fashion. On a semi-log paper (see the graph paper of Fig. 2.5) the horizontal scale is an ordinary one, in which the large divisions are divided into tenths and each division has the same size. The vertical scale is a logarithmic scale (it automatically takes logarithms of data plotted), in which each power of ten or decade (also called frequency) corresponds to the same length of scale. In each decade, the divisions become progressively compressed towards the upper end. Now in the semi-log graph paper we plot the data of Table 1. We obtain a straight line as shown in Fig. 2.5. If you compare Figs. 2.4 and 2.5 you will see that the points plotted on semi-log paper are distributed in just the same way as the logarithms of the corresponding datum would be distributed on a linear graph paper. A question may strike in your mind that how to calculate the slope of the

straight line of Fig. 2.5? Also what is the equation of the straight line? Let $\log M$ be represented by y and day by t , then we have a straight line graph of y against t . Let the equation be represented as

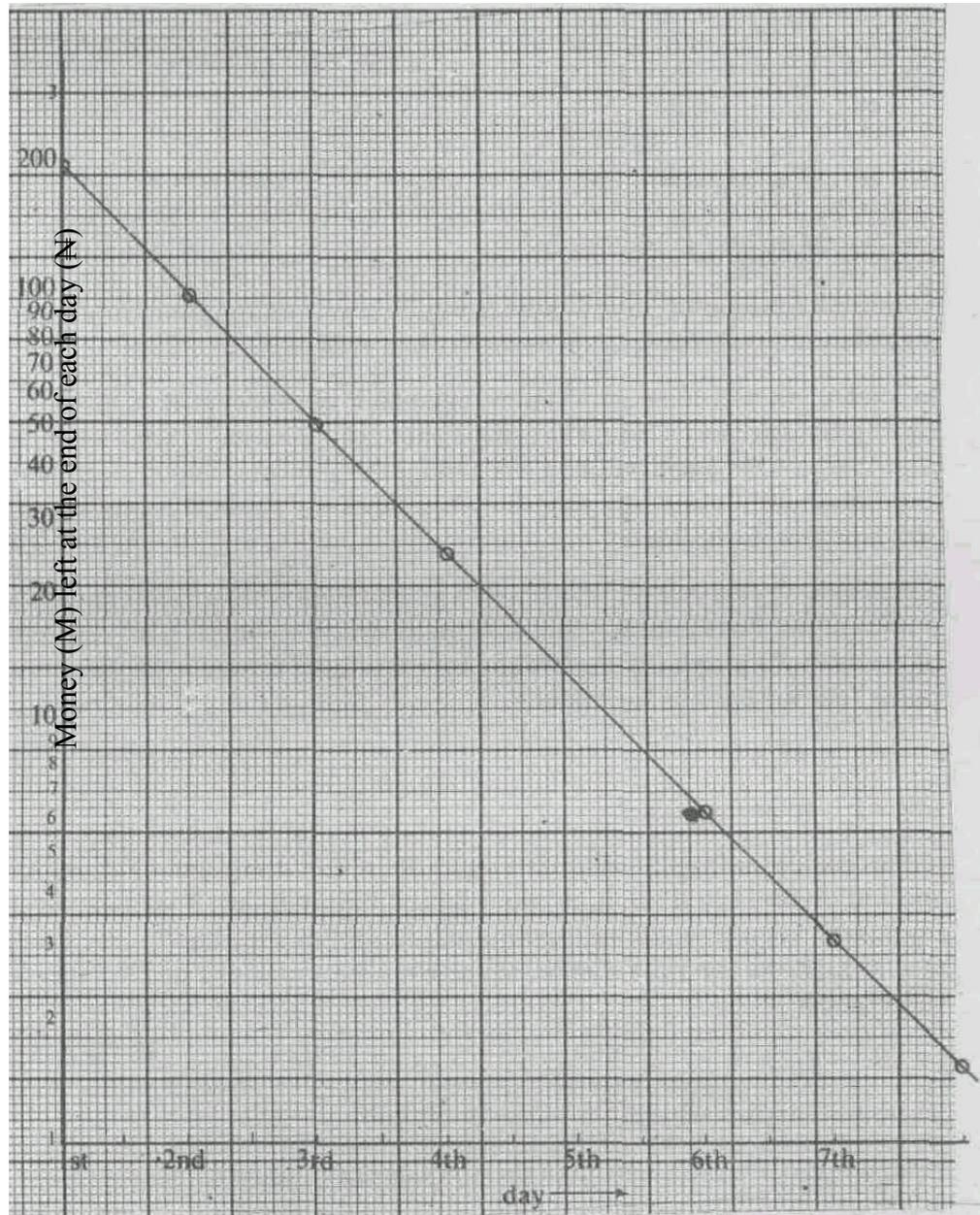


Fig. 2.5: Representation of Table 5 graphically on a semi-log paper

Where b is the intercept of the line on the y -axis, and k the slope of the line. We can find the values of b and k from the graph as follows. When $t = 0$, $M = 200$ then $\log M = \log 200 = 2.30 = y$

$$2.30 = b + 0 \quad \text{or} \quad b = 2.30$$

$$\therefore y = 2.30 + kt$$

when $t = 7$ th day, $M = 1.56$ and $\log M = \log 1.56 = 0.193 = y$

Putting these values in Eq. (1) we get

$$0.193 = 2.30 + 7k$$

$$\therefore \text{The slope } k = -\frac{2.1}{7} = -0.3$$

and the equation of the straight line is

$$y = 2.3 - 0.3t$$

From the graph of Fig. 2.5 or in other words from the Eq. (2) can you find the equation of the curve plotted in Fig. 2.3?

Let the value of M at $t = 0$ be denoted as M_0 then Eq. (2) becomes

$$\log M = \log M_0 + kt$$

or $\log M - \log M_0 = kt$

or $\log \frac{M}{M_0} = kt$

or $\frac{M}{M_0} = 10^{kt}$

or $M = M_0 10^{kt}$

or $M = 200 \times 10^{-0.3t}$

This is the equation of the curve plotted in Fig. 2.3. It tells us that the money is decreasing logarithmically (also called exponentially) with each day.

In science you will come across many logarithmic or exponential relations of the form of Eq. (3). In such cases it would be convenient to plot the data on semi-logarithmic graph paper because the graph will be a straight line if the relationship is logarithmic. Also the slope of the line (which may give you the value of any physical constant) can be read simply and directly from the graph.

Sometimes we find that we wish to plot a graph where both variables range over several powers of ten. For example, you know that according to Kepler's law, the semi-major axis of the orbit of a planet (R) is related to its period (time for one revolution around the sun) T by the following power-law relation:

$$R^3 = kT^2 \quad (4)$$

where k is another constant.

If you consider the experimental data which show how the quantity T depends on quantity R you will observe that R varies by two orders of magnitude and T varies by three orders of magnitude. In other words the experimental data follows the Eq. (4). For a moment, suppose you do not know the exact relationship between the variables T and R . Then you can suppose that

$$R = kT^n \quad (5)$$

where n is another constant.

Using the conventional method to find the value of n , you will take logarithm of Eq. (5) as follows:

$$\log R = \log k + n \log T$$

Now you will plot $\log R$ vs. $\log T$ on a linear graph paper. The slope of straight line obtained will give the value of exponent n . But again, as mentioned above, taking logarithm of each experimental data is rather tedious so it would be convenient to plot both the variables T and R on a logarithmic scale where the lines on both the axes are drawn in a logarithmic fashion. A log-log graph is shown in Fig. 2.6. The points lie upon a straight line. The slope of the straight line will give the exponent (n) of the power-law relation and hence the exact relationship between R and T will be found out.

To determine the error in the value of the slope of the straight line drawn in any graph paper (linear or semi-log or log-log) we draw two dashed lines representing the greatest and least possible slopes which reasonably fit the data as shown in Fig. 2.2. Thus the error in the slope is defined as

$$\text{error in slope} = \frac{\text{maximum slope} - \text{minimum slope}}{2}$$

Thus from the graph we get the error in the slope as

$$\delta a = \frac{0.23 - 0.19}{2} = 0.02 \text{ m/s}^2$$

Thus the experimental value of acceleration from the graph is $a \pm \delta a = 0.20 \pm 0.02 \text{ m/s}^2$.

Similarly the error in intercept =

intercept of minimum slope line - intercept of maximum slope

$$\delta V_0 = \frac{0.45 - 0.17}{2} = 0.14 \text{ m/s}$$

Thus the velocity $V_0 = 0.32 \pm 0.14 \text{ m/s}$

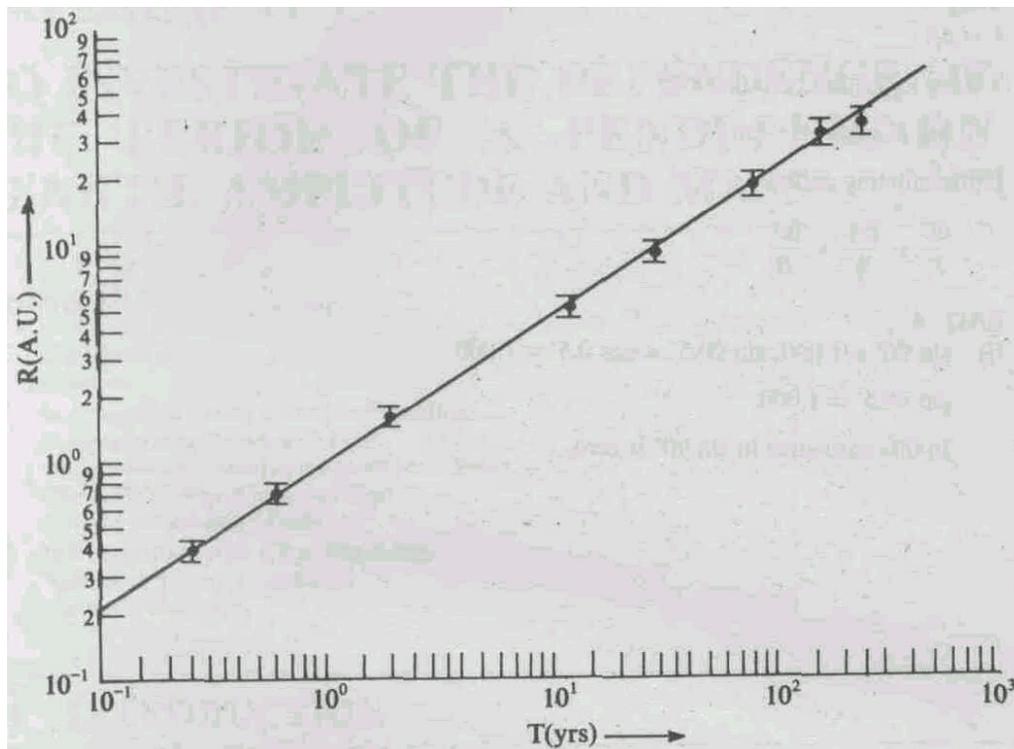


Fig. 2.6 : log-log graph

3.4 Use of π

It appears that most of our students are under the impression that the value of n is equal to $22/7$ exactly. Unfortunately many book writers also have contributed to perpetuate and establish this false idea by setting many numerical problems with data so cooked up that using $\pi = 22/7$, the factor 7 always happily cancels out and the simplification becomes very easy. However, in the real world the values of actual physical quantities are not such as to facilitate cancellation with 7. Also, we may as well acknowledge that the value of π cannot be expressed exactly in terms of any whole number. The value of $\pi = 22/7$ is one of the many approximations that can be used. In fact, a better approximation is $355/113 = 3.1415928$. Compare this with the calculator value $\pi = 3.141592654$ and $22/7 = 3.14286$. It may be noted that the value of $22/7$ deviates from the more accurate value from the calculator in the third decimal place; if we round it off to 5-digit accuracy, $\pi = 3.1415$ (from calculator), whereas the approximations $355/113 = 3.1416$ and $22/7 = 3.1429$. For practical purposes at the undergraduate level, the

most convenient and comparatively more accurate thing to do will be to remember

$$\pi = 3.142; \log \pi = 0.4972;$$

$$\pi^2 = 9.870; \log \pi^2 = 0.9943$$

Wherever the value of it is to be used in our calculations, the above values may prove fruitful.

4.0 CONCLUSION

In the earlier unit, you have been introduced errors and after going through this unit, you would have learnt to distinguish among all types of errors and to compute errors in measurements of various quantities. Graphs, interpretation of graphs, and determination of physical quantities have also been explained.

5.0 SUMMARY

- There are two main types of errors- random and systematic errors. Systematic errors can be eliminated to some extent
- Errors in physical quantities are additive
- The values of certain physical quantities can be determined from the slopes of straight line graphs
- Functional relationships can be determined by plotting some characteristic graphs.

6.0 TUTOR MARKED ASSIGNMENTS

- 1(a) Distinguish between random and systematic errors. State how to some extent, systematic errors in measurements can be corrected.
- (b) The measurement of the length of an object using a metre rule yielded the following values: $L_1 = 165.0$ cm; $L_2 = 163.5$ cm; $L_3 = 166.0$ cm and $L_4 = 165.5$ cm.

Calculate,

- (a) the average length;
- (b) the precision index;
- (c) How does the precision index compare with the possible error?
- (d) Express the final average length with the error.

2. In an experiment to determine the velocity of a moving object, the distance covered was 0.54 ± 0.05 m within a time frame of 1.82 ± 0.01 s. Express the velocity indicating the error involved.

7.0 REFERENCES/FURTHER READINGS

IGNOU (1997) Physics Laboratory I, Physics PHE-03, New Delhi, India

UNIT 3 **EXPERIMENT 1** **TO INVESTIGATE THE DEPENDENCE OF** **THE PERIOD OF A PENDULUM ON LENGTH,** **AMPLITUDE AND MASS**

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Investigations with a Simple Pendulum
 - 3.1.1 Dependence of the Period on the Length
 - 3.1.2 Dependence of the Period on the Amplitude of Swing
 - 3.1.3 Effect of Mass of the Bob on the Period
 - 3.1.4 Damping and Relaxation Time
 - 3.2 Investigations with a Bar Pendulum
 - 3.2.1 Variation of the Period with Length
 - 3.2.2 The Radius of Gyration
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor Marked Assignments
- 7.0 References/Further Readings

1.0 INTRODUCTION

In your school you must have worked with a simple pendulum. A simple pendulum essentially consists of a heavy metallic bob suspended from a rigid support by means of a weightless and inextensible string. It can freely oscillate to and fro about the point of suspension. The maximum displacement of the bob on either side of its equilibrium position is called the *amplitude* of oscillation. The time taken by the pendulum to complete one oscillation is called the *period*. As we examine the motion of a simple pendulum, some questions that immediately come to our mind are:

1. How do the material, shape and size of the bob affect the period of the pendulum?
2. How does the period change with amplitude of the swing?
3. Does the length or thickness of the string change the period?
4. How does the air dragged by the bob influence the period of the pendulum?

We will investigate some of these questions here. You may think that this experiment is far too simple to perform at your level. But our purpose of having a simple and familiar arrangement is to help your understanding of simple harmonic motion and also to give you experience of planning an experiment, taking measurements and analysing results. That is, we intend to give you training in scientific method of learning and develop your investigative skills.

Such a system was first envisaged by Galileo on observing the vibrations of a chandelier at a banquet. He calculated its period by his pulse rate. (You can make a simple pendulum by tying a piece of stone to a 70 to 100 cm long thread.) A modification of this arrangement is used in wall clocks. You may also know that nowadays most precise time measurements are done by atomic clocks, where caesium atoms act like a pendulum.

You may now think that a simple pendulum is an ideal arrangement for time measurements. But it is not so; a simple pendulum has some inherent drawbacks. For example, the bob drags air, the string is not strictly inextensible, motion about the point of suspension may have rotational component, etc. Some of these can be eliminated by using a compound pendulum. A compound pendulum is a rigid body capable of oscillating freely about a horizontal axis passing through it. In your laboratory, you will find it in the form of a metallic bar having a series of holes. These holes allow us to make the pendulum oscillate freely when suspended from a knife-edge. The pendulum executes simple harmonic motion.

Oscillatory motion is a universal phenomenon. Like simple and compound pendulums, a spring-mass system also executes simple harmonic motion and may be used to determine the spring constant. You will learn to do it in the next experiment.

2.0 OBJECTIVES

After doing this experiment, you should be able to:

- determine whether two parameters are related by a power law;
- establish the relation between the period and the length of a simple pendulum;
- discover the dependence of the period on the amplitude of swing and the mass of the bob;
- compute relaxation time
- compare the values of acceleration due to gravity using a simple and a bar pendulum; and
- compute the radius of gyration.

3.0 MAIN CONTENT

3.1 Investigations with a Simple Pendulum

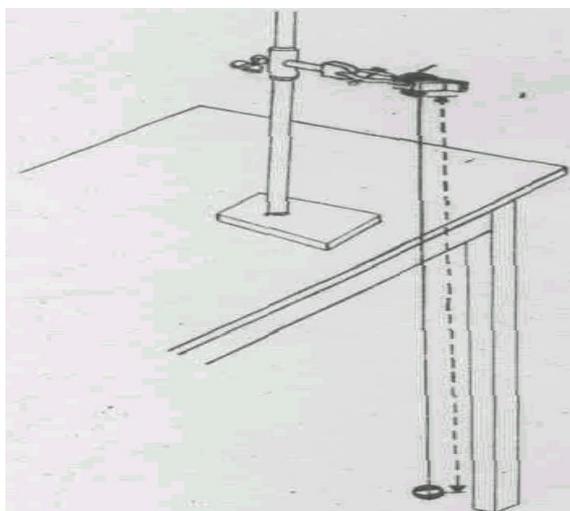


Fig. 1.1: A simple pendulum

In the first part of the investigations with a simple pendulum you are required to investigate the dependence of the period of the simple pendulum on its length, the amplitude of swing and mass of the bob. Since we are interested to know the way in which *three* different parameters affect the period, it makes sense to vary only one parameter at a time, keeping the other two constant. Then any change in period can be attributed to the change in the parameter that has been altered. (If all three parameters were changed simultaneously, we would have no way of knowing how much of the change in period is due to one particular parameter.) Therefore, we shall like you to make investigations in three steps. The apparatus with which you will work is listed below.

Apparatus

Three identical bobs of different materials, protractor, strings of varying lengths, stop watch, metre rod, clamp stand, cork pads, and vernier callipers.

Take a long piece of string nearly 2m long, and tie it to the pendulum bob. Fix the top of the string between cork pads placed in the jaws of the clamp as shown in Fig. 1.1. Displace the bob to one side and then release it. It begins to oscillate. You should ensure that the bob neither spins nor experiences jerks. That is, the pendulum executes free oscillations. Now your set-up is ready and you can begin your investigations.

But before proceeding, we would like you to spend a few minutes trying to predict how changes in three parameters can change the period of oscillation. Record your predictions and verify them after completing your investigations.

Predictions for the dependence of time period on length of pendulum, mass of bob and amplitude of swing.

1. _____
2. _____
3. _____

3.1.1 Dependence of the Period on the Length

Make a reference mark, using a pointer at the equilibrium position of the bob as well as at the maximum displacement of oscillation. You should keep the amplitude constant in each observation and it should be such that at no time the small angle approximation is violated ($\theta \leq 10^\circ$). That is, the motion is simple harmonic. This may be ensured by using a protractor. (If a protractor is not available, you can make your own on a cardboard. It may be fixed by using drawing pins on the edge of the table on which you are working so that 0° line coincides with the equilibrium position of the pendulum.) Otherwise, the motion will not be simple harmonic. Note the least count of the stopwatch and record it in Observation Table 1.1. Now set the bob in motion by displacing it slightly aside. To count the number of oscillations you can choose your reference point in two ways, as shown in Fig. 1.2. We prefer the second option (Fig. 1.2b) because in this case the reference point does not change.

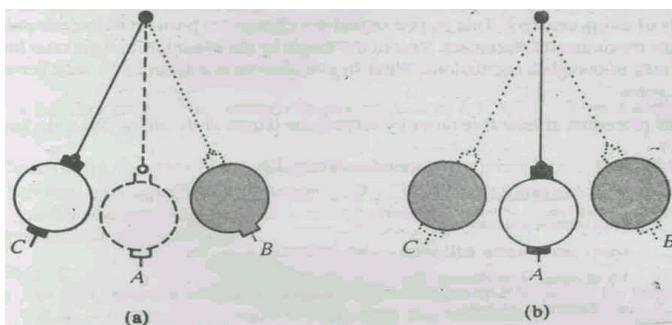


Fig. 1.2: Two different ways of counting the number of oscillations

If two students are working together, then one can count while the other keeps time. The 'counter' should begin countdown two, one, "go", one, two ... and so on. This gives the timekeeper a warning about the 'Go' signal. The end of counting may be indicated by saying 'stop'.

Begin your counting through the equilibrium position of the bob. It is important to simultaneously start the stopwatch. Usually, there is time lag between the starting/stopping the watch and the oscillation count. This is called *reaction time* and is, on an average, 0.3s. This can introduce some error in the value of time period (T). An important point to consider here is to know the degree of accuracy that is necessary. Another point is to measure a time interval in which the amplitude of swing does not diminish significantly. To see this you can note time for 1, 10, 20, 30, 50, 70, 100 oscillations and record your readings in Observation Table 1.1. Calculate the period of oscillation. To decide on the optimum number, observe the variation in the value of T . When the difference between two successive values of T is less than 0.1 per cent, it is acceptable. We expect the optimum number of oscillations to be 50.

The reaction time is the time interval between the input stimulus and its response.

An ordinary stopwatch has a least count of 0.1 s. So whenever we have to measure time of the order of one second or so, we use a more accurate automatic switching device, such as digital timer.

Observation Table 1.1: Determination of optimum number of oscillations

Least count of stopwatch = s

S.No.	No. of oscillation	Time (s)				T (s)
		(i)	(ii)	(iii)	(mean)	
1.	1					-
2.	10					
3.	20					
4.	30					
5.	50					
6.	70					
7.	100					

Conclusion: The optimum number of oscillations is _____

You have now decided on the number of oscillations (N) to be counted. Measure the diameter of the bob using vernier callipers. Record your readings in Observation Table 1.2 (a). Take observations in different directions. Calculate the radius. The length of the string plus the radius of the bob defines the length of the pendulum.

Note the time for N complete oscillations. Repeat this observation at least three times. Measure the length of the string from the point of suspension to the point of attachment to the bob using a metre scale. Enter your data in Observation Table 1.2(b).

Change the length of the pendulum by about 25cm and repeat the experiment, keeping the amplitude of swing constant. That is, you should not change the position of the reference mark at the maximum displacement. Record the length of the pendulum and the time for the same number of complete oscillations. What do you observe in respect of the time period as length changes?

Repeat the procedure at least five times by varying the length of the string. What do you conclude?

Observation Table 1.2

Least count of the stopwatch = _____ s

Least count of metre scale = _____ cm

Least count of vernier callipers = _____ cm

No of complete oscillations (N) = _____

a. Diameter of bob

S. No.	Diameter (cm)	Radius (cm)
1.		
2.		
3.		

Mean radius = _____ cm.

b. Effect of length on the period of the simple pendulum

S.No.	Length of Pendulum (m)	Time for N Complete Oscillations (s)				Time period (s)
		(i)	(ii)	(iii)	(Mean)	
1.						
2.						
3.						
4.						
5.						

Conclusion: The period of the pendulum _____ as length increases.

To investigate the exact relation between the time period and the length of the pendulum, you find out whether T increases or decreases as length increases. (An increase in time period suggests that T is directly proportional to the length and vice versa.) A variation in T suggests its connection with the length of the pendulum. That is, $T \propto l$. From your observations you can't directly quantify this proportionality. To know the exact dependence of T on l , we write

$$T = Al^n \quad (1.1)$$

where A is constant of proportionality and n is some constant.

On taking logarithm to the base e , we get

$$\ln T = n \ln l + \ln A \quad (1.1a)$$

This is the equation of a straight line.

Now you may plot $\ln T$ versus $\ln l$. The slope of the curve will give you the value of n . Theory predicts that n should be $\frac{1}{2}$. Compare the two values and discuss reasons for the difference, if any.

The equation of a straight line is $y = mx + c$

Thus we can write

$$T = Al^{1/2} \quad (1.2)$$

You can also arrive at this relation by plotting $T^{1/2}$ vs l , T vs $l^{1/2}$, T^2 vs l and so on till you get a straight line.

Theoretically, the slope of the straight line obtained on plotting T^2 vs l should be $4\pi^2 / g$. Therefore, by computing the slope from your graph, you can easily calculate acceleration due to gravity. Compare your value of g with the standard value at your place and compute the percentage error in your result.

SELF ASSESSMENT EXERCISE 1

- i. In your observations, you are required to record time with respect to the reference mark at the equilibrium position of the bob. Why is it necessary?

- ii. Why is it necessary to add the radius of the bob to the length of the string to know the length of the pendulum?

- iii. Can we use a metre scale or a micrometer screw to measure the radius of the bob? Justify your answer.

- iv. Read time periods from your graph for lengths of 100cm and 125cm. Calculate the ratio of time periods.

- v. Relate g to the y - intercept in $\ln T$ vs l graph.

3.1.2 Dependence of the Period on the Amplitude of Swing

To study the effect of amplitude on the period of the pendulum, we have to keep the length of the string and the mass of the bob constant. You may work with a length of about 1.5 m and in the beginning take angular amplitude in the range $2-10^\circ$. This ensures SHM. Fix a

protractor, as shown in Fig. 1.3. Note time for N oscillations and record it in Observation Table 1.3.

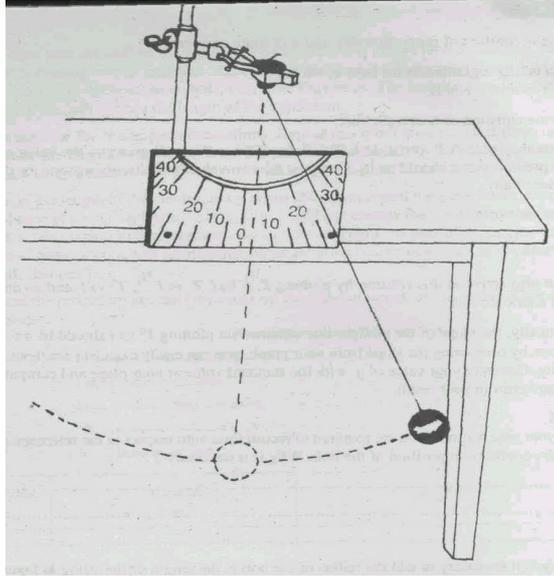


Fig. 1.3: Dependence of the period on amplitude of swing

Compute the period of oscillation and compare your observations. Are they different? Next take larger angular amplitudes of say, 30° , 40° , 50° and 60° and note the time period in each case. Is it different from that in the small angle approximation? If so, quantify the difference by calculating the relative change. What do you infer about the motion of the pendulum?

Observation Table 1.3: Variation of time period with angular amplitude

No. of complete oscillations counted each time (N) = _____

Length of the pendulum = _____ m

S. No.	Angular Amplitude (degree)	Time for N Oscillations (s)				Time period (s)
		(i)	(ii)	(iii)	(Mean)	
1.						
2.						
3.						
4.						
5.						
6.						

Conclusion:

1. For small angular amplitudes the period of the simple pendulum is....
2. For large angular amplitude, the motion of the pendulum is

3.1.3 Effect of Mass of the Bob on the Period

To determine whether or not the period of pendulum depends upon the mass of the bob, we take three bobs of different materials. These should be identical (in shape and size) so that (i) the air-drag experienced by every bob is the same and (ii) the length of the pendulum is same in all cases. Can you suggest any alternative arrangement to study this effect? Is it possible to work with a plastic table tennis ball? Yes, we can. Different amount of sand may be poured in the ball to vary its mass. Comment on your observations. Note that we have to ensure constant length of pendulum and the amplitude of swing.

Note the time for 30 complete oscillations. Repeat the procedure for at least two other bobs of same size but different materials. Record your readings in Observation Table 1.4. Compute the period. Is it influenced by the mass of the bob? If yes, how much? To quantify this change, calculate the difference between the values of time period for bobs of minimum and maximum masses. Theoretically, we do not expect any change in the time period as the mass of the bob is varied. Discuss it with your counsellor and point out the possible reasons.

Observation Table 1.4: Variation of time period with mass of the bob

Length of the pendulum = m

No. of complete oscillations counted each time (N) =

S. No.	Mass of bob (g)	Time for N Oscillations (s)				Period (s)
		(i)	(ii)	(iii)	(Mean)	
I.						
2.						
3.						
4.						
5.						

Conclusion: The period of pendulum within experimental error limits is s

3.1.4 Damping and Relaxation Time

You must have observed that the amplitude of oscillations of the pendulum bob *does not* remain constant with time. It gradually becomes smaller and smaller. This is because the pendulum loses energy due to air resistance. Such a motion is said to be *damped*. In practice, every oscillating system experiences damping to a varying extent. We can know the amount of damping once relaxation time is known. So in the second part of the investigations with a simple pendulum you are required to calculate this quantity.

A systematic way of introducing damping in case of a simple pendulum is to put a fan on and let the pendulum oscillate. We assume that frictional force F_d is small and take it to be linearly proportional to velocity. That is, we write $F_d = \gamma v$.

If $x(t)$ is the displacement at any time t , then the motion of a damped oscillator is described by the equation

$$\frac{d^2x}{dt^2} + 2b \frac{dx}{dt} + \omega_0^2 x = 0 \quad (1.3)$$

where $\omega_0 = \sqrt{g/l}$ is the angular frequency of undamped oscillations and $b = \gamma/2m$ is a measure of damping experienced by a bob of mass m . It has dimensions of frequency. The inverse of this quantity, $b^{-1} = \frac{2m}{\gamma} = \tau$ is called the **relaxation time**. So values of τ will be less for a heavily damped system.

When damping is small, the solution of Eq. (1.3) is

$$x(t) = a_0 \exp(-t/\tau) \cos(\omega_d t + \phi) \quad (1.4)$$

where $\omega_d = \sqrt{\omega_0^2 - b^2}$ is the angular frequency of damped oscillations and ϕ is the initial phase. $a_0 \exp(-t/\tau)$ is the amplitude of oscillation in the presence of damping. (a_0 is amplitude of oscillation in the absence of damping). Note that Eq. (1.4) represents a periodic motion but it is not simple harmonic. After n oscillations, the amplitude will be

$$a_n = a_0 \exp(-nT_d/\tau)$$

where T_d is the period of damped oscillations. Taking logarithms, we get

$$\ln a_n = \ln a_0 - \left(\frac{T_d}{\tau}\right)n \tag{1.5}$$

This equation shows that if we measure a_n and we plot a graph between $\ln (a_n)$ versus n , the curve will be a straight line. Its intercept on the y -axis gives $\ln a_0$. The slope of the straight line gives T_d / τ . This means that the relaxation time can readily be calculated once T_d is known for a given length of the pendulum.

To measure a_n you should fix a scale on the table. Displace the bob to one side and release it. Note the amplitude after 10, 20, 30, oscillations and record it in Observation Table 1.5. (In case it is not convenient to do so in one go, you can do it in steps. But in each case the initial amplitude of swing should be kept the same.)

Observation Table 1.5: Variation of amplitude with number of oscillations

Length of the pendulum = m

Period of the pendulum = s

Mass of the bob = g

S. No.	n	a_n (cm)	$\ln a_n$
1.	10		
2.	20		
3.	30		
4.	40		
5.	“		
	“		
	“		
	“		
	“		
10.	“		

Result: The relaxation time of the given pendulum vibrating in a viscous medium (air) iss.

SELF ASSESSMENT EXERCISE 2

Name a physical system where linear damping model holds.

3.2 Investigations with a Bar Pendulum

We know that a simple pendulum suffers from the drawback that some air is always dragged by the bob. Similarly, the string may not be perfectly inextensible leading to non-planar oscillations. These sources of error sometime lead to a variation in the value of T . Can you suggest a way to overcome these problems? The remedy lies in the use of a compound pendulum. A compound pendulum is a rigid body capable of oscillating freely about a horizontal axis. In the physics laboratory, it is in the form of a bar of length nearly one metre and width about 2 cm. A series of circular holes 5-6 mm in diameter are drilled symmetrically about its centre of gravity (C.G), i.e. along the length of the bar. (You can make a bar pendulum by taking a metre scale and drilling equidistant holes in it, as shown in Fig. 1.4.) The centres of any two consecutive holes are at equal distances of about 2 cm. These holes allow the bar to be suspended from a knife-edge. Usually, two movable knife-edges are provided with the bar pendulum. These can be fitted successively in various holes, one on each side of C.G and at equal distances from it (Fig. 1.4).

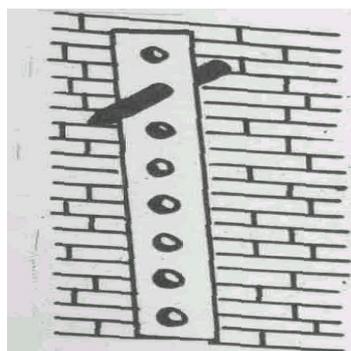


Fig. 1.4: A bar pendulum

As the bar pendulum is made to oscillate about a horizontal axis, its motion is simple harmonic and the time period is given by

$$T = 2\pi \sqrt{\frac{k_r^2 + l^2}{lg}} \quad (1.6)$$

where l is the distance between the point of suspension and C.G and k_r is the radius of gyration of the body about an axis passing through the C.G and parallel to the axis of rotation. The radius of gyration is defined as the distance between the axis of rotation and the point at which the whole mass of the body could be considered to be placed without any change in its moment of inertia about that axis.

We define

$$L = \sqrt{\frac{k_r^2}{l} + l}$$

and call it the length of an equivalent simple pendulum. Combining this result with Eq. (1.6), we get

$$T = 2\pi \sqrt{L/g} \quad (1.7)$$

In this part of the experiment, you are required to investigate how the period of oscillation varies with the distance between the point of suspension and *C.G* of the bar pendulum. The apparatus with which you will work is listed below.

Apparatus

Bar pendulum, stop watch, metre scale.

3.2.1 Variation of the Period with Length

Fix one knife-edge in the hole nearest to one end of the pendulum. The other knife-edge is fixed in the hole nearest to the other end so that the two knife-edges are equidistant from the *C.G* of the bar. Now suspend the pendulum vertically by resting it on one of the knife-edges on a horizontal rigid support. As before, put a reference mark to denote the mean position of the pendulum. Displace the bar slightly aside and let it oscillate. You should ensure free oscillations in the vertical plane. Now you are ready to perform the experiment.

Measure the distance between the point of suspension and the *C.G* of the bar (centre of the hole). This gives us l . Now measure the time for N ($=30$) complete oscillations. Record your readings in Observation Table 1.6. Invert the pendulum and note the time for the same number of oscillations. Now insert the knife-edges in the adjacent holes so that they are symmetrical about *C.G*, as before. You will note that now the length of the pendulum has changed. So you will find that the time for N oscillations is different from the preceding value. Repeat observations by inserting the knife-edges in different holes. At all times, the knife-edges should be symmetrical about *C.G*. What happens as you approach the centre of the bar? You will observe that the time for N oscillations first decreases, takes a minimum value and then increases. As you near the *C.G* of the bar, it becomes very large. Can you measure the period by putting the knife-edge at the central hole? It is not possible to do so because the bar will not oscillate; it just gets stuck up on one side.

Plot a graph between T and l . You will get two curves which are symmetrical about the *C.G* of the bar (Fig. 1.5). Now you draw a line parallel to the x-axis. At how many points it cuts these curves? The

number of points should be four, say at J, K, M and N , as shown in Fig. 1.5.

Observation Table 1.6: Variation of time period with distance of hole from C.G.

Least count of the stopwatch = s

No. of complete oscillations counted each time (N) =

No. of hole from one end	Distance of the point of suspension from C.G, l (cm)	Time for N Oscillations (s)	T (s)	lT^2 (cm s ²)

Result: The plot of T versus l is

At all these points, the period of the pendulum is the same. Measure distances JM and KN . How do you interpret these? Each of these distances represents the length of an equivalent simple pendulum, L . Using Eq. (1.7), you can compute the acceleration due to gravity.

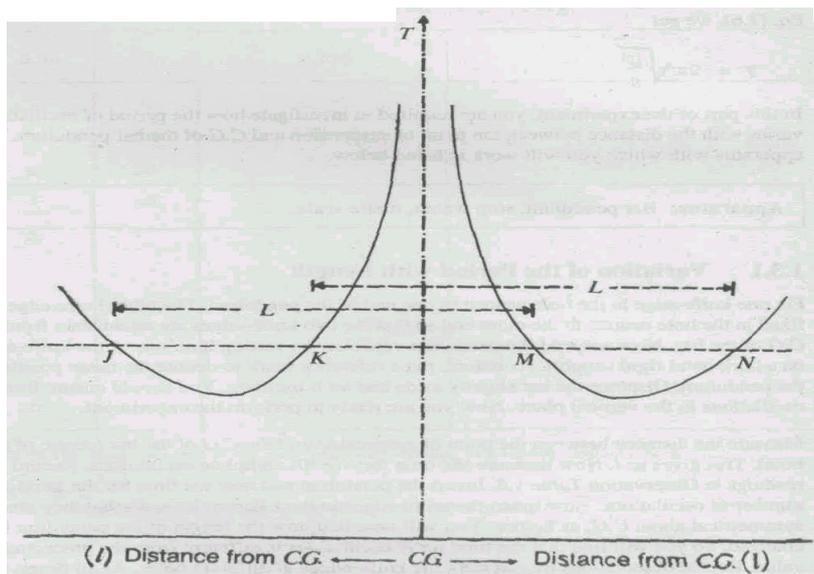


Fig. 1.5: Plot of time period with distance of hole from C.G.

3.2.2 The Radius of Gyration

How will you calculate the radius of gyration? To answer this question, we rewrite Eq. (1.6)

$$l^2 = \frac{g}{4\pi^2} lT^2 - k_r^2$$

This equation suggests that if you plot l^2 versus lT^2 , you will obtain a straight line, which on extrapolation will meet the y -axis. The intercept on the y -axis gives k_r^2 . How do you interpret the slope of the curve? It is $g/4\pi^2$. Hence, you can calculate the value of g also from this graph. Compare this value with that obtained using a simple pendulum. Which one is more accurate?

Result:

- (i) The radius of gyration of the bar pendulum about-an axis passing through C.G and parallel to the axis of rotation is m
- (ii) The acceleration due to gravity ism/s²

SELF ASSESSMENT EXERCISE 3

- i. Why is it necessary to keep the knife-edges symmetrically about C.G?

- ii. Is your bar pendulum oscillating about a horizontal axis or a vertical axis?

- iii. Name two sources of error in your experiment.

4.0 CONCLUSION

Learners are required to arrive at a conclusion from the results and observation made during the experiments.

5.0 SUMMARY

In this unit, you have learnt how the two parameters are related by a power law. The investigations with a simple and bar pendulum are made in this experiment. It is investigated that how the followings affect the period of pendulum:

- length or thickness of the string
- amplitude of swing
- size and mass of the bob

You have also learnt to compute experimentally the radius of gyration of the bar pendulum about-an axis passing through C.G and parallel to the axis of rotation and the acceleration due to gravity

Finally, learn to compare the values of acceleration due to gravity using a simple and bar pendulum.

6.0 TUTOR MARKED ASSIGNMENT

As given by the facilitator

7.0 REFERENCES/FURTHER READINGS

IGNOU (1997) Physics Laboratory I, Physics PHE-03, New Delhi, India

IGNOU (1997) Elementary Mechanics, Physics PHE-01, New Delhi, India

UNIT 4 **EXPERIMENT 2** **OSCILLATIONS OF A SPRING-MASS SYSTEM** **AND A TORSIONAL PENDULUM**

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Determination of Spring Constant Using a Spring-Mass System
 - 3.1.1 Static Method
 - 3.1.2 Dynamical Method
 - 3.2 Determination of Torsional Rigidity of a Wire using a Torsional Pendulum

- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor Marked Assignments
- 7.0 References and Further Reading

1.0 INTRODUCTION

We know that spiral springs find various uses. In a transistor set and a pocket calculator, springs hold dry cells in proper position. Springs are used as shock absorbers in automobiles and railway wagons. You may have also used yourself a bull-worker or seen body-builders using it. Do you know that it essentially consists of springs? In ammeters, voltmeters and a wristwatch, springs control oscillations of the system. In all these cases, the basic difference in the springs being used is in their spring constants. So to decide upon the type of a spring for a particular purpose, we must know its spring constant. In a physics laboratory we can determine the value of spring constant in two different ways:

1. by knowing extension in the spring for a given load (static method), and
2. by determining the period of harmonic oscillations of the spring-mass system (dynamical method).

We come across many instruments in the physics laboratory which involve torsional oscillations. The most familiar of these are the torsional pendulum (used to calculate modulus of rigidity), inertia table (used to determine moment of inertia) and the moving coil galvanometer (used to measure charge and current). When wire in such a torsional system is twisted, due to elasticity a restoring torque is set up within the wire (fibre). It tends to oppose twisting of the wire. The restoring couple per unit radian is known as *torsional rigidity or torsional constant*. While choosing the suspension wire (fibre) for a specific purpose we should have a prior knowledge of torsional rigidity. In this experiment you will learn to measure torsional rigidity by a simple experiment.

2.0 OBJECTIVES

After doing experiments with a spring-mass system and a torsional oscillator, you will be able to:

- acquire skills of measuring small thickness with precision using a micrometer screw;
- measure extension of the spring for a given load and calculate the spring constant (k) for the given spring (static method);

- measure the period of oscillation of a spring-mass system for different loads and calculate k (dynamical method);
- compare the accuracies of static and dynamical methods;
- compute torsional rigidity (k_t) and modulus of rigidity of the given wire; and
- predict the material of the wire.

3.0 MAIN CONTENT

3.1 Determination of Spring Constant Using a Spring-Mass System

In the preceding experiment you investigated the question: What determines the value of T for a simple and a bar pendulum? You may now ask: Can we make similar investigations for a spring-mass system? It makes sense and you can do so along lines outlined in Experiment 1. But now we intend to calculate the spring constant of a spring in two different ways: (i) by knowing extension for a given load, and (ii) by measuring the period of harmonic oscillations of a spring-mass system. The apparatus required for this purpose is listed below:

Apparatus

A spiral spring, slotted weights in multiples of 100g, stop watch, a laboratory stand and a 50 cm scale.

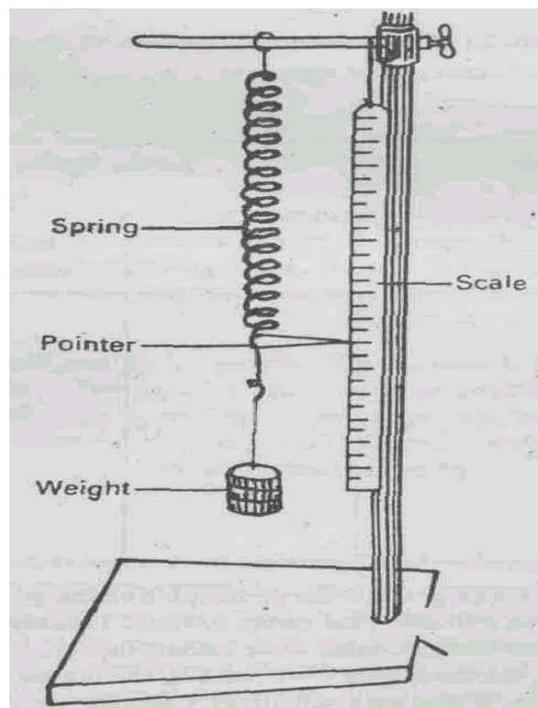


Fig. 2.1: A Spring-mass system

Suspend a spring and a metre scale in the stand side by side, as shown in Fig. 2.1. Fix a sharp-tipped pointer (needle) at the lower end of the spring. (In case you do not get a needle, you can make a pointer of cardboard by cutting it in the shape of an isosceles/equilateral triangle.

When a body is subjected to an external force, it tries to maintain its shape and size. As applied force is removed, it tends to recover its original configuration. This property is known as elasticity. The magnitude of applied force up to which a specimen retains its elastic property defines the elastic limit. Beyond the elastic limit the applied force produces plastic (permanent) deformations, i.e. the body will not recover its original shape and size even if the applied force is withdrawn.

Then you have to attach its base to the straight end of the spring so that its vertex moves in contact with the scale. This helps in minimising parallax error also.) Suspend a hanger (which itself is a known weight equal to any other slotted weight) in the hook of the spring. (Alternatively, you can tie a pan to the lower end of the spring and put weights.) Normally, it is advisable to put an initial load on the hook as it will take care of the kinks and other such inhomogeneities in the spring. This implies that the choice of the initial position really does not matter. Stretch the spring by pulling the hanger downwards through a small distance and then let it go. The spring-mass system will execute vertical oscillations. Ensure that the pointer does not stick anywhere and the oscillations are free. Now your apparatus is ready and you can start your experiment. But before you do this, do spend a few minutes making qualitative observations as to how extension/period changes when the mass is changed within **elastic limits**. This limit will be different for different springs. So you better consult your counsellor before putting a load on the spring.

3.1.1 Static Method

Load the spring by putting a weight. Due to elasticity, a restoring force is set up in the spring. It tends to oppose the applied force and bring the system back to its original state. If extension is small compared to the original length of the spring, the magnitude of restoring force exerted by the stretched spring on the mass is given by

$$F = -kx \quad (2.1)$$

where x is extension in the spring and k is *spring constant*.

From Eq. (2.1) it is clear that once you know extension as a function of load, k can easily be calculated. It is with this purpose that we attach a pointer to the lower end of the spring. This method of determining k is known as *static method*.

Note the new equilibrium position of the pointer on the scale and regard it as initial observation. Record your reading in Observation Table 2.1. Now increase the load in steps by adding equal weights each time. For each load, record the position of the pointer. Before taking a reading, you should wait for some time so that the pointer comes to rest. Take at least six observations.

Observation Table 2.1: Measurement or extension of the spring: Static Method

Least count of metre scale =cm

S. No.	Load on the spring (g)	Reading of pointer on the metre scale (cm)		
		Load increasing	Load decreasing	Mean
1.				
2.				
3.				
4.				
5.				
6.				

To ensure that you are working within the permissible elastic limit, you may record the position of the pointer by unloading the spring in steps. Tabulate your observations. Do these readings differ from those recorded while loading the spring? If observations for a given weight are nearly the same, both while loading and unloading, you can be sure that you are certainly working within the elastic limit. Calculate the mean extension for a given load.

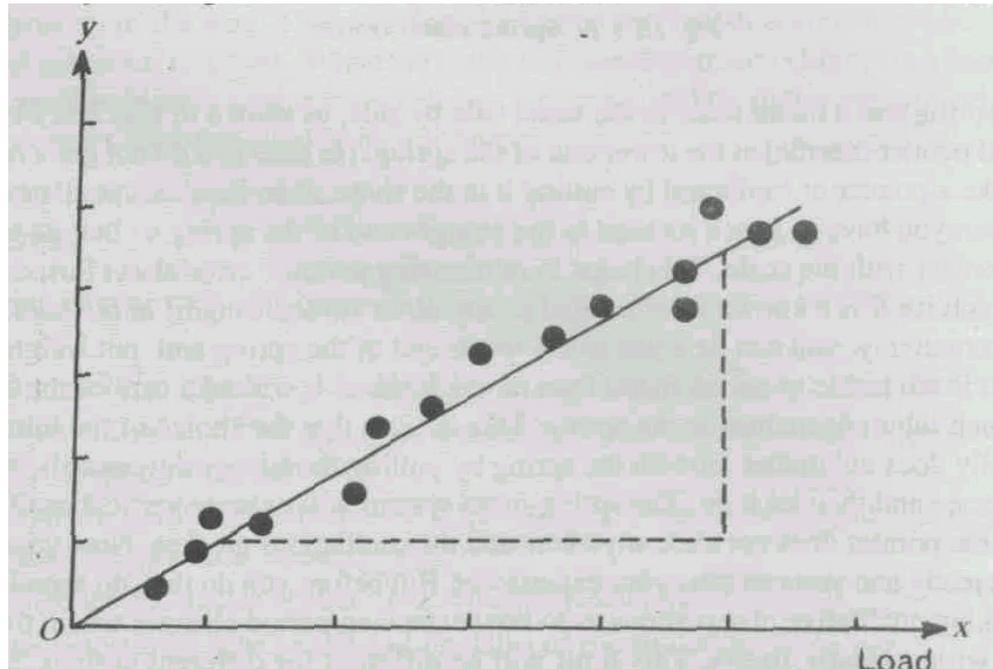


Fig. 2.2: Best fit curve through observed points

Now you should plot a graph between load and the corresponding extension. Conventionally, we plot the independent variable along the x -axis and the dependent variable along y -axis. Which physical quantity will you plot for this experiment along x -axis? Draw the best fit through observed points as shown in Fig. 2.2. (For a good steel spring, we expect the graph to be linear.) Does your straight line pass through the origin? The inverse of the slope of the straight line is a measure of the spring constant. To calculate the slope, you should use two widely separated points on the straight line. Use $g = 9.8 \text{ m/s}^2$ and express your result in SI units.

Error Analysis

Find the change in slope of the straight line caused by drawing lines of maximum and minimum slopes. This gives maximum error in the slope. Using $g = 9.8 \text{ m/s}^2$, calculate the error in k in SI units.

Conclusion: The spring constant of the given spring =
 \pm N/m.

SELF ASSESMENT EXERCISE 1

- i. Name the factor(s) on which k depends.

- ii. From your graph, note extension for a load of 2N.

3.1.2 Dynamical Method

In the preceding section you learnt a method of measuring extension of spring as a function of load. This information was used to compute the spring constant. You may now ask: Is there some other method also for determining k ? Yes, there is. We can use the so-called *dynamical method*. It is based on observing the period of harmonic oscillations of the spring-mass system.

On seeing a spring-mass system oscillating, you may like to know: Is this motion different from that of a simple pendulum? Though these two systems are physically different, both execute SHM, provided the extension is not large. Another question that comes to our mind immediately is: Does gravity affect the frequency of oscillations? Gravity has no effect on the frequency of oscillations. The period of oscillation is given by

$$T = 2\pi \sqrt{m/k} \quad (2.2)$$

This relation shows that we can compute k by knowing the period of oscillations for a given mass. In case you get to know the standard value of k (from your counsellor or a book) for the material of spring, you can judge whether the dynamical method is more accurate than the static method or not. So you will be required to measure the period of simple harmonic oscillations. You must ensure that oscillations of the system hanging vertically are longitudinal. That is, there should not be any lateral oscillations. Otherwise, the motion will not be simple harmonic.

Put a load on the hanger and take the position of the pointer on the scale as the equilibrium position. Now stretch the spring by pulling the hanger downward and then release it. For small displacement, the system will execute SHM.

Note the least count of the stopwatch and record it in Observation Table 2.2. Now set the spring-mass system into oscillations. Allow the first few oscillations to pass so that there is no unharmonic component. Begin your counting through the equilibrium position and simultaneously start the stopwatch. Note the time for N , say 30 complete oscillations. To minimise error in T , it is desirable to take 50 or more oscillations. However you must ensure that the amplitude of swing does not decay significantly. Draw your Observation Table and enter your readings. Add more weights in the hanger and repeat the procedure at

least five times. Tabulate your observations. How does the time period change?

As before, the procedure may be repeated by decreasing the load in steps. Calculate the mean period for each load.

Plot T^2 versus m . Which variable will you plot along y -axis and why? Draw the best possible straight line as shown in Fig. 2.3. Does it pass through the origin? From the slope of the straight line, you can easily compute k . Check if this value agrees with that obtained by the static method. The two values should be same or nearly equal.

Observation Table 2.2: Measurement of Time Period: Dynamical Method

Least count of stop watch = s

No. of complete oscillations counted each time (N) =

S. No.					
1.					
2.					
3.					
4.					
5.					
6.					

Result: The spring constant of the given spring =
 \pm Nm

As before, you can compute error in k by drawing lines of maximum and minimum slopes. What is the relative change in the value of k ?

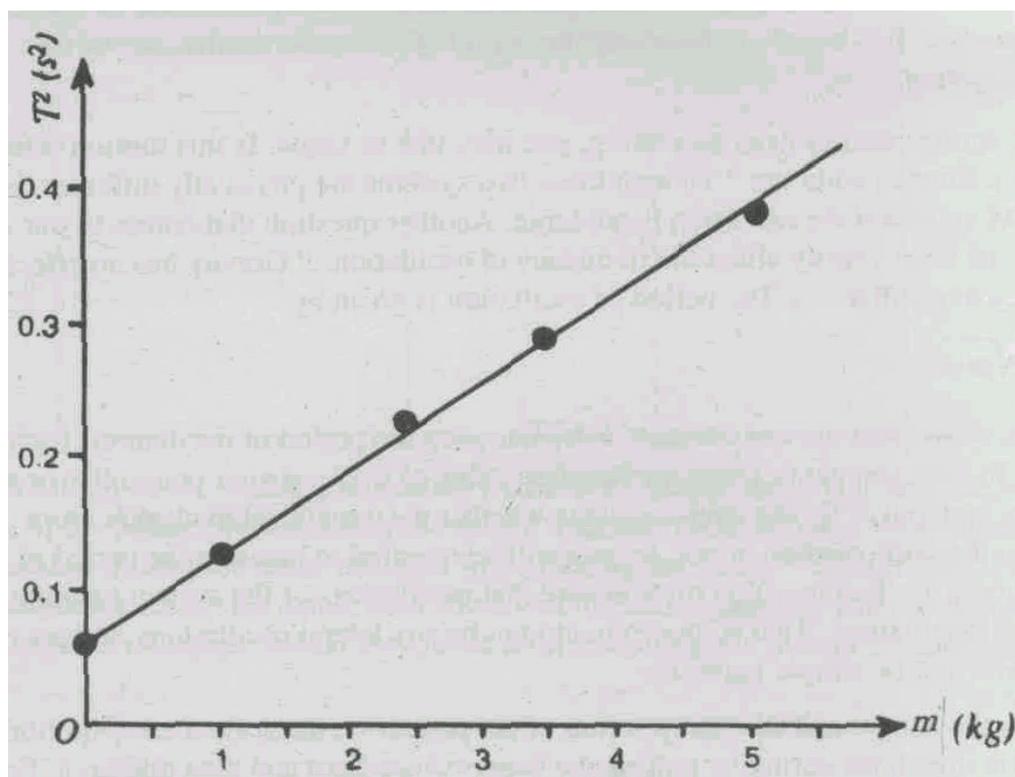


Fig. 2.3: Expected plot of T^2 versus m

SELF ASSESSMENT EXERCISE 2

- i. Extrapolate the graph between T^2 and m backward till it meets the m -axis. Interpret the intercept on m -axis.

- ii. Use your graph to determine T for a load of 3N.

3.2 Determination of Torsional Rigidity of a Wire Using a Torsional Pendulum

As mentioned before, you are required to use a torsional pendulum shown in Fig. 2.4 to measure torsional rigidity. All necessary apparatus required for this purpose is listed below:

Apparatus

Torsional pendulum (inertia table), stopwatch, rigid circular cylinder, vernier callipers, micrometer screw, spirit level, physical balance and a weight box.

In a torsional pendulum one end of a long and thin metallic wire is clamped to a rigid support. The other end of the wire is fixed to the centre of a projection coming out of the central portion of the circular disc. Normally, this disc is made of aluminium or brass. You can observe concentric circles on the upper face of the disc and a groove near the circumference. The concentric circles facilitate symmetrical loading. The concentric groove helps in setting the disc horizontal by placing balancing weights. The iron table below the disc is provided with three levelling screws.

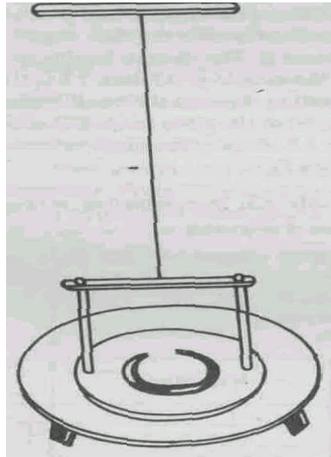


Fig. 2.4: A torsional pendulum

Think of what happens when a cylindrical wire (rod) is clamped at one end and the other end is twisted in a plane perpendicular to its length. Due to elasticity, an equal and opposite torque is developed in the wire. The restoring torque per unit radian, k_t , is given by

$$k_t = \frac{\pi n r^4}{2l} \quad (2.3)$$

when n is the modulus of rigidity and r is radius of the wire of length l . (In many textbooks, the restoring torque per unit radian is denoted by the symbol C .) In the apparatus given to you, if you rotate the disc in a horizontal plane (keeping the wire vertical) and then release it, the system executes torsional oscillations in the horizontal plane. These torsional oscillations are simple harmonic. The period of oscillations is given by

$$T_0 = 2\pi \sqrt{\frac{I_0}{k_t}} \quad (2.4)$$

where I_0 is the moment of inertia about the axis of rotation.

If an auxiliary body of known moment of inertia I is placed on the disc such that its centre coincides with the centre of the disc, the period of oscillations changes. Do you know why?

It is because of the redistribution of mass about the axis of rotation. If we denote the period of the system now by T , we can write

$$T = 2\pi \sqrt{\frac{I_0 + I}{k_t}} \quad (2.5)$$

Now, square equations (2.4) and (2.5) and subtract the former from the latter. This gives an elegant expression for torsional rigidity:

$$k_t = \frac{4\pi^2 I}{T^2 - T_0^2} \quad (2.6)$$

On combining this result with Eq. (2.3) we get an expression for the modulus of rigidity

$$n = \frac{8\pi I l}{(T^2 - T_0^2) r^4} \quad (2.7)$$

Equations (2.6) and (2.7) show that we can readily calculate k , and n once T , T_0 , I , l and r are known. Let us now determine T and T_0 . To do so, you should first level the iron table by the levelling screws. You should test this using a spirit level. Next you should adjust the balancing weights in the groove of the disc so that the disc is horizontal. To ensure this, you should again use a spirit level. You should also make sure that the suspension wire is free from kinks. Now place a vertical pointer in front of the disc and just put a mark on the disc when the latter is at rest. This denotes the equilibrium position and reference for counting the number of oscillations. Next, rotate the disc slightly in a horizontal plane so that the wire is twisted and then release it. The system begins to oscillate. How are these oscillations different from those of the simple pendulum? Let the first few, say 5, oscillations to pass. Begin your counting through the equilibrium position and simultaneously start the stopwatch. Note the time for N (20 or 30) oscillations. Record your readings in Observation Table 2.3. Repeat the observations at least five times. Calculate the mean period. This gives T_0 .

Observation Table 2.3: Determination of T_0 and T

Least count of stop watch = s.

No. of oscillations counted each time (N) =

S. No.	Time for N oscillations (s)		Time period (s)	
	No cylinder	With cylinder	T_0	T
2.				
3.				
4.				
5.				

Now place a right circular cylinder at the centre of the disc such that its axis coincides with the axis of suspension of the wire. Do you know why is it necessary to place the cylinder like this?

Now record the time for the same N number of oscillations at least five times. Calculate the period of oscillations. This gives us T .

From Eq. (2.6) we note that to calculate k_t , we must know I also. The moment of inertia of a right circular cylinder of mass M and radius R about an axis passing through its centre is given by

$$I = \frac{MR^2}{2}$$

This shows that I can be calculated if we know M and R . The mass may be known by weighing the cylinder in a physical balance.

Measure its diameter using vernier callipers. Record your readings in Observation Table 2.4. Take at least five readings. Calculate the mean value.

Table 2.4: Radius of Cylinder

Least count of vernier callipers = cm

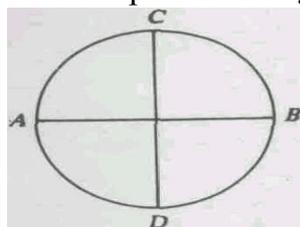
S. No.	Diameter of the cylinder (cm)	Radius (cm)
1.		
2.		
3.		
4.		
5.		
6.		

Mean radius of cylinder = cm

Mass of right circular cylinder = kg

Moment of inertia of right circular cylinder = kgm^2 **Result:** The torsional rigidity of the material of the given wire is Nm.

Once k_t is known, n may be computed if you measure the length of the wire and its radius. To find r , use a micrometer screw. Take readings at several points along the length of the wire and record these in



Observation Table 2.5. By doing so we can account for any non-uniformity in the diameter of the wire. For greater accuracy, measure the diameter of the wire in two mutually perpendicular directions. From measured value of n you should be able to predict the material of the wire by consulting some practical

physics textbook.

Table 2.5: Diameter of Wire

Least count of micrometer screw = cm.

S. No.	Diameter of the Wire (cm)			Radius (cm)
	<i>AB</i>	<i>CD</i>	Mean	
1.				
2.				
3.				
4.				
5.				

Mean radius of the wire = m.

Length of the wire = m.

Compute log error following the procedure outlined in Unit 2 on error analysis. Do your results differ from standard values of k and n within these error limits only? If not, then you should discuss the reasons of deviation with your counsellor,

If time permits, you can investigate the relation between it, and the radius of the wire by choosing another wire of the same material. Similarly, you may study dependence of k_t on the material of wire. For this you should take another wire of different material but having the same radius.

SELF ASSESSMENT EXERCISE 3

- i. Name at least two sources of error in this experiment.
- ii. Why is it necessary to coincide the centre of the circular cylinder with the axis of the suspension wire?
- iii. Can you determine the M.I. of an irregular body with this apparatus? If yes, how?

4.0 CONCLUSION

Learners are required to write conclusions based on their results and observations during the experiment.

5.0 SUMMARY

From the experiments you would have obtained values for the following parameters:

- (a) k , spring constants from static and dynamic methods
- (b) k_t torsional rigidity and modulus rigidity for the given wire.

Learners would have acquired skills in the use of micrometer screw and the prediction of the materials of wire.

6.0 TUTOR MARKED ASSIGNMENTS

As given by the facilitator

7.0 REFERENCES/FURTHER READINGS

IGNOU (1997) Physics Laboratory I, Physics PHE-03 (L), New Delhi, India.

IGNOU (1997) Elementary Mechanics, Physics PHE-01, New Delhi, India.

UNIT 5 EXPERIMENT 3

A STUDY OF ENERGY AND MOMENTUM CONSERVATION PRINCIPLES

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Verification of the Principle of Conservation of Mechanical Energy
 - 3.1.1 Description of Apparatus
 - 3.1.2 Procedure
 - 3.2 Verification of the Principle of Conservation of Linear Momentum
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- 6.0 Tutor Marked Assignments
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1.0 INTRODUCTION

In the preceding experiments you worked with systems executing simple harmonic motion (SHM). An important characteristic of the system executing SHM is that in the absence of dissipative forces, the energy of the system remains constant, i.e. it is conserved. Recall the oscillations of the bob of a simple pendulum. When the bob is displaced from its equilibrium position, it gains potential energy. At the extreme position, all its energy is potential in form. On being released, its potential energy gradually changes to kinetic energy. At the mean position, its energy is wholly kinetic. As the bob crosses the mean position, its kinetic energy begins to transform to potential form. But at anytime the total mechanical energy, which is the sum of the kinetic and potential energies, remains constant. This is known as the principle of conservation of mechanical energy. This principle is also valid for other forms of energy such as chemical, thermal, electrical, and nuclear. This analogy suggests that we can state a general principle of conservation of energy.

This principle is perhaps the most fundamental and elegant principle of physics. It is observed in all natural processes from radioactive decay to the motion of planets around the sun. Do you know of any situation where this law is violated? Probably there is not a single exception to it so far, though it has been challenged many times.

The principle of conservation of energy does not provide satisfactory solutions to problems where details of interactions between different bodies are not known. For such situations we require a conservation principle involving a vector quantity like linear momentum. This principle finds wide applications ranging from nuclear reactions to rocket propulsion. There are many other conservation principles in physics. But in this experiment you will learn to verify the principles of conservation of mechanical energy and linear momentum using simple arrangements.

In general, the conservation principles help us in discovering new phenomena as well as clarifying the less understood ones. You may recall from your school science course how the principles of conservation of momentum and energy led Pauli to predict the existence of the neutrino. Many a time, these principles forewarn us of the non-occurrence of some phenomenon.

2.0 OBJECTIVES

After performing this experiment you should be able to:

- acquire skill of removing parallax
- use a plumb line
- translate a vector parallel to itself
- verify the principle of conservation of mechanical energy
- verify the principle of conservation of linear momentum.

3.0 MAIN CONTENT

3.1 Verification of the Principle of Conservation of Mechanical Energy

The principle of conservation of energy is stated as follows:

Energy can neither be created nor destroyed. It may be transformed from one form to another; the total energy in a system remaining constant.

Its verification demands that we must be able to measure energy very precisely. Since it is most convenient to measure mechanical energy, we will verify the principle of conservation of energy with particular reference to *conservation of mechanical energy*.

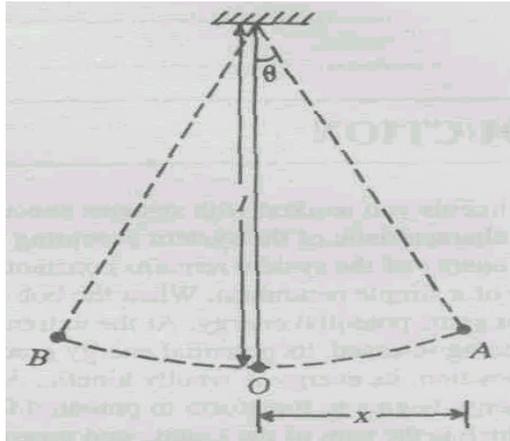


Fig. 3.1: Oscillations of a simple pendulum

Consider the oscillations of a simple pendulum (Fig. 3.1). Referring to this figure we note that at A , the maximum of the swing, the energy of the bob will be wholly potential (kinetic energy zero). But at the mean position O , the energy of the bob will be wholly kinetic. Therefore, to verify that mechanical energy is conserved, we must show that at any point between A and B , the sum of potential energy and kinetic energy of the bob remains constant. Since it is more convenient to measure potential than kinetic energy, we intend to measure the energy of the bob at A . The maximum value of potential energy (U) is given by

$$U_{\max} = (1/2)mg l \theta^2 \quad (3.1)$$

where m is the mass of the bob, l is the length of pendulum, θ is the maximum angular displacement and g is the acceleration due to gravity.

If θ is small, we can write

$$\theta \cong \frac{x}{l}$$

where x is the amplitude of oscillation.

Using this result in Eq. (3.1), we get

$$U_{\max} = \frac{mg}{2l} x^2 \quad (3.2)$$

This equation tells us that the maximum potential energy of a bob is directly proportional to the square of the amplitude of oscillation and inversely proportional to the length of the pendulum. So, to verify the principle of conservation of mechanical energy, we must show that at A or at B ,

$$\frac{x^2}{l} = K \quad (3.3)$$

where K is constant.

Let us pause for a minute and ask: What is implied by Eq. (3.3)? It tells us that for a given length of a simple pendulum (l fixed), the amplitude of swing, on either side of the mean position, should remain constant for the principle of conservation of mechanical energy to hold. You can check this by releasing the bob and measuring x on either side of the mean position. (Alternatively, you can measure the vertical heights by which the bob rises above the equilibrium position.) However, a more convincing way to verify Eq. (3.3) will be to have l and x in such a way that they are different at the extremities of the same oscillation. You will realise that in a simple pendulum it is not possible to vary both x and l simultaneously. So to achieve this we have designed a special pendulum, which we call *two-in-one pendulum*. It is a modified form of simple pendulum and is similar to that used by Galileo to study the principle of conservation of energy. (His experiment is known as pin and pendulum experiment.) We will describe the two-in-one pendulum in the paragraphs that follow. But we first list all the apparatus with which you will work.

Apparatus

Two-in-one pendulum, a heavy bob with a pointer, inextensible weightless string.

3.1.1 Description of Apparatus

The two-in-one pendulum consists of a specially designed stand fixed on a flat base, which carries a mirror strip fitted with a scale, as shown in Fig. 3.2. The mirror strip helps us in avoiding parallax while taking readings of the displacements of the bob. The bob is tied to a string and suspended from clamp A , which is fixed so that AX is about 1.5m. Clamp B is movable and can be made to slide vertically in a graduated groove. This clamp must be smooth and have a sharp end, like a pin so that it slightly interrupts the swing when the bob reaches its mean position. What will happen if the end is not sharp? In such a situation, appreciable energy loss may occur.

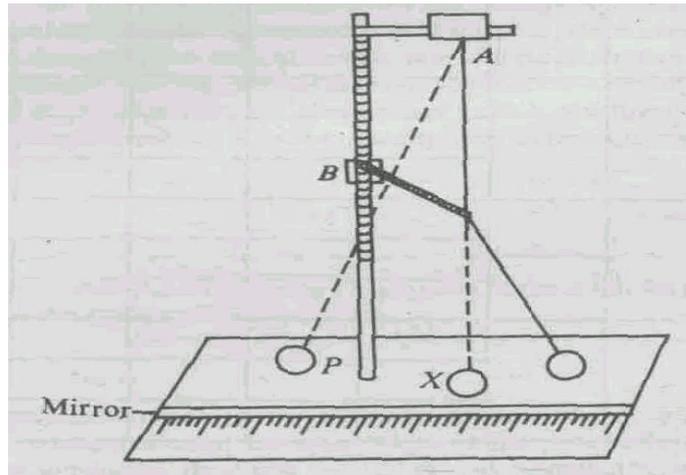


Fig. 3.2: A two-in-one pendulum apparatus

3.1.2 Procedure

Note the least count of vernier callipers and record it in Observation Table 3.1 (a). Now measure the diameter of the bob in different directions. Take at least five readings. Calculate the mean radius, r .

Observation Table 3.1

a. Diameter of bob

Least count of vernier callipers = cm

Zero error (if any) = cm

S. No.	Diameter (cm)	Radius (cm)
1.		
2.		
3.		
4.		
5.		

Mean radius (r) = cm

Now, take a thread of length (l) about 1.5 m and firmly tie its one end to the bob carrying a pointer. The distance between the point of suspension of the pendulum and centre of gravity (C.G) of the bob defines the length of the pendulum (l_1) = $l + r$. Record it in Observation Table 3.1(b). Now displace the pendulum to one side fixing x_1 (Fig. 3.3). While doing this you must make sure that the angular amplitude is small. This means that $\frac{x_1^2}{l}$ is now fixed.

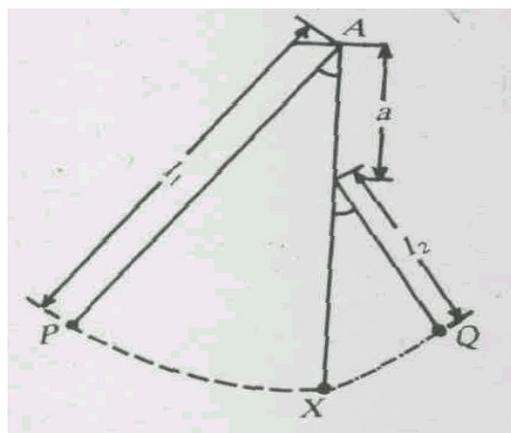


Fig. 3.3: Schematic Depiction of two-in-one pendulum

b. Verification of the Principle of Conservation of Mechanical Energy

Least count of metre scale = cm

S. No.	$l_1 = l + r$ (cm)	x_1 (cm)	$\frac{x_2^2}{l_2}$ (cm)	$l_2 = l_1 - a$ (cm)	x_2 (cm)				$\frac{x_1^2}{l_2}$ (cm)
					(i)	(ii)	(iii)	(mean)	
1									
2									
3									
4									
5									

Now you release the bob. As the Clamp B obstructs the swing, you get another pendulum two-in-one pendulum of smaller length BQ , with B as the point of suspension. This is illustrated in Fig. 3.3.

Note the extreme point of swing on the right-hand side of the mean position. This gives us x_2 . Record your reading in Observation Table 3.1(b). Repeat the procedure at least three times. Do you get the same value every time? Now calculate the average value. Compute $x_2^2 \div l_2$,

where $l_2 = l_1 - a$; a being the distance between clamps A and B . Does Eq. (3.3) hold? If not, then you should look for energy dissipative mechanisms.

Next, you vary l_2 by sliding the clamp B in the groove. Take at least three values of l_2 for one value of l_1 . You should make sure that l_2 is never less than 0.5m. For, otherwise the assumption $\sin\theta \approx \theta$ may not hold.

Next, you should change l_1 by about 20 cm and repeat the above-said steps. Is energy conserved? Comment on your findings.

SELF ASSESSMENT EXERCISE 1

- i. At which position in the two-in-one pendulum can energy loss occur?

- ii. Do your results show a departure from conservation of energy as l_2 is reduced? If so, calculate the maximum deviation.

- iii. List at least two sources of error in your experiment.

3.2 Verification of the Principle of Conservation of Linear Momentum

Let us consider as to what happens when a bullet is fired from a gun. The principle of conservation of energy tells us that the kinetic energies of the bullet and the recoiling gun, along with the heat and sound energies will be equal to the chemical energy of the detonated explosive. However, it does not tell us how this total energy is distributed amongst the bullet, the gun and the surrounding environment. Moreover, since energy is a scalar quantity, its conservation does not even suggest that the gun will recoil. In fact, the law of conservation of energy does not rule out the reverse process – recoiling of the bullet – which we know never occurs. So it is obvious that in such situations we require a conservation principle involving a vector quantity such as linear momentum. This principle may be stated as follows:

When there is no external force acting on a system of particles, the total linear momentum of the system is conserved.

We know that momentum is a vector quantity. Its conservation demands that it should be conserved both in magnitude and direction. When a bullet is fired by a gun the momentum is conserved in one dimension. Can you give an example where two bodies go off in different directions after collision? In a three dimensional space this principle should hold for all three components. But an experiment in two dimensions, rather than three, will be easy to perform and enough to demonstrate the vector nature of the principle of conservation of momentum. This demands that we should know the momenta of the colliding bodies before and after the collision in two dimensions (2-D).

We know that momentum is a product of mass and velocity. Of these, mass of a body can be accurately determined using a physical balance. But to measure velocities of the colliding bodies, we have specially designed a 2-D collision apparatus. We will describe this apparatus in the paragraphs that follow.

Apparatus

2-D Collision apparatus, steel balls, sheet of paper, carbon paper, drawing-board and board pins, plumb line, ruler, protractor and physical balance.

3.2.1 Description of Apparatus

The two-dimensional collision apparatus consists of a curved channel ABC , which may be held with the help of a stand or clamps (Fig. 3.4). The right end of the channel is horizontal. When a steel ball B_1 is released from some point in the channel, it shoots off with zero vertical component of velocity. S is an adjustable support with a flat tip, where another steel ball B_2 may be placed. The support can be moved horizontally so that the two balls can be placed at any desired distance. Moreover, we can adjust this support so that the centres of B_1 and B_2 lie in one horizontal plane, called the *collision plane*.

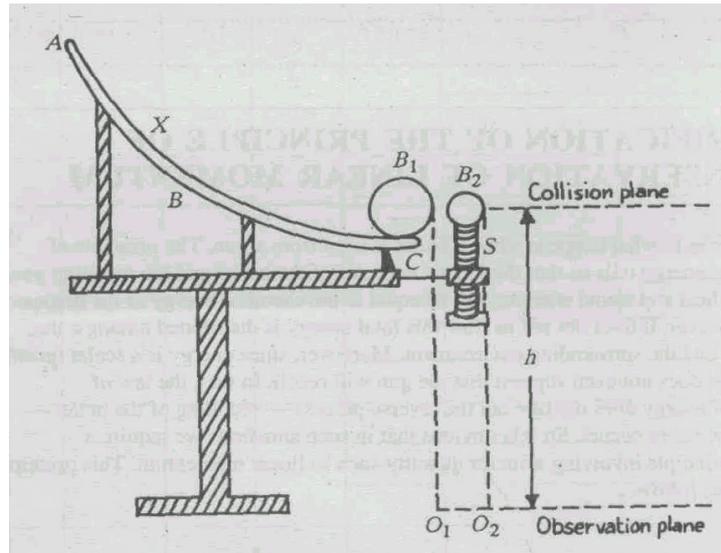


Fig. 3.4: A two dimensional collision apparatus

The floor level forms our *observation plane*. A sheet of paper is laid on the floor with a carbon paper – carbon side down over it. When the ball falls, it will leave a mark on the paper. However, an important point here is to ensure that the floor is smooth. In case the floor is not smooth you should place the white sheet and the carbon paper on a drawing board.

3.2.2 Procedure

Choose two identical balls B_1 and B_2 . Weigh them carefully in a physical balance. Set the apparatus as shown in Fig. 3.4. Mark the point O_1 , on the floor directly below the edge (point C) using a plumb line. Release B_1 from a particular position marked as X on the channel ABC. The ball will fall on the paper, say at P_1^0 . Our knowledge of projectile motion tells us that $O_1P_1^0$ is a measure of velocity of ball B_1 . Repeat this observation ten or fifteen times and encircle the distribution of points on the paper. To what degree is the velocity always the same? Mark the point which is most reproducible.

Collisions between particles of equal masses find most important application in the design of nuclear reactors. We find that neutron energy is most efficiently reduced in collisions with hydrogen nuclei. That is why water is used as moderator.

A projectile motion is characterised by (i) a constant horizontal velocity component, and (ii) a constant downward acceleration, which is the same as that of a freely falling body. The horizontal distance travelled by a projectile is proportional to the horizontal velocity component.

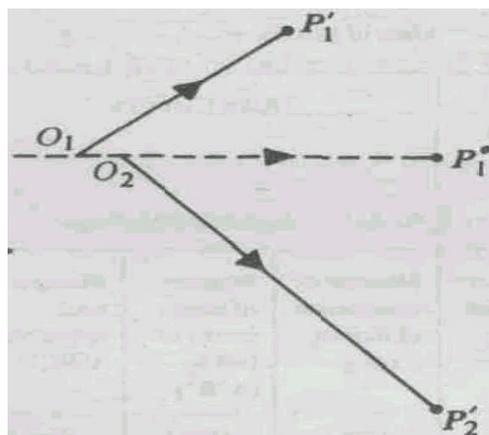


Fig. 3.5: Vectorial representation of a 2-D Collision

Now put ball B_2 on support S . Adjust its position so that the line joining the centres of B_1 and B_2 is a little inclined to the initial line of motion of B_1 . This ensures that the collision is two-dimensional. (A head-on collision is essentially one-dimensional.) For such a collision, the distance between the edge C and the centre of the target ball may be kept at about 2.5 radii. Mark the vertical projections O_1 and O_2 of balls B_1 and B_2 . Now, release ball, B_1 from the same position X . The balls B_1 and B_2 collide and fall on the paper at P_1' and P_2' . You must ensure that B_1 has a smooth trajectory after the collision and its motion is not hampered due to the support holding B_2 . Now remove the carbon paper and draw vectors $O_1P_1^0$, O_1P_1' and O_2P_2' as shown in Fig. 3.5. Then O_1P_1' and O_2P_2' are measures of velocities of B_1 and B_2 after the collision, whereas $O_1P_1^0$ is a measure of velocity of B_1 before collision. Since the masses of the balls are equal, the velocity vectors represent the momenta of the balls.

When two bodies moving along the same line but in opposite directions collide, the collision is said to be head-on. For a head-on collision the distance between the edge of the channel and centre of stationary ball (B_2) should be three radii.

To know the total momentum of colliding balls after the collision, we should add vectors O_1P_1' and O_2P_2' . We know that translation of a vector parallel to itself does not alter it. So you may choose O as the reference point and generate the vector diagram for the momenta following the procedure outlined below:

Shift O_1P_1' parallel to it so that $OA' \parallel O_1P_1'$. Similarly, shift O_2P_2' so that $A'B' \parallel O_2P_2'$ as shown in Fig. 3.6. You should also ensure that the tail of $A'B'$ should fall on the head of OA' . Next draw OB' parallel to $O_1P_1^0$. The triangle law of vectors tells us that OB' denotes the vector sum of

OA' and $A'B'$. According to the principle of conservation of momentum, OB' and OB° should coincide. Comment on your findings on the relationship between OB' and OB° as regards its magnitude and the direction. Compute the error, if any.

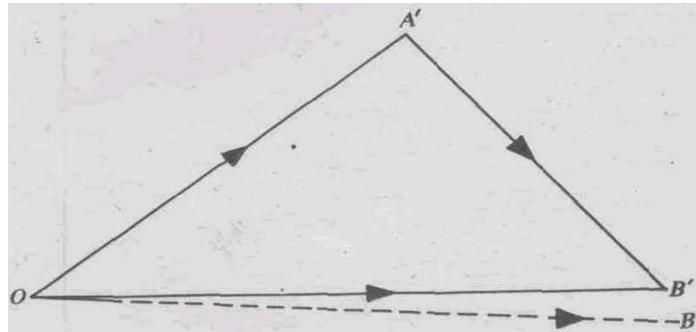


Fig. 3.6: Computation of resultant vector using the triangle law

Repeat the above procedure by releasing B_1 from other positions. Take at least three observations by varying the position X . Record your findings in Observations Table 3.2.

Observation Table 3.2: Conservation of Linear Momentum

Mass of ball $B_1 = \dots\dots\dots$ g

Mass of ball $B_2 = \dots\dots\dots$ g

S. No.	Position on the channel from where the ball B_1 is released (cm)	Before Collision	After Collision			Difference between OB' and OB°	Comments
		Measure of momentum of Ball B_1 (OB^0) (cm)	Measure of momentum of Ball B_1 (OA') (cm)	Measure of momentum of ball B_2 ($A'B'$) (cm)	Measure of total momentum (OB') (cm)		

Repeat the experiment using balls of unequal mass but of the same size. Which one should you use as incident ball? The lighter ball should be the target. Make your own observation table. Is momentum conserved? What do you conclude about the principle of conservation of momentum?

SELF ASSESSMENT EXERCISE 2

- i. Do balls B_1 and B_2 fall through the same height?

- ii. Can we verify this principle by taking B_1 to be lighter than B_2 ? If not, why?

- iii. Can we use parallelogram law to compute the resultant of $\mathbf{O}_1\mathbf{P}'_1$ and $\mathbf{O}_2\mathbf{P}'_2$? If yes, how?

- iv. A shell lies on the ground at rest when it explodes into two equal fragments. How will the fragments move?

- v. Does the friction in the channel play any role in momentum conservation? If so, what?

- vi. List chief sources of error in the second part of your experiment.

For each collision involving balls of equal masses, calculate the square of the velocities before and after the collision. How do they compare? Does this suggest that something else is conserved? Make the same calculation for balls of unequal masses. Is the square of velocities conserved now? Multiply the squares of velocities in the later case by the respective masses and compare your values. What else do you think is conserved besides the momentum? Is it kinetic energy? Comment on the nature of collisions.

4.0 CONCLUSION

The collisions are of elastic/inelastic. Learners are required to arrive at a conclusion based on their experimental observation

5.0 SUMMARY

From the experiments, the principles of conservation of mechanical energy and linear momentum have been verified. Also learners acquired skills in the use of plumb line and removal of parallax.

6.0 TUTOR MARKED ASSIGNMENT

As given by the facilitator.

7.0 REFERENCES/FURTHER READINGS

IGNOU (1997) Physics Laboratory II, Physics PHE-03(L), New Delhi, India.

IGNOU (1997) Elementary Mechanics Physics, PHE-01, New Delhi, India.

MODULE 2

Unit 1	Experiment 4 A Study of Coupled Oscillations
Unit 2	Experiment 5 Relation between Wavelength and Frequency of Stationary Waves
Unit 3	Experiment 6 Young's Modulus for a Material by Bending of Beams
Unit 4	Experiment 7 Measurement of Low Resistance using Carey Foster's Bridge
Unit 5	Experiment 8 Variation of Thermo-E.M.F. with Temperature

UNIT 1 EXPERIMENT 4
A STUDY OF COUPLED OSCILLATIONS

CONTENTS

1.0	Introduction
2.0	Objectives
3.0	Main Content
	3.1 Measurement of the Period of Normal Modes
	3.2 Frequency of Energy Transfer
4.0	Conclusion
5.0	Summary
6.0	Tutor Marked Assignment
7.0	References/Further Readings

1.0 INTRODUCTION

In Experiments 1 and 2 you made measurements with isolated (single) oscillating systems such as a pendulum (simple or compound), a spring-mass system and a torsional oscillator. In nature we come across many examples of coupled oscillators. For example, atoms in a solid are coupled by inter-atomic forces. In molecules, say the water molecule, two hydrogen atoms are coupled to an oxygen atom while in a oxygen molecule, two oxygen atoms are coupled to one another. Although we cannot quantify the coupling in atoms, yet it is important to realise that coupling influences oscillations of individual atoms in a molecule or a solid. In a continuous medium, coupling leads to the phenomenon of wave motion. In the next experiment, you will establish a relation between frequency and wavelength of a wave.

When two (identical or different) atoms are coupled together, the coupled system executes oscillations which are different from the oscillations of independent atoms. In radio and TV transmission, we use coupled electrical circuits. It is therefore important to study oscillations of a coupled system. In general, individual oscillators of a coupled system may or may not all be identical. But in this experiment you will work with two identical mechanical oscillators in the form of metallic strips (Hacksaw blades), which may be coupled by a rubber band, a spring or a pair of bar magnets.

You will observe that the oscillations of the coupled system are no longer simple harmonic. But there are two modes in which motion is simple harmonic and each has a definite frequency. These are called *normal modes*. In this experiment you will learn to measure the period of normal modes. Moreover you will observe that individual oscillators exchange energy repeatedly back and forth. What is the rate of energy exchange? You will discover answer to this question also here. In fact, after doing this experiment you will realise that you can understand all good physics involved in the study of a coupled system through very simple equipment.

2.0 OBJECTIVES

After performing this experiment, you should be able to:

- demonstrate the effect of coupling in the behaviour of individual oscillators
- measure the period of normal modes
- plot a graph between angular frequency and position of rubber-band from the fixed end of oscillators
- compute the frequency of energy transfer.

3.0 MAIN CONTENT

3.1 Measurement of the Period of Normal Modes

We know that an isolated system vibrates with its natural frequency. What happens when two such isolated systems are coupled together? The presence of coupling affects its amplitude and frequency of oscillation. We expect that the motion may not remain simple harmonic. Does this mean that for a coupled system we cannot define the period of oscillation? To answer this and other related questions we consider a system of two identical coupled oscillators. The apparatus needed for this purpose is listed below:

Apparatus

Two identical hacksaw blades (1/2" or 1" width and 12" length), two vices, rubber bands/soft springs/a pair of strong bar magnets, and a stopwatch.

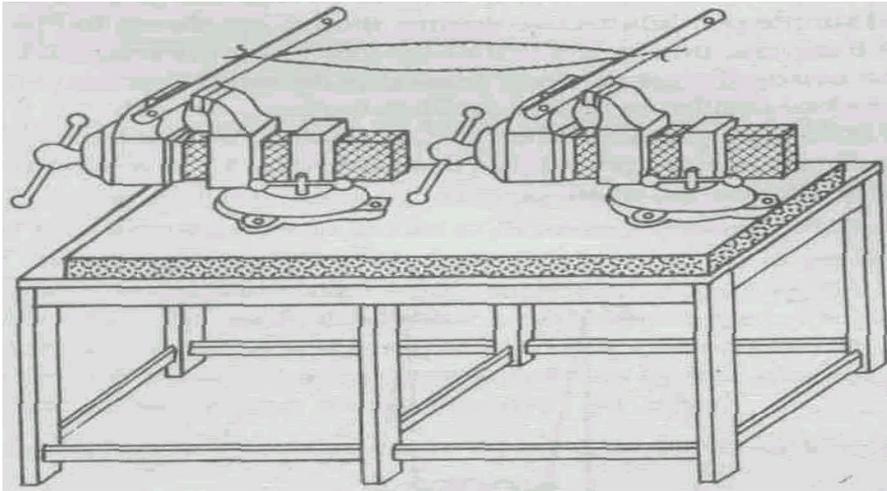


Fig. 4.1: A coupled oscillator system

Set the apparatus as shown in Fig. 4.1. For the success of this experiment you should note that both oscillators (in this case hacksaw blade) should be in the same plane and act as identical oscillators. That is, the time periods for both these oscillators should be the same. To ensure this, you should use a stopwatch with good accuracy. Note the least count of the stop watch and record it in Observation Table 4.1. Next displace one of them from its respective mean position and then release it. It begins to oscillate. You should ensure that the oscillations are free. To begin with, you should count time for 10 oscillations. Enter your data in Observation Table 4.1. Calculate its time period by dividing the measured time by N , the total number of oscillations counted.

Observation Table 4.1: Time Period of Isolated Oscillators

Least count of stopwatch = s

S. No.	No. of Oscillations (N)	Time for N Oscillations (s)		Time Period (s)	
		1st Oscillator	2nd Oscillator	1st Oscillator	2nd Oscillator
1	10				
2	20				
3	30				
4	40				
5	50				

Repeat the same procedure for the other hacksaw blade. Compare their time periods. Are they same? We expect these to be same. If not, then load the blade with larger time period with wax. Alternatively you can file the blade which has smaller time period. You will require considerable experimental skill of measuring time and practice to achieve exactly same values of time periods. You should repeat this process till you get identical time periods. In case you fail to do so repeat the procedure till the difference between these periods is not more than 0.1 %. Next you should measure time for 20 oscillations and repeat the above-said procedure. To get more precise results you can work with 30, 40, 50 or more oscillations. Let us denote the time period by T_0 .

Now couple these two oscillators by putting a rubber band or a spring near the fixed end. In this way you obtain a mechanically coupled system. Alternatively you can use a pair of strong bar magnets. Is there any difference between these two types of couplings? We expect that the system will display similar behaviour in both cases. You can, therefore, use either of these arrangements for this experiment.

In the course Oscillations and Waves, you will learn that the motion of a coupled system is not simple harmonic. However it can be analysed in terms of normal modes. For two coupled simple pendulums two normal modes are shown in Fig. 4.2. Consider the transverse motion and first excite the *in-phase* normal mode by equally displacing the two oscillators (hacksaw blades) in the same direction (Fig.4.2a). You

should ensure that the two oscillators always oscillate in phase. As such, this is somewhat tricky and you will need some practice. When you are finally convinced, measure time for 30 oscillations and calculate the period. Let us denote it by T_1 . It is important that the amplitude of oscillations be small.

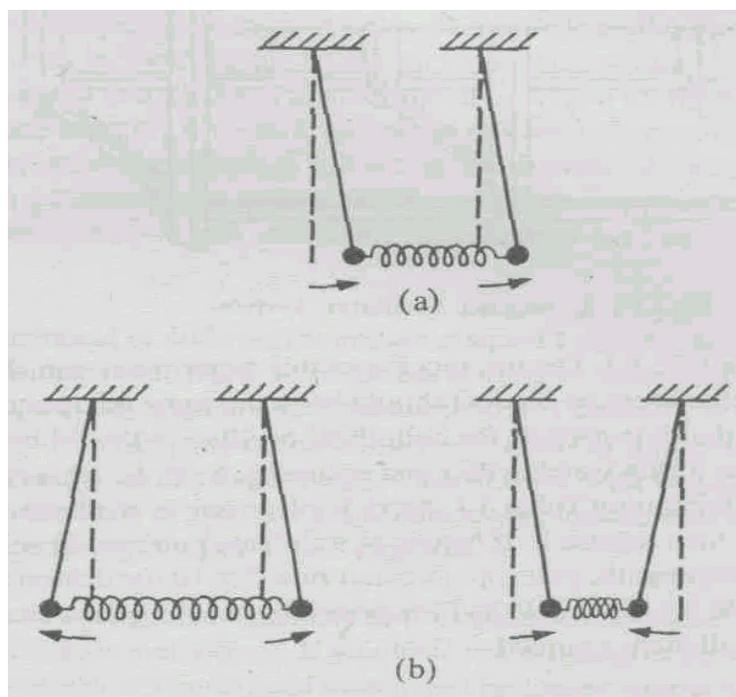


Fig. 4.2: (a) In-phase, and (b) out of phase normal modes

Now you make the system to vibrate in the *out of phase* normal mode without changing the position of the coupling system (spring/magnet/rubber-band). This can be done in two ways, as shown in Fig 4.2(b). You can choose to work with the case in which two oscillators are drawn closer. Repeat the above procedure and determine the time period for this case. Let it be T_2 . Are T_0 , T_1 and T_2 the same? We expect them to be different. What do you conclude from this? This only means that coupling is effective.

Next, you move the coupling arrangement away from the fixed end by 1 cm. This will bring about a change in the coupling. Another way of changing the coupling strength will be to change the quality of rubber band or take springs of different spring constants. Repeat the above procedure and record your data in Observation Table 4.2.

Table 4.2: Effect of Coupling on the Period of Normal Modes

Least count of stop watch =s

No. of oscillations (N) =

S. No.	Distance of rubber band from the fixed end (cm)	Time for N Oscillations (s)		T_1 (s)	T_2 (s)
		Ist Normal Mode	IInd Normal Mode		
1.					
2.					
3.					
4.					
5.					
6.					

Are time periods influenced by changing the position of the rubber-band?

Repeat the experiment for other positions of the rubber band and enter your readings in Observation Table 4.2.

Calculate the corresponding angular frequencies using the relation, $\omega = 2\pi / T$. (The difference in the frequencies of two normal modes is known as *frequency splitting*. We denote it by the symbol $\Delta\nu$ and it is given by $(\omega_2 - \omega_1) / 2\pi$). Do these frequencies vary as position of the rubber band is changed? A variation in their values suggests that coupling has an influence on the motion of the system. To clarify this further, you can plot angular frequency as a function of the distance of the rubber band from the fixed end. Is the relation linear? Discover the functional dependence between the two quantities by following the procedure outlined in Experiment 1. Discuss your results with your counsellor.

Conclusion: The angular frequency varies..... with the distance of the rubber band from the fixed end.

SELF ASSESSMENT EXERCISE 1

Is there any damping in the system? How will you account for it?

SELF ASSESSMENT EXERCISE 2

Choose two widely separated points on your angular frequency versus distance of rubber-band graph and correlate frequency splitting to the coupling constant.

3.2 Frequency of Energy Transfer

So far you have seen that for a coupled system angular frequencies of normal modes differ due to the presence of coupling. Another manifestation of coupling is exchange of energy. In this part of the experiment you will study how frequently energy transfers from one oscillator to another. Keep the rubber band (or spring) nearest to the clamped point so that the coupling is minimum. Then displace one of the oscillators without disturbing the other. Observe the change that occurs in the second oscillator. You will note that the second oscillator starts oscillating and gradually gains displacement. What happens to the first oscillator? It begins to lose amplitude and ultimately comes to a stop momentarily. But soon you will observe that it begins to gain displacement. Do you know the reason for these periodic changes? This is brought about by the presence of coupling and implies that total energy flows back and forth between the two oscillators.

To measure the periodicity of energy exchange, again displace the oscillator from its mean equilibrium position. Measure the time in which one cycle is completed. If it happens very rapidly, then measure time for 5 or 10 cycles of energy transfer. Make your own Observation Table and record your readings in it. Calculate the time period. Repeat the procedure for several positions of the rubber band along the hacksaw blades i.e., for different values of coupling:

Do you get the same time period for every position? We expect it to be different. The inverse of time period gives the frequency of energy transfer.

Observation Table 4.3: Frequency of Energy Transfer

--

4.0 CONCLUSION

The frequency of energy transfer depends on.....

Learners are required to provide conclusion from their observations and calculations.

SELF ASSESSMENT EXERCISE 3

Does air damping affect the frequency of energy transfer? Justify your answer.

5.0 SUMMARY

Learners have demonstrated

- (a) The effect of coupling on the behaviour of individual oscillator
- (b) The graphical relationship between frequency and the position of rubber band from the fixed end of the oscillator
- (c) From data acquired, the frequency of energy transfer was computed and the period of normal modes was measured.

6.0 TUTOR MARKED ASSIGNMENTS

As provided by the facilitator.

7.0 REFERENCES/FURTHER READINGS

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IGNOU(1997) Elementary Mechanics, Physics PHE-01, New Delhi, India.

UNIT 2 EXPERIMENT 5

RELATION BETWEEN WAVELENGTH AND FREQUENCY OF STATIONARY WAVES

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 To Set up Stationary Waves in a Stretched Wire
 - 3.2 Variation of Wavelength with Tension
 - 3.3 Variation of Wavelength with Mass per Unit Length
 - 3.4 Relation between Wavelength and Frequency
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor Marked Assignments
- 7.0 References/Further Readings

1.0 INTRODUCTION

You all must have enjoyed the pleasing music produced by stringed instruments like guitar, violin, etc. at a concert or on a radio or a television. Do you know how stringed instruments produce music? When the string of such an instrument is plucked, bowed or struck, it begins to vibrate and produces sound. The quality of sound thus produced depends upon the frequency of vibration of the stretched string. Now the question arises: What factors determine the frequency of vibration of the string? How are these factors related to frequency? In this experiment you would discover answer to these questions.

You may have observed that in an orchestra a violinist ties up or loosens the pegs of his instrument while tuning with other musicians. (As the peg is tied or loosened, a portion of the string is either wound or unwound round the peg.) As a result, tension in the string changes. This means that the frequency produced by the string of the violin depends on the tension in it. Can you think of other parameters which may influence the frequency of vibration of the string? What happens if you take strings of different thicknesses or strings of different materials but same thickness? Well, we expect that the frequency of vibration of the string in each case will differ. This means that the mass per unit length of the string also influences its frequency of vibration. That is why the strings of guitars and pianos are wrapped with a metal winding.

You may have seen a harp or veena. In these instruments, strings of unequal lengths are tied between two fixed ends. You may have also seen that once a musician has tuned his instrument, he moves his fingers

along its string to produce music. In this way he varies the vibrating length in order to produce different notes. This suggests that the frequency of vibration of the string depends on its vibrating length as well. Since the length of the vibrating segment of the string is related to the wavelength of the stationary wave set up in it, we expect that there exists a definite relationship between the wavelength and frequency.

The aim of this experiment is to know how frequency of vibration of a stretched string depends on tension, mass per unit length and its vibrating length. You would recall from your investigations with a simple pendulum (Experiment 1) that when a physical quantity depends on more than one parameter it makes sense to vary only one parameter at a time. So in this case any change in the frequency can be attributed to the change in that particular parameter. It is possible to set up waves of known wavelength in a wire. But it is easier to make a wire vibrate with a known frequency. So we would discover the effect of tension and mass per unit length of the wire on the wavelength, keeping the frequency constant. Therefore, we would like you to do this experiment in three parts. In the first part you will investigate as to how wavelength changes with tension in the wire while the frequency of vibration of the wire and its mass per unit length are kept fixed. In the second part you will investigate how the wavelength varies when wires of different thickness (but same material) or different materials (but same thickness) are used. That is, you will learn how the wavelength varies with mass per unit length of the wire when tension in the wire and frequency are held constant. In the third part you will establish the relation between frequency and wavelength.

2.0 OBJECTIVES

After doing this experiment you should be able to:

- set up stationary waves in a stretched string
- investigate the dependence of wavelength of stationary waves on the tension in a string and its mass per unit length
- establish the relationship between wavelength and frequency
- establish the expression for velocity of transverse stationary waves on a string.

3.0 MAIN CONTENT

3.1 To Set up Stationary Waves in a Stretched Wire

The measurement of tension (T) and mass per unit length (μ) is a rather easy exercise. But to make a precise determination of wavelength, we set up *stationary waves*. Stationary waves are formed by superposition

of two identical progressive waves moving in opposite directions. These waves do not move with time in either direction. (For this reason, they are also sometimes referred to as *standing waves*.) Stationary waves can be produced in air columns as well as stretched strings. Here we intend to set up stationary waves in a sonometer wire.

A wave which transports energy as it propagates in space is said to be progressive. In a stationary wave no energy is transported.

A sonometer consists of a hollow wooden box with a peg at one end and a pulley on the other. One end of a wire is fixed to the peg and the other end, passing over a smooth pulley, carries a hanger (in place of hanger you can also use a pan). By placing weights on the hanger, the string can be stretched. The wire is made to pass over two bridges B_1 and B_2 as shown in Fig. 5.1. While performing experiments with a sonometer, the string is made to vibrate in *unison* with the source of sound, which may be a tuning fork or an electromagnet. To achieve this, the vibrating length B_1B_2 of the wire is adjusted by sliding the bridges between the peg and the pulley. This condition (of unison) is ensured when a V-shaped paper rider placed in the middle of the wire between the bridges falls down.

The sonometer wire is said to vibrate in unison with the source of sound when the natural frequency of the wire is equal to the frequency of the source.

The vibrations are said to be forced vibrations when a body vibrates with the frequency of the applied periodic force. In this condition the energy fed from outside equals the energy lost by the body.

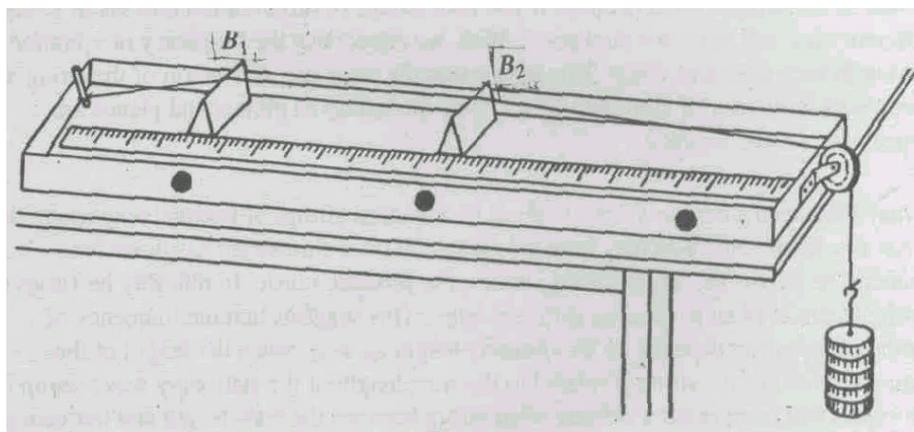


Fig. 5.1: A Sonometer

When a vibrating tuning fork is placed on the sounding board of the sonometer, the wire executes forced vibrations and transverse waves are

set up in it. In the region B_1B_2 , these waves are reflected at the fixed points B_1 and B_2 . As a result we obtain a set of incident and reflected waves travelling in opposite directions, which give rise to transverse stationary waves. The wire between the bridges then vibrates in one or more well-defined segments as shown in Fig. 5.2. You will observe that there are some points at which the wire remains motionless at all times. On the other hand, at some other points, the waves reinforce strongly and the wire vibrates vigorously. The points corresponding to zero amplitude of vibration are called **nodes** (N) whereas points with maximum amplitude are called **antinodes** (A). In the fundamental mode, the wire within fixed ends vibrates in one loop. The fixed points act as nodes with an antinode in the middle.

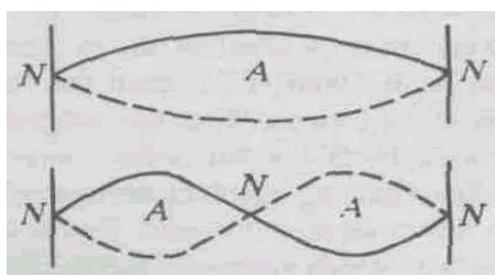


Fig. 5.2: Stationary waves set up in the wire fixed at both ends

The apparatus required for this experiment is listed below:

Apparatus

4 iron wires of different thicknesses (Alternatively 4 wires of different magnetic materials), sonometer, hanger, slotted weights, an electromagnet with a 6 volt A.C. transformer, six tuning forks of known frequencies, rubber pad, metre scale, screw gauge or a chemical balance with weight box.

3.2 Variation of Wavelength with Tension

You now know that in this part of the experiment you have to keep mass per unit length of the wire and its frequency of vibration constant. The former of these can be accomplished by working with a wire of known material. To achieve the latter you can use either a tuning fork or an electromagnet. Of these two, an electromagnet is preferred because with its help the wire can be made to execute sustained vibrations.

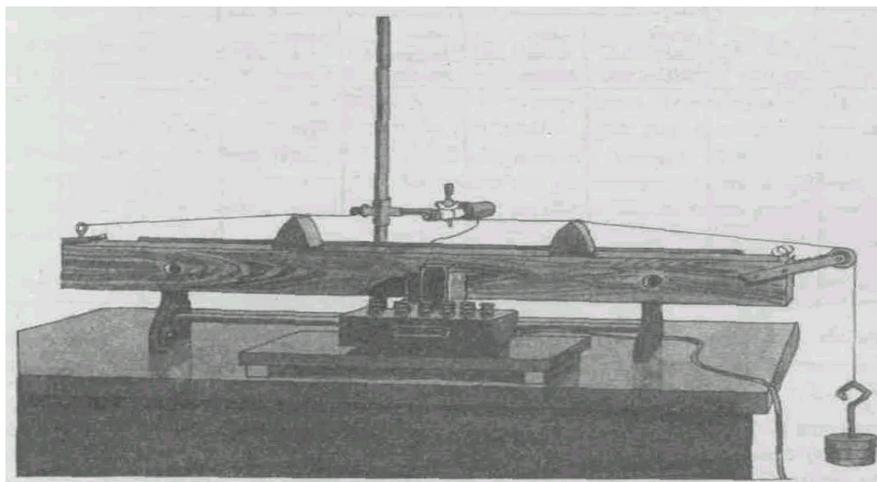


Fig. 5.3: Experimental arrangement for setting up transverse stationary waves in a sonometer wire

The experimental arrangement is shown in Fig. 5.3. Connect the electromagnet to a 6 V transformer and place the electromagnet near the middle of the wire. When an alternating current (ac) is sent through the electromagnet, in each cycle the core is magnetised twice with opposite polarities. As a result, the sonometer wire is attracted by the electromagnet twice in each cycle and it begins to vibrate. Since the frequency of ac is 50 Hz, the wire will vibrate with a fixed frequency of 100 Hz.

SELF ASSESSMENT EXERCISE 1

Suppose that the electromagnet is connected to a source of direct current. Will the wire vibrate? If yes, what will be its frequency of vibration?

Stretch the wire by putting weight of 0.5 kg in the hanger. (If a weight of M kg is used for stretching the wire then the tension in the wire will be $T = mg$ Newton, where g is acceleration due to gravity. You can use $g = 10 \text{ m/s}^2$.) Keep the bridges of the sonometer at a distance of about 25 cm. As soon as the current is switched on, the electromagnet is energised and you will observe that the wire begins to vibrate. This means that the apparatus is in working order and you can begin your investigations. When the sonometer wire vibrates in the fundamental mode, the distance between the two nodes is equal to half the wavelength of the stationary wave in the wire. The vibrating length of the wire will therefore be a measure of the wavelength of stationary waves set up in the wire. That is why we are interested in determining that length of the wire which vibrates in the fundamental mode with a frequency of 100 Hz. First of all, make the wire vibrate in one single

loop. Then to achieve unison, you first place a rider on the wire. Fix one of the bridges, say B_1 and move the other bridge B_2 towards it. What do you observe? Does the amplitude of vibration of the wire decrease? If so, then move the bridge B_2 away from B_1 . Continue to move it away from B_1 till the amplitude becomes maximum. In this position, the rider will fall down. Measure the distance between the bridges accurately and record it in Observation Table 5.1. Next, you repeat the above procedure by keeping the bridges closer, separated by 10 cm. Move bridge B_2 away from bridge B_1 and note the length of the wire between the bridges at which the rider is again thrown off. Enter your reading in Observation Table 5.1,

Observation Table 5.1: Dependence of Wavelength on Tension

Frequency of Vibration of the wire = 100 Hz

Least count of metre scale =cm.

S. No.	Weight placed on the hanger (kg)	Tension $T=Mg$ (N)	Length (l) of the wire between two bridges in unison with electromagnet (cm)				Mean l (cm)	Wave-length $\lambda = 2l$ (m)	In T	In λ
			load increasing		load decreasing					
			when bridges are far apart	when bridges are closer	when bridges are far apart	when bridges are closer				
1										
2										
3										
4										
5										

Now you change the tension in the wire by adding weights on the hanger in equal steps of, say, 0.5 kg and measure the resonating lengths of the wire in each case. Enter your data in Observation Table 5.1. You should not load the wire beyond its elastic limit.

To check that you are working within the permissible range, you should repeat the above — said procedure by unloading the wire in equal steps. Tabulate your observations in each case. Do these lengths differ from those measured while loading the wire? We expect these to be almost

the same. If they differ significantly, you should discuss with your Counsellor. Calculate the mean length for a given tension.

From the table you will observe that λ changes with T . The variation in λ suggests that it is related to tension. Mathematically, we can write

$$\lambda \propto T$$

Can you give an exact relation between these variables by looking at your observations? Probably you cannot. To discover the exact relationship between λ and T , you can proceed along the lines suggested in the experiment on simple pendulum. That is, you may plot λ vs $T^{1/2}$, λ vs. T , λ vs T^2 and soon. One of these plots will be a straight line. For example, if λ vs $T^{1/2}$ plot is a straight line passing through the origin and the slope of the line is k_1 , the exact relation between λ and T is given by $\lambda = k_1 T^{1/2}$. Alternatively you can arrive at this relation as follows: Let $\lambda \propto T^a$,

$$\text{or } \lambda = k_1 T^a \quad (5.1)$$

where k_1 is the constant of proportionality and a is another constant. Taking logarithms to the base e on both sides, we get

$$\ln \lambda = \ln k_1 + a \ln T \quad (5.2)$$

So if you plot $\ln \lambda$ along the y -axis and $\ln T$ along the x -axis, the graph will be a straight line.

On comparing Eq. (5.2) with the equation of a straight line namely

$$y = mx + c,$$

we find that the intercept on the y -axis gives $\ln k_1$, while the slope gives the value of a . Calculate the slope by using two well separated points on the straight line. We expect the value of a to be $1/2$. What is your result? Calculate the error in the slope by drawing lines of maximum and minimum slope.

Then the relation between λ and T is:

$$\lambda = k_1 \sqrt{T} \quad (5.3)$$

SELF ASSESSMENT EXERCISE 2

Plot a graph between λ and $T^{1/2}$. For $T_1 = 64$ N and $T_2 = 324$ N, calculate the ratio of wavelengths from your graph.

3.3 Variation of Wavelength with Mass per Unit Length

To investigate the dependence of wavelength on mass per unit length of the wire, we take four wires of different thicknesses but of the same material. If it is not possible to get wires of different cross-sections, you can take wires of same cross-section but different materials. For each wire, you first determine the mass per unit length (μ). To do so you have to measure their diameters. For this you should use a screw gauge. Note its least count and observe whether there is any zero error. Measure the diameter at several places. In this way you can account for the inhomogeneity, if any, in the wire. Record your readings in Observation Table 5.2(a).

Observation Table 5.2 (a): Determination of Mass per unit Length of the Wire

	Diameter $d(\text{cm})$					
	(i)	(ii)	(iii)			
<i>A</i>						
<i>B</i>						
<i>C</i>						
<i>D</i>						

If you know the density (ρ) of the material of the wire from a text on physical data, you can easily compute mass per unit length for any wire using the relation $\mu = \frac{\pi d^2 \rho}{4}$. Alternatively, you can determine μ , for a wire by weighing a known length of it.

In this part of the experiment, you should keep the tension constant, say 20N (i.e. $M = 2$ kg). As soon as the current is switched on, the electromagnet is activated and the wire begins to vibrate with a frequency of 100 Hz. Keep the bridges at a distance of, say, 25 cm. As mentioned earlier, you should adjust the distance between bridges so that the wire vibrates in one single loop with maximum amplitude. As before, this should be tested by placing a paper rider. Measure the

distance between the bridges accurately and record it in Observation Table 5.2(b). Repeat the process by putting the bridges closer, say, at a distance of 10 cm and moving one of these bridges away from the other. Record the resonating length in Observation Table 5.2 (b). Calculate the mean length.

Repeat this procedure for other wires, keeping the tension in the wire constant. Tabulate your observations in Observation Table 5.2 (b).

Observation Table 5.2 (b): Dependence of Wavelength on Mass per unit Length

Frequency = Hz

Tension in the Wire = N

S. No.	Mass per unit length μ (kg/m)	Length (l) of the wire between two bridges in unison with the electro-magnet (cm)		Mean length l (cm)	Wavelength $\lambda = 2l$ (m)	$\ln \mu$	$\ln \lambda$
		(i)	(ii)				
		when the bridges are far apart	when the bridges are closer				
1.							
2.							
3.							
4.							
5.							
6.							

Does the wavelength vary with μ ? Does λ decrease or increase as μ increases? A decrease in the value of λ suggests inverse dependence on μ . To quantify this dependence we write,

$$\lambda = k_2 \mu^b,$$

where k_2 a constant of proportionality and b is another constant. Taking logarithms to the base e on both sides, we get

$$\ln \lambda = \ln k_2 + b \ln \mu$$

If you plot $\ln \lambda$ versus $\ln \mu$, you will obtain a straight line. Is the slope of the straight line negative? Of course it should be. This signifies that as μ increases, λ decreases. The slope of the straight line gives us the

exponent b . We expect $b = -0.5$. What is your value of b ? Calculate the maximum error by taking lines of maximum and minimum slopes. Thus we can write

$$\lambda \propto \sqrt{1/\mu} \quad (5.4)$$

On combining the results of the two investigations done so far, you can write

$$\lambda \propto \sqrt{\frac{T}{\mu}},$$

or

$$\lambda = k \sqrt{\frac{T}{\mu}}, \quad (5.5)$$

where k is a constant of proportionality.

SELF ASSESSMENT EXERCISE 3

- i. What will happen if the wire stretched on the sonometer is hollow?

- ii. Suppose you have adjusted the length of the string (of iron) in unison with a tuning fork. Now you replace the string with a similar one of nickel. Will the same length of the string be in unison with the fork?

3.4 Relation between Wavelength and Frequency

To establish the relation between wavelength and frequency for a given wire, the tension in the wire is kept fixed. To vary the frequency, you would require a set of tuning forks of different frequencies. (Electromagnet will not do because it makes the wire to vibrate with only one single frequency.) Stretch the wire with a constant tension of 20N ($M = 2\text{kg}$).

Put the bridges B_1 and B_2 at a distance of about 25cm. As before, place a V-shaped paper rider in the middle of the portion B_1B_2 . Strike one of the prongs of a tuning fork with rubber pad. The tuning fork should be struck gently on the rubber pad. This will ensure that vibration will correspond to only the fundamental mode. Press the stem of the tuning

fork on the sounding board. You should not touch the tuning fork anywhere on its U part. (If you do so, the vibrations will become damped). The vibrations of the tuning fork are transmitted to the wire, which in turn begins to vibrate and stationary waves are set up in it. Now slowly move the bridge B_2 towards the bridge B_1 until the paper rider falls off. This means that the wire and the tuning fork are in unison. Measure this length of the wire carefully and record it in Observation Table 5.3.

Observation Table 5.3: Dependence of Wavelength on Frequency

Tension in the string = N

S.No	Frequency (f) of the tuning fork (Hz)	Length of the wire between two bridges in unison with tuning fork (m)		Mean length l (m)	Wavelength $\lambda = 2l$ (m)
		when bridges are far apart	when bridges are closer		
1					
2					
3					
4					
5					

Now place the bridges B_1 and B_2 at about 10cm and repeat the above procedure by moving one of the bridges away from the other. As before measure the length which resonates with the tuning fork. Enter your readings in Observation Table 5.3. Calculate the mean length.

Keeping the tension fixed, repeat the procedure for other tuning forks. Measure the length each time and record it in Observation Table 5.3.

How does frequency influence the wavelength? We expect it to decrease. Mathematically, we express it as

$$f = k_3 \times \frac{1}{\lambda^c} \tag{5.6}$$

Here k_3 is a constant of proportionality and c is some other constant. Plot f vs $\lambda^{-1/2}$, f vs λ^{-1} , f vs λ^{-2} , and so on. We expect that the plot of f vs λ^{-1} will be a straight line. The slope of this straight line gives you the value of k_3 . Compare this value of k_3 with the ratio $\sqrt{T/\mu}$ for this wire. Are the two values the same? Theoretically they should be. It

implies that frequency and wavelength of stationary waves on a string are connected by the relation

$$f = \frac{1}{\lambda} \sqrt{\frac{T}{\mu}} \quad (5.7)$$

The dimensions of the product $f\lambda$ are those of the velocity. This means that the velocity of transverse stationary waves on stretched strings is controlled by its mass per unit length and tension in the wire.

SELF ASSESSMENT EXERCISE 4

What will be the change in frequency if the length of the string between the bridges is doubled?

4.0 CONCLUSION

Learners are required to provide conclusion from their observations and calculations during the performance of the experiment.

5.0 SUMMARY

Investigations have been performed to

- (a) Determine the dependence of wave length on the tension in a string and its mass per unit length
- (b) Establish the relationships between wave length and frequency
- (c) Establish the expression for velocity of transverse stationary waves on a string.

6.0 TUTOR MARKED ASSIGNMENTS

As provided by the facilitator.

7.0 REFERENCES/FURTHER READINGS

IGNOU (1997) Physics Laboratory I, Physics PHE-03 (L), New Delhi, India.

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UNIT 3 EXPERIMENT 6

YOUNG'S MODULUS FOR A MATERIAL BY BENDING OF BEAMS

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Depression of a Beam Supported at the Ends and Loaded at the Centre
 - 3.1.1 Cantilever
 - 3.1.2 Bending Moment
 - 3.1.3 Depression at Free End of a Cantilever
 - 3.2 Measurement of Depression in a Beam using a Microscope
 - 3.3 Measurement of Depression in a Beam using a Telescope and an Optical Lever
 - 3.4 Comparison of Accuracies of above Methods by Determining Young's Modulus
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor Marked Assignments
- 7.0 References/Further Readings

1.0 INTRODUCTION

As a child, while playing you may have pressed a rubber ball or a piece of sponge and observed that the shape of the ball/sponge undergoes a change. Now if you stop pressing it, you will observe that the ball regains its original shape. In fact, all bodies can, more or less, be deformed by a suitably applied force and when the deforming force is removed, the bodies tend to recover their original state. The simplest case of deformation can be observed in a wire which is fixed at its upper end with a weight suspended at its lower end. The weight at its lower end brings about a change in its length. When the suspended weight is removed from the wire, it tends to come back to its original length. This property of the wire is called **elasticity**. It is by virtue of this property that a body opposes any change being produced in its shape and/or size by an external force and tends to regain its original shape and/or size after the removal of the external force. The greater the force necessary to produce deformation in the body, the more elastic it is.

Whenever a body is subjected to a deforming force, a force of reaction comes into play within it. This internal force is termed as **restoring force**. It tends to resist the applied force and restores the original shape and/or size of the body. In equilibrium state, the restoring force is equal

to the applied external force. The restoring force per unit area set up inside the body is called **stress**. The fractional change in its length, volume or shape relative to original state of the body is termed as **strain**. For example, when a wire is stretched by applying a force along its length, i.e. normal to its cross-sectional area, the change occurs in its length. Then restoring force developed per unit cross-sectional area of the wire is known as longitudinal stress. The change in length per unit original length of the wire is called **longitudinal strain**. The ratio of longitudinal stress to longitudinal strain, within the elastic limit, is called **Young's modulus**. Its value depends on the nature of the material and not on the dimensions of the sample.

*The maximum stress a material can sustain without undergoing permanent deformation is termed as **elastic limit**.*

Knowledge of Young's modulus is of great importance in bridge design. Its value is one of the pieces of information which must be known to calculate accurately the deformation (depression) that will occur in a loaded structure and its parts. When a beam bends, the surface is compressed and the other is stretched as in Fig. 6.1, so that Young's modulus is involved. Similarly, Young's modulus enables us to calculate the stress which a given body, say the connecting rod or piston of a steam engine or a girder, can bear. You must have observed that the girders and beams used in bridges and steel frame buildings are manufactured with their cross-sections in the form of the letter I. Also that in a beam of rectangular cross-section, the longer side is used as depth. In fact, 'beam theory' – one of the foundation stones of structural engineering – gives us all the above mentioned information. In this experiment you will learn to determine Young's modulus of a material by the method of bending of beams.

A beam is a bar of uniform cross-section (circular or rectangular) of a homogeneous, isotropic (which have the same properties at all points and in all directions) elastic material.

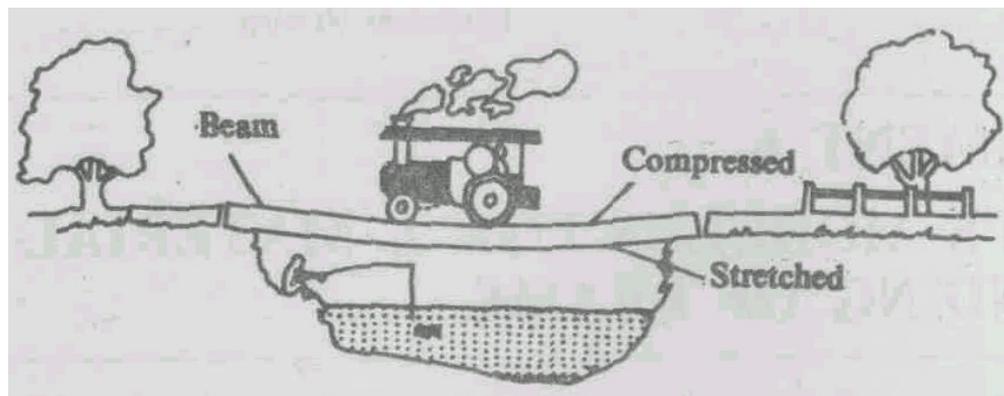


Fig. 6.1: A railway engine (of early days) moving over the iron railway bridge causes the beam to depress so that one of its surface is compressed while the other is stretched.

2.0 OBJECTIVES

After doing this experiment, you should be able to:

- focus a microscope and a telescope on a given object
- remove parallax error
- measure small depressions
- compare accuracies of the methods used for the measurement of the depression of the beam using (i) microscope and (ii) telescope and optical lever arrangement
- compute Young's modulus of elasticity.

3.0 MAIN CONTENT

3.1 Depression of a Beam Supported at the Ends and Loaded at the Centre

When a beam is supported near its two ends and loaded at the centre, it shows maximum depression at the loaded point. Usually, the depression produced is very small. Suppose a beam is supported on two knife-edges at A and B near its two ends, as shown in Fig. 6.2.

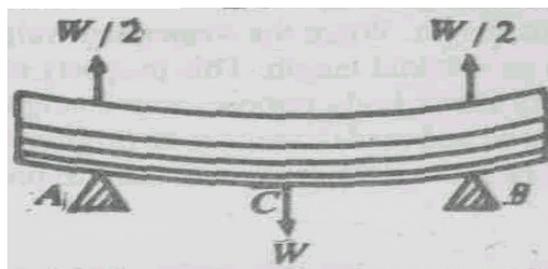


Fig. 6.2 Depression of the beam supported at the two ends and loaded at the centre with a weight W

Let it be loaded in the middle at C with a weight W . The reaction of each knife edge will clearly be $\frac{W}{2}$ in the upward direction. In this position, the beam may be considered as equivalent to two inverted cantilevers, fixed at C . The bending in these two cantilevers will be produced by the load – acting upwards at A and B . Therefore, it is important for us to know how the bending is produced in a cantilever and on what factors does the bending depend.

A Cantilever is a beam fixed horizontally at one end.

3.1.1 Cantilever

Consider the cantilever shown in Fig. 6.3. Let us put a weight W_1 , at the free end. As soon as the beam is loaded, it bends.

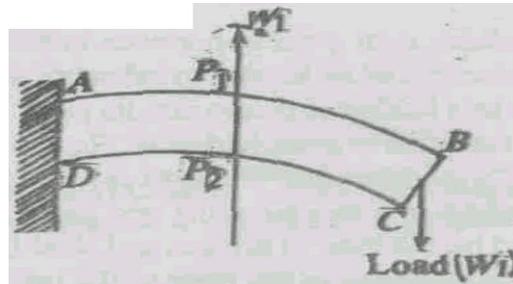


Fig. 6.3: When a beam $ABCD$ is fixed at the end AD it forms a cantilever. When loaded at the free end it bends

Do you know why? To answer it let us consider the section P_1BCP_2 of the beam. Since the load W_1 is applied at the free end of the beam, the force of reaction, which is of the same magnitude as W_1 must act vertically upwards along P_2P_1 . These two forces, being equal and opposite, will form a couple. You will recall that the tendency of a couple acting on a body is to rotate it. Do you expect the cantilever to rotate? It will not rotate because its one end is fixed. Therefore, in this case the tendency of the couple is to bend the beam in the clockwise direction. (This is indicated by the dashed arrow.) For this reason, this couple is called **bending couple** and the moment of this couple is called **bending moment**.

Now you may wonder that a couple acts on the beam, yet the beam is in equilibrium. It can happen only if a balancing couple is also acting on the beam. To understand how this balancing couple is formed, let us see what happens in the interior of the beam when its free end is loaded. For this purpose you can imagine the beam to be made up of a large number of small elements placed one above the other. Let us call these small elements as **filaments**. When the beam is loaded the filaments in the upper half of the beam get stretched and the filaments in the lower half are compressed. However, there is a surface (or filament) in the middle which neither extends nor contracts. This surface is known as **neutral surface**. These features are illustrated in Fig. 6.4.

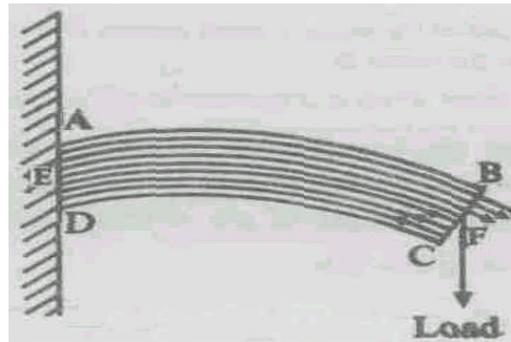


Fig. 6.4: When the beam gets bent under the action of the couple due to the load applied, the upper surface of beam gets stretched and the lower surface gets compressed. EF represents the neutral surface. The lengths of the arrows indicate, in rough proportion, the extent of extensions and contractions of the filaments in the upper and the lower halves of the beam respectively.

As the filaments above the neutral surface are extended, restoring forces are developed in the filaments as shown in Fig. 6.5. These forces act towards the fixed end of the beam and tend to oppose extensions.



Fig. 6.5: When the beam is bent then in the upper half, the restoring forces opposing extensions in the filaments act inwards towards the fixed end. While, in the lower half, the restoring forces opposing contractions act outwards. The moments of these two sets of forces about the neutral axis are directed in the anticlockwise (indicated by dotted arrows) direction and thus oppose the bending of the beam.

On the other hand, because of filament contractions below the neutral surface, restoring forces developed in the lower-half act towards the loaded end and oppose further contractions. You will note that these two sets of forces act in opposite directions. Yet their moments about the neutral surface are directed in the same i.e. anticlockwise direction (indicated by dashed arrows). This direction is opposite to that in which the beam has been bent due to the bending couple acting on it. Hence the above-said set of forces tends to restore the beam to its original condition. This set of forces constitutes a couple called the **balancing couple or restoring couple**. The moment of the couple is referred to as the moment of the resistance to bending. When the beam is in equilibrium, the moment of the resistance to bending is equal to the bending moment. You may now like to know the factors on which the bending moment or the moment of the restoring couple depends.

3.1.2 Bending Moment

Let us consider a small portion of the beam shown in Fig. 6.6(a). It is bent in the form of an arc subtending an angle θ at the centre of curvature O . Let R be the radius of curvature of the part ab of the neutral surface. Then the length of portion $a'b'$ of a filament which is at a distance z from the neutral surface (filament), will be given by

$$a'b' = (R + z)\theta$$

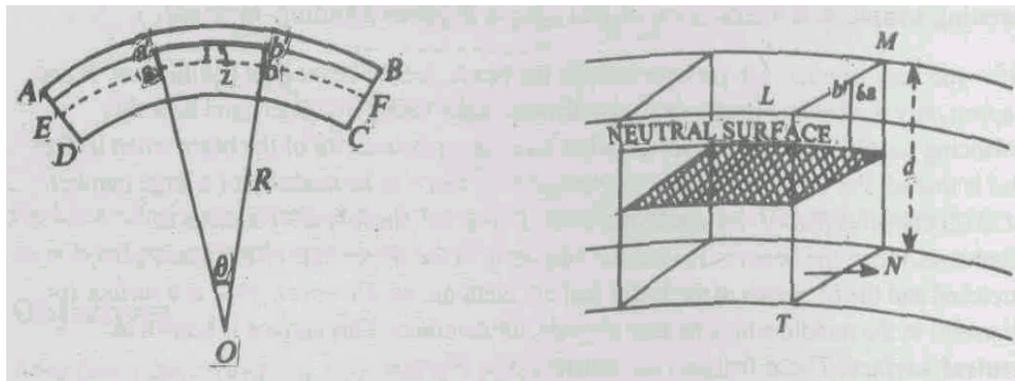


Fig. 6.6: (a) In the strained condition of the beam, a small portion of it is considered to be bent in the form of a circular arc subtending an angle θ at the centre O . (b) $LMNT$ is a cross-section of the beam which is perpendicular to the length and the plane of bending of the beam.

When the beam was not bent, the length of this filament was equal to the length of the neutral filament. Since the length of the neutral filament does not change even after the bending of the beam, the original length of $a'b' = \text{length of } ab = R\theta$.

\therefore increase in length of $a'b' = a'b' - ab$

$$= (R + z)\theta - R\theta = z\theta \quad (6.1)$$

Since the original length of $a'b' = R\theta$, we have

$$\text{Longitudinal strain} = \frac{\text{increase in length}}{\text{original length}} = \frac{z\theta}{R\theta} = \frac{z}{R} \quad (6.2)$$

Let us consider a cross-section $LMNT$ of the beam, which is perpendicular to its length and the plane of bending as shown in fig. 6.6(b). In this cross-section $LMNT$, if we consider a small area a , which

is at a distance z from the neutral surface, then the strain produced in the filament passing through this area will be $\frac{z}{R}$.

As explained in the previous sub-section, whenever the length of a filament increases, a force acts on the filament towards the fixed end of the beam. You can calculate this force by noting that

$$Y = \frac{\text{stress}}{\text{longitudinal strain}}$$

or Stress = $Y \times$ longitudinal strain,

where Y is the Young's modulus for the material of the beam. This shows that stress at this area,

$$a = Y \frac{z}{R} \quad (6.3)$$

And, therefore,

$$\text{force on area } a = aY \frac{z}{R} \quad (6.4)$$

The moment of this force about the neutral surface

$$\begin{aligned} &= Ya \frac{z}{R} z \\ &= Ya \frac{z^2}{R} \end{aligned} \quad (6.5)$$

Since the moments of the forces acting on both upper and lower halves of the cross-section are in the same direction, the total moment of the forces acting on all the filaments in the section $LMNT$ (or in the beam) is given by:

$$\sum \frac{Yaz^2}{R} = \frac{Y}{R} \sum az^2 = \frac{Y}{R} I_g \quad (6.6)$$

where $I_g = \sum az^2$ is the geometrical moment of inertia of the beam. Thus, the bending moment of the beam (or moment of the restoring couple) = $\frac{Y}{R} I_g$.

$\sum az^2$ is the geometrical moment of inertia of the section of the beam about the neutral surface. Therefore, it is equal to Ak^2 , where A is the

whole area of the surface LMNT of the beam and k its radius of gyration about the neutral surface. For a rectangular cross-section,

$$A = b \times d \quad \text{and} \quad k^2 = \frac{d^2}{12}$$

where b is the length and d the width of the rectangular portion.

$$\therefore I_g = \sum az^2 = \frac{bd^3}{12}$$

For a circular cross-section, $A = \pi r^2$ and $k^2 = \frac{r^2}{4}$, where r is its radius.

$$\therefore I_g = ak^2 = \frac{\pi r^4}{4}$$

You will now like to know how the moment of the restoring couple is related to the depression at the free end of the cantilever.

3.1.3 Depression at the Free End of a Cantilever

Consider a cantilever of length l . Let us choose x -axis along its length and y -axis vertically downwards, as shown in Fig. 6.7. When the free end of the cantilever is loaded with a load W_1 , the maximum depression occurs at the free end. Consider a section P of the beam at a distance x from the end A . Due to the load, W_1 , the bending moment acting on this section is given by $W_1 \times PB = W_1(l - x)$. Since the beam is in equilibrium, this must be equal to

$$W_1(l - x) = \frac{YI_g}{R} \quad (6.7)$$

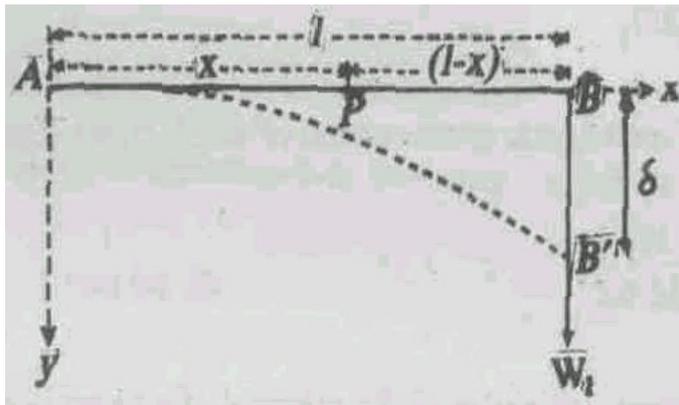


Fig. 6.7: Cantilever loaded at the free end. AB represents the neutral axis of a cantilever of length l . When loaded at B the neutral axis takes up the position AB' and the end B is depressed by δ .

Since the neutral surface remains flat, the radius of curvature (R) of the neutral surface at any given point is given by the relation, $\frac{1}{R} = \frac{d^2y}{dx^2}$.

Substituting for R in equation (6.7), we have

$$W_1(l-x) = YI_g \frac{d^2y}{dx^2},$$

or
$$\frac{d^2y}{dx^2} = \frac{W_1}{YI_g}(l-x) \quad (6.8)$$

Integrating equation, (6.8) twice with respect to x we get the depression (δ) at the free end as:

$$\delta = \frac{W_1 l^3}{3YI_g} \quad (6.9)$$

Refer to any elementary book on differential calculus for the complete expression:

$$\frac{1}{R} = \frac{(d^2y/dx^2)}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

Since $\frac{dy}{dx} \ll 1$ due to small bending,

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

Integrating Eq. (6.8) with respect to x , we get

$$\frac{dy}{dx} = \frac{W_1}{YI_g}(lx - x^2) + C_1$$

where C_1 is a constant of integration.

When $x = 0$, $\frac{dy}{dx} = 0$. Hence, $C_1 = 0$.

$$\therefore \frac{dy}{dx} = \frac{W_1}{YI_g} \left(lx - \frac{x^2}{2} \right)$$

Again integrating, we have

$$Y = \frac{W_1}{YI_g} \left(\frac{lx^2}{2} - \frac{x^3}{6} \right)$$

At the free end of the beam, $x=l =$ (length of the beam), $y = \delta$ (depression). Hence,

$$\delta = \frac{W_1}{YI_g} \left(\frac{l^3}{2} - \frac{l^3}{6} \right),$$

or

$$\delta = \frac{W_1 l^3}{3YI_g}$$

SELF ASSESSMENT EXERCISE 1

By looking at eq. (6.9), name the factors on which depression of the free end of the cantilever depends.

Let us look back at Fig. 6.2. If the length of the beam AB be L , then the length of each cantilever AC or BC will be $L/2$. Since the reaction at each knife-edge is $\frac{W}{2}$, we can regard that each cantilever (AC or BC) is loaded at the free end by a load $\frac{W}{2}$. Then Eq. (6.9) can be used to compute the elevation of A or B above C by substituting $W, = W/2$ and $l = L/2$. This gives elevation of A or B above $C = \frac{W \left(\frac{L}{2} \right)^3}{3YI_g}$

$$= \frac{WL^3}{48YI_g}$$

The elevation of A or B above C is the same as the depression of C below A and B .

Therefore, depression (δ) at the centre of the beam is $\delta = \frac{WL^3}{48YI_g}$ and

$$\delta = \frac{WL^3}{48\delta I_g}.$$

If the beam is of rectangular cross-section of width b and depth d , we can write WL^3

$$Y = \frac{WL^3}{4\delta b d^3} \quad (6.10)$$

To determine Young's modulus of the material of a beam using Eq. (6.10) you have to measure the depression at its centre when loaded with a known weight. This depression, being very small, has to be measured very accurately. For this purpose the most suitable instrument is travelling microscope. What will you do if you are given a telescope instead of a microscope? Can you still measure depression in the beam with the same accuracy? Of course, by using the optical lever method you can measure depression. But to discover an answer to the second question, you have to measure depression in the beam using (i) a microscope and (ii) a telescope with an optical lever arrangement.

The apparatus required for this purpose are the following:

Apparatus

Rectangular beam, two knife-edges, a hook (or a stirrup) for hanging weights at the centre of the beam, a travelling microscope, a pin, an optical lever, a telescope, metre scale, a hanger, a set of half-kilogram weights, vernier callipers and a screw gauge.

3.2 Measurement of Depression in a Beam using a Microscope

Place the given beam horizontally on the knife-edges, as shown in Fig. 6.8. See that equal (but small) portions of the beam project beyond the knife-edges and the smaller side of its cross-section is vertical. Suspend a hanger (either a hook or a stirrup with the hook) for loading the beam, exactly at the centre, between the two knife-edges. Attach a small pin (vertically) at the centre of the beam with wax for reading the position of the beam. Focus the microscope on the pin and coincide its horizontal cross-wire with the tip of the pin. If you are not able to focus the microscope on the pin you should seek the help of the counsellor. Before you start taking observations, you should calculate the least count of the microscope. For this purpose find the value of the smallest division of the main scale of the microscope. Next find the value of a division of the verniers scale. The difference between the value of one smallest division of the main scale and value of one division of verniers scale will give its **least count**. Suppose, 10 divisions of the verniers coincide with 9 smallest divisions of the main scale, each of which is 1 mm. Then, we can write

$$10 \text{ verniers divisions} = 9 \text{ mm,}$$

$$\text{or } 1 \text{ verniers division} = \frac{9}{10} \text{ mm}$$

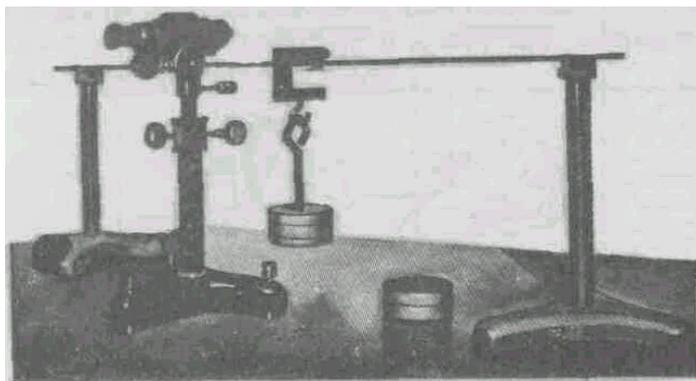


Fig. 6.8: Experimental arrangement for measuring the depression of the beam using a *Microscope*

\therefore least count = 1 main scale div. - 1 verniers div.

$$= \left(1 - \frac{9}{10}\right) \text{mm} = \frac{1}{10} \text{mm}$$

$$= \frac{1}{100} \text{cm} = 0.01 \text{cm}$$

SELF ASSESSMENT EXERCISE 2

Suppose in verniers there are 50 divisions equal to 49 mm on the main scale. Find out the least count of this scale.

Now read the main scale and the verniers scale readings. This is the reading when no load is placed in the hanger. Record it in Observation Table 6.1. Next, without disturbing anything at all, place a weight of half-a-kilogram in the hanger. Is the tip of the pin visible in the field of view of the microscope? If so, does the tip of the pin coincide with the horizontal cross wire? We expect that it will not because the beam has been depressed. You will observe that a gap appears between the tip of the pin and the horizontal cross wire. Slightly move the microscope vertically downward so that the tip of the pin again coincides with the cross wire of the microscope. Again note the main scale and the verniers scale readings, Record this in Observation Table 6.1.

Increase the load in equal steps of half-a-kilogram. Note the position of the pin by coinciding it with the horizontal cross-wire in each case. Now remove the weights gently in the same steps and note the microscope

readings. This is to be repeated till there is no weight on the hanger. **The weight should be placed or removed from the hanger very gently.**

S. No.	Load (W) placed on the hanger (g)	Microscope reading when the tip of the pin coincides with the horizontal cross-wire			Depression (δ) (cm)
		with load increasing (cm)	with load decreasing (cm)	Mean (cm)	
1	0				
2	500				
3	1,000				
4	1,500				
5	2,000				
6	2,500				
7	3,000				
8	3,500				

Observation Table 6.1: Measurement of depression using a microscope

Value of 1 small division of the main scale of the microscope = cm

Value of 1 verniers scale division = cm

Least count of the Microscope = cm

SELF ASSESSMENT EXERCISE 3

Why is it necessary to take reading with decreasing load as well?

This will give you two readings for each load – one when the load was increasing and the other when the load was decreasing. Compute the mean of these two readings for a given, load. Calculate the depression produced in the beam for each load by subtracting the initial mean reading from the mean reading for that particular load.

Plot a graph between the load (along the x -axis) and depression (along the y -axis). Draw a smooth best straight line passing as closely as possible through the points, as shown in Fig. 6.9. Calculate the slope of the straight line by choosing two widely separated points. The slope will give you the value of δ / W .

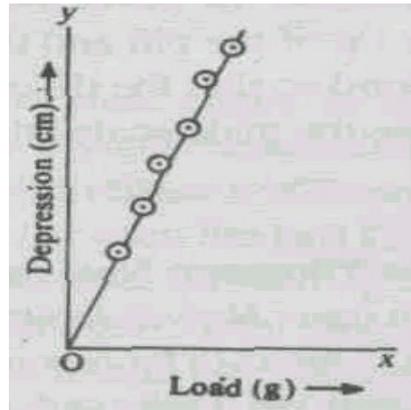


Fig. 6.9: Graph of load (W) vs. depression (δ)

3.3 Measurement of Depression in a Beam using a Telescope and an Optical Lever

To measure the depression of the beam using a telescope you will require an optical lever and a lamp and scale arrangement. (An optical lever consists of a plane mirror mounted on a tripod stand.) First place the beam as in the previous part of this experiment. Remove the vertical pin and replace it by an optical lever such that the two legs supporting the mirror 'M' rest on the fixed horizontal base F behind the beam and the third leg L rests on the beam at its centre C , as shown in Fig. 6.10. What will happen if you place the two legs supporting the mirror on the beam and the third leg on a base? If you do so the depression will not correspond to the one at the centre. It is important to adjust the mirror so that it is vertical and parallel to the length of the beam.

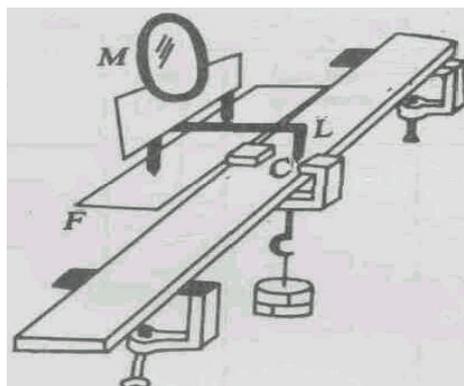


Fig. 6.10: Experimental arrangement for measuring the depression of the beam using a telescope and optical lever

When a load is placed on the hanger, depression is produced in the beam. As a result, the leg of the optical lever, touching the centre of the beam, goes down. This tilts the mirror, forward. So, once you measure the angle through which the mirror tilts, you will be able to find out the depression. This requires the use of a telescope and scale. Let us see how.

Fix a vertical scale in front of the mirror at a distance of about one metre on a rigid stand. Place the telescope close to the scale and at the same height as the mirror. Focus the eyepiece so that the horizontal cross wire of the telescope is distinctly visible. Now focus the telescope on the image of the scale in the mirror. For this focussing you may have to turn the mirror slightly about its horizontal axis. If you are not able to focus the image of the scale clearly, you should not waste time. You can consult your **counsellor**. Note the position of the horizontal cross-wire on the image of the scale and record it in Observation Table 6.2.

What does the position of the horizontal cross-wire signify? Let us observe Fig. 6.11. Here M_1 is the position of the plane mirror. Division A of the scale is seen in the telescope after reflection from plane mirror. This means that what you have recorded in fact is division A of the scale.

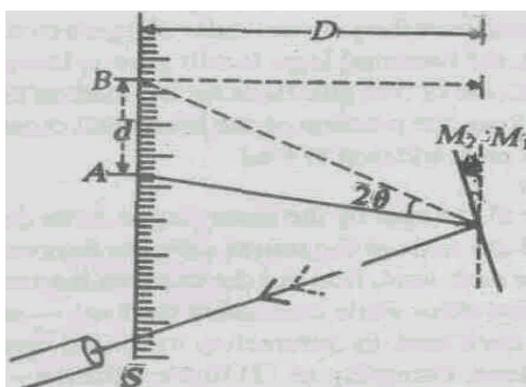


Fig. 6.11: Illustrating the principle underlying the use of optical lever

Now gently place a load of 500 gm on the hanger. This would depress the beam slightly. As a result of this the mirror will tilt forward through an angle, say, θ . We know that when a beam of light falls on a plane mirror, which is turned through an angle θ about a vertical axis in its plane then, the reflected ray turns through angle 2θ . Hence now instead of division A of the scale the division B (see Fig. 6.11) is seen in the telescope after reflection from the plane mirror. Record it in Observation Table 6.2.

Observation Table 6.2: Measurement of depression using a telescope and an optical lever

Distance D of scale from mirror = cm

Distance x of the front foot of the optical lever from the line joining the other two = cm

S. No.	Load (W) placed on the hanger (g)	Position of the horizontal cross-wire of the telescope			d (cm)	$\delta = \frac{xd}{2D}$ (cm)
		with load increasing	with load decreasing	Mean (cm)		
1.	0					
2.	500					
3.	1,000					
4.	1,500					
5.	2,000					
6.	2,500					
7.	3,000					
8.	3,500					

If the distance between the two divisions A and B on the scale is represented as d and if D is the distance between the mirror and scale then

$$2\theta = \frac{d}{D}$$

If the third leg is at a distance of x from the hind legs P and Q , then the depression, δ , of the beam is given by

$$\begin{aligned} \delta &= x\theta \\ &= \frac{xd}{2D} \end{aligned} \quad (6.11)$$

From this relation we find that once x , d and D are known, δ can be readily computed. Measure the distance D between the mirror and the scale. To measure x , place the optical lever on a sheet of paper and by pressing it lightly produce impressions of its feet on it. From these impressions determine the perpendicular distance of the front foot of the optical lever from the line joining the two hind legs. It will give x . Using Eq. (6.11) find out the depression of the beam for load of 500 gm. Increase the load on the hanger by equal steps of half-a-kilogramme. Note down the position of the horizontal cross-wire of the telescope on the image of the scale after each addition of load.

Next, decrease the load on the hanger by the same stages. Note down the position of the cross-wire on the image of the scale in the mirror after the removal of each load. Record it in Observation Table 6.2. For each load, find out the mean of the two readings – one taken while increasing

the load and other while decreasing the load – of the cross-wire thus obtained. Calculate d , for each load, by subtracting the initial mean reading from the mean reading for that particular load. Using Eq. (6.11) find out the depression of the beam for each load and record it in Observation Table 6.2. Plot a graph between load (W) along the x -axis and depression (δ) along y -axis. Calculate the slope of the straight line thus obtained.

3.4 Comparison of Accuracies of above Methods by Determining Young's Modulus

To know Young's modulus your must measure the thickness and width of the beam and its length between the knife edges. To measure the length of the beam between the knife edges, you can use a metre-scale. Using different parts of the scale, repeat the measurement several times and get the mean value. Record your reading in Observation Table 6.3(a).

Observation Table 6.3: Length (L) of the beam between the knife-edges A and B

S. No	Scale reading for the knife-edge A x (cm)	Scale reading for the knife-edge B y (cm)	Length ($y-x$) (cm)	Mean length L (cm)
1.				
2.				
3.				
4.				
.				
.				
.				
.				

SELF ASSESSMENT EXERCISE 4

Instead of measuring total length of the beam you are measuring the length of the beam between the two knife edges. Why?

Use a screw gauge to measure the thickness of the beam at several places along its length. Make your own Observation Table 6.3 (b) and calculate the mean thickness. Similarly take a number of readings to

measure the width of the beam with vernier callipers at several places. Record the readings in Observation Table 6.3 (c). Calculate the mean value.

Observation Table 6.3(b): Measurement of thickness (*d*)

Least count of the screw gauge = cm

Zero error (if any) = cm

Zero correction (if there is zero error) = cm

Mean Value = cm

Corrected value (if zero correction is made) = cm

Observation Table 6.3 (c): Measurement of width (*b*)

Least count of the screw gauge = cm

Zero error (if any) = cm

Zero correction (if there is zero error) = cm

Mean Value = cm

Corrected value (if zero correction is made) = cm

Knowing L , b , d and the slope of the straight line obtained in the Section 6.3, you can easily calculate Young's modulus of the material of the beam using the Eq. (6.10) as follows:

$$Y = \frac{L^3}{4bd^3} \times \frac{1}{\text{slope}} = \dots\dots\dots \text{dynes cm}^{-2}$$

$$= \dots\dots\dots \text{Nm}^{-2}$$

Result: Young's modulus of the material of the given beam using microscope

$$= \dots\dots\dots \text{Nm}^{-2}$$

Next, using the slope of the straight line obtained in the Section 6.4 in the relation

$$Y = \frac{L^3}{4bd^3} \times \frac{1}{\text{slope}} \text{ find the value of Young's modulus}$$

Result: Young's modulus of the material of the given beam using telescope and optical lever

$$= \dots\dots \text{N/m}^2$$

The accuracy to which the depression is measured using a microscope is equal to the least count (L.C.) of the microscope.

Suppose, L.C. of microscope = 0.001cm.

In the case of optical lever arrangement, the least count of vertical scale is, suppose, 0.1 cm.

This is multiplied by the factor x/D (see observation Table 6.2). If $D = 1\text{m} = 100\text{ cm}$ and

$$x = 3\text{ cm}, \text{ then } \frac{x}{D} = \frac{3}{100} = 0.03.$$

Hence the least count of measurement of depression by the optical-lever-arrangement = $0.1 \times 0.03 = 0.003\text{ cm}$.

$$\text{Ratio} = \frac{\text{L.C. (microscope)}}{\text{L.C. (optical lever)}} = \frac{0.001}{0.003} = \frac{1}{3}$$

This shows that measurement of depression and hence of Y is about three times more accurate with microscope than with an optical-lever

arrangement. But an optical lever method can also give better results than microscope method. For this you have to think of ways to improve upon the least count of the measurement of depression by optical-lever arrangement. You may, for instance, use a half-millimetre scale instead of metre scale. The least count of the measurement of depression with the optical-lever arrangement depends on (i) x , the length of tilting arm of the optical lever and on (ii) D , the distance between the mirror and the scale. Try to adjust these factors so that the optical-lever method is more accurate than the microscope method.

4.0 CONCLUSION

Learners are to come up with conclusion based on their observations and calculations after performing the experiments.

5.0 SUMMARY

During the experiment, we achieved,

- (a) Removal of parallax errors and measured depressions
- (b) Compared accuracies of different instruments- microscope, telescope etc
- (c) Young's modulus of elasticity was determined.

6.0 TUTOR MARKED ASSIGNMENTS

As provided by the facilitator.

7.0 REFERENCES/FURTHER READINGS

IGNOU (1997) Physics Laboratory I, Physics PHE-03 (L), New Delhi, India.

UNIT 4 EXPERIMENT 7

MEASUREMENT OF LOW RESISTANCE USING CAREY FOSTER'S BRIDGE

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Whetstone's Bridge
 - 3.1.1 Carey Foster's Bridge
 - 3.2 Setting Apparatus
 - 3.3 Procedure
 - 3.3.1 Determination of Resistance per Unit Length
 - 3.3.2 Determination of Unknown Resistance
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor Marked Assignments
- 7.0 References/Further Readings

1.0 INTRODUCTION

You must have handled electrical appliances like electric heater, electric iron, and geyser at your home. Have you ever thought as to how electric current flows through these appliances? Which material is used in the heating element and why? We know that every material offers some resistance to the flow of current. How does this resistance arise and what factors determine it? Is it the same for all materials? You must have learnt answers to these questions in your school physics course.

Suppose we wish to regulate the flow of current in an electric circuit. All we need to know is the resistance of the circuit. Similarly, to produce the desired heating effect we should know the resistance of the heating element. Depending on our requirement we have to design resistors of different values from several million ohms to a fraction of an ohm. One metre of copper wire, normally used in electric connections in a physics laboratory, has a resistance of about $0.02\ \Omega$. When a very delicate electrical instrument like ballistic galvanometer is used in an electric circuit, a shunt in the form of wire of low resistance ($\sim 0.1\ \Omega$ or less) is used. In power transmission also, it is desirable to use cables having low resistance so that power loss is less. On the other hand, when we wish to regulate current in a circuit, a variable or a constant high resistance is used. In commercially produced resistors, resistance is provided by a thin layer of carbon. These are commonly used in radio and T.V. circuits. This raises a very important question. How to measure

resistance over the entire range from $10^6 \Omega$ down to $10^{-3} \Omega$? To be able to answer this question, you should first know to measure resistance.

Ohm's law states that current flowing through a conductor is directly proportional to the potential difference across it, provided temperature and other physical conditions like pressure, shape and size remain the same.

The resistances of the order of a few ohms ($1-100\Omega$) can be measured by methods which depend on the direct application of the ohm's law. You must have used this law in your earlier classes to measure resistance. For a low resistance, these methods are not reliable. So we have to look for alternative methods. Usually we measure a low resistance using methods based on the principle of Whetstone's bridge. These include a post office box, a meter bridge and a Carey Foster's bridge. In this experiment you will learn to use a Carey Fosters bridge. Now you may logically ask: Why do we prefer it? You will be able to answer this and other related questions after doing this experiment. In fact our basic purpose of asking you to perform this experiment is (i) to reinforce your knowledge of the concepts involved in resistance measurements, (ii) to make you familiar with the instruments used in electrical circuits in a physics laboratory, and (iii) to develop in you the skills and confidence required in making measurements with electrical equipments.

2.0 OBJECTIVES

After doing this experiment you should be able to:

- make electrical connections on the basis of circuit diagrams;
- acquire the skills of making measurements using null (no deflection) methods;
- appreciate the role of contact resistances (or loose connections) in electrical circuits; and
- measure a low unknown resistance.

3.0 MAIN CONTENT

3.1 Whetstone's Bridge

A Whetstone's bridge circuit diagram is shown in Fig. 7.1. Here P , Q , R and S are resistances in the arms AB , BC , AD , and CD respectively, and G , connected between B and D , is a galvanometer. You will note that a galvanometer has no positive and negative terminals marked on it. It shows deflection on both sides of the zero mark, which is in the centre of the scale.

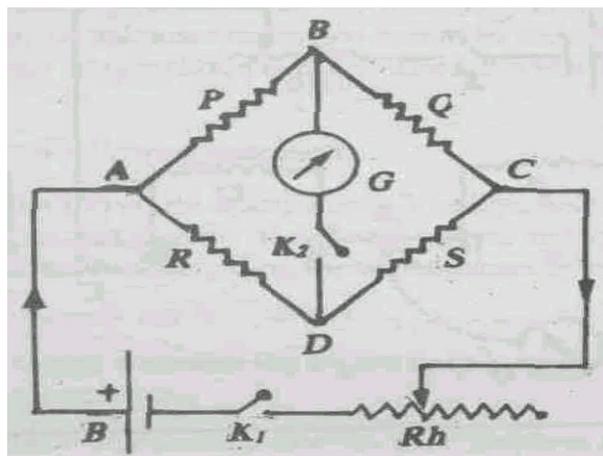


Fig. 7.1: Circuit diagram of Whetstone's bridge

When junctions B and D are at the same potential, no current will flow through the galvanometer. This is evidenced by zero deflection in the galvanometer. The bridge is then said to be balanced. For a balanced bridge, the following condition holds good:

$$\frac{P}{Q} = \frac{R}{S} \quad (7.1)$$

From this equation it is clear that an unknown resistance can be found if we know the other three. However, for maximum sensitivity it is important to ensure that all four resistances are preferably of the same order of magnitude. This means that if the unknown resistance is low, the bridge will be most sensitive when other resistances are also low.

The principle of Whetstone's bridge forms the basis of many experiments/instruments in a physics laboratory. The more familiar of these instruments are the Post Office Box, the slide-wire bridge (also called the metre bridge), the Carey Foster's bridge and the potentiometer. In your physics laboratory, you will get an opportunity to work with the last two.

Let us now pause for a minute and ask: Why do we use Whetstone's bridge to measure low resistance? This is essentially because it is a null method. This means that when the bridge is balanced, the detector, a galvanometer in this case, registers no current. That is, there is no deflection in the galvanometer and the pointer remains stationary at zero.

SELF ASSESSMENT EXERCISE 1

- i. Write True or False against the following statements

- (a) The key k_1 , in Fig. 7.1 should always be kept inserted in the circuit
- (b) If we know the values of resistances in any three arms of a Whetstone's bridge, the fourth one can always be found irrespective of whether the bridge is balanced or unbalanced.
- (c) For maximum accuracy, P , Q , R and S should preferably be of the same order of magnitude.

3.1.1 Carey Foster's Bridge

The Carey Foster's bridge works on the principle of the Whetstone's bridge. The Carey Foster's bridge along with its electrical connections is shown in Fig. 7.2. It has four gaps in the copper strip. Two known resistances P and Q (preferably equal and small) are inserted in the inner gaps at x and y . A galvanometer is attached between the terminal B and the sliding tapping key (or the jockey) at D . One known (preferably a standard) resistance R and an unknown (low) resistance S are introduced in the outer gaps at m and n , respectively. A battery, a key and a rheostat are inserted between the terminals A and C . A one metre long uniform resistance wire EF is mounted alongside a metre rod, and is soldered to the two ends of the copper strip.

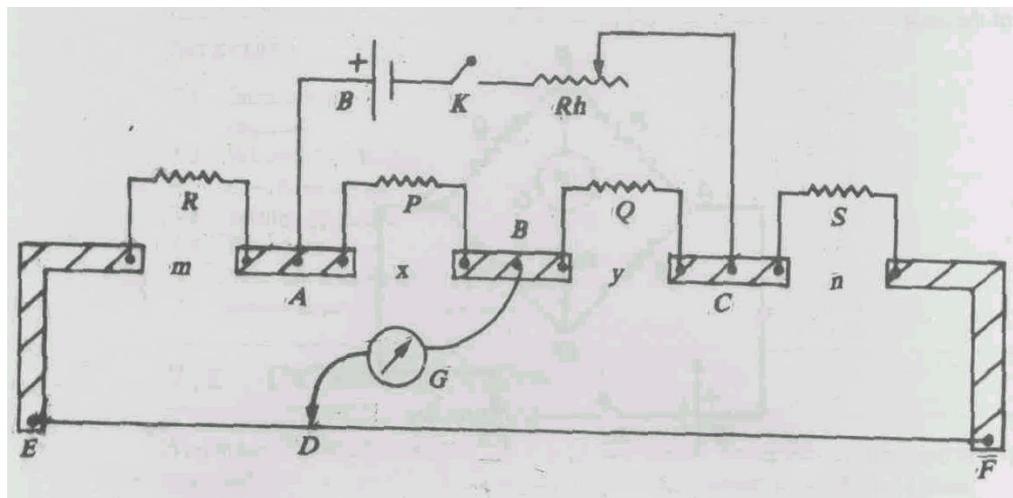


Fig. 7.2: Carey Foster's Bridge

You may note that point D is variable. It can be anywhere between E and F . It marks the position at which there is *no deflection* in the galvanometer. It is located by moving the tapping key over the wire.

When the contact between the two wires (or between a screw and a wire) is not good, the area of cross section at the contact becomes very small. This introduces a significant resistance in the circuit which we call the contact resistance. In the slide wire and Carey Foster's bridge, the contact resistances are usually referred to as end-resistances or

end-errors. Usually, these are low, of the order of a milli-ohm. The contact resistances assume importance only in low resistance measurements. It is for this reason that (i) you should clean the heads of all connecting wires with sand paper and (ii) the connection must be tight.

Since the wire EF is uniform, we can assume that it has a constant resistance per unit length. Let us assume this to be r . Then resistance between E and D is equal to $l_1 r$ where l_1 the length of the wire ED measured from E . Similarly, the resistance between D and F is equal to $(100 - l_1) r$.

The points A , B , C and D here correspond exactly to those of the Whetstone's bridge shown in Fig. 7.1. The Carey Foster's bridge is thus effectively a Whetstone's bridge. Then we may write the condition of balance as:

$$\frac{P}{Q} = \frac{R + \alpha + l_1 r}{S + \beta + (100 - l_1) r}, \quad (7.2)$$

where, α and β are the end-corrections at the left and the right ends.

Next, resistances R and S are interchanged i.e., the resistance in gap m is put in gap n , and vice versa. Let us assume that the balance point is obtained at a distance l_2 from E . You will note that the errors at the ends E and F stay the same, irrespective of the resistances in the gaps m and n .

Then the condition of balance can be written as

$$\frac{P}{Q} = \frac{R + \alpha + l_2 r}{S + \beta + (100 - l_2) r} \quad (7.3)$$

Equating Eqs. (7.2) and (7.3), we get

$$\frac{R + \alpha + l_1 r}{S + \beta + (100 - l_1) r} = \frac{R + \alpha + l_2 r}{S + \beta + (100 - l_2) r}$$

Adding one on both sides and simplifying the terms, we obtain

$$\frac{R + S + \alpha + \beta + 100r}{S + \beta + (100 - l_1) r} = \frac{R + S + \alpha + \beta + 100r}{R + \beta + (100 - l_2) r} \quad (7.4)$$

You will note that in this equation the numerators are equal. So the denominators must also be equal. Therefore, we can write

$$S + \beta + (100 - l_1)r = R + \beta + (100 - l_2)r$$

giving,

$$R - S = (l_2 - l_1)r \quad (7.5)$$

Let us pause for some time and ask: How does this relation enable us to determine the value of a low resistance accurately? It shows that the difference between the known and the unknown resistance is equal to the resistance of the bridge wire between the two balance points. Once we know $(l_2 - l_1)$, r and R , the unknown resistance can easily be determined. Now you may like to know: Is there any limitation of this method? Yes, there is one. The difference between the known and the unknown resistance cannot be more than the total resistance of the bridge wire. When this condition is not satisfied, the method fails.

SELF ASSESSMENT EXERCISE 2

- i. Write True or False against the following statements
 - (a) To find the balance point (D) on the Carey Foster's bridge, we slide the tapping key along the bridge wire, tap it gently at different points, and look for the position at which deflection in the galvanometer becomes zero.
 - (b) The bridge wire may or may not be uniform.
 - (c) If the soldering of the bridge wire with the copper strip is weak, the contact resistance is large.....
 - (d) The Whetstone's bridge is more sensitive when the resistances in the four arms are nearly equal.....
- ii. Write the reason for your answer for l(b)

Before we describe the procedure to determine the value of low resistance, let us list the apparatus with which you will work.

Apparatus

Carey Foster's bridge, two resistors; each of about $2\ \Omega$ (or two resistance boxes), thick copper strips, standard low resistances (or a fractional resistance box), a battery, a one-way key, a rheostat, a sensitive galvanometer, and an unknown low resistance.

3.2 Setting Apparatus

1. Place Carey Foster's bridge apparatus on the table and keep it in such a way that the gaps in copper strip are away from you.
2. Clean the ends of the connecting wires with sand paper.
3. Identify and mark the various terminals of the Carey Foster's bridge by comparing them with Fig. 7.2.
4. Connect the galvanometer between B and the sliding key at D .
5. Connect the given resistance coils (or the resistance boxes) in gaps x and y .
6. Connect the standard resistance (or the fraction resistance box), and the unknown low resistance in gaps m and n .
7. Connect a battery, a one-way key and a rheostat between A and C . For connecting the rheostat, you should use one of its lower terminals and an upper one.
8. Check that connections (and keys in resistance boxes, if used) are tight.
9. Move the slider of the rheostat towards its lower terminal which is connected to the key and the battery.
10. If resistance boxes are used, take out resistances of, say, $2\ \Omega$ each from the boxes in the inner gaps at x and y .
11. If fractional resistance box is used, take out a resistance of, say, $0.1\ \Omega$ from this box.
12. Insert the key in the battery circuit. Gently tap the jockey near the end E . The galvanometer will show deflection on one side of the zero mark. Now move the sliding key to the other end (F) and again tap it. The galvanometer should show deflection on the other side of the zero mark. Only when you have ensured this,

can you be sure that your circuit connections are correct and you can begin to take observations. If you do not get deflection on both sides, you should check your connections again and repeat the above procedure. If you succeed, fine. Otherwise, you should seek help from your facilitator without spending any further time. (Only when you are fully convinced, should you proceed further.)

3.3 Procedure

This experiment is to be done in two parts. In the first part, you have to find r , the resistance of the bridge wire per unit length. In the second part, you determine the lengths l_1 , and l_2 . These two measurements then give us the unknown low resistance.

3.3.1 Determination of Resistance per Unit Length

1. Connect a fractional resistance box (or the standard resistance coil) in the right outer gap n and a thick copper strip in the left outer gap m . (If a fractional resistance box is used, you take out a resistance of 0.1Ω .) Let us denote it as $R \Omega$.
2. Locate the balance point by moving the sliding key over the bridge wire and tapping it gently at different points. The deflection will become zero at some point on the wire. (At the balance point, the galvanometer needle should not move at all.)
3. Note the position of the balance point (with the help of the metre scale mounted along the bridge wire) and record your observations in Observation Table 7.1. This gives us l_1 .
4. Now, interchange the positions of the copper strip and the fractional resistance box (or the standard resistance coil) and again obtain the balance point. Record it again. This gives us l_2 . Calculate resistance per unit length of the wire using the relation

$$r = R/(l_2 - l_1)$$

5. Repeat the procedure at least four times by taking out different values of the resistance from the fractional resistance box (or by using different standard resistance coils). Compute the mean value of r .

**Observation Table 7.1: Determination of Resistance per unit.
Length of the Wire**

s. No.	Fractional Resistance $R(\Omega)$	Balancing lengths when R is in gap (cm)		Difference in balancing lengths $l_2 - l_1$ (cm)	$r = \frac{R}{l_2 - l_1}$ (Ω / cm)
		n	m		
		l_1 (cm)	l_2 (cm)		
1.					
2.					
3.					
4.					
5.					
.					
.					
.					

Mean value of $r = \dots\dots\dots \Omega / \text{cm}$.

3.3.2 Determination of Unknown Resistance

1. First remove the copper strip and insert the unknown resistance in one of the outer gaps.
2. Repeat the entire sequence of steps given in Subsection 7.4.1. Make your own Observation Table and record the data. Compute R using Eq. (7.5).

You are bound to face difficulty in locating the balance point if the difference in the known and unknown resistances is more than the resistance of the bridge wire. In such a situation, you should change the known resistance in small steps.

Observation Table 7.2

Mean value of $R = \dots\dots\dots \Omega$

Do you get consistent values of R in each case? We expect so. Estimate the instrumental error.

Result: The low resistance determined by using a Carey Foster's bridge = $\dots\dots\dots \Omega$

4.0 CONCLUSION

Learners are required to provide Conclusion based on their observations and calculations.

5.0 SUMMARY

The experiments demonstrated

- (a) The basis of circuit diagrams for electrical connections
- (b) Measurements using null deflections
- (c) The role of contact resistance in electrical circuits.

We now want you to think and answer the following questions:

SELF ASSESSMENT EXERCISE 3

Can you determine r by plotting a graph with the measurements recorded in Table 7.1? If yes, plot this graph. If not, say why? If you have plotted the graph, what is the shape of the curve obtained and how does your graphical value compare with the one found above? Does Eq. (7.5) indicate anything about the nature of this curve?

SELF ASSESSMENT EXERCISE 4

Imagine that the values marked on standard resistances are not correct. What possible errors do you expect in your result?

SELF ASSESSMENT EXERCISE 5

Suppose that the total resistance of the bridge wire is $0.2\ \Omega$. You have two standard resistance coils of $0.3\ \Omega$ and $0.4\ \Omega$, respectively. You are asked to make an unknown resistance using these two coils and perform the experiment. How would you do that?

6.0 TUTOR MARKED ASSIGNMENTS

As provide by the facilitator

7.0 REFERENCES/FURTHER READINGS

IGNOU (1997) Physics Laboratory I, Physics PHE-03(L), New Delhi, India.

UNIT 5 EXPERIMENT 8

VARIATION OF THERMO-E.M.F. WITH TEMPERATURE

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Potentiometer
 - 3.2 Fabrication of Thermocouple
 - 3.3 Measurement of Thermo-e.m.f. of a Thermocouple and its variation with Temperature
 - 3.3.1 Procedure
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor Marked Assignments
- 7.0 References/Further Readings

1.0 INTRODUCTION

You all must be familiar with the principle of conservation of energy. Conservation of energy implies that energy can neither be created nor destroyed. Only the form of the energy changes from one to another. For example, in an electric cell the chemical energy is converted into electrical energy; and in an electric heater, the electrical energy is converted into heat energy. Is it possible to convert heat energy back into electrical energy? Yes.

In 1821, T. J. Seebeck found that if wires of two different metals, such as copper and iron, are joined together to form a closed loop and if one junction is kept at a different temperature from the other, an electric current will flow in the closed loop. This phenomenon is called thermo-electric effect or Seebeck effect. The two metals comprising the circuit are referred to as a thermo-couple. The existence of a current implies that there is e.m.f. (electromotive force) acting in the circuit. This e.m.f. is known as thermo-electric-e.m.f and the electric current produced in this way is called thermo-electric current. The direction of the current and magnitude of the e.m.f depend upon the kind of materials used and the difference of temperature between the two junctions.

The conversion of heat into electricity by metal thermo-couples is not a very efficient process because the e.m.f. produced is very small. But its efficiency is improved by employing better thermo couples, now available, based on alloys and semiconductors. On account of their reliability, long life and low cost, these are suitable as small power

supply units in space satellites, weather ships etc. Thermo-couples are extensively used as the thermometers particularly for measuring varying temperature.

In this experiment you will learn to use a thermocouple as a thermometer. In other words you will investigate how the thermo-e.m.f. varies with temperature.

2.0 OBJECTIVES

After doing the experiment, you will be able to:

- appreciate that a small potential difference of the order of micro volt can be measured with the help of potentiometer with some modification
- fabricate thermo-couple
- make the necessary experimental set up for the measurement of thermo- e.m.f.
- plot the graph between thermo-e.m.f. and the temperature.

3.0 MAIN CONTENT

3.1 Potentiometer

You all know that for the accurate measurement of e.m.f. of a cell or potential difference between any two points of a circuit the most suitable instrument is potentiometer. Yet for your convenience we will briefly discuss how the potentiometer works.

It is a device which is used to measure an unknown e.m.f. or potential difference by comparing it with a variable known potential difference. In its simplest form it consists of a long piece of uniform wire of fairly high resistance (usually manganin or constantan wires are used) stretched over a scale of equal divisions. The ends of the wire are connected to a battery or an accumulator so as to maintain a perfectly steady e.m.f. between the ends of the wire. The e.m.f. of the battery must always be greater than the e.m.f. or potential difference to be measured. If the battery in the potentiometer circuit is not of greater e.m.f., the potential difference between the ends of the potentiometer wire will be less than the e.m.f. to be measured and consequently null point will not be detected. The potential difference per unit length of the wire produced in this way is called **potential gradient** and can be calculated by dividing the e.m.f of the battery by the total length of the wire.

To understand the principle of working of a potentiometer let us consider the flow of an electric current along a conductor AB shown in Fig. 8.1a as a result of a potential difference between A and B , the potential at A being higher. If at any two points C and D between the ends of the conductor, a branch conductor CPD is connected to the conductor AB , then the current flowing along AC will divide at C into two portions: one along CD and the other along the new path CPD . The greater the distance between C and D the greater will be the potential difference tending to urge the current along the branch conductor. Now let us suppose that a cell of constant e.m.f. is inserted in the branch CPD as shown in Fig. 8.1b. Let P be the positive pole of the cell. Since the potential at P is higher than at N , the current will flow through the branch conductor due to potential difference between P and N in the direction PCD .

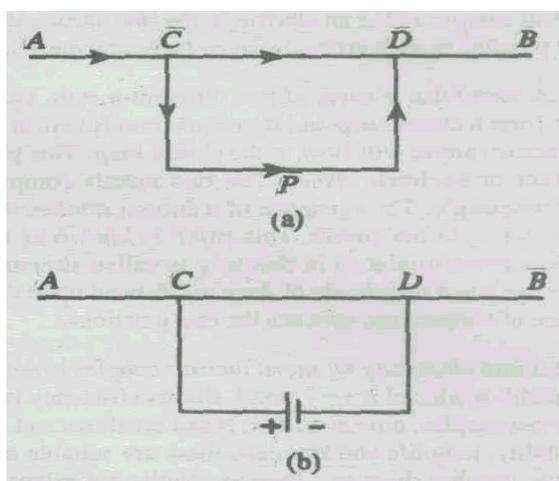


Fig. 8.1: Illustrating the principle of working of a potentiometer

If the potential difference between P and N is smaller than that between C and D , the current in the branch conductor will flow in the direction CPD . When the potential difference between P and N is greater than that between C and D the current will flow in the opposite direction i.e. PCD . But if there is no flow of current through the branch conductor CPD , then the tendency for the current to flow in the direction CPD is neutralised by the tendency for the current to flow in the direction PCD . This means that the potential difference between C and D , which urges the current to flow in the direction CPD , is exactly balanced by (or equal to) the potential difference between P and N which urges the current in the opposite direction. The absence of the current in the branch conductor can be shown by a galvanometer inserted in the branch CPD .

Suppose you are asked to measure the e.m.f. of a given cell E_1 with the help of a potentiometer. For this purpose, connect a battery E of higher e.m.f. e across the uniform resistance wire AB of length L as shown in

Fig. 8.2. Then a potential gradient $\frac{e}{L}$ will be developed on the wire. Let the e.m.f. of the given cell E_1 be e_1 , then it may be balanced with the potential difference across a certain length AC of the wire. For balancing it, the cell E_1 is connected through a galvanometer G such that the positive terminals of both the cells meet at a common point A and the negative terminal of the cell E_1 has a variable contact at the point C on the wire AB . Two electric currents, one due to the cell E_1 and the other due to the potential difference between A and C , will flow in opposite directions through the galvanometer. Under the condition when no current flows through the galvanometer i.e. at null point, the e.m.f. of the cell E_1 will be equal to the potential difference across the portion AC of length l of the potentiometer wire.

\therefore e.m.f. of the cell $E_1 = \text{potential gradient} \times \text{length of AC}$

$$\text{or, } e_1 = \frac{e}{L}l = kl \quad (8.1)$$

where, k is the potential gradient on the wire.

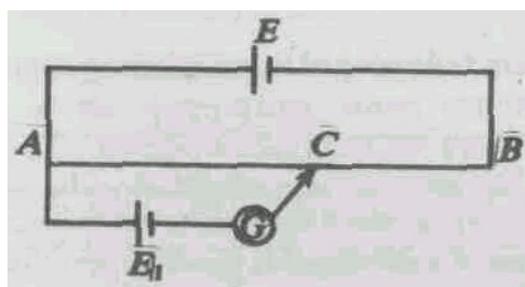


Fig 8.2: Illustrating the principle of determination of e.m.f. of a cell by means of a potentiometer

3.2 Fabrication of a Thermocouple

When wires of two different materials are either twisted together or welded at their ends so as to form a closed circuit, then, on heating one of the junctions, a current flows round the circuit as shown in Fig. 8.3. The pair of materials combined in this way is called thermocouple. In physics laboratory, usually, you may be given a thermocouple but even if you are asked to fabricate a thermocouple, there is no need to worry. You can easily fabricate a thermocouple. Which two different material you will choose? In the laboratory you can find plenty of connecting wires, which are actually copper wires. The wire in sonometer is made of iron. Hence by using copper and iron wires you can easily fabricate a thermocouple by following the method given below.

Pass one end of the iron wire down a thin glass tube G as shown in Fig. 8.4 and join it to a copper wire AJ at J . This will ensure that the wires

are in contact at the junction only. Now to ensure good contact at J , place the combination in a test tube containing a small amount of mercury and dip the junction J into the mercury.

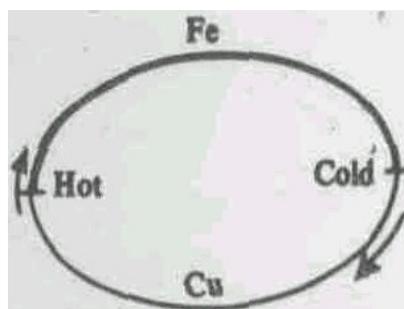


Fig. 8.3: Thermocouple

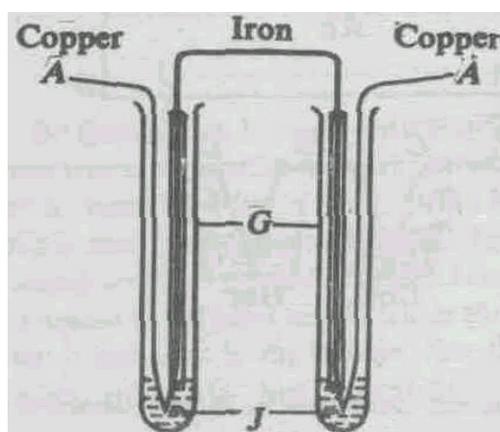


Fig. 8.4: Fabrication of copper iron thermocouple

Similarly, pass the other end of the iron wire down another thin glass tube G and prepare another junction J of iron and copper. While doing the experiment, keep one of the junctions in ice contained in a beaker and the other junction in a beaker containing water so that it may be heated to different temperatures.

3.3 Measurement of Thermo-E.M.F. of a Thermocouple and its Variation with Temperature

As the magnitude of thermo-e.m.f. is very small, generally of the order of a few milli-volts, it cannot be measured with the help of potentiometer in the usual way. However, with certain modification, the ordinary potentiometer may be used for measuring thermo-e.m.f. This modification is such that it enables production of a potential gradient of the order of microvolt on the potentiometer wire. The apparatus required for this experiment is as follows.

Apparatus

A potentiometer, a battery of steady e.m.f., a standard cadmium cell, a rheostat, a resistance box, a high resistance of 1.5,000 ohms, copper (Cu) - Iron (Fe) thermocouple, a sensitive galvanometer, two single-way plug keys, a two-way key, thermometer, ice, beaker, tripod, gauge, Bunsen burner, multimeter and connecting wires.

SELF ASSESSMENT EXERCISE 1

Can you use a voltmeter to read the thermo-e.m.f. developed in your thermo-couple directly?

In order to measure thermo-e.m.f., make a circuit as shown in Fig. 8.5. Connect a high resistance R (of the order of 1000 ohm) in series with the potentiometer wire AB and then connect this combination to a battery S_1 of steady e.m.f. with a rheostat Rh . The current enters the combination of R and potentiometer wire at M and leaves at B . Let a current I flow through them from the battery S_1 . Next, join the positive terminal of the standard cadmium cell S to the higher potential terminal M of the resistance R . The low potential terminal i.e. point D of the $Cu - Fe$ thermocouple is connected to the jockey so that the hot junction of the thermo-couple is towards the jockey. The negative pole of the standard cadmium cell and the higher potential terminal i.e. point C of the thermo-couple is connected to two similar terminals of the two-way key K_3 . The third terminal of the two-way key is connected to one terminal of the galvanometer G . The other terminal of which is joined to the lower potential N of the resistance R . Now if the standard cell circuit be closed by means of the two-way key and after closing the one way key K_1 , the rheostat Rh so adjusted that there is no deflection in the galvanometer, then the potential difference across the resistance R will be balanced by e.m.f. E of the standard cadmium cell. We have

$$E = IR \quad (8.2)$$

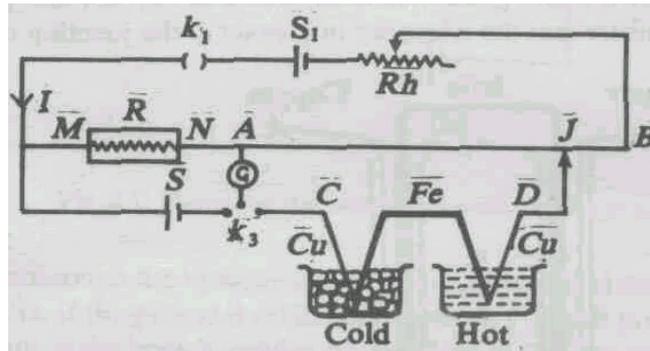


Fig. 8.5: Experimental arrangement for the measurement of thermo-e.m.f.

Now let the standard cell circuit be broken and the cold junction of the thermo-couple be connected to the galvanometer. Then if J is the position of the null point on the potentiometer wire, the thermo-e.m.f. of the copper-iron thermo couple is equal to the potential difference between A and J . If e is the thermoelectric e.m.f. and r is the resistance of the portion of the wire between A and J , then

$$\begin{aligned} e &= Ir \\ \text{or, } e &= I\rho l \end{aligned} \quad (8.3)$$

where ρ the resistance per unit length of the potentiometer wire and l is its length between A and J . Using Eqs. (8.2) and (8.3),

$$e = \frac{\rho E}{R} l \quad (8.4)$$

Knowing (i) the e.m.f. E of the standard cell, (ii) resistance per unit length of potentiometer wire, (iii) high resistance from resistance box required to produce the null point with the standard cell and (iv) length of the potentiometer wire at the null point with the thermocouple, we can determine the thermo-e.m.f. with the help of Eq. (8.4). If the temperature of the hot junction of the thermocouple is changed, the thermo-e.m.f. is also changed. By measuring thermo-e.m.f. at various temperatures (T) of the hot junction, you can draw a graph between e and T . Thus you can observe the variation of thermo-e.m.f. with temperature by following the procedure given below.

3.3.1 Procedure

- (1) Measure the resistance and the length of the potentiometer wire using a multimeter and a metre scale respectively. Calculate its resistance per unit length (ρ). Record it in Observation Table 8.1.

- (2) Make the electrical connection as shown in Fig. 8.6 and as described above. **The ends of the connecting wires should be clean and the connections should be firmly made.** Here the positive terminal of standard cadmium cell S is connected to the higher potential terminal M of the resistance box R through a high resistance R_1 of about 15,000 ohms. This is done to protect the standard cell because this will prevent large currents from being taken from the cell. Connect a plug key K_2 across R_1 .

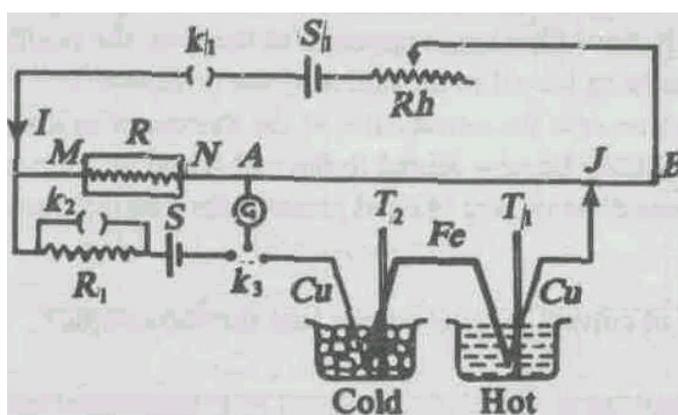


Fig. 8.6: Circuit diagram for studying the variation of thermo-e.m.f. with temperature

SELF ASSESSMENT EXERCISE 2

Will the resistance R_1 affect the position of balance while measuring thermo-electric e.m.f.?

- (3) To investigate how the thermo-e.m.f. varies with temperature, you have to keep one junction of the thermocouple at constant temperature whereas the other junction should be heated to different temperatures. Insert one junction of the thermo-couple into the pieces of ice kept in a beaker. Its temperature is 0°C (cold). Dip the other junction into a large beaker containing water. When the beaker is heated by a burner, the junction of the thermo-couple is heated to different temperatures (hot). Insert a sensitive thermometer into the water kept in the beaker. **See that the bulb of the thermometer is very near the hot junction.**
- (4) If the cold junction of copper-iron thermocouple is at 0°C and the hot junction at 100°C , then the thermo-e.m.f. developed will be about 1300 microvolt. To measure it, a potential difference of 1000 microvolt has to be produced between the points A and B of the potentiometer wire. In other words a potential difference of 1 microvolt per centimetre of the wire has to be produced. For this purpose, a suitable resistance is to be put in box R so that the potential difference per cm. of the wire is $1\ \mu\text{V}$. Adjust the

resistance in the box R to about a thousand ohms, preferably to 1018.3 ohms. Record it in Observation Table 8.1. Shunt the galvanometer. Close K_1 but keep K_2 open and then connect the negative terminal of the standard cell to the galvanometer by closing the key K_3 (towards the standard cell side). Adjust the rheostat Rh until there is practically no deflection in the galvanometer. Then remove the shunt from the galvanometer, close K_2 and adjust the rheostat Rh finally until there is no deflection in the galvanometer. This exactly balances the potential difference across R by the e.m.f. of the standard cell. This means that the potential difference across R is equal to the e.m.f. of the standard cell. You know e.m.f. of the standard cell is 1.0183 volts. Therefore, the current flowing through R is 1mA (Current = $\frac{1.0183}{1018.3} = 1\text{mA}$). This means that the current through the potentiometer wire is also 1mA (R and potentiometer wire being in series). If the resistance of the potentiometer wire is exactly one ohm then the potential difference across the wire will be 1 mV since the current flowing in the wire is 1mA. As there are 1000 divisions on the measuring scale the potential gradient will be exactly equal to one microvolt as desired.

SELF ASSESSMENT EXERCISE 3

A shunt is connected across the galvanometer. But while determining the exact position of the null point it is removed from the galvanometer. Why?

- (5) Check whether the positive end of the thermocouple is joined to the end A of the potentiometer wire. To do this, make $R = 0$, allow the jockey to touch the beginning of the wire and note the galvanometer deflection. Now bring the jockey in contact with the end of the wire. If the deflection is opposite to the first, the positive end of the thermocouple has been joined to the end A of the potentiometer wire. If the deflection is not opposite, then reverse the connection of the thermo-couple i.e. the terminal connected to the jockey be now joined to the end A and vice versa. Then put resistance in the box R as was done in step (4) and proceed for making observations.

SELF ASSESSMENT EXERCISE 4

What is the direction of current in your copper- iron thermocouple?

-
-
- (6) Now the hot junction is immersed in water at room temperature. Note this temperature and record it in Observation Table 8.1. Since one junction of the thermocouple is at $0\text{ }^{\circ}\text{C}$ and the other is at room temperature, a thermo-e.m.f. will develop. If you can find the length l of the potentiometer wire across which the potential difference exactly balances the thermo-e.m.f. then, using Eq. (8.4) you can easily calculate the thermo-e.m.f. at room temperature. For this purpose open K_2 and shunt the galvanometer again. Close the two-way key K_3 on the thermocouple side so that the cold junction of the thermocouple is connected to the galvanometer. Obtain an approximate position of the null point (i.e., the point where the galvanometer shows no deflection) on the wire by sliding the jockey on the wire. **Jockey should be pressed momentarily and it should not be slid over the potentiometer wire.** Now remove the shunt from the galvanometer and determine the exact position of the null point of the potentiometer wire. Note down the number of full wires and the length of the potentiometer wire between the point A and the jockey in the Observation Table 8.1. Find the equivalent length of the potentiometer wire. Then calculate the value of thermo-e.m.f. from Eq. (8.4). It will give the value of thermo-e.m.f. at room temperature.
- (7) Heat the water containing the hot junction of the thermo-couple to a high temperature (i.e. close to the boiling point of water) by means of burner. After heating, allow it to cool. As it cools, measure the thermo-e.m.f. after an interval of temperature of about $10\text{ }^{\circ}\text{C}$ until the temperature of the hot junction has fallen to about room temperature. For different temperatures of the hot junction, observe and record the position of the null point in the Observation Table 8.1. **Remember that reading of length should be taken first and then that of temperature.** During experiment, the cold junction of the couple should always remain at $0\text{ }^{\circ}\text{C}$. For this purpose it is necessary that the mass of ice should be poked from time to time. For each observation calculate the value of thermo-e.m.f. and record it in Observation Table 8.1.

Observation Table 8.1: Variation of thermo-e.m.f. with temperatureE.M.F. of the standard cell, E = voltResistance per unit length of the potentiometer wire, ρ = Ω /cmHigh resistance R = Ω

S. No.	Temperature of hot junction (T) ($^{\circ}\text{C}$)	Length of the potentiometer wire balanced by the thermo-couple			Thermo-e.m.f. in microvolt ($e = \frac{\rho E}{R} l$)
		No. of full wires	Position of the null point (cm)	Total length (cm)	
1	(room temperature)				
2					
3					
4					
.					
.					
.					
.					

Draw a graph between the temperatures of the hot junction of the thermo-couple and the thermo-e.m.f. developed. Plot temperature along the x-axis and the thermo-e.m.f. along the y-axis. The graph, in general, should be a parabola as shown in Fig. 8.7a. But within a short range of temperature as in the present case ($0^{\circ}\text{C} - 90^{\circ}\text{C}$ or 100°C), the graph will be a straight line as shown in Fig. 8.7b. The straight line actually is the straight portion of the parabola. This graph can be used to determine any unknown temperature (within this range).

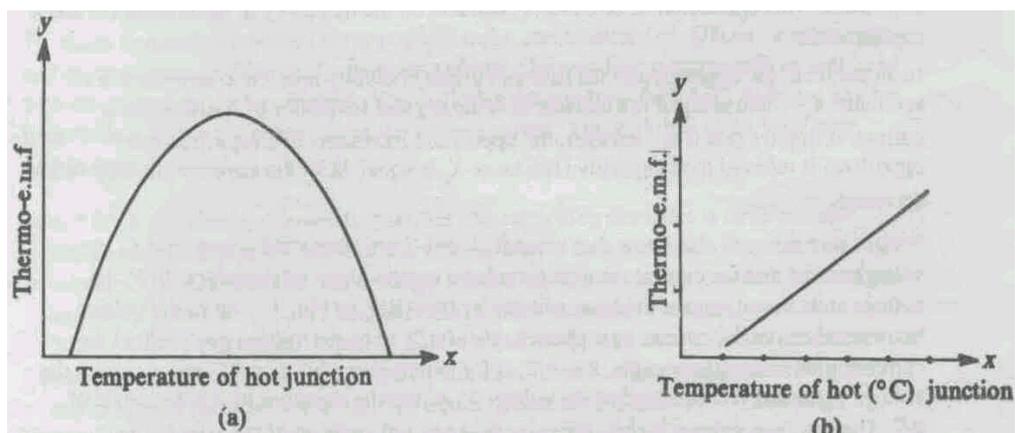


Fig. 8.7: Variation of thermo-e.m.f. with temperature (a) expected (b) experimentally observed.

4.0 CONCLUSION

Learners are expected to write conclusion based on their observation and calculations during the performance of their experiments.

5.0 SUMMARY

The experiment exposed learners to potential difference of the order of micro-volts. They also learnt how to fabricate thermo couple. Experimental set up for measuring thermo-emf was demonstrated, while the relationship between thermo-emf and temperature was established.

6.0 TUTOR MARKED ASSIGNMENTS

As provided by the facilitator.

7.0 REFERENCES/FURTHER READINGS

IGNOU (1997) Physics Laboratory I, Physics PHE-03 (L), New Delhi, India.

MODULE 3

Unit 1	Experiment 9 Frequency Response of A.C. Series Circuits
Unit 2	Experiment 10 Zener Diode Characteristics And Zener as A Voltage Regulator
Unit 3	Experiment 11 A Study of Transistor Characteristics

UNIT 1 EXPERIMENT 9
FREQUENCY RESPONSE OF A.C. SERIES
CIRCUITS

CONTENTS

1.0	Introduction
2.0	Objectives
3.0	Main Content
3.1	To Study the Frequency Response of a Resistor, an Inductor and a Capacitor
3.2	To Study the Frequency Response of RL and RC Series Circuits
3.3	To Study the Frequency Response of LCR-series Circuit
3.4	To Determine the Quality Factor (Q) of a LCR-series Circuit
4.0	Conclusion
5.0	Summary
6.0	Tutor Marked Assignment
7.0	References and Further Reading

1.0 INTRODUCTION

The three important components of an alternating current circuit are a resistor, a coil and a capacitor. These are denoted by:

Resistor: by its opposition (R) to current

Coil: by its co-efficient of self inductance (L)

Capacitor: by its charge storing capacity (C).

All these components are an integral part of the modern electronic devices like radio, TV etc. which we use in our home. Though these components find some use in direct current circuits, their uses in *A.C.* circuits are enormous. They find application in almost any electronic circuit and telecommunication system.

To acquaint you with these components, we would like you to investigate their frequency response, which is of help during fabrication and designing of various electronic circuits.

If a coil and a resistor appear in a series in a circuit, it is called an *RL*-series circuit. If on the other hand, *L* and *C* appear in series, it is called a *LC*-series circuit. If, however, all the three components appear in a series, it is called an *LCR*-series circuit.

You already know how these components affect the flow of current in an *A.C.* circuit. The opposition offered to the flow of current by a resistor (*R*) is independent of the frequency of the current. This opposition is, however, dependent on the frequency in an inductor (*L*) and a capacitor (*C*).

In an inductor, the opposition to the flow of current is usually referred to as inductive reactance X_L which is equal to $L\omega$. Here ω is the angular frequency of the alternating current. It implies that if ω increases, the opposition increases, in a capacitor, this opposition is referred to as capacitive reactance X_c is equal to $1/C\omega$. If ω increases, the opposition decreases.

We are sure that you also know that when *R*, *L* and *C* are connected across an *A.C.*, the voltage across and the current through them have certain phase relationships. In *R*, the voltage and current remain in phase with each other (Ref. to Fig. 9.1). In *L*, the voltage becomes ahead of the current by a phase angle of $\pi/2$. In *C*, the voltage goes behind the current by the same phase angle, i.e., $\pi/2$. If for example, we have an *RC*-series circuit, the voltage V_R in *R* becomes ahead of the voltage V_C across the capacitor by a phase angle of $\pi/2$. The resultant voltage is then given by the Vectorial addition of the two. Using the Pythagoras theorem, the resultant voltage $V = \sqrt{V_R^2 + V_C^2}$. Similarly, for an *LR*-series circuit, $V = \sqrt{V_R^2 + V_L^2}$. However, if instead we have an *LCR* series circuit, the voltage V_L and V_C get out of phase by an angle of π . Furthermore, both V_L and V_C are also out of phase from V_R by $\pi/2$. In such a case the resultant voltage V is given by

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

The voltage across R is in phase with the circuit current, since current and voltage are in phase in pure resistive circuits.

The voltage across L leads $\pi/2$ radians since current lags the voltage by $\pi/2$ degrees in purely inductive circuits.

The voltage across C lags the circuit current by $\pi/2$ degrees since the current lead by $\pi/2$ degrees in purely capacitive circuit.

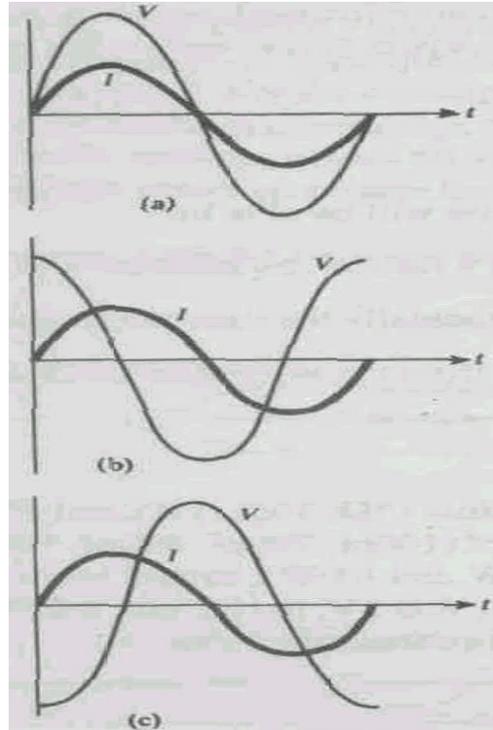


Fig 9.1: RLC-Series circuit showing phase relationship in individual component

If the total voltage V in an *A.C.* series circuit is divided by the total current in the circuit, we get the opposition offered to the current in the circuit. This opposition is usually referred to as impedance and is denoted by Z . For various series circuits, the following are the impedances:

$$LR \text{ circuit: } \sqrt{R^2 + (L\omega)^2}$$

$$RC \text{ circuit: } \sqrt{R^2 + 1/C^2\omega^2}$$

$$LCR \text{ circuit: } \sqrt{R^2 + (L\omega - 1/C\omega)^2}$$

In this experiment you will observe yourself as to how the three components L , C and R behave individually or in combination when the frequency of the applied current is varied with the help of an oscillator. When the current frequencies are plotted against the voltages across these components, the voltages are found to depend on the frequencies

of the applied currents. These curves are called the frequency response curves.

To obtain frequency response curves you will make combinations of different components and apply currents of different frequencies to them. The resulting voltages will be measured with the help of an *A.C.* voltmeter. You will plot the resulting data, viz., voltages against frequencies and see the behaviour of the various curves. You will then make your own conclusions.

In an *LCR* series circuit, at lower frequencies, the capacitive reactance is large and the inductive reactance is small. Most of the voltage drop is then across the capacitor. At high frequencies, the inductive reactance is large and the capacitive reactance is low. Most of the voltage drop is then across the inductance. In between these two extremes, there is a frequency called the resonant frequency f_r , at which the capacitive and inductive reactance are exactly equal and neutralise each other. In this case, there is only the resistance R in the circuit to oppose the flow of current. The current at resonant frequency is equal to the applied voltage divided by the circuit resistance, and is thus very large if the resistance is low.

At resonance the current is maximum. Locate the points on your graph where the current is .707 times that of the maximum current. These two points on either side of f_r may be called the half power points. The frequency difference Δf between these points is known as the band width of the resonance curve. In terms of f_r and the band width Δf , we can define a new term Q , known as the quality factor. This is equal to $f_r / \Delta f$. The Q is usually used in designing electronic circuits and the communication engineering.

In the introductory part of your laboratory manual, we have demonstrated the use of semi-log graph paper. In this experiment we expect you to use such a graph paper. This will help you in appreciating its use.

2.0 OBJECTIVES

After performing this experiment you will be able to:

- show the frequency response of a resistor, an inductor and a capacitor
- select the scale and plot experimentally the data using semi-log and log-log graph paper.
- calculate the quality factor (Q) from the resonance curve of a *LCR*-series circuit.

Apparatus

Oscillator (10-100KHz, 20V), resistors ($5\ \Omega$, $10\ \Omega$, $15\ \Omega$ and $20\ \Omega$ -2W), inductors (5mH, 10mH, 15mH and 20mH), capacitors (100pF, 200pF, 500pF, 100mf- 20V), carbon resistances ($500\ \Omega$, $1k\ \Omega$, $5k\ \Omega$ - 1/4W and 1/2 W), digital multimeter or digital microvolt meter or a.c. voltmeter (0-1V, 0-5V, 0-10V and 0-20V etc.) and a.c. ammeter (0-1mA, 0-10mA, 0-50mA etc.) and connecting wires.

3.0 MAIN CONTENT

3.1 To Study the Frequency Response of a Resistor, an Inductor and a Capacitor

Procedure: You arrange the circuit connections shown in Fig. 9.2. Connect the main lead of the oscillator to the *A.C.* mains. Connect the resistor R along with the external resistance R_s and *A.C.* ammeter across the output terminals of the oscillator. The resistance R serves the purpose of limiting the value of current in the circuit. Hence its value is of the order of a few hundred ohms. Connect the *A.C.* voltmeter across the resistor R .

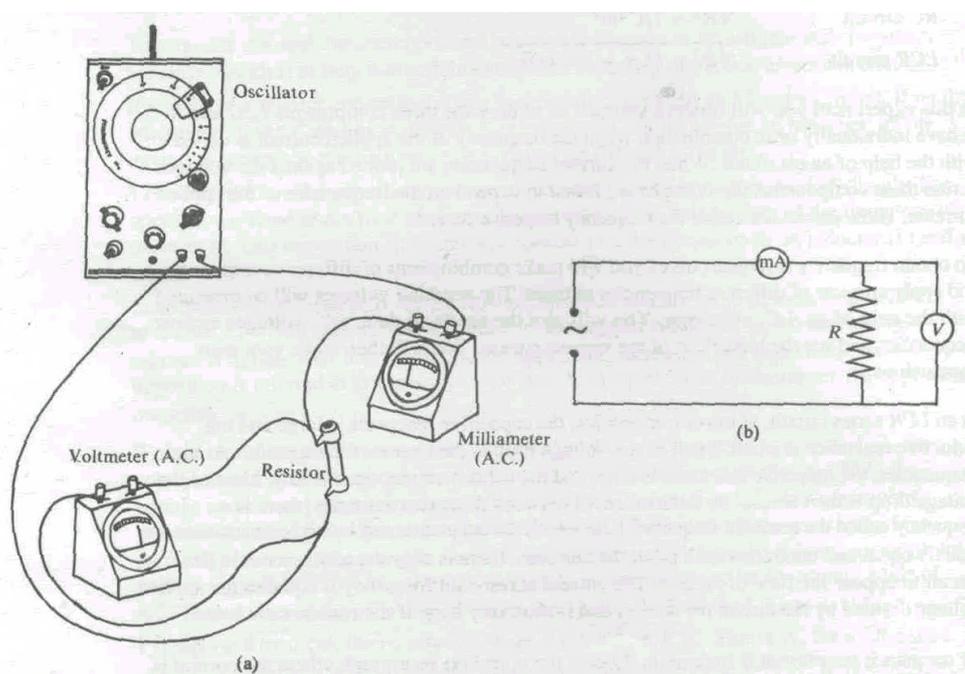


Fig. 9.2: A.C. series circuit containing only resistance (a) Actual diagram (b) Circuit diagram

Switch on the oscillator at least half an hour before performing the experiment so that it gives you a stable output. Keep the output of the oscillator at 10V with the help of output varying knob marked as K_1 in Fig. 9.2. You can change the frequency of the oscillator with the help of

two knobs K_2 and K_3 . The knob K_2 is known as range selector, and knob K_3 as frequency selector. Select the frequency of the oscillator, say at 100 Hz, and measure the potential drop V_R across the resistor R . For the measurement of accurate voltage select the proper range of voltmeter. For difference readings change the frequency with the help of knobs K_2 and K_3 and measure the voltages across the resistor R . Record your data in Observation Table 9.1. Repeat the above procedure for different values of the resistance.

Observation Table 9.1 Frequency response of a resistor

Current across the ResistormA

S. No.	Frequency f (Hz)	Voltage across the resistor (..... Ω) in volts	Voltage across the resistor (..... Ω) in volts

Now, plot a graph between V_R and f for each value of R on a semi-log graph paper. Semi-log graph paper is being used to accommodate a large frequency range along the x -axis.

A graph between V_R and f

With the help of these graphs explain your results i.e., the frequency dependence of a resistor in the space provided below.

Now replace the resistor R with a capacitor of 100pF and repeat the same procedure. Record your data in Observation Table 9.2 for different values of capacitors.

Observation Table 9.2 Frequency response of a capacitor

Current across the capacitor = mA

S. No.	Frequency f (Hz)	Voltage across the capacitor (.....pf) in volts	Voltage across the capacitor (.....pf) in volts

Now, you plot a graph of V_C vs. frequency f on a log-log graph paper. (If you face any difficulty in the use of log-log graph paper, consult your counsellor present in the laboratory.)

A graph between V_C and f

With the help of your graphs outline your results in the space provided below:

Now you replace your capacitor with an inductor of 5mH. Repeat the above procedures and record your data in the Observation Table 9.3 for different values of the inductor.

Observation Table 9.3 Frequency response of an inductor

Current across the Inductor = mA

S. No.	Frequency f (Hz)	Voltage across the inductor (..... mH) in Volts	Voltage across the inductor (..... mH) in volts

Now, plot a graph between V_L and frequency f on a log-log graph paper.

A graph between V_L and frequency f

Discuss your results on the basis of above graphs in the following space.

SELF ASSESSMENT EXERCISE 1

Calculate the inductive and capacitive reactance of any inductor or capacitor, which you have used in the above experiment for a frequency of 1 KHz.

SELF ASSESSMENT EXERCISE 2

What is the value of X_L as the frequency approaches zero or infinity?
What will happen when such inductors are used in *A.C.* Circuits?

SELF ASSESSMENT EXERCISE 3

Can you use a *D.C.* voltmeter instead of an *A.C.* voltmeter in your experiment? If not, why?

3.2 To Study the Frequency Response of RL and RC Series

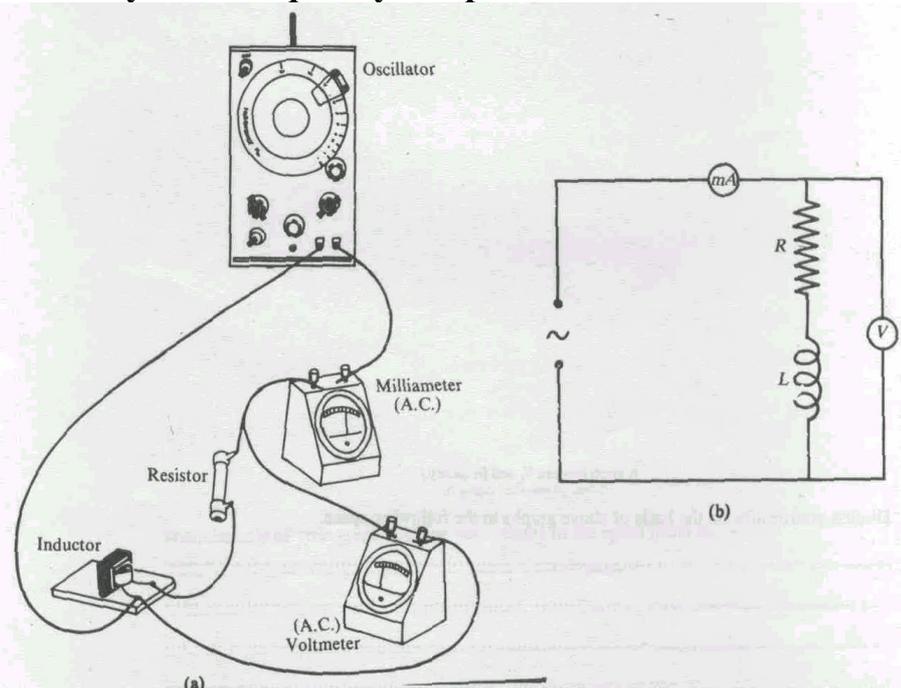


Fig 9.3: RL -Series circuit (a) Actual diagram; (b) Circuit diagram

Now connect a resistor and an inductor in a series between the source points where you had earlier connected the three components individually. This is shown in Fig. 9.3. Repeat the same procedure again and record your data in the following Table 9.4.

Observation Table 9.4 Frequency response of a RL -circuit

S. No.	Frequency f (Hz)	Total Voltage V across RL -series circuit

Now plot a graph between voltage V and frequency f .

A graph between voltage V and frequency f

Record your conclusions in the space provided below.

Now, you replace the inductor by a capacitor and record your data in the Observation Table 9.5.

Observation Table 9.5 Frequency response of a RC-Circuit

S. No.	Frequency f (Hz)	Total Voltage V across RC -series circuit

Plot a graph between voltage V and frequency f on a semi-log graph paper.

A graph between voltage V and frequency f

Record your results in the space given below:

SELF ASSESSMENT EXERCISE 4

Calculate the impedance of an RL -series circuit.

SELF ASSESSMENT EXERCISE 5

On the basis of your results, what do you think is the difference between RL series and RC series circuits?

3.3 To Study the Frequency Response of an LCR-Series Circuit

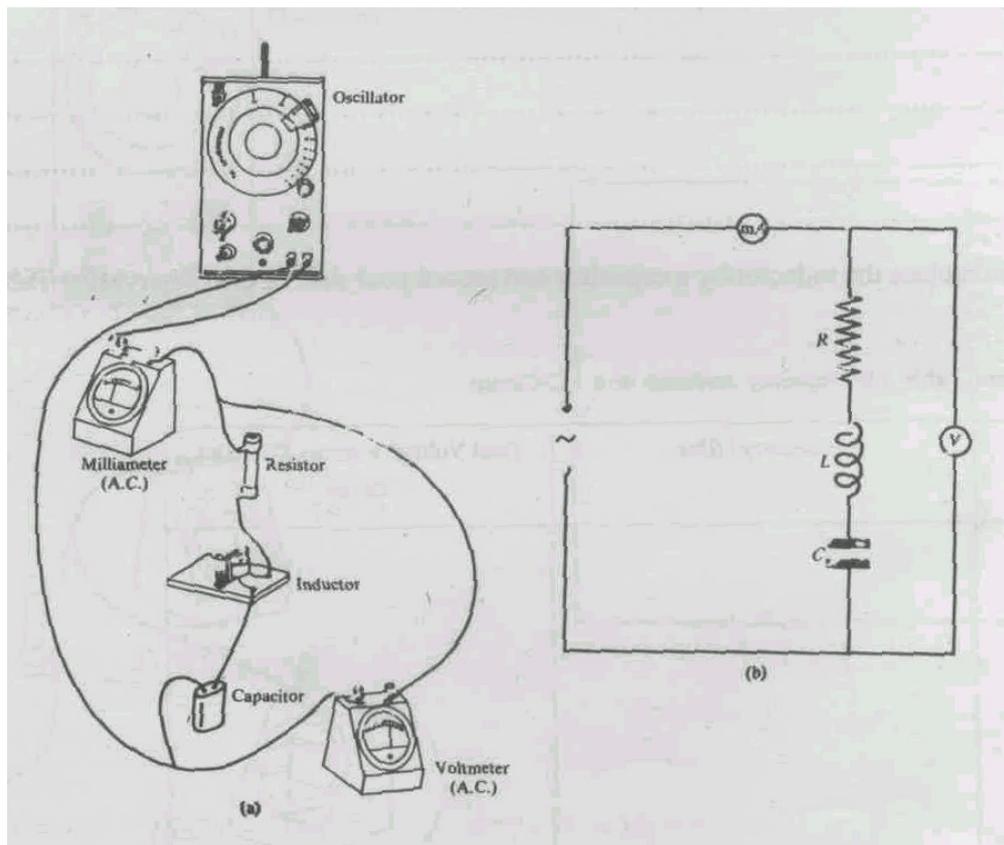


Fig 9.4: RLC-Series circuit (a) Actual diagram (b) Circuit diagram

Procedure: In the last part you had connected a resistor and a capacitor in series. Now, in between the same end points, you add an inductor as well to make an LCR series circuit as shown in Fig. 9.4. Following the same procedure record your data in the Observation Table 9.6.

Observation Table 9.6: Frequency response of a RLC series circuit

S. No.	Frequency f (Hz)	Total Voltage V across LCR -series circuit

Now, plot a graph between voltage V and frequency f

Graph between voltage V and frequency f

Do you observe that this graph is quite different from the previous graphs? Do you know why? Give your reasons in the lines below. If you are unable to answer the above, consult the last part of the introduction for this experiment.

3.4 To Determine the Quality Factor (Q) of an LCR-Series Circuit

Procedure: Now, you repeat the above-experiment and measure the current in a LCR -series circuit with the help of an $A.C.$ ammeter for different values of the frequency. Record your data in the Observation Table 9.7.

Observation Table 9.7: Frequency response of a LCR circuit

S. No.	Frequency f (Hz)	Current I in the LCR -series circuit

From above data, plot a graph between Current I and frequency f .

A graph between current I and frequency f

Explain your result on the basis of the above your graph in the following lines.

The curve obtained above by you is known as the resonance curve. The frequency at which the current is maximum is known as the resonant frequency f_r . The points on the graph where the current reduces to .707 times that of the maximum value are known as the half power point. The frequency difference between these two points denoted by Δf is called the band width. Now, with the half f_r and Δf , calculate the value of the quality factor Q .

SELF ASSESSMENT EXERCISE 6

Explain the difference between voltage vs frequency and current vs frequency curves in a LCR series circuit which you have plotted in the above experiment.

4.0 CONCLUSION

Learners are required to arrive at a conclusion based on observations and calculations during the experiments.

5.0 SUMMARY

The frequency responses of resistors, inductors and capacitors were established. Values for quality factor, Q , from the resonance curve of a LCR-series were obtained graphically.

6.0 TUTOR MARKED ASSIGNMENTS

As provided by the facilitator.

7.0 REFERENCES/FURTHER READINGS

IGNOU (1997) Physics Laboratory I, Physics PHE-03 (L), New Delhi, India.

UNIT 2 EXPERIMENT 10

ZENER DIODE CHARACTERISTICS AND ZENER AS A VOLTAGE REGULATOR

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Introductory Information
 - 3.1.1 Electronic Configuration of Germanium and Silicon
 - 3.1.2 Crystal Lattice of Germanium and Silicon
 - 3.1.3 N-Type Semi-conductor
 - 3.1.4 P-Type Semi-conductor
 - 3.1.5 Current through Semi-conductor
 - 3.1.6 *P-N* Junction
 - 3.1.7 *P-N* Junction in Forward-Bias
 - 3.1.8 *P-N* Junction in Reverse-Bias
 - 3.1.9 Zener Diode
 - 3.1.10 Testing of a *P-N* Junction
 - 3.2 Voltage-Current Characteristics of a Zener Diode
 - 3.3 Zener Diode as a Voltage Regulator
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor Marked Assignments
- 7.0 References and Further Reading

1.0 INTRODUCTION

On the basis of their resistivity values, materials can be broadly classified into – metal, insulator and semi-conductor. The resistivity of a metal is of the order, of 10^{-8} ohm-cm and an insulator is of the order of 10^{22} ohm-cm. The resistivity of a semi-conductor lies in between those of a metal and an insulator. Germanium and silicon are the most commonly used semiconductors. At absolute zero, i.e., -273°C , the semi-conductor would be a near perfect insulator. As the temperature increases, the conductivity of the semi-conductor increases. This change in the conductivity with an increase in the temperature varies for different semi-conducting materials. For example, with an increase in the temperature by 10°C , the conductivity increases twice in germanium and thrice in silicon.

A semi-conductor has limited device-possibilities. For instance, it is used in photo-cell and temperature-sensitive resistor etc. In order to increase the device applicability of the semiconductors, impurities are added to the semi-conductors to make them *p*-type and *n*-type. In *p*-type

materials the impurity is of the acceptor type whereas in n -type the impurity is donor type. When p and n semi-conductors are fused and the thickness between them is 10^{-4} cm, a p - n junction is formed. In practice, a p - n junction may be formed from a pure semiconductor by doping part of it with acceptor impurities and the remainder with the donor.

A p - n junction performs essentially the same jobs that an electron tube (vacuum diode) does in the electronic equipment. The p - n junction becomes very important in electronics because of their many advantages over electron tubes. It is smaller in size and lighter in weight. This makes the equipment small in size and lighter in weight. The equipments which were heavy, bulky and permanently mounted now can become portable and miniaturised. Another advantage of p - n junction is that it need not be heated as in the case of electron tubes. In this way the power supply equipment and the circuit components can be made smaller and cheaper. It has become all the more important because many other solid-state devices contain several such junctions. If the mechanism of current flow in a simple p - n junction is understood, then it becomes easier to understand the mechanism of the operation of the more elaborate structures.

Zener diode is a special kind of p - n junction diode. The voltage-current characteristics of a Zener diode are the same as those of a p - n junction. But in Zener diode, there is a small change in the current when the voltage across the Zener is increased. In this way a Zener diode differs from a p - n junction. This is a very important property of the zener diode, which enables us to use it as a voltage regulator in power supplies and voltage reference standards.

Here in the first part of the experiment, we will plot the voltage-current characteristics of a Zener diode in the forward-bias which are similar to the characteristics of a p - n junction. In the second part of the experiment, we will plot voltage-current characteristics in the reverse direction. In the third part of the experiment we will show how a Zener diode is used as a voltage regulator.

In the next unit we will perform experiments on transistor characteristics. A transistor is regarded as a combination of the two p - n junction diodes in different ways.

2.0 OBJECTIVES

After doing this experiment, you should be able to:

- draw the voltage-current characteristic curves of a given zener diode in forward and reverse biases
- determine whether zener diode is made up of silicon and / or germanium from its voltage-ampere characteristic curves
- measure the effects of line and load change on the output of a zener diode
- construct a zener voltage regulator and experimentally determine the range over which the zener maintains a constant output voltage.

Apparatus

Zener diode BZ-146, BZ-147, CZ-6, IN-753, IN-3020 or any other zener diode (1 W, 10 V -20 V), a variable regulated a.c./d.c. power supply (0-30 V), transformer (12-0-12 Volt), capacitor (0-100 μ F-25 Volt), ammeter (0-10 μ A, 0-50 μ A, 0-100 μ A, 0-30 mA, 0-50 mA, 0-100 mA), voltmeter (0-10 V, 0-25 V, 0-50 V), resistances (0-1 K Ω , 0-10 K Ω , 0-25 K Ω , 0-100 K Ω $\frac{1}{2}$ W and 1 W), multimeter and connecting wires, soldering wire, soldering paste and solder (20 W) etc.

3.0 MAIN CONTENT

3.1 Introductory Information

You have read about semi-conductors in your school science courses. You have also read about *p*-type and *n*-type semi-conductors. Let us now recapitulate what we know about semiconductors. Let us now recapitulate what we know about semi-conductors.

A semi-conductor is a material whose electrical conductivity lies between that of a metal and an insulator. Germanium and silicon are the most commonly used semi-conductors. A pure semi-conductor is also known as intrinsic semi-conductor. They have crystalline structure.

The conductivity of a pure-semiconductor can be increased by adding minute quantities (1 part in 10⁸) of certain impurities to the semi-conducting crystal. This process of adding controlled quantities of certain impurities to the pure crystals of germanium and silicon is known as doping.

3.2.1 Electronic Configuration of Germanium and Silicon

A pure atom of germanium has 32 electrons. Out of 32 electrons, 28 are tightly bound to the nucleus, whereas the remaining four revolve in the outermost orbit. The electrons in the outermost orbit are called valence electrons. The electronic configuration is shown in Fig. 10.1a.

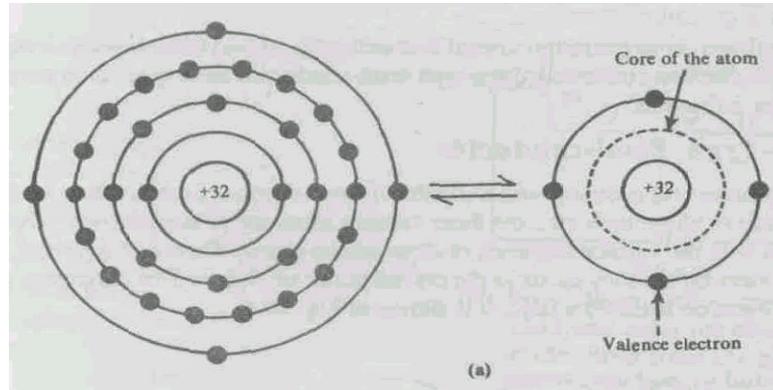


Fig. 10.1. (a) Electronic configuration of germanium

Silicon has 14 electrons. The electronic configuration of silicon is shown in Fig. 10.1(b)

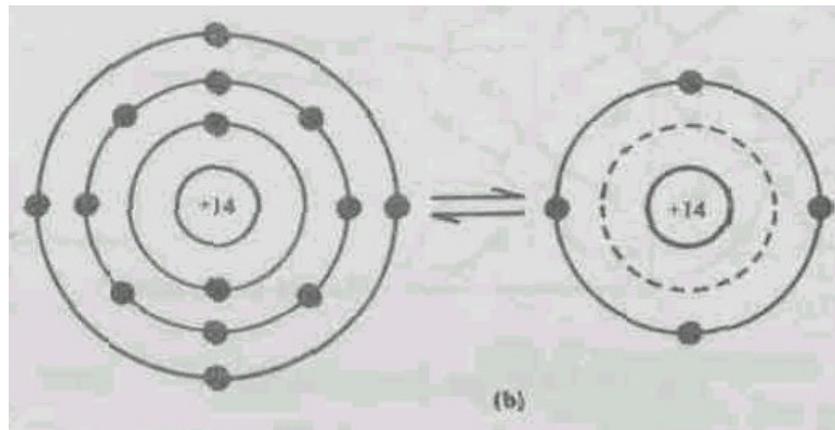


Fig. 10.1(b) Electronic configuration of silicon

3.2.2 Crystal Lattice of Silicon (or Germanium)

In a crystal lattice, each atom shares its outermost or valence electrons with those of the neighbouring atom forming what are known as an electron-pairs or covalent bonds between atoms as shown in Fig. 10.2.

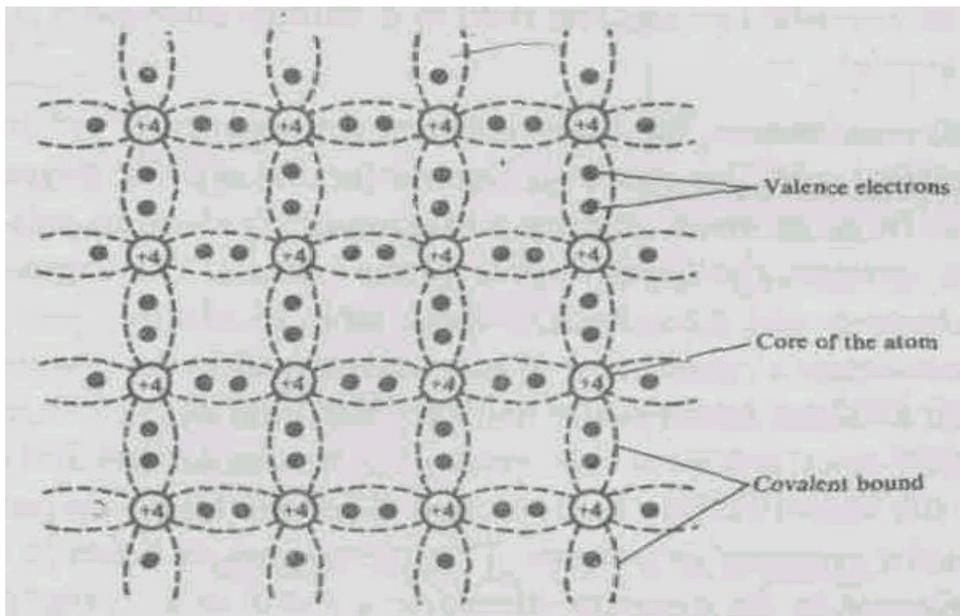


Fig. 10.2 Crystal lattice of silicon (or germanium)

3.2.3 *n*-Type Semi-conductor

When silicon (or germanium) in its pure form is doped with a pentavalent (five electrons in the outermost orbit) atom like arsenic or antimony, four out of its five valence electrons form covalent bonds with the valence electrons of four silicon atoms, but the fifth valence electron of arsenic remains unattached and becomes a free electron. It is shown in Fig. 10.3.

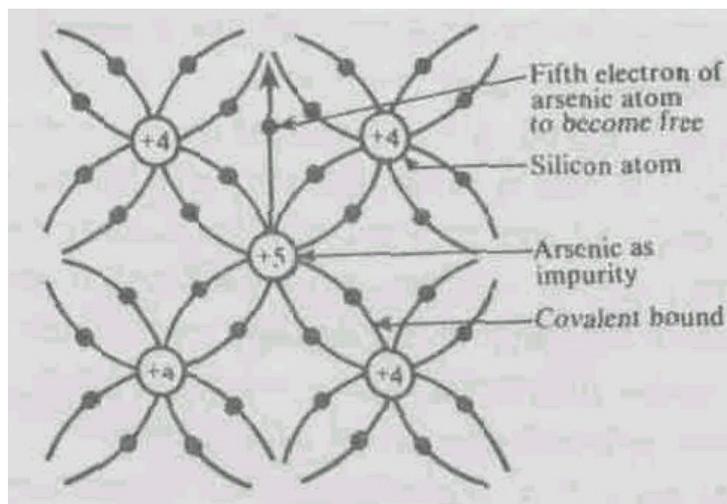


Fig. 10.3 N-Type Semiconductor

Thus, when a silicon (or germanium) crystal is doped with arsenic (or antimony) it develops an excess of free electron and is called an *n*-type semi-conductor. Such types of impurities are known as donor impurities.

3.2.4 *p*-Type Semi-conductor

If silicon (or germanium) is doped with a trivalent (three electrons in the outermost shell) atom like indium or aluminium etc., the three valence electrons of impurity atom, form covalent bonds with the valence electrons of three silicon atoms, There is a deficiency of one electron or an electron vacancy exists in the crystal lattice of silicon. This deficiency or absence of an electron is called a hole. It is shown in Fig. 10.4.

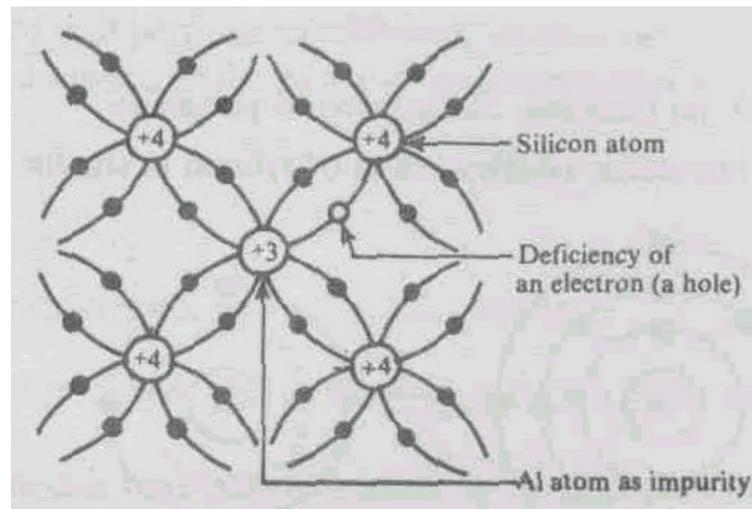


Fig. 10.4 *p*-Type Semi-conductor

The semi-conductor so formed has a deficiency of electrons or an excess of holes and is called a *p*-type semi -conductor. Such types of impurities are known as acceptor impurities because they can accept an electron from silicon atoms.

3.2.5 Current through Semi-conductor

You have learnt that an *n*-type semi-conductor has an excess of free electrons. These free electrons act as current carriers when an electric field or a voltage difference is applied across an arsenic doped silicon crystal.

In the case of *p*-type semi-conductors, the holes indicate the absence of electrons. These holes behave like positively charged particles when an electric field is applied across the crystal. Under the influence of the field, an electron from a neighbouring electron pair bond breaks loose and falls into a hole towards the positive pole of the battery. This creates a new hole that can accept another electron which has broken loose from its electron pair-bond. This process continues and constitutes a movement of electrons towards the positive pole of the battery and the movement of holes towards the negative terminal of the battery. As the

holes reach the negative terminal, electrons from this terminal enter the crystal and neutralise these holes. At the same time, the loosely held electrons that filled the holes are pulled away from the positive terminal thereby creating new holes. This movement of holes in one direction and the movement of electrons in the opposite direction constitute a current flow in the same direction. Thus in a semi-conductor, the current conduction is the result of the movement of holes inside the crystal and the movement of electrons through the external circuit and the battery. The current flow in pure, *n*-type and *p*-type semi-conductors (silicon) is shown in Fig. 10.5.

- Electrons
- Hole (Dangling bond)
- A hole that has captured an electron

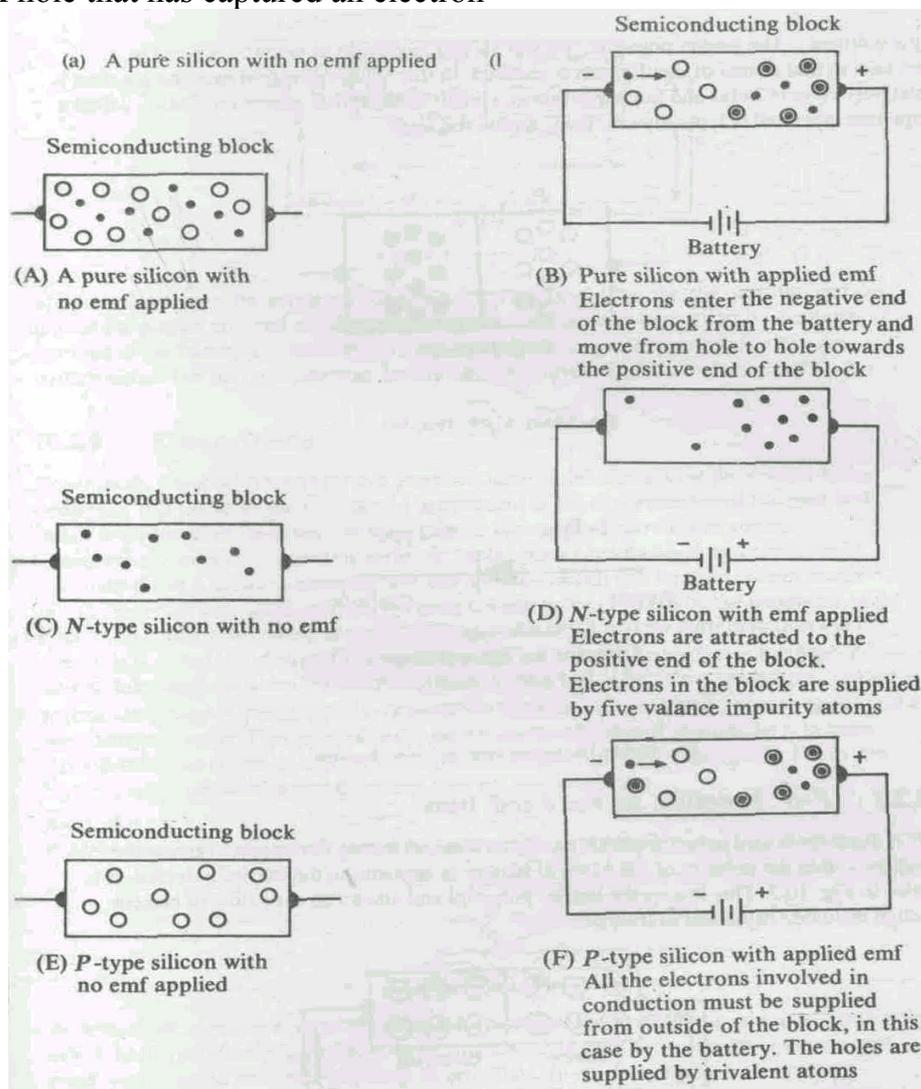


Fig. 10.5 Current through a Semi-conductor

3.2.6 *p-n* Junction

A *p-n* Junction diode is formed by combining a *p*-type semi-conductor with an *n*-type semiconductor. The *p-n* junction so formed exhibits the interesting and useful property of offering a low resistance to current flow in one direction. The *p-n* junction has the same rectifying characteristics as a vacuum diode. A *p-n* junction cannot be formed by simply putting together a *p*-type and an *n*-type semi-conductor. The construction of a *p-n* junction diode can be done as follows. Take a dot of indium and fuse it on a germanium wafer of, *n* type at a suitably high temperature. This produces a *p*-type germanium immediately below the surface resulting in the formation of a *p-n* type junction between the *p*-region and the body of the *n*-type germanium. It is shown in Fig. 10.6a. The junction is formed because of the concentration gradient. Then holes from the *p*-side diffuse into the *n*-side and recombine with free electrons. Similarly, the electrons from the *n*-type diffuse to the *p*-side and recombine with the holes. Such an exchange of mobile carriers occurs mainly in a narrow region around the junction. This region is called the depletion layer or space-charge layer, as it becomes depleted of the free charge carriers. It leaves behind the unneutralised space charge due to positive ions on the *n*-side and negative ions on the *p*-side as shown in Fig. 10.6b. Such a space charge causes an electric field in the depletion region and a potential difference called the junction barrier potential develops across the *p-n* junction, making the *p*-side negative with respect to *n* -side. This barrier potential cannot be measured by a voltmeter. The barrier potential opposes further migration of electron across the junction so that a state of equilibrium is reached. In this state, the region near the junction is relatively clear of holes and free electrons as a result of the initial migration. This is called a depletion layer and is typically less than one micron wide.

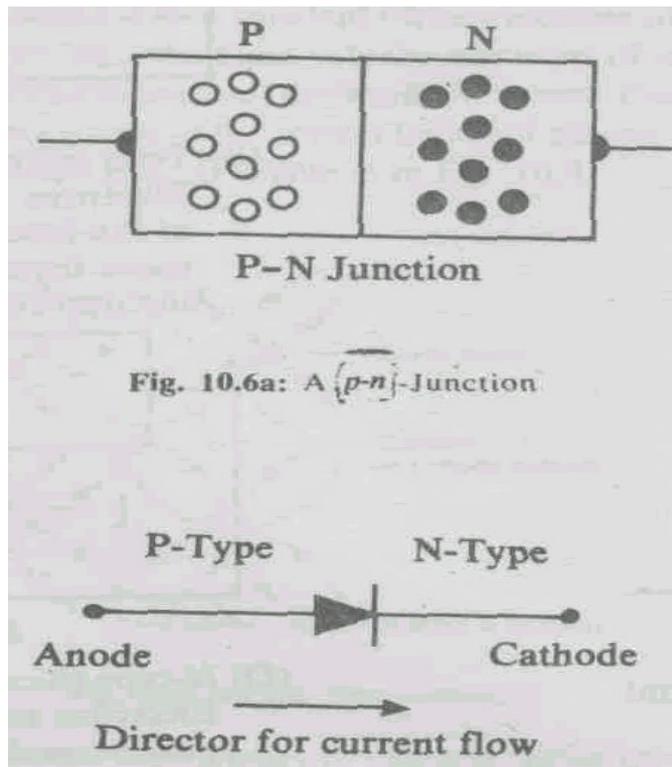


Fig. 10.6.b: Depletion layer in a p - n Junction

3.2.7 p - n Junction in Forward Bias

A p - n Junction is said to be forward biased if an external battery is connected across the junction so that the polarity of the external battery is opposite to the barrier potential. It is shown in Fig. 10.7. This lowers the barrier potential and allows an easy flow of current through the diode explained as follows:

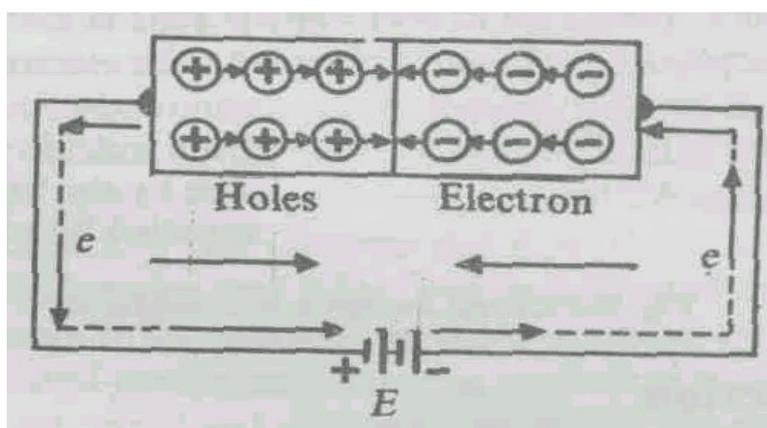


Fig. 10.7 A p - n Junction in Forward Bias

Free electrons from the negative terminal of the battery repel the free electrons in the n-type material. These free electrons move towards the p - n junction. The holes in the p -type material are also repelled by the

positive terminal of the battery and move towards the junction. At the junction the free electrons and holes combine and are lost in the process. However, the current carriers lost in these combinations are replenished by new current carriers, resulting from the separation of electron hole pairs. The free electrons produced in the p -type material are attracted by the positive terminal of the battery and flow in the external circuit as shown in Fig. 10.7. This is a continuous process and constitutes a current flow by electrons in the external circuit but inside the junction, both holes and electrons carry currents.

3.2.8 p - n Junction in Reverse Bias

A p - n junction is said to be reverse biased when the external voltage or battery connected across it aids the barrier potential as shown in Fig. 10.8. Since the barrier potential is actually raised by the reverse-bias, there is practically no flow of current through the diodes.

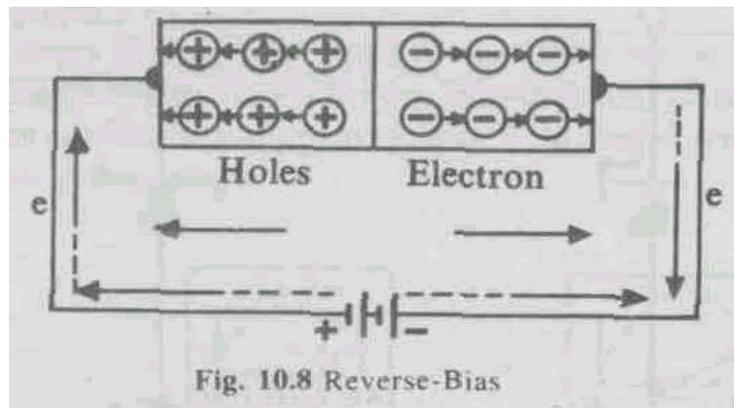


Fig. 10.8 Reverse-Bias

The free electrons in the n -type material are attracted away from the p - n junction and the hole in the p -type material are similarly attracted away from p - n junction by the negative terminal of the battery and there are practically no holes or electron carriers left in the neighbourhood of the p - n junction. In this case the current flow stops completely.

3.2.9 Zener Diode

Zener diode is a special type of a p - n junction diode which operates in the reverse-bias condition. It is manufactured by careful adjustment of the concentration of acceptor and donor impurity atoms near the junction. Unique reverse-bias current and voltage characteristics provide completely different applications from those of the crystal diode. When the diode is forward-biased, it acts like a closed switch and forward current increases with an increase in applied voltage. Forward current is then limited by the parameter in the circuit. When the Zener diode is reverse biased, and a small reverse current I_z flows in the circuit. It is

called the saturation current. It is relatively constant despite an increase in reverse bias, until the Zener breakdown voltage V_z reached. After Zener breakdown voltage V_z reverse current starts rising rapidly. A zener for this reason is used as a voltage regulator at a predetermined value. This value depends on the choice of material conductivity. In zener diode, breakdown occurs at reverse bias from about three volt to several hundred volts. But higher value zener diodes are rare because they are very expensive.

SELF ASSESSMENT EXERCISE 1

Explain why an ordinary diode cannot be used as a Zener diode?

In zener diodes which are operated below 6 volts, the breakdown of the junction is due to zener effect. In this mechanism, the breakdown is initiated through a direct rupture of covalent bonds owing to the existence of strong electric field. In the diodes which are operated between several volts to a few hundred volts, the breakdown is due to both the zener effect and the avalanche breakdown.

SELF ASSESSMENT EXERCISE 2

What do you mean by Zener breakdown voltage?

In avalanche breakdown, the minority charge carriers (holes in n -type and electrons in p -type) acquire sufficient energy from the applied reverse voltage to produce new carriers by removing valence electrons from the covalent bonds. The new carriers in turn produce additional carriers and the process **multiplies to give a large reverse current. The diode is then said to be in the region of avalanche breakdown.** However, in general, all semi-conductor diodes, which are operated in the breakdown region of their reverse characteristics – whether zener breakdown or avalanche breakdown are known as zener diodes. The circuit symbol of a zener diode is shown in Fig. 10.9. The symbol is similar to that of an ordinary diode with the change that the bar is replaced by the letter Z.

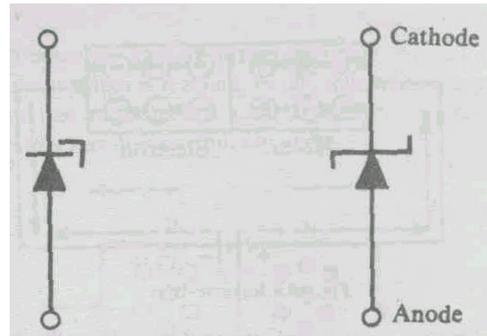


Fig. 10.9 Symbol of a zener diode

SELF ASSESSMENT EXERCISE 3

How many breakdowns are there in a zener diode? What are they?

The breakdown voltage of a zener diode is limited by an external circuit to a suitable value such that the power dissipation across the junction is within its power handling capacity. Zener breakdown need not result in the destruction of the diode. As long as current through the diode is limited by the external circuit to a level within its power-handling capabilities, the diode functions normally. Moreover by reducing reverse bias below the zener voltage, the diode can be brought out of its breakdown level and restored to the saturation current level. This process of switching the diode between its zener and non zener current states can be repeated again and again without damaging it. If the limit of power-handling capacity is exceeded, then a large current may cause damage to the diode.

Zener diodes are used as voltage regulators and as voltage reference standards.

3.2.10 Testing of a $p-n$ Junction

The property of a junction is that it represents a low resistance in forward bias and a high resistance in reverse bias. This property can be used to test a junction with the help of a multimeter. When a diode is tested with a multimeter then it should be kept in the range of $10k\Omega$. When a multimeter is used as an ohm-meter, then it has a battery connected inside the meter. When a diode is connected across the testing lead of the multimeter, then it is conducting either in forward direction or reverse direction, depending upon the negative or the positive terminal connected to the cathode lead of the junction. Now reverse the connection of the junction. Check whether junction is conducting in reverse or forward direction. On changing the direction (i.e., the

terminal of the diode), if the conduction changes from reverse to forward or forward to reverse, then you can say that the diode is in proper working condition. It is shown in Fig. 10.10.

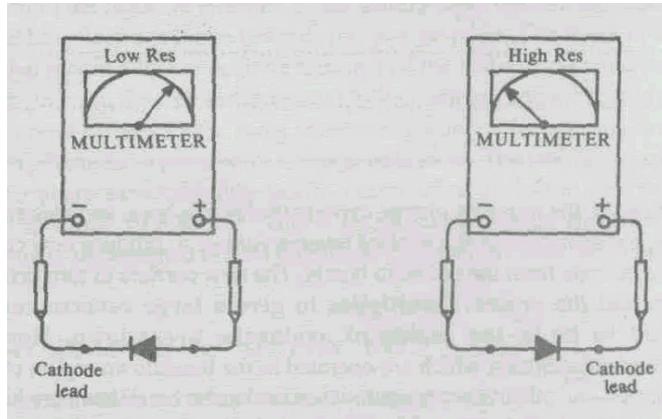


Fig. 10.10 Testing of a diode

If the multimeter shows direct continuity or conduction of the junction, does not change by changing the polarity of the terminals. Then the junction is said to be defective. So, with the help of this, we can detect whether a *p-n* junction diode is in proper working condition or not. The testing is shown in Fig. 10.11.

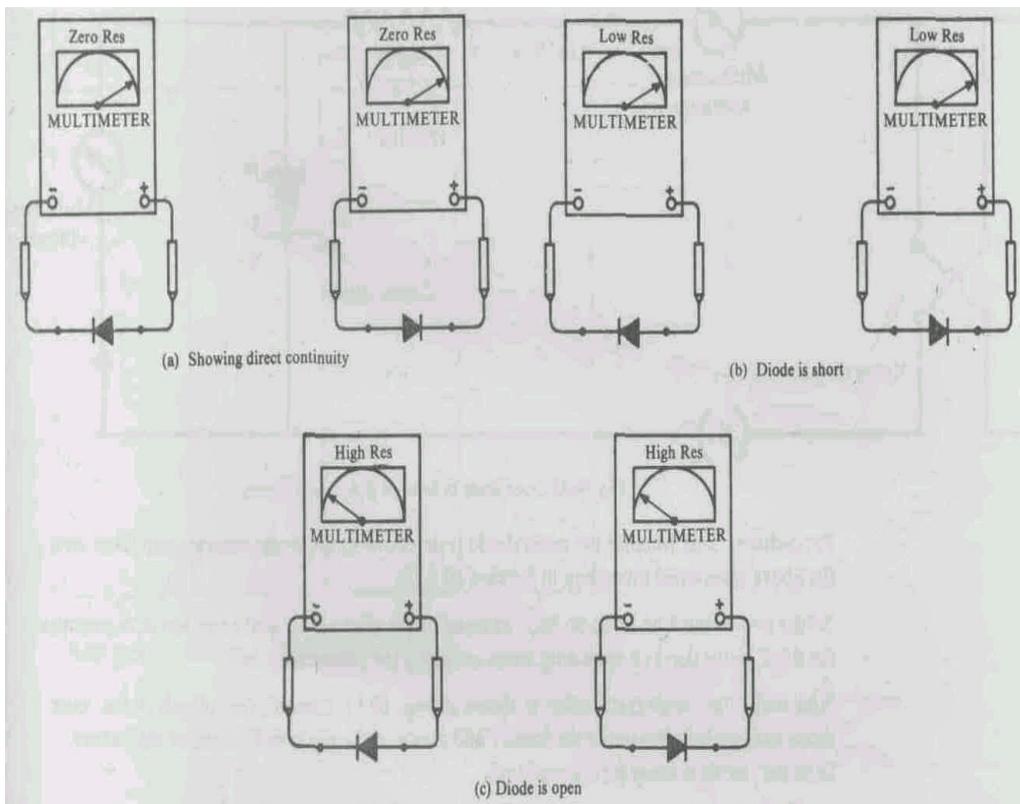


Fig. 10.11 Testing of a *p-n* Junction

This procedure can also be used to identify the leads of an unmarked junction diode or when markings on the diode are not clearly visible. For this the polarity of the multimeter is marked on it or it can be checked with a d.c. voltmeter or another multimeter. Then the lead of the diode which shows low resistance when connected to the negative lead of the meter is cathode lead, and the other one is anode lead. It is shown in Fig. 10.12.

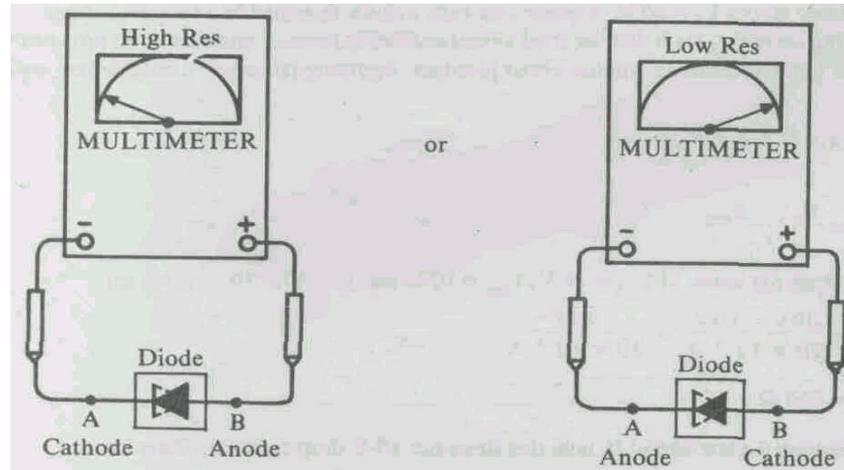


Fig. 10.12 Identification of anode and cathode in a diode

It is found that in some multimeters the terminal marked negative (-) on the meter is actually connected to the positive terminal of the battery inside. That is why you are instructed to know the polarity of multimeter with the help of a voltmeter.

3.3 Voltage-Current Characteristics of a Zener Diode

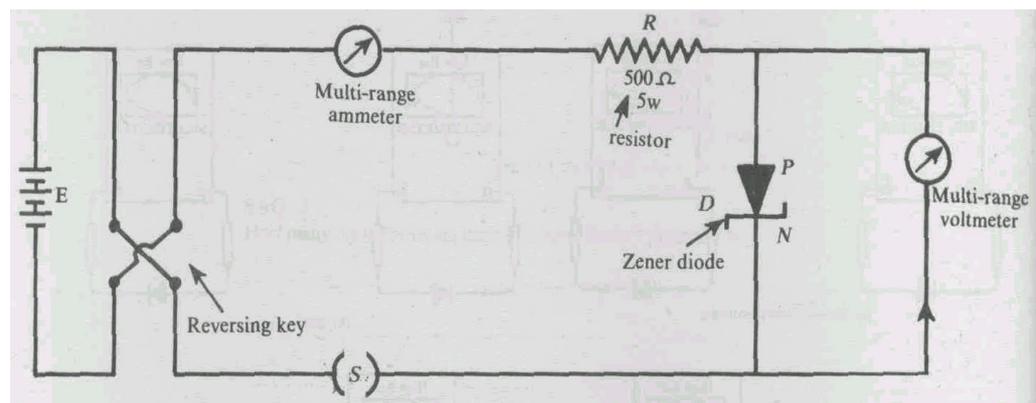


Fig. 10.13 Zener diode in forward and reverse direction

Procedure: Test whether the zener diode is defective or in proper working condition with the above mentioned procedure.

Solder two connecting leads on each terminal of the zener diode and again test it. Sometimes the diode bums due to overheating when soldering the connecting lead.

Now make the circuit connection as shown in Fig. 10.13. Connect the cathode of the zener diode to negative terminal of the battery and anode to the positive terminal of the battery. Now the circuit is ready for forward bias.

In this circuit, the power supply is of the range of 0-30 V. Here V is a voltmeter of range 0-100mV or 0-20 V and A is a milli-ammeter of range 0-50 mA or 0-100mA depending on the requirement. R is a decade resistance, whose value is calculated as follows:

Calculation of the value of R .

The value of R depends on zener and the requirement of the circuit. Consider that a constant 10 V (± 0.7) output V_{out} is required for a load whose current I_L may vary from 5 to 30 mA. Power is supplied to the circuit from a constant 30 V dc source. It is required to design a regulating circuit which will achieve this.

Let us consider that Fig. 10.13 will meet the specification of the problem. We can select a zener diode whose $V_z=10$ V. Assume that such a diode is available which will pass a regulating current I_z such that the total circuit current I_T remains constant at 30 mA over the range of the load current variation in our problem. Applying Kirchhoff's voltage law, we can write

$$V_{AA} = I_T \times R + V_{out} ,$$

and

$$R = \frac{V_{AA} - V_{out}}{I_T}$$

Substituting the value of $V_{AA} = 30$ V, $V_{out} = 10$ V and $I_T = 30 \times 10^{-3}$ A, we get

$$\begin{aligned} R &= \frac{30 - 10}{30 \times 10^{-3}} = \frac{20}{30 \times 10^{-3}} \\ &= 666 \Omega \end{aligned}$$

To determine the wattage of R , note that there is a 10-V drop across it. Therefore,

$$\text{Wattage} = \frac{V^2}{R} = \frac{10^2}{666} \approx \frac{1}{6} \text{ W}$$

Increase the voltage in step of 0.1 V and measure the diode current. Remember not to exceed the maximum allowed forward current in the diode.

Record the reading in Observation Table 10.1.

Observation Table 10.1 Forward - direction (bias)

S. No.	Voltmeter Reading in Volts (V)	Current Reading in milli-ampere			Forward resistance in ohms
		when increasing	when decreasing	Mean	
1.	0.0				
2.	0.1				
3.	0.2				
4.					
5.					
.					
.					
.					
.					

Now plot the data on a graph paper.

Now change the direction of the current with the help of reversing key as shown in Fig. 10.13. After changing the direction of current, the zener diode is in reverse direction of current, (i.e., the zener diode is in reverse bias). Now change the voltmeter of range (0-2 V) to a voltmeter of range (0-30V) and milli-ammeter to micro-ammeter of range (0-100 μ A). Increase the voltage in steps of 0.5 V and measure the current. Record the data in observation table 10.2.

Observation Table 10.2 Reverse Bias

S. No.	Voltmeter Reading in Volts (V)	Current Reading in micro-amperes I_r		
		when increasing	when decreasing	Mean
-1.	0			
2.	2			
3.	4			
4.				
5.				
.				
.				
.				
.				

Now plot the data on a graph paper.

Discuss your results on the basis of the above graphs in the space given below:

SELF ASSESSMENT EXERCISE 4

Observe the forward-characteristic curve of the diode carefully. You will find that the current starts flowing in the diode only when the applied voltage is more than 0.6 V in the case of silicon diode and 0.25 volt in case of germanium diode. Explain the reason?

You may wonder how the above voltage can be used to find whether the diode is made of germanium or of silicon.

3.4 Zener Diode as a Voltage Regulator

Procedure

For voltage regulation, the zener diode is used in the reverse bias. The circuit arrangements are shown in Fig, 10.14, R_L is the load across which the voltage is to be stabilised. The voltmeter V_1 measures the supply voltage and the voltmeter V_2 measures the voltage across the load R_L .

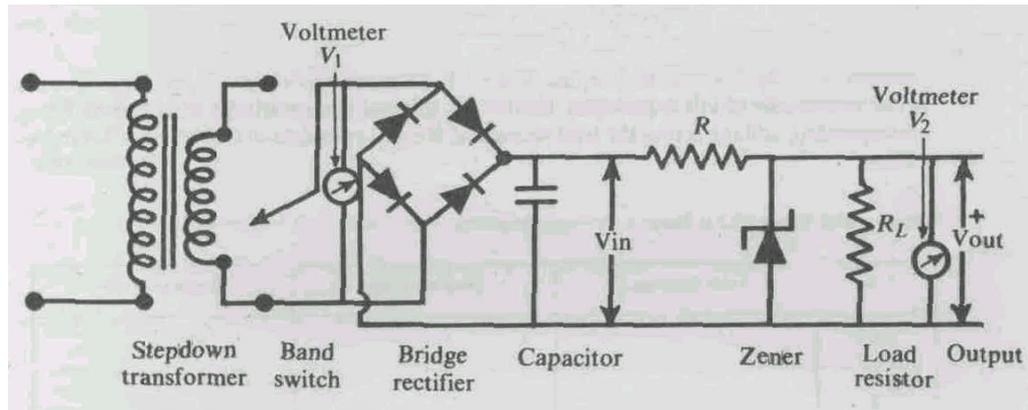


Fig. 10.14 Zener diode as a voltage regulator

Now set the input voltage such that it is in the neighbourhood of 10% of the breakdown voltage. Close the switch S and measure the voltage across the load resistance R_L .

Slowly increase the input voltage in steps of 0.2 Volt and measure the voltage across the load resistance. Record your data in Observation Table 10.3.a.

Observation Table 10.3a: Zener as Voltage Regulator

S. No.	Input Voltage (V_1)	Load Voltage (V_2)	Remarks if any
1.			
2.			
3.			
4.			
5.			
6.			
7.			
.			
.			
.			
.			

Now plot a graph between input voltage and the voltage across the load resistance.

In the second part of this experiment, change the value of load resistance and measure the, corresponding voltage across the load resistance. Record your data in Observation Table No. 10.3.b.

Observation Table 10.3.b: Zener as Voltage Regulator

S. NO.	Load Resistance	Output Voltage	Remarks if any
1			
2			
3			
4			
5			
6			
7			
.			
.			
.			

Now, plot a graph between load resistance and output voltage.

Explain your results on the basis of your data.

Results:

If you have enough time, then remove the zener diode from the circuit and repeat experiment No. 10.5.

Now compare the result of experiment No. 10.5 with and without zener diode on a separate graph paper. Draw conclusions on the basis of the data of these two experiments.

SELF ASSESSMENT EXERCISE 5

When the load resistance is changed from 100 ohm to 1 kilo-ohm, the load voltage changes only slightly. Explain why this is important in a voltage regulation.

SELF ASSESSMENT EXERCISE 6

When the line voltage was changed from 15 V to 25 V, the load voltage changed by a much smaller amount. Explain the importance.

SELF ASSESSMENT EXERCISE 7

Explain the operation of the regulator circuit of Fig. 10.14.

4.0 CONCLUSION

Candidates are required to provide conclusions based on observations and calculations during the experiments.

5.0 SUMMARY

The experiment demonstrated

- (a) The voltage-current characteristics of Zener diodes
- (b) The material composition of Zener diodes
- (c) The characteristic changes of line and load on the performance of zener diodes
- (d) Learners were equipped with capabilities to construct zener voltage regulator and to experimentally determine its range of performance.

6.0 TUTOR MARKED ASSIGNMENTS

- 1.
 - a. What are semi-conductors?
 - b. Describe what is known as: (i) Doping.(ii). n-type semi-conductors. (iii). p-type semi-conductors.
- 2.
 - a. Give the characteristics of Zener diode
 - b. Give reasons why an ordinary diode cannot be used as a Zener diode.

- c. Distinguish between Zener and avalanche breakdowns.
3. Using Fig 10.13 and given that, $E = 12.5\text{v}$, $V_z = 5.6\text{v}$, $R_L = 500$ ohm, $R_S = 100$ ohm.
Calculate,
i) the current through R_S .
ii) the current through the Zener diode.
iii) the power dissipated in diode, R_L .

7.0 REFERENCES/FURTHER READINGS

IGNOU (1997) Physics Laboratory I, Physics PHE-03(L), New Delhi, India.

IGNOU (1997) Electronics I, Physics PHE-10, New Delhi, India

UNIT 3 EXPERIMENT 11

A STUDY OF TRANSISTOR

CHARACTERISTICS

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 A Junction Transistor Revisited
 - 3.2 Transistor Characteristics in CE Configuration
 - 3.2.1 Input Characteristics
 - 3.2.2 Output Characteristics
 - 3.2.3 Transfer Characteristics
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor Marked Assignments
- 7.0 References/Further Readings

1.0 INTRODUCTION

In the preceding experiment, you plotted the characteristics of a $p-n$ diode. A diode permits current to pass through it in only one direction. That is why its applications are limited mostly to rectification and detection. A more useful semiconductor device is a junction transistor. It can be looked upon as two diodes connected back to back. Transistors find so many and so varied uses in our daily life—ranging from gas lighter and toys to amplifiers, radio-sets and TV-video games. In fact, their use is consistently increasing. In the form of switching device, these are used to regulate vehicular traffic on our roads. They form the key elements in computers, space vehicles, satellites, communication and power systems. In a sense, transistors have brought about a technological revolution. It is therefore important to know how a transistor works.

The practical use of a semiconductor device in electronic circuits depends on the current and voltage ($I-V$) relationship. Such a relationship depicted graphically constitutes what we call $I-V$ characteristics. These characteristics give vital information to a circuit designer as well as a technician. Therefore, the first thing of interest is: How does a transistor respond to voltage applied to it? Is the response linear? In your school you may have learnt that for a resistor, the characteristic curve ($I-V$ plot) is a straight line passing through the origin. This is manifestation of ohm's law. Do you get a similar curve for $p-n$ junction or zener diode? In this experiment, you will plot

characteristic curves for a transistor in the common-emitter mode and compute current gain, input resistance and output admittance.

2.0 OBJECTIVES

After performing this experiment, you will be able to:

- study the variation of the base current with potential difference between the base and the emitter (input characteristics)
- study the variation of collector current with potential difference between the collector and the emitter (output characteristics)
- examine the relationship between the collector current and the base current in the common-emitter configuration (transfer characteristics)
- compute current gain, input resistance and output admittance.

3.0 MAIN CONTENT

3.1 A Junction Transistor Revisited

The junction transistor is a three terminal device. These terminals are connected to layers which are in the $p-n-p$ or in the $n-p-n$ configuration (Fig. 11.1). The first letter designates the emitter (E), the middle letter designates the base (B) and the last letter designates collector (C). You will note that (i) the base is sandwiched between the emitter and the collector, (ii) the emitter and the collector are of the same type ((p or n) of material, and (iii) the base and the emitter (or collector) are of different materials. Does this mean that we can interchange the collector and the emitter at will? We cannot do so because the collector and the emitter differ in their levels of doping apart from geometry. (The doping level in the emitter is more than that in the collector.)

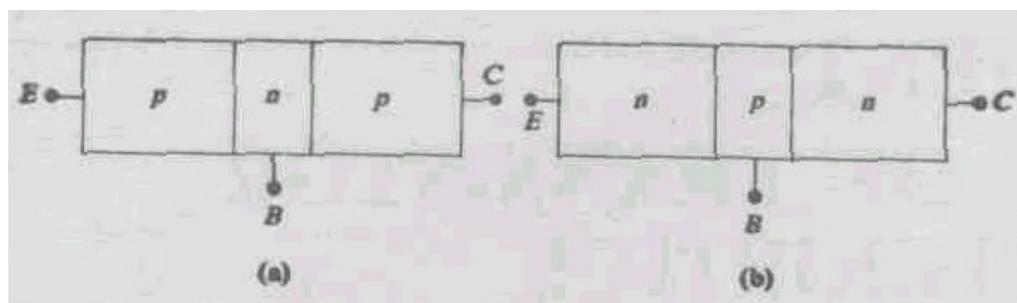


Fig. 11.1: $n-p-n$ and $p-n-p$ transistors

You may now ask: How is a transistor symbolised in a circuit? The circuit symbols for $p-n-p$ and $n-p-n$ transistors are shown in Fig. 11.2. The element with the arrow is the emitter and its symmetrical counterpart is the collector.

In the $p-n-p$ transistor, the emitter arrow points to the base whereas in the $n-p-n$ transistor the arrow points away from the base. The arrow mark signifies the direction of the conventional current when the emitter junction is forward biased.

When the tail of the arrow is connected to the positive terminal of the battery, the emitter base junction is said to be forward biased.

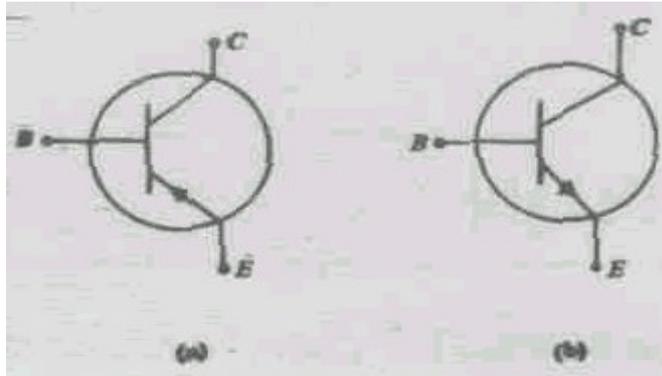


Fig. 11.2 : (a) Circuit symbol for a $p-n-p$ transistor (b) Circuit symbol for an $n-p-n$ transistor

Semiconductor diodes in general and transistors in particular are designated two letters followed by a serial number. The first letter gives an indication of the material: A is used for devices using material with a band gap of 0.6 eV to 1.0 eV such as germanium. B is used for devices using material with a band gap of 1.0 eV and 1.3 eV such as silicon. The second letter indicates the main application: A is for detection diodes, B for variable capacitance diodes, C for transistors for audio frequency applications, D for power transistors, E for tunnel diodes, F for transistors for radio frequency applications, Y for rectifying diodes, and Z to denote voltage reference or Zener diode. The serial number consists of digits. For example, AC125 represents germanium transistor for AF application and BC107 represents silicon transistor for AF application. How will you interpret AD149, BY127 and BZ148? The first of these is a germanium power transistor, the second a silicon rectifier diode and the last one is a silicon Zener diode.

Now you may like to know: How to connect a transistor in a circuit? A transistor can be connected in a circuit in one of the three ways:

- (i) when emitter is common to both input and output circuits – CE configuration
- (ii) when base is common to both input and output circuits – CB configuration

- (iii) when collector is common to both input and output circuits – CC configuration.

In each of these configurations, the transistor characteristics are unique. The CE configuration is used most widely because it provides voltage, current and power gain. In the CB configuration, the transistor can be used as a constant current source while the – CC configuration is frequently used in impedance matching. Does this mean that the CE configuration is superior?

For each configuration, we can plot three different characteristics. These are: (a) characteristic between input quantities called the **input characteristics**, (b) characteristic between output quantities called **the output characteristic** and (c) characteristic between an input quantity and an output quantity called the **transfer characteristic**. Table 11.1 gives various quantities related to each of these characteristics in all the three configurations and the transistor constants of interest.

Table 11.1: Related quantities in the characteristics of a transistor

Configuration	Input characteristic	Output characteristic	Transfer characteristic	Important transistor constant
CE	V_{BE} and I_B with V_{CE} as parameter	V_{CE} and I_C with I_B as parameter	I_B and I_C	Current amplification factor, β
CB	V_{EB} and I_E with V_{CB} as parameter	V_{CB} and I_C with I_B as parameter	I_E and I_C	Current amplification factor, α
CC	V_{BC} and I_B with V_{EC} as parameter	V_{EC} and I_E with I_B as parameter	I_B and I_E	

As mentioned earlier, here we wish you to work in the CE configuration. The apparatus required for this purpose is listed below.

Apparatus

Two low range variable dc power supply (0-15V), a multirange micro-ammeter, two multirange milli-ammeters, two multirange voltmeters, a multimeter, a BC107 *npn* transistor (or any other given transistor) with a socket, two 2.5 K Ω /2W potentiometers, and leads (or a transistor characteristics kit with these provisions.)

3.3 Transistor Characteristics in CE Configuration

First of all you should identify the base, emitter and collector. Then you should check that your transistor is in working order. To do so, you can measure the resistance between the emitter and base, and the collector and base using a multi-meter following the steps given in Appendix-A. Having tested your transistor, you should use the following procedure to be able to plot transistor characteristics of interest.

3.3.1 Input Characteristics

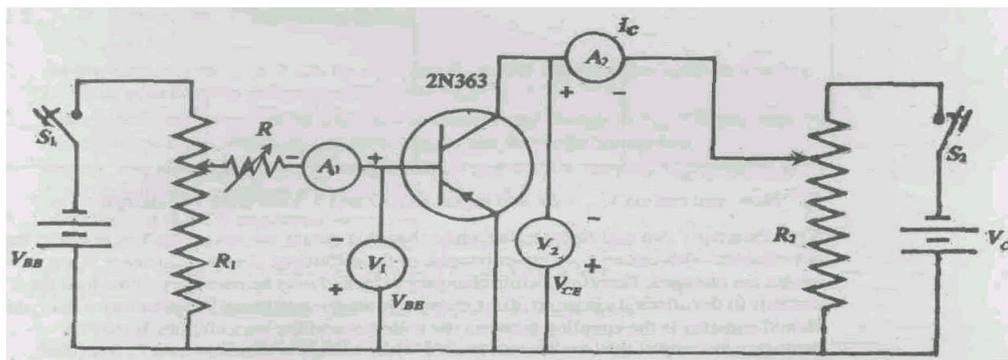


Fig. 11.3: Circuit diagram for investigating CE characteristics

- (1) Make the connections as shown in Fig. 11.3. V_{BB} and V_{CC} are the base and collector supply batteries (0-15V). R_1 and R_2 are $2.5 \text{ k}\Omega / 2\text{W}$ potentiometers, V_1 and V_2 are multirange voltmeters, A_1 , and A_2 are multirange micro and milliammeter, respectively and R is a variable resistance.

In case a transistor kit is given to you, you should ensure that the basic circuit given in Fig. 11.3 is in operation. If you are given an $n-p-n$ transistor, you should reverse the polarities of batteries and various meters. By means of potentiometers R_1 and R_2 you can adjust the base current and the collector current, respectively.

- (2) Keep collector to emitter voltage (V_{CE}) at zero volts.
- (3) Choose the range 0-1 V for base to emitter voltage (V_{BE}).
- (4) Adjust the base current to a low value say $20 \mu\text{A}$. Vary it in steps of $20 \mu\text{A}$ up to $200 \mu\text{A}$. We expect that V_{BE} will also change. In each case, measure the base to emitter voltage. Record your readings in Observation Table 11.1. In case the base to emitter voltage does not change, check the circuit again. If you cannot locate the fault, seek help from your **counsellor**. You may be having a faulty component or transistor kit.

Observation Table 11.1: Input characteristicsLeast count of micro-ammeter = μ A

Least count of voltmeter =V

S. No.	I_B (μ A)	Base to emitter voltage V_{BE} (V)	
		$V_{CE} = 0.0$ V	$V_{CE} = 2.0$ V
1.	20		
2.	40		
3.	60		
4.	..		
..	..		
..	..		
..	..		
..	..		
..	..		
..	200		

- (5) Now, you can set $V_{CE} = 2$ V and repeat steps 3 and 4. How does V_{BE} change now? Very frequently you will observe that while changing ranges on current-meters, readings may not coincide. This discrepancy arises because of the difference in meter resistance when ranges are changed. Therefore, while changing ranges, it may be necessary to readjust the controls in the affected circuit to offset changes in meter-resistance. Another factor that you should consider is the coupling between the collector and the base circuits. It may be necessary to readjust the base current control when voltage is varied to hold I_B at a fixed value.

Plot I_B along the x -axis and V_{BE} along the y -axis for each value of V_{CE} . Draw best-fit curves. These are referred to as **input characteristics**. Select a suitable point in the linear portion of the curve and compute the slope at that point. This will give you the input resistance defined as $h_{ie} = \frac{\Delta V_{BE}}{\Delta I_B}$, where ΔV_{BE} and ΔI_B denote small changes in base to emitter voltage and base current, respectively.

Result: The input resistance h_{ie} for the given transistor is..... ohm

SELF ASSESSMENT EXERCISE 1

Plot I_B along the y -axis and V_{BE} along the x -axis. What is the nature of the graph?

3.3.2 Output Characteristics

1. Fix the base current I_B at $20\ \mu\text{ A}$ by adjusting R_1 and R .
2. Vary the collector to emitter voltage V_{CE} by varying R_2 from 0 to 10 V in steps of 0.5 V.
3. Note the collector current I_C in each case and record it in Observation Table 11.2.
4. Repeat steps 2 and 3 for $I_B = 40\ \mu\text{ A}$, $60\ \mu\text{ A}$, $80\ \mu\text{ A}$ and $100\ \mu\text{ A}$.
5. Plot V_{CE} versus I_C for different values of I_B . Which Quantity will you plot along the x-axis? Draw smooth curves for each I_B . These are referred to as output characteristics.

Compute output admittance (h_{oe}) using the relation $h_{oe} = \frac{\Delta I_C}{\Delta V_{CE}}$.

Observation Table 11.2: Output characteristics

Least count of voltmeter = V

Least count of milliammeter = $\mu\text{ A}$

S. No.	$V_{CE}(\text{V})$	Collector current I_C (mA)				
		$I_B = 20\ \mu\text{ A}$	$I_B = 40\ \mu\text{ A}$	$I_B = 60\ \mu\text{ A}$	$I_B = 80\ \mu\text{ A}$	$I_B = 100\ \mu\text{ A}$
1						
2						
3						

Result: The output admittance for the given transistor is =

3.3.3 Transfer Characteristics

1. Set V_{CE} at 5.0V
2. Set the base current I_B at $20\ \mu\text{ A}$ by adjusting R_1 and R . Measure the collector current I_C . Enter your reading in Observation Table 11.3.
3. Change the base current to $40\ \mu\text{ A}$. Do you observe any change in V_{CE} ? If yes, then you should adjust R_2 to restore V_{CE} at 5.0V. Again note the collector current.
4. Repeat your observations for $I_B = 60\ \mu\text{ A}$, $50\ \mu\text{ A}$ and $100\ \mu\text{ A}$, keeping V_{CE} constant at 5.0 V. Record your readings in Observation Table 11.3.

5. Next fix V_{CE} at 6.0V and repeat steps 2 - 4.
6. If you have time, you may repeat the above procedure by keeping V_{CE} at 4.0V.
7. Now plot I_B along the x -axis and I_C along the y -axis for all values of V_{CE} .

Observation Table 11.3: Transfer characteristics

Least count of micrometer = μ A)

Least count of voltmeter = V

S. No.	I_B (μ A)	Collector current I_C (mA)	
		$V_{CE} = 5.0$ V	$V_{CE} = 6.0$ V
1	20		
2	40		
3	60		
..	..		
..	..		
..	..		
..	..		
..	..		
..	200		

Draw a smooth best fit curve through the observed points. This curve is referred to as transfer characteristic curve. Compute current amplification factor β using the relation

$$\beta = \frac{\Delta I_C}{\Delta I_B}$$

Result: The current amplification factor for the given transistor is =

SELF ASSESSMENT EXERCISE 2

What other equipment would you require to demonstrate the use of a transistor as amplifier?

SELF ASSESSMENT EXERCISE 3

Draw a circuit diagram to show amplifier/switching action of a transistor.

4.0 CONCLUSION

Learners are required to provide conclusion based on their observations and calculations.

5.0 SUMMARY

The experiment demonstrated;

- (a) The variation of base current with potential difference between the base and the emitter
- (b) The variation of collector current with potential difference between the collector and the emitter.
- (c) The transfer characteristics
- (d) The computation for current gain, input resistance and output admittance.

6.0 TUTOR MARKED ASSIGNMENTS

As provided through the facilitator.

7.0 References/Further Readings

IGNOU (1997) Physics Laboratory I, Physics PHE-03 (L), New Delhi, India.

IGNOU (1997) Electronics, Physics, PHE-10, New Delhi, India.

APPENDIX-A

1. Measurement of resistance by a multi-meter

Insert the black and red cords in the sockets marked 'common' and (Ω V A') respectively. Place the selector in the position for measurement of resistance. This is normally available in three scales ($\times 10$), ($\times 100$) and ($\times 1000$). Now keep the test ends of the cord apart- In this position, the resistance placed across it is infinite. The pointer must indicate ∞ in the topmost scale (i.e. the ohm-scale). If it does not indicate so, bring the pointer to the ∞ -mark by turning the infinity adjustment knob. (If your multimeter does not have the infinity adjuster, the infinity-error, when present, cannot be corrected.) Now make the zero-adjustment, which is very vital. For this, make the test ends of the cord to touch. The pointer should indicate zero for the selector in each scale. If it does not do so, bring the pointer to zero-mark by turning the zero-adjustment knob. The multimeter is now ready for the measurement of resistance.

Now place the selector in the position ($\times 1$). Connect the test ends of the cords across the terminals of the resistance to be measured. Note the reading of the position of the pointer corresponding to the ohm-scale. If the pointer stops at 50, the resistance is $50\ \Omega$. If the pointer overshoots the scale, place the selector at ($\times 10$) and repeat the above procedure. If even now the pointer overshoots the scale, you should place the selector at ($\times 100$) or ($\times 1000$) marks. In these cases you will have to multiply the observed reading by 10 or 100 or 1000, depending on the position of the selector.

2. Identifying emitter, base and collector

Turn the transistor upside down. The three terminals are accommodated roughly within a semi-circle (Fig. A.1). The emitter (E) and the collector (C) are diametrically opposite. The collector is near the notch (N). The third junction is obviously the base.

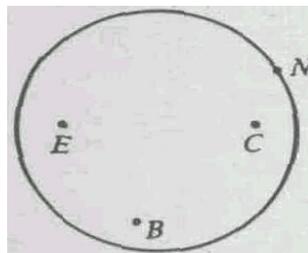


Fig. A.1

The emitter-base and collector-base are two separate $p-n$ junctions. So you can determine their types by measuring the resistance between the ends of these diodes using a multimeter. For this, the multimeter has to be kept in the resistance measurement mode. In this mode, a battery

placed inside the multimeter is operative. The 'common' terminal is the positive end of the battery, whereas the (Ω VA) terminal is its negative end. Now let us indicate the two ends of the diode by '1' and '2'. Measure the resistance between these ends by connecting 1 with the black cord and 2 with the red cord. Repeat the measurement by interchanging 1 and 2. The measured resistance will not be same. The arrangement for which the resistance is smaller will be the case of connecting the diode in forward bias. Corresponding to that, the end connected to the black cord will be the p -side.